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Relation Between Residual and Hoop Stresses and Rolling Bearing Fatigue Life

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Abstract

Rolling-element bearings operated at high speed or high vibration may require a tight interference fit between the bore of the bearing and shaft to prevent rotation of the bearing bore around the shaft and fretting damage at the interfaces. Previous work showed that the hoop stresses resulting from tight interference fits can reduce bearing lives by as much as 65 percent. Where tight interference fits are required, case-carburized steel such as AISI 9310 or M50 NiL is often used because the compressive residual stresses inhibit subsurface crack formation and the ductile core inhibits inner-ring fracture. The presence of compressive residual stress and its combination with hoop stress also modifies the Hertz stress-life relation. This paper analyzes the beneficial effect of residual stresses on rolling-element bearing fatigue life in the presence of high hoop stresses for three bearing steels. These additional stresses were superimposed on Hertzian principal stresses to calculate the inner-race maximum shearing stress and the resulting fatigue life of the bearing. The load-life exponent *p* and Hertz stress-life exponent *n* increase in the presence of compressive residual stress, which yields increased life, particularly at lower stress levels. The Zaretsky life equation is described and is shown to predict longer bearing lives and greater load- and stress-life exponents, which better predicts observed life of bearings made from vacuum-processed steel.

Introduction

Classical rolling-element fatigue is the process by which repeated cycles of a concentrated compressive surface load initiate subsurface cracks in the zone of maximum shearing stresses that propagate into a crack network resulting in a spall on the surface of the running track. In some cases, cracks can propagate radially into the structure causing inner-ring fracture and a catastrophic failure.

It is generally accepted that if a rolling-element bearing is properly designed, manufactured, installed, lubricated and maintained, "classical" rolling-element fatigue is the only failure phenomenon that limits bearing life (Ref. 1). Rolling-element fatigue is extremely variable but is statistically predictable depending on the steel type, processing and heat treatment, bearing manufacturing process and type, and operating conditions. This type of failure is a cycle-dependent phenomenon resulting from repeated stress under rolling-contact conditions and is considered high-cycle fatigue. Sadeghi et al. (Ref. 2) provide an excellent review of rolling-element fatigue failure.

Bearing dynamic load ratings were developed by Lundberg and Palmgren (Refs. 3 and 4). The dynamic load capacity is defined as the radial load for a life of 1 million inner-race revolutions with a 90-percent probability of survival. Load ratings were adopted in bearing load and life standards such as ANSI/ABMA-9:1990 (Ref. 5) and ANSI/ABMA–11:1990 (Ref. 6).

Life computation in the rating standards is based on the effect of the Hertzian loading only. Any other effects, including accounting for improvements in bearing materials, manufacturing processes,

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factors is that they are presented as constants rather than being related to loading, clearance, or the interaction of stress due to loading with the residual stress in the material.

The ISO 281 International Standards (Ref. 7) initially followed the ANSI/ABMA life standards (Refs. 5 and 6). However, in a 2000 amendment (Ref. 8) to the 1990 standard and continuing with the 2007 revision (Ref. 7), ISO incorporated the concept of a fatigue limit, with the assumption that a bearing made from high-quality steel with good manufacturing quality and lubrication may have infinite life if contact stresses are kept below this limit.

Compressive residual stress in the tangential (hoop) direction improves the life of rolling-element bearings because it reduces the effective shearing stress beneath the contacting surfaces of the bearing, inhibiting subsurface crack formation, thus delaying the onset of fatigue failure and extending bearing life. Most bearings are made from through-hardened steels, such as AISI 52100 or AISI M-50. While these give satisfactory service in many applications, the hardened core can be susceptible to fracture.

Bamberger and Kroeger (Ref. 9) investigated a carburizing-grade modified version of AISI M-50 steel for use as a race material for rolling-element bearings. This steel, designated M50 NiL (AMS 6278), has reduced carbon content (from 0.8 to 0.13 percent) to improve the fracture toughness of the core and 3.5 percent nickel is added to stabilize the austenite and prevent the formation of excessive amounts of ferrite and retained austenite. The case (surface) hardness was increased from 38 to 60.6 HRC by carburizing to 0.81 percent carbon to an effective case depth of 1.9 mm (0.075 in.), which produces compressive residual stress to a depth of at least 1.5 mm (0.060 in.). The induced compressive residual stress significantly increased the rolling-element fatigue life of the M50 NiL steel over the conventional AISI M-50 steel.

Parker and Zaretsky (Ref. 10) performed tests in a five-ball fatigue tester using 12.7-mm- (0.5-in.-) diameter vacuum-degassed AIS1 52100 balls. Their tests were run at four levels of maximum Hertz stress ranging from 4500 to 6000 MPa (650 to 875 ksi). They found that the L_{10} life (10-percent fatigue life) is inversely proportional to maximum Hertz stress raised to the power of 12, instead of the ninth power as assumed in ball bearing life standards. With the results corrected for temperature effects, the stress-life exponent was reduced slightly from 12 to 11.5.

Jalalahmadi and Sadeghi (Ref. 11) conducted a study of the scatter in the critical shearing stress, depth to this stress and the Weibull modulus (slope) of rolling contacts, using Voronoi tessellation methods to simulate the random grain structure of bearing steels. Their results generally agree with the Lundberg-Palmgren theory. They also show that steels without initial flaws in the grain structure have greater Weibull modulus (slope) and thus longer lives, particularly at higher reliability levels. Jalalahmadi and Sadeghi (Ref. 11) did not investigate the effect of variations in Hertzian loading on rolling-element fatigue life.

Kotzalas (Ref. 12) investigated changes in the residual stress during operation on the full subsurface stress field in rolling-element bearing fatigue life prediction, concluding that instantaneous values of the material constants in most stress-field-based life equations are incalculable and thus the pre-fatigue residual stress field should be used in fatigue life calculations.

Rosado et al. (Ref. 13) reported fatigue test results with thrust-loaded, 208-size angular-contact hybrid ball bearings made from AISI M-50 and M50 NiL (AMS 6278) steel with silicon nitride (Si_3N_4) balls. Their maximum Hertz stress was reported to be 3100 MPa (450 ksi). The AISI M-50 bearing tests produced only 1 failure out of 12 tests. M50 NiL produced 4 failures in 16 tests.

Rosado et al. (Ref. 13) obtained seven times higher life than predicted by STLE life analysis (Ref. 1) for the AISI M-50 steel but only 65 percent of the life predicted by STLE life analysis for the M50 NiL steel. However, they reported that a TiN coating from the M50 NiL cage-land surface had delaminated with hard particles from the coating causing raceway surface damage that reduced life of the M50 NiL raceways.

Townsend and Bamberger (Ref. 14) reported results of fatigue tests on spur gears at 1710 MPa (248 ksi) and rolling-contact bars at a maximum Hertz stress level of 4830 MPa (700 ksi). The tests compared AISI 9310 and M50 NiL operating under line contact. These fatigue tests were obtained on a test machine that applied load to half of the gear face width in order to obtain a high surface fatigue load without generating tooth-bending failures. They obtained fatigue lives 4.5 times higher on the spur gear tests and 13.2 times higher for the rolling-contact tests for the M50 NiL steel compared to AISI 9310.

Oswald et al. (Refs. 15 and 16) investigated the effect of hoop stress due to interference fits on the lives of roller and ball bearings, showing that these hoop stresses can reduce bearing lives by as much as 65 percent. This analysis did not consider the beneficial effects of residual stress from case-carburized steels.

The objective of the work reported herein is to examine the effect on fatigue life from combined residual stress and hoop stress over a range of applied Hertz stress on rolling-element bearings. An analysis was carried out at four levels of Hertz stress ranging from 1380 to 2215 MPa (200 to 350 ksi) to calculate the unfactored bearing lives for cylindrical roller bearings and angular-contact ball bearings. Then these lives were adjusted to account for material life factors and tensile hoop stress from a heavy interference fit of the inner ring and for the beneficial effect of compressive residual stress in carburized steel.

Nomenclature

b	semi-width of Hertzian contact area in direction of rolling, mm (in.)
c	shear stress-life exponent
D	diameter at the location of the maximum shear stress beneath the surface of the inner race, mm (in.)
E	Young's modulus of elasticity, MPa (ksi)
f	ratio of ball bearing raceway groove radius to ball diameter (conformity)
F_M	material life factor
h	exponent in Equations (1) and (B1)
k	constant in Equations (18) and (B1) to (B3), $\text{mm}^{h/m}$ ($\text{in.}^{h/m}$)
l	roller length, mm (in.)
L	life, millions of inner-race revolutions or hr
L_{10}	10-percent life: life at which 90 percent of a population survives, millions of inner-race revolutions or hours
LF	life factor: ratio of computed life to life at reference Hertz stress without hoop stress or residual stress
LR	life ratio for one race of a bearing
m	Weibull slope
n	Hertz stress-life exponent
p	load-life exponent
p_i	contact pressure between shaft and inner ring due to interference fit, MPa (psi)
P_N	normal load
RL	relative life
S	stress, MPa (ksi)
$S_{\max}$	maximum Hertz stress, MPa (ksi)
$(S_{\max})_{\text{ref}}$	reference value of maximum Hertz stress, MPa (ksi)
V	stressed volume, mm^3 (in.^3)
y	transverse direction
z	distance below surface to maximum shear stress due to Hertizan load, mm (in.)
Δ	diametral interference, mm (in.)
ν	Poisson's ratio

σ	stress, MPa (ksi)
τ	shear stress, MPa (psi)
τ_{max}	maximum shear stress, MPa (psi)
τ_o	maximum orthogonal shearing stress, MPa (psi)
$(\tau_{max})_{ref}$	reference value of maximum shear stress, MPa (psi)
$(\tau_{max})_{rh}$	maximum shear stress modified by residual and hoop stress, MPa (psi)

Subscripts:

adj	adjusted life
eff	effective, used to adjust the outside diameter of the inner ring in Equations (7) and (8)
eq	equivalent
h	hoop stress (in tangential or x-direction)
IR,OR	inner or outer races of bearing
LP	refers to Lundberg-Palmgren life equation (Eq. (1))
max	refers to maximum hertz stress or maximum shear stress
o	refers to maximum orthogonal shearing stress
R	roller
r	residual stress
RE	rolling-element set
S	shaft and inner-ring bore
x	tangential direction
y	transverse direction
Z	refers to Zaretsky life equation (Eq. (3))
z	normal direction

Enabling Equations

A representative cylindrical roller bearing is shown in Figure 1(a). The bearing comprises an inner and outer ring and a set of rollers interspersed between the two rings and positioned by a cage or separator. Profiles for flat (uncrowned cylindrical) and aerospace roller profiles are shown in Figure 1(b) and typical stress profiles across the rollers in Figure 1(c). Flat rollers have no crowning except for a small edge break at the ends of the rollers. The aerospace profile is defined by a flat portion in the center of the roller and a crown radius at each end.

Figure 2(a) is a schematic of the contact of a cylindrical roller on a race. Figure 2(b) shows the principal stresses at and beneath the surface. From these principal stresses the shearing stresses can be calculated. Three shearing stresses can be applied to bearing life analysis: the orthogonal, the octahedral, and the maximum shearing stress.

In 1947, Lundberg and Palmgren (Ref. 3) related the fatigue life L of a radially loaded bearing to the critical shear stress τ_o, stressed volume V and depth to the critical shear stress z:

$$L \sim \left(\frac{1}{\tau_o}\right)^{c/m} \left(\frac{1}{V}\right)^{1/m} (z)^{h/m} \tag{1}$$

where c, m, and h are exponents chosen to fit available experimental data. Zaretsky et al. (Ref. 17) show that Equation (1) is based on earlier work by Weibull but with the addition of the term involving z, the depth to critical shear stress. The rationale for the term involving z is that a significant portion of the fatigue life represents the time required for a crack to propagate to the surface and produce a fatigue spall. A more important reason is that by adding this term, Equation (1) better fits experimental bearing fatigue data available at that time.

The Lundberg-Palmgren life equation can be summarized by Equation (2), where S_{max} is the maximum Hertz stress, exponent $n = 8$ for roller bearings with line contact, and $n = 9$ for ball bearings with point contact.

$$L \sim \left(\frac{1}{S_{max}} \right)^n \tag{2}$$

Zaretsky et al. (Ref. 17) modified the Lundberg-Palmgren life equation to better fit post-1960 life data for bearings made from vacuum-processed steel, which have much longer lives, particularly at light load.

$$L \sim \left(\frac{1}{\tau_{max}} \right)^c \left(\frac{1}{V} \right)^{1/m} \tag{3}$$

The Zaretsky life equation (Eq. (3)), which does not include the term involving the depth to the critical shearing stress, results in the value $n = 10$ for the Hertz stress-life exponent n in Equation (2) for roller bearings with line contact and approximately $n = 11$ to 12 for ball bearings with point contact. Zaretsky et al. (Ref. 17) suggest that these larger values are more appropriate for contemporary steels.

Lundberg and Palmgren (Ref. 3) chose the orthogonal shearing stress as the critical stress in Equation (1). The orthogonal shearing stress is not affected by hoop or residual stress, therefore, Equation (1) is not appropriate where these additional stresses are present. The Zaretsky life equation employs the maximum shearing stress in Equation (3). For the analysis reported herein, the maximum shearing stress is considered. There is also a difference in the stressed volume between Equations (1) and (3) because the depth to the maximum shearing stress is greater than the depth to the orthogonal shearing stress.

For a frictionless contact, the maximum in-plane shearing stress is one-half the maximum difference between principal stresses:

$$\tau_{max} = \frac{\sigma_z - \sigma_x}{2} \tag{4}$$

From Reference 1, the maximum shear stress due to Hertzian loading for a cylindrical roller bearing with line contact is

$$\tau_{max} = -0.300 S_{max} \tag{5a}$$

where S_{max} is the maximum Hertz stress. Although S_{max} is compressive, it is represented here as a positive number. For a ball bearing with point contact and with typical race conformity $f = 0.52$, the corresponding relation is (Ref. 16)

$$\tau_{max} = -0.317 S_{max} \tag{5b}$$

The fatigue life of a bearing race is inversely related to the magnitude of the maximum shearing stress, τ_{max} created by the loading on the bearing to the exponent c. For calculation purposes, c can be taken as 9. The maximum shearing stress τ_{max} may be modified by the presence of hoop stress from an interference fit and/or by residual stress from heat treatment of the bearing race (Refs. 1 and 16). The residual and hoop stresses both occur in the σ_x direction of Figure 2(a). We designate the shearing stress modified by hoop and residual stress as $(\tau_{max})_{rh}$.

$$\left(\tau_{\max}\right)_{\text{rh}} = \tau_{\max} - \frac{1}{2}(\sigma_r + \sigma_h) \tag{6}$$

The pressure p_i at the interface from a bearing shrunk on a solid shaft is given by Equation (7) (adapted from Juvinall (Ref. 18))

$$p_i = \frac{E\Delta\left(D_{\text{eff}}^2 - D_S^2\right)}{2D_S D_{\text{eff}}^2} \tag{7}$$

where E is Young's modulus, Δ is the diametral interference, D_S is the common diameter of the shaft and bearing bore, and D_{eff} is the effective thick-wall cylinder diameter of the inner ring.

For a bearing without a shoulder on the inner race (as is typical for a roller bearing), the effective inner-ring outside diameter D_{eff} is simply the outside diameter of the inner race. For a ball bearing with an inner-race shoulder, D_{eff} can be calculated by adding the area of the shoulders of the bearing race to the "shoulderless" inner race and dividing by the width of the ring. This procedure is described in Reference 16.

The hoop or tangential stress σ_h in a bearing ring is calculated at the location of maximum shear stress D beneath the surface of the inner race. The hoop stress is given by Equation (8) (Ref. 16) as adapted from Reference 18

$$\sigma_h = \frac{p_i D_S^2}{D_{\text{eff}}^2 - D_S^2}\left[1 + \left(\frac{D_{\text{eff}}}{D}\right)^2\right] \tag{8}$$

In Equations (4) to (6), the maximum shear stress $\tau_{\max}$ is defined to be negative. The hoop stress σ_h is normally positive (tensile) and compressive residual stress σ_r is negative. The life L may be represented as an inverse function of $\tau_{\max}$ to the power c, where c, for convenience, can be taken to be 9 although other values can be used, such as 9.3 from Lundberg and Palmgren (Ref. 3) or 10.3 from Shimizu et al. (Ref. 19).

$$L \sim \frac{1}{\left|\tau_{\max}\right|_{\text{rh}}^c} \tag{9}$$

The life ratio for a bearing race can be taken as the ninth power of the ratio of $\tau_{\max}$ to $(\tau_{\max})_{\text{rh}}$, where $\tau_{\max}$ and $(\tau_{\max})_{\text{rh}}$ are defined by Equations (5) and (6). The life ratio in Equation (10) becomes unlimited when $\tau_{\max} = ½(\sigma_r + \sigma_h)$ and thus $(\tau_{\max})_{\text{rh}}$ approaches zero.

$$LR = \frac{(L)_{\text{rh}}}{L} = \left[\frac{\tau_{\max}}{\left(\tau_{\max}\right)_{\text{rh}}}\right]^c \tag{10}$$

Using Equations (5) to (10), we can calculate a life factor for a bearing race. The factor must be calculated separately for the inner and outer races because the loading conditions are not equal. In this article, we are using life results from a database (described below) taken at a reference value of maximum Hertz stress, $(S_{\max})_{\text{ref}}$ = 1710 MPa (248 ksi). From this value, the reference maximum shearing stress $(\tau_{\max})_{\text{ref}}$ is calculated from Equations (5a) or (5b).

$$(LF)_{\text{rh}} = F_M\left[\frac{\tau_{\max}}{\left(\tau_{\max}\right)_{\text{rh}}}\right]^c\left[\frac{\left(\tau_{\max}\right)_{\text{ref}} - (\sigma_r/2)}{\left(\tau_{\max}\right)_{\text{ref}}}\right]^c \tag{11}$$

where the material life factor F_M is based on experimental fatigue data, the maximum shear stress τ_{max} is calculated from Equations (5a) or (5b) and the maximum shearing stress modified by hoop and residual stress is calculated from Equation (6).

Lundberg and Palmgren (Ref. 3) expressed the bearing system fatigue life in terms of the lives of the inner and outer races. However, the race lives L_{IR} and L_{OR} in Equation (12) implicitly include the life of the rolling-element set.

$$\frac{1}{L^m} = \frac{1}{L_{IR}^m} + \frac{1}{L_{OR}^m} \tag{12}$$

In order to consider the effect of residual stress and hoop stress on the various components of a bearing, the life of the rolling-element set must be separated from the race lives. This can be done by employing Zaretsky's Rule (Refs. 1 and 20) to calculate adjusted race lives designated $(L_{IR})_{\mathrm{adj}}$ and $(L_{OR})_{\mathrm{adj}}$. The adjusted lives in Equation (13) are greater than the corresponding race lives of Equation (12). Application of Zaretsky's Rule is described in detail by Oswald et al. (Refs. 15 and 16) and in Appendix A.

The life factor computed from Equation (11) must be calculated and applied separately for the inner and outer races and for the rolling elements. From Zaretsky's Rule, (Appendix A), the life of the rolling-element set of a cylindrical roller bearing or deep groove ball bearing is assumed equal to the life of the outer race. Likewise, the life of the ball set of an angular-contact ball bearing is assumed equal to the life of the inner race (Refs. 20 and 21). The life L of the bearing is given by Equation (13).

$$\frac{1}{L^m} = \frac{1}{L_{IR-\mathrm{adj}}^m} + \frac{1}{L_{RE}^m} + \frac{1}{L_{OR-\mathrm{adj}}^m} \tag{13}$$

where m is the Weibull modulus (slope) and L_{RE} is the life of the rolling-element set (balls or rollers). Zaretsky's Rule does not change the bearing life L; it simply redistributes the failure probabilities. By using Zaretsky's Rule in conjunction with a life factor as defined by Equation (11), we can apply the effect of hoop stress and residual stress to bearing life calculated by the Lundberg-Palmgren life equation (Eq. (1)).

Bearing Material Database

Three bearing steels were considered in this study: through-hardened AISI M-50 and case-carburized AISI 9310 and M50 NiL (AMS 6278). All of these materials were VIM-VAR (double-vacuum) processed, where VIM designates vacuum induction melted and VAR means vacuum arc remelted.

AISI M-50 is a molybdenum-based through-hardened tool steel that is often used for rolling-element bearings in turbine engines. The near-surface microstructure (Fig. 3(a)) shows course bands of white carbides against a dark martensitic matrix. The AISI M-50 used in the test gear database was processed to limit retained austenite to less than 1 percent.

AISI 9310 is a nickel-chromium-molybdenum steel with finer carbides (Fig. 3(b)) and no banding. The carbide structure of M50 NiL (Fig. 3(c)) shows very fine carbides dispersed evenly within the microstructure. Case carburizing produces a beneficial compressive residual stress in bearings made from AISI 9310 and M50 NiL.

NASA spur gear fatigue data from References 14 and 22 were used to develop material life factors for these three steels. Spur gears produce line contact conditions, similar to a cylindrical roller bearing. The tooth profile in the axial direction is crowned and is finished identical to that for bearing rollers. Because the test gear ratio is one to one, that is both gears in contact are the same size and identical, the Hertzian stressed volume for each gear tooth is always in contact with the same and opposite stressed volume on the second gear. This leads to a more consistent and uniform contact and less scatter in the fatigue results manifested in higher Weibull modulus or slope.

The failure morphology for spall formation in these gears is identical to that of classical rolling-element fatigue in ball and roller bearings. For the gears, crack initiation begins at or just above the pitch line of the gear tooth, in the subsurface region of the maximum resolved shear stress. The largest value of the maximum Hertzian stress occurs at the pitch line of the gear teeth in contact. At this location, nearly pure rolling occurs. There are no significant traction forces present necessary to modify the subsurface critical shearing stresses. The crack or cracks form a network that propagates to the surface resulting in classical fatigue spalls (Ref. 1).

Gear fatigue data for the three steels from a maximum Hertz stress level of 1710 MPa (248 ksi) are plotted in Figure 4 and summarized in Table 1. Using the method of Johnson (Ref. 23), the 90 percent confidence band was determined for the L_{10} life of each steel. Based upon the Weibull slope m and the number of fatigue failures for each steel reported, statistically 90 percent of the time value of the L_{10} life will fall between the values calculated in Table 1.

X-ray diffraction was used to determine the magnitude of compressive respective residual stress as a function of depth below the surface. Electro-polishing removed surface material to each depth value required for these measurements. The resulting measured residual stress below the surface for each steel is plotted in Figure 5.

Weibull slopes for rolling-element bearings generally range from 1.0 to 2.0, while for gears; the Weibull slope is normally between 2.0 and 3.0. The Weibull slope is subject to statistical variability relating to the size of the database (the number of specimens failed) (Refs. 20 and 24). However, the data of Table 1 for the AISI M-50 steel reflects a higher than expected Weibull slope of 3.8. According to Jalahamadi and Sadeghi (Ref. 11), this could be reflective of steel, without initial flaws in the grain structure. However, it can result in quantitative differences at different levels of reliability. Accordingly, we have restricted our analysis and conclusions to a 90-percent probability of survival or L_{10} life.

The material life factors from the gear fatigue experiments as shown in the right column of Table 1 were calculated by dividing the L_{10} life for each steel by the reference L_{10} life for AISI M-50. These material factors F_M were applied to the lives of the individual bearing components (inner and outer races and rolling elements).

In calculating relative rolling-element bearing lives, adjustments were made to account for the effect of residual stress from case carburization of the steel and the effect of hoop stress due to an interference fit. In order to account for the effect of compressive residual stress, the reported gear life values were normalized to a maximum Hertz stress of 1710 MPa (248 ksi).

Example Calculations

COBRA AHS (Advanced High Speed), a commercial bearing analysis code (Ref. 25), was used to calculate the inner- and outer-race fatigue lives, maximum Hertz stresses, and depth below the surface to the maximum shearing stress for radially loaded 210-size cylindrical roller bearings and thrust-loaded 210-size angular-contact ball bearings. Unfactored lives (without life adjustment factors) were used in this analysis.

Roller bearings were modeled with zero internal clearance and an aerospace crown on the rollers (Fig. 1(b)) with 61.5 percent flat length and with the crown radius equal to 100 times the roller length. Angular-contact ball bearings were modeled with a 25° free contact angle, with race conformity $f = 0.52$, with shoulder height of 20 percent of the ball diameter on one side of the inner race and relieved shoulders on the other side of the race.

Bearing tolerances are typically specified by an RBEC or ABEC number, which is an odd integer between 1 and 9. RBEC and ABEC stand for Roller (and Annular) Bearing Engineering Committee of the American Bearing Manufacturers Association (Ref. 26). In this paper, we assume RBEC or ABEC–5, a medium tolerance level.

Four load cases were chosen to produce particular values of inner-race maximum Hertz stress, S_{max}: 1380, 1710, 1900, and 2415 MPa (200.2, 248.0, 275.6, and 350.3 ksi). The 1710 MPa (248 ksi) maximum Hertz stress was designated the reference level of maximum Hertz stress, $(S_{max})_{ref}$ because this was the stress level used for the gear fatigue tests in our database (Refs. 14 and 22). The reference value of maximum shear stress, $(\tau_{max})_{ref}$ = –513 MPa (–74.4 ksi) was calculated from $(S_{max})_{ref}$ using Equation (5a).

The maximum shear stress τ_{max} for ball bearings as calculated from the maximum Hertz stress S_{max} in Equation (5b) differs by about 6 percent from the value from Equation (5a) for roller bearings. We did not adjust $(S_{max})_{ref}$ for ball bearings to account for this difference. The error in the calculated relative life from not making this adjustment for ball bearings is about 1 percent.

The procedure can best be illustrated through examples. The following is similar to examples given in References 15 and 16 except the bearing radial load and fit are different and here we add the effect of residual stress.

Life of 210-Size Roller Bearing at Reference Hertz Stress With M50 NiL Inner Race and m6 Fit

Consider a 210-size cylindrical roller bearing, as shown in Figure 1, with bore diameter D = 50 mm (1.9685 in.). With no shoulders on the inner ring, the effective inner-ring outside diameter is equal to the inner-race diameter. Thus D_{eff} = 57.65 mm (2.2697 in.). The elastic modulus E = 205,878 MPa (29.86×10^6 psi) and Poisson's ratio ν = 0.3. For this example, we have chosen an m6 fit; the tightest fit recommended for a 50-mm-bore bearing.

If the inner ring is made to RBEC 5 tolerance and mounted with an m6 fit at the tight end of the tolerance band (Ref. 15), it will have an interference of 0.033 mm (0.0013 in.). Assuming both the bearing bore and shaft have a fine ground finish, the interference will be reduced by 0.004 mm due to asperity smoothing (Ref. 15), which means the actual interference fit is 0.029 mm (0.0011 in.).

The applied radial load of 15,770 N (3525 lbf) was chosen to produce an inner-race maximum Hertz stress of S_{max} = 1710 MPa (248 ksi). From Equation (5a), τ_{max} = –0.3(1710) = –513 MPa (–74.4 ksi). Note that for this case, $S_{max} = (S_{max})_{ref}$ and $\tau_{max} = (\tau_{max})_{ref}$.

From the analysis code (Ref. 25), the depth to maximum shear stress z = 0.127 mm (0.0050 in.). Thus, the diameter to the maximum shear stress: $D = D_{eff} - 2z$ = 57.396 mm (2.2597 in.). From Equations (7) and (8), the fit produces an interface pressure p_i = 14.794 MPa (2.146 ksi) and the hoop stress at diameter D is σ_h = 90.22 MPa (13.09 ksi). We assumed that the bearings have initial internal clearance such that with the interference fit applied the operating internal clearance is zero.

The analysis code (Ref. 25) gave the unfactored life of the bearing as L_{10} = 118.4 million rev with inner- and outer-race lives of L_{IR} = 131.8 and L_{OR} = 816.99 M rev. These lives implicitly include the effect of the life of the rolling-element set. Zaretsky's Rule (Appendix A, Refs. 1, 15, and 16) was used to separate the life of the roller set from the lives of the races. The result, with a Weibull slope m = 1.125 is $L_{IR\text{-adj}}$ = 145.05 M rev, $L_{OR\text{-adj}} = L_{RE}$ = 899.14 M rev. The L_{10} life of the bearing is not changed by this procedure.

Next, we adapted the adjusted inner-race life to account for effects of residual and hoop stress. We assumed the inner ring of the bearing is made from case-carburized M50 NiL with a residual stress of σ_r = –400 MPa (–58 ksi) and a material life factor F_M = 3.6. We assumed the outer ring and rolling elements are made from through-hardened AISI M-50 with no residual stress and with F_M = 1.0. For the inner ring, using Equation (6), $(\tau_{max})_{rh}$ = –513 –½(–400+90.22) = –358.1 MPa.

Using Equation (11), we calculated the life factor for the inner race:

$$(LF)_{rh} = 3.6\left[\frac{-513}{-358.1}\right]^9\left[\frac{-513-(-400/2)}{-513}\right]^9 = 1.07 \tag{14}$$

Thus, the modified life of the inner race is $(L_{IR})_{\text{rh}}$ = 1.07(145.05) = 155.2 M rev. The lives of the outer race and roller set do not change. From Equation (13) by substituting $(L_{IR})_{\text{rh}}$ for $L_{IR\text{-adj}}$, the life of the entire bearing is

$$\frac{1}{L^{1.125}}=\frac{1}{155.2^{1.125}}+\frac{1}{899.14^{1.125}}+\frac{1}{899.14^{1.125}} \tag{15a}$$

$$\left(L_{10}\right)_{\text{rh}}=125 \text{ M rev} \tag{15b}$$

This bearing, if made with M50 NiL inner race with an m6 fit and AISI M-50 outer race and rollers and operating at the reference load that produces a maximum Hertz stress, S_{max} = 1710 MPa, has a relative life of 125.0/118.4 = 1.06. At this Hertz stress, the beneficial effect of the M50 NiL steel is slightly greater than the detrimental effect of hoop stress from the m6 fit of the inner race on its shaft.

Without the effect of the material factor and residual stress, the fit alone produces relative life for the entire bearing of 0.52. In the absence of an interference fit, the use of M50 NiL steel in the inner race alone, with a residual stress of 400 MPa (58 ksi) reduces the effective inner-ring maximum shearing stress from 513 to 313 MPa (74.4 to 46.0 ksi). This combined with the material factor produces a relative life for the inner ring of 307 and for the entire bearing of 2.3.

Modifying Life Results for a Different Value of Hertz Stress

The example above can be modified for a different load (thus a different stress level) by either performing a new analysis with the bearing code or, as shown here, by using Equation (16), which is based on Equation (2), to re-compute the adjusted lives of the inner and outer races. From the Lundberg-Palmgren life theory (Refs. 3 and 4) for roller bearings with line contact, the exponent n = 8.

$$L_2=L_1\left(\frac{S_{\text{max}_1}}{S_{\text{max}_2}}\right)^n \tag{16}$$

With a smaller applied load such that the inner-race maximum Hertz stress is reduced from S_{max} = 1710 to 1380 MPa (200 ksi), the adjusted inner-race life will be $L_{IR\text{-adj}} = 145.05(1710/1380)^8$ = 806.2 M rev. In performing the analysis for this paper, we have observed that the ratio L_{OR}/L_{IR} remains nearly constant at 6.20 for this bearing even with significant changes in the loading and stress. Therefore, $L_{OR\text{-adj}} = L_{RE}$ = (6.20) 806.2 = 4998 M rev.

The life of the bearing from Equation (13) is L_{10} = 658 M rev. The relative life of the bearing (compared to the life at the reference stress) is RL = 658/118.4 = 5.6. The L_{10} life is 3.5 percent greater than the value obtained through a new analysis with the code. Later we show that for the crowned rollers modeled here, the Hertz stress-life exponent, n = 8.2, rather than 8.0. With exponent n = 8.2, the error in the L_{10} life is –0.7 percent.

The analysis code results included the depth below the surface to the maximum shear stress z = 0.1024 mm (0.00403 in.). From z we calculated the diameter to the maximum shear stress $D = D_{\text{eff}} - 2z$ = 57.445 mm (2.262 in.). The interface pressure was unchanged from the previous example at p_i = 14.79 MPa (2.146 ksi) and the hoop stress at diameter D is σ_h = 90.14 MPa (13.07 ksi).

To complete this example, in addition to the hoop stress, we assumed as above that the inner ring of the bearing is made from M50 NiL with a residual stress of σ_r = –400 MPa (–58 ksi) and a material life factor F_M = 3.6. We also assume the outer ring and rolling elements are made from through-hardened AISI M-50 with no residual stress and with F_M = 1.0. From Equation (6), $(\tau_{\text{max}})_{\text{rh}}$ = –414.0 –½(–400+90.14) = –259.1 MPa (–37.6 ksi).

Using Equation (11), where $(\tau_{max})_{ref}$ = 513 MPa (74.4 ksi), the life factor for the inner race is

$$(LF)_{rh} = 3.6\left[\frac{-414}{-259.1}\right]^9\left[\frac{-513-(-400/2)}{-513}\right]^9 = 2.87 \tag{17}$$

The adjusted life of the inner race is $(L_{IR})_{rh}$ = 2.87(806.2) = 2311 M rev. From Equation (13) by substituting $(L_{IR})_{rh}$ for $L_{IR\text{-adj}}$, the life of the entire bearing is $(L_{10})_{rh}$ = 1344 M rev. This bearing made with M50 NiL inner race with an m6 fit and AISI M-50 outer race and rollers and operating at the load that produces a maximum Hertz stress S_{max} = 1380 MPa has a relative life RL = 1344/118.4 = 11.4. Note that the relative life is given with respect to the L_{10} life computed at the reference stress.

Life Results With the Zaretsky Life Equation

The computer code and the procedure illustrated above implicitly employ the Lundberg-Palmgren life equation (Eq. (1); Lundberg and Palmgren (Refs. 3 and 4)), which assumes a Hertz stress-life exponent n = 8 for line contact and n = 9 for point contact. The procedure can be modified for the Zaretsky life equation (Eq. (3)) by removing the term involving the depth to critical shear stress in Equation (1) and by adjusting for the difference in critical shear stress and in the stressed volume.

In Appendix B, we derived Equation (18) to convert the adjusted inner- and outer-race roller bearing lives from the Lundberg-Palmgren life equation to the Zaretsky life equation, where L_Z is the L_{10} life from the Zaretsky Equation and L_{LP} is the life from the Lundberg-Palmgren equation.

$$L_Z = k\left(L_{LP}\right)(0.1254)\left(\frac{1}{0.5b}\right)^{2.071} \tag{18}$$

The constant k incorporates the dimension of $1/b^{2.071}$, where b is the depth to the critical shear stress. The value of k is unknown. The constant k will vary for different units of b. We assumed that k is unity for b expressed in millimeters. Appendix B includes a similar expression for converting lives of ball bearings.

For the inner race of our roller bearing example, the contact half width b = 0.1614 mm (from analysis code). Thus, $(L_{IR\text{-adj}})_Z$ = (1)(145.05)(0.1254) $[1/(0.5 \cdot 0.1614)]^{2.071}$ = 3340 M rev. For the outer race, the analysis code yields, b = 0.1945 mm. Thus, $L_{OR\text{-adj}} = L_{RE}$ = 899.14 (0.1254) $[1/(0.5 \cdot 0.1945)]^{2.071}$ = 14,074 M rev.

From Equation (13) the life of the entire bearing is $(L_{10})_{rh} = [1/(L_{IR\text{-adj}})_Z{}^m + 2/(L_{IR\text{-adj}})_Z{}^m]^{-(1/m)}$ = 2482 M rev. This life is 21 times the value calculated by the analysis code, which implements the Lundberg-Palmgren life equation (Refs. 3 and 4).

The process to adjust the life of the inner race for M50 NiL steel and for an m6 interference fit is similar to the procedure described above. The inner-race life factor is the same as calculated in the previous section: $(LF_{IR})_{rh}$ = 1.07, thus the life of the inner race becomes 1.07 (3340) = 3579 M rev. The roller-set and outer-race lives are unchanged at 14,074 M rev. The life of the bearing from Equation (13) becomes 2607 M rev. The relative life RL for the Zaretsky life equation (as compared to the analysis code unfactored life) is RL = 22.

Modifying Zaretsky Life Equation Results for a Different Hertz Stress

To convert the life results in the section above for a new value of maximum Hertz stress, we used Equation (16) but with exponent n = 10. $L_{IR\text{-adj}}$ = 3340 $(1710/1380)^{10}$ = 28,503 M rev. Likewise, $L_{OR\text{-adj}} = L_{RE}$ = 14,074 $(1710/1380)^{10}$ = 120,115 M rev. The adjusted outer-race life can also be obtained directly from the inner-race life as above except that the ratio L_{OR}/L_{IR} for roller bearings with the Zaretsky equation is 4.21 instead of the 6.20 obtained above. The life of the bearing from Equation (13) is

L_{10} = 21,182 M rev. The relative life (compared to the unfactored life at the reference stress) is RL = 21,182/118.4 = 179.

The L_{10} life found here is 5.2 percent less than the value obtained by applying Equation (18) to the results of the analysis code for this new stress value. Later we show that for the crowned rollers modeled here, the stress-life exponent is 10.3, rather than 10.0. With an exponent of 10.3, the error in the L_{10} life is +1.3 percent. The more accurate value was used in the results below.

To calculate the effect on bearing life from using M50 NiL in the inner race and with an m6 fit, we note that much of the data from the second example above does not change, including S_{max}, τ_{max}, b, z, diameter D, and life factor $(LF_{IR})_{rh}$. The inner-race life is $(L_{IR})_{rh}$ = 2.87(28,496) = 81,690 M rev. $L_{OR\text{-adj}}$ = L_{RE} = 120,085 M rev (unchanged from just above).

Using Equation (13), the life of the bearing from the Zaretsky life equation (Eq. (3)) with M50 NiL inner race and m6 fit and AISI M-50 outer race and rollers without residual or hoop stress is L_{10} = 39,013 M rev. The relative life as compared to the analysis code unfactored life at the reference stress and without residual stress or hoop stress is RL = 39,013/118.4 = 330. We showed above that using the Lundberg-Palmgren life equation, RL = 11.4.

Results and Discussion

Effect on Hertz Stress-Life Relation

The procedure illustrated above was applied to radially loaded, 210-size cylindrical roller bearings with zero operating internal clearance and to thrust-loaded angular-contact ball bearings with a free contact angle of 25°, race conformity f = 0.52 and 20 percent shoulder height on one side of the race. Properties for the bearings are summarized in Table 2. Three steel materials were compared: (1) AISI M-50 through-hardened steel; (2) AISI 9310 case-carburized steel; and (3) M50 NiL (AMS 6278) case-carburized steel.

The relation between the relative life of the inner race alone and the maximum Hertz stress for a cylindrical roller bearing with uncrowned (flat) rollers is shown in Figure 6(a) for the three steels described in Table 1. Life was calculated, using the bearing analysis code, which is based on the Lundberg-Palmgren life equation and then adjusted by Equation (11) and shown relative to AISI M-50 steel at the reference stress level $(S_{max})_{ref}$ = 1710 MPa (248 ksi). The stress-life exponent n is the expected value 8.0 for AISI M-50 steel. For AISI 9310 steel, with 200 MPa compressive residual stress, exponent n increases to 10.1. For M50 NiL (AMS 6278), with 400 MPa compressive residual stress, n becomes 13.5.

The corresponding relative life of the inner race alone of angular-contact ball bearings is given in Figure 6(b). The stress-life exponent n for AISI M-50 is 9.0, increasing to 11.1 and 14.5 for AISI 9310 and M50 NiL steels, respectively.

The relative lives calculated using both the Lundberg-Palmgren and Zaretsky life equations for 210-size cylindrical roller bearings and angular-contact ball bearings at the reference level of maximum Hertz stress $(S_{max})_{ref}$ = 1710 MPA (248 ksi) are given in Table 3. Relative lives are shown for both the no interference fit case and for an m6 interference fit at the tight end of the RBEC/ABEC tolerance class.

Because 1710 MPa equals the reference stress level $(S_{max})_{ref}$, and AISI M-50 is the reference material, the relative life for both ball and roller bearings from the Lundberg-Palmgren equation equals 1.0 by definition for AISI M-50 steel at zero hoop stress. One might expect that the no hoop stress relative life for AISI 9310 would equal the value of F_M = 0.8. However, S_{max} at the outer race is less than $(S_{max})_{ref}$. Therefore, the life is greater at the outer race and thus the relative life of the entire bearing is slightly greater than 0.8. Likewise, for bearings made from M50 NiL, the relative life is slightly greater than the value of F_M = 3.6 for this steel.

The lives shown in Table 3 for the Zaretsky life equation are about 22 times as high as the corresponding values for the Lundberg-Palmgren equation for roller bearings and 15 times as high for ball bearings. The difference in lives is greater for lower stress levels and less for higher stress because of the change in the value of the stress-life exponent n. The addition of a heavy m6 fit significantly reduces the life for all of the materials, by as much as a factor of 5 for roller bearings and a factor of 3 for ball bearings.

The specific life value for the Zaretsky life equation will depend on the value of the conversion constant k in Equation (18), for which we have arbitrarily assigned a value of unity. Test data are required to establish the actual value of this constant. With $k = 1.00$, the life results give approximately the values we expect.

The relative lives of both radially loaded cylindrical roller bearings and thrust-loaded, angular-contact ball bearings as predicted by the Lundberg-Palmgren (designated “LP model”) and Zaretsky (designated “Zaretsky model”) life equations are plotted in Figures 7 and 8. The Lundberg and Palmgren (Refs. 3 and 4) theory assumes a stress-life exponent $n = 8$ for roller bearings with line contact and $n = 9$ for ball bearings, with point contact. The curves marked “Zaretsky model” involve the conversion based on Equation (B1), which produces a stress-life exponent $n = 10.3$ for roller bearings with aerospace crown and 11.1 for ball bearings. With either model, the relationship is altered by the presence of any effects that change the subsurface shearing stress.

Each plot includes results for both no hoop stress (solid lines) and for an m6 interference fit between the bearing bore and shaft (dashed lines). The dotted lines indicate a relative life of 1.0 for the reference stress $(S_{\max})_{\text{ref}} = 1710$ MPa (248 ksi). The curves are located horizontally relative to the reference steel AISI M-50 by the material life factor, F_M, which was taken from the database described above.

For roller bearings with no interference fit, the Hertz stress-life exponent $n_{LP} = 8.2$ for AISI M-50 roller bearings, according to the Lundberg-Palmgren (LP) model shown in Figure 7(a). The exponent differs slightly from the expected value $n = 8.0$ because the rollers were modeled with crowning. If flat rollers are modeled (as in Fig. 6(a)), then the stress-life exponent for roller bearings will be $n = 8.0$.

For roller bearings made from AISI M-50 the life with the Zaretsky (Z) model is 21 times the life for the Lundberg-Palmgren (LP) at the reference stress. In addition, the stress-life exponent n_Z increased to 10.3. With an m6 fit, the life is reduced by almost 50 percent with either model. The stress-life exponent is reduced to 7.6 for the LP model and to 9.7 for the Zaretsky model.

Similar results were obtained for roller bearings made with AISI 9310 steel (Fig. 7(b)), which has a compressive residual stress $\sigma_r = -200$ MPa (–29 ksi). The relative life at the reference stress is slightly lower than for M-50 ($RL = 0.86$ for the LP model and $RL = 19$ for the Zaretsky model) while the exponent n_{LP} increases to 10.3 for the LP life model and to $n_Z = 12.4$ for the Zaretsky model. An m6 fit reduces life by nearly 60 percent with either model.

The analysis predicts that bearings made from M50 NiL steel (AMS 6278), which has residual stress $\sigma_r = -400$ MPa (–58 ksi) (Fig. 7(c)) will have much higher life, roughly four times the life of M-50 at the reference stress level. In addition, with no interference fit, the stress-life exponent n_{LP} increases to 13.8 and to n_Z to 15.9, which means that life at light loads increases much more. The curves are not straight: at lower Hertz stress the effect of the residual stress becomes more important. If the Hertz stress becomes small enough, the denominator of the first fraction in Equation (11) will approach zero and the predicted life will become unlimited. An m6 fit reduces life by nearly 70 percent; however, the life is still longer than with M-50 steel with no fit.

For critical bearing applications where large interference fits are required between the inner ring and shaft, case carburized steels having fracture toughness and high compressive residual stresses should be considered, particularly for the inner ring. Figure 7(d) shows the behavior of roller bearings made from a combination of M50 NiL for the inner ring and AISI M-50 for the outer ring and rolling elements. The life and stress-life exponent lie between what was observed above for the AISI M-50 and M50 NiL used alone.

Figure 8 shows similar results for thrust-loaded angular-contact ball bearings. For AISI M-50 steel, which does not have residual stress and without interference fit, $n_{LP} = 9.0$ and $n_Z = 11.1$ for the LP and Zaretsky models, respectively (Fig. 8(a)). With an interference fit, the stress-life exponents become 8.6 and 10.7, respectively.

The relative lives between bearings made from different steels is a function of a combination of the Hertz stress level, inner-ring interference fit and any compressive residual stress in the ring material. With the case-carburized steels, the life at light loads (thus low values of $S_{\max}$) is much longer than with AISI

M-50 through-hardened steel. This effect makes the stress-life exponent much greater than would otherwise be expected.

To illustrate the effect of residual stress without hoop stress from an interference fit on a radially-loaded roller bearing, we compare the effect of substituting case carburized M50 NiL steel for AISI M-50 on the inner ring alone. The M50 NiL has a residual stress of 400 MPa (58 ksi), while AISI M-50 generally has no residual stress.

Consider first a lightly loaded bearing with inner-ring maximum Hertz stress, S_{max} = 1380 MPa (200 ksi). The residual stress reduces the maximum shearing stress, τ_{max} from 414 to 214 MPa (60 to 31 ksi). The change in shearing stress in addition to the material life factor F_M for M50 NiL (Table 1), increases the life of the inner race by a factor of 16 and increases the life of the entire bearing by 3.56 times.

However, for a heavy load case, where S_{max} = 2415 MPa (350 ksi) the effect on life is different because of the greater stress-life exponent of M50 NiL. Although the residual stress reduces the maximum shearing stress, τ_{max} from 724.5 to 524.5 MPa (105 to 76 ksi), the life of the inner race is lower than the life of an AISI M-50 race by 23 percent and the life of the entire bearing is lower by 19 percent.

Effect on Load-Life Relation

From Hertz theory as given by Jones (Ref. 27), the maximum contact (Hertz) stress in a rolling-element bearing can be expressed as a function of the load. For line contact in roller bearings, the relationship is given by Equation (19a) and for point contact in ball bearings, by Equation (19b).

$$S_{\text{max}} \sim \sqrt{P} \tag{19a}$$

$$S_{\text{max}} \sim \sqrt[3]{P} \tag{19b}$$

Combining Equation (2) with Equation (19) allows us to relate the life to the applied bearing load P to the power p, or

$$L \sim \left(\frac{1}{P}\right)^p \tag{20}$$

Thus, for roller bearings with line contact $p = n/2$ and for ball bearings with point contact $p = n/3$. Using the Lundberg-Palmgren life equation (Eq. (2)), for roller bearings, where n = 8 results in

$$p = n/2 = 4 \tag{21a}$$

and for ball bearings, where n = 9 results in

$$p = n/3 = 3 \tag{21b}$$

Using the Zaretsky life equation (Eq. (3)) for roller bearings, where n = 10 results in

$$p = n/2 = 5 \tag{22a}$$

and for ball bearings, where n = 11.1 results in

$$p = n/3 = 3.7 \approx 4 \tag{22b}$$

The load-life exponent p will be affected by factors such as residual and hoop stresses in the bearing steel and in roller bearings, by roller crowning. The analysis was extended to calculate the load life relation for ball and roller bearings. Results for the four material combinations with and without a heavy interference fit are given in Tables 4 and 5.

For roller bearings without residual or hoop stress effects and with either the Lundberg-Palmgren or Zaretsky life equation, the load-life exponent p was found to equal $n/2.3$ rather than the expected value of

$n/2$. Likewise, for angular-contact ball bearings p was found to equal $n/3.2$ rather than the expected value of $n/3$.

In analyzing angular-contact ball bearings, we maintained the free contact angle constant at 25°, allowing the loaded contact angle to increase with the applied load, which is consistent with normal use of such bearings. For the load that produced the reference stress S_{max}= 1710 MPa (248 ksi) the loaded contact angle was 28.8°.

We also tried modeling a constant loaded contact angle, rather than constant free contact angle. For the reference stress case, if the free contact angle is reduced from 25° to 20.5°, then the loaded contact angle will be 25°. We also reduced the applied thrust load so that S_{max} remained constant at 1710 MPa (248 ksi). With these two changes, the life did not change (within computational accuracy) and thus the stress-life exponent n did not change. However, since the load changed, the load-life exponent p is greater than what is shown in Table 5. For this constant loaded contact angle case, the load life exponent $p = n/3 = 9/3$, which agrees with Equation (21b) and the Lundberg-Palmgren theory.

Effect of Retained Austenite on Compressive Residual Stresses

In general, for a given through-hardened bearing steel the amount of retained austenite increases with increasing material hardness. Experience has also shown that test rollers made from AISI 52100 of Rockwell C hardnesses greater than 63 will have sufficient austenite-to-martensite transformation during rolling contact to alter the surface waviness and cause early surface spalling (Ref. 28).

In 1982 Johnston et al. (Ref. 29) studied the effect of the decomposition of retained austenite and the inducement of compressive residual stress as a result of bearing operation. What is unique about their data is that the magnitudes of the compressive residual stresses are directly proportional to the decomposition of retained austenite (Ref. 28).

Voskamp and Mittemeijer (Ref. 30) in 1997 reported tests that they performed at a maximum Hertz stress of 3.3 GPa (479 ksi) with 6309-size deep-groove ball bearings made from electroslag remelted AISI 52100 bearing steel with 10 to 15 percent retained austenite by volume. Their tests showed the accumulation of plastic deformation during cyclic stressing under rolling-contact loading conditions that resulted in a complex state of residual stress in the subsurface volume of the deep-groove bearing inner races. They also cite unpublished test results that showed evidence of residual stress development in bainitically hardened bearing rings with less than 1 percent retained austenite. Hence, they suggest that the build up of residual stresses are not solely caused by the transformation of retained austenite (Ref. 30).

Changes in microstructure (phase transformations) have been reported to occur in the same areas as the maximum induced residual stress (Refs. 31 and 32). Under some conditions of extremely high contact stresses, nonmicrostructural alteration was apparent after significant residual stresses had been induced in a few cycles (Ref. 32). H. Muro and N. Tsushima (Ref. 33) proposed that the induced residual stresses and the microstructural alterations are independent phenomena (Ref. 28).

Research performed by D. Zhu et al. (Ref. 34) in 1985 on carburized rollers suggested that the structural change in the zone of maximum resolved shearing stresses observed by Jones (Ref. 35) in 1947 and later by Carter (Ref. 36) in 1960 as well as others is a manifestation of retained austenite transforming to martensite under cyclic Hertzian stress conditions. A combination of thermal and strain energy and time is believed to cause this change (Ref. 28).

In 1972, Parker and Zaretsky (Ref. 37) made residual stress measurements on several AISI 52100 207-size, deep-groove ball bearings that were run for different times suitable for inducing significant compressive residual stresses in the inner-race–ball groove. Compressive residual stresses greater than 0.7 GPa (100 ksi) were included in the maximum shear stress region of the bearing inner race, which was run for 25 hr at a maximum Hertz stress of 3.3 GPa (480 ksi) and a shaft speed of 2750 rpm. Twenty-seven bearings were prestressed for 25 hr at this condition and fatigue tested at a maximum Hertz stress of 2.4 GPa (350 ksi). The results of these tests were compared with the results of baseline tests without a prestress cycle at identical test conditions. The L_{10} life of the prestressed ball bearings was greater than twice that of the baseline bearings. Additionally, after 3000 to 4000 hr of testing the differences between the measured residual stresses in the prestressed and baseline bearings were small. This result would suggest that the early presence of compressive residual stresses is a requirement for longer lived bearings. However, the time dependence of residual stress buildup with different materials under varying load may be an additional factor affecting bearing life. This factor has not been studied.

In 1982, Lorosch (Ref. 38) published results of fatigue tests on three groups of vacuum-degassed 7205B-size AISI 52100 inner races at maximum Hertz stresses of 2.6, 2.8, and 3.5 GPa (370, 406, and 500 ksi), respectively (Ref. 38). These were very highly loaded bearings. From these tests, Lorosch (Ref. 38) concluded that "Under low loads and with elastohydrodynamic lubrication, there is no material fatigue, thus indicating that under such conditions bearing life is practically unlimited."

Zwirlein and Schlicht (Ref. 39) in a companion paper published concurrently in 1982 with that of Lorosch (Ref. 38) and using the same 7205B-size bearing inner races, reported large amounts of compressive residual stress due to the transformation of retained austenite into martinsite (Ref. 39).

Lorosch (Ref. 38) and Zwirlein and Schlicht (Ref. 39) failed to account for the significantly large presence of these induced compressive residual stresses in their bearing raceways. Instead they assumed that the large increases in life that they reported were due to a 'fatigue limit.' Zwirlein and Schlicht (Ref. 39) concluded that "contact pressures (maximum Hertz stresses) less than 2.6 GPa (370 ksi) do not lead to the formation of pitting within a foreseeable period (Zwirlein and Schlicht (Ref. 39)). This corresponds to 'true endurance strength' …" However, their observation is not supported by rolling-element fatigue data in the open literature for maximum Hertz (contact) stress levels under 2.6 GPa (370 ksi). If Lorosch (Ref. 38) and Zwirlein and Schlicht (Ref. 39) were correct, virtually no bearing in rotating machinery applications would fail by classical rolling-element fatigue since maximum Hertz stress levels in the range of 1.2 GPa (175 ksi) to 1.9 GPa (275 ksi) are typical.

Summary of Results

An analysis was performed using the COBRA-AHS rolling-element bearing code, to calculate the unfactored bearing lives for 210-size cylindrical roller bearings and angular-contact ball bearings at four levels of maximum Hertz stress ranging from 1380 to 2215 MPa (200 to 350 ksi). These lives were adjusted to account for (1) material life factors; (2) the beneficial effect of compressive residual stress from heat treatment of carburized steel; and (3) the detrimental effect of tensile hoop stress from a heavy interference fit. Results are given for both the traditional Lundberg-Palmgren life equation and for the Zaretsky life equation. The following results were obtained:

1. Relative lives between bearings made from different steels is a function of a combination of the Hertz stress level, inner-ring interference fit, and any compressive residual stress in the ring material.
2. For critical bearing applications, particularly where heavy interference fits between the inner-ring bore and shaft are required, case carburized steels with high fracture toughness and high compressive residual stress should be considered as the material of choice for the inner ring.
3. For both ball and roller bearings, the Hertz stress-life exponent n and the load-life exponent p are functions of the combination of the compressive residual stresses in the zone of resolved maximum shearing stresses below the Hertzian contact and the tensile hoop stress resulting from the interference fit between the shaft and bearing inner-ring bore.
4. The Zaretsky life equation predicts much longer lives and greater stress- and load-life exponents than the Lundberg-Palmgren life equation without resorting to life adjusting factors. The predicted lives from the Zaretsky life equation better match experimental life data for bearings made from vacuum-processed steel.

Appendix A—Zaretsky's Rule on Determining Rolling-Element System Life

Lundberg and Palmgren (Refs. 3 and 4) did not directly calculate the life of the rolling-element (ball or roller) set of the bearing. However, through benchmarking of the equations with bearing life data by use of a material-geometry factor f_{cm}, the life of the rolling-element set is implicitly included in the life calculation where

$$\frac{1}{L^m} = \frac{1}{L_{IR}^m} + \frac{1}{L_{OR}^m} \tag{A1}$$

Equation (A1) is identical to Equation (13) of the text.

The rationale for not including the rolling-element set in Equation (A1) appears in the 1945 edition of Palmgren's book (Ref. 40) wherein he states that, "…the fatigue phenomenon which determines the life (of the bearing) usually develops on the raceway of one ring or the other. Thus, the rolling elements are not the weakest parts of the bearing …"

The rolling bearing fatigue data that Palmgren used to benchmark his and later the Lundberg-Palmgren equations were obtained under radially loaded conditions. Under these conditions, the life of the rolling elements as a system (set) will be equal to or greater than that of the outer race. As a result, failure of the rolling elements in determining bearing life was not initially considered by Palmgren. Had it been, Equation (A1) would have been written as follows:

$$\left(\frac{1}{L_{10}}\right)^m = \left(\frac{1}{L_{IR}}\right)^m + \left(\frac{1}{L_{RE}}\right)^m + \left(\frac{1}{L_{OR}}\right)^m \tag{A2}$$

where the Weibull slope m is the same for each of the components as well as for the bearing as a system. Equation (A2) is identical to Equation (13) of the text.

Comparing Equation (A2) with Equation (A1), the value of the L_{10} bearing life will be the same. However, the values of the L_{IR} and L_{OR} between the two equations will not be the same, but the ratio of L_{OR}/L_{IR} will remain unchanged.

The fraction of failures due to the failure of a bearing component is expressed by Johnson (Ref. 23) as

$$\text{Fraction of inner-race failures } = \left[\frac{L_{10}}{L_{IR}}\right]^m \tag{A3a}$$

$$\text{Fraction of rolling-element failures } = \left[\frac{L_{10}}{L_{RE}}\right]^m \tag{A3b}$$

$$\text{Fraction of outer-race failures } = \left[\frac{L_{10}}{L_{OR}}\right]^m \tag{A3c}$$

From Equations (A3a) to (A3c), if the life of the bearing and the fractions of the total failures represented by the inner race, the outer race and the rolling element set are known, the life of each of these components can be calculated. Hence, by observation, it is possible to determine the life of each of the bearing components with respect to the life of the bearing.

Equations (A3a) to (A3c) were verified using radially loaded and thrust-loaded 50-mm-bore ball bearings. Three hundred and forty virtual bearing sets totaling 31,400 bearings were randomly assembled and tested by Monte Carlo (random) number generation (Ref. 41). From the Monte Carlo simulation, the percentage of each component failed was determined and compared to those predicted from Equations (A3a) to (A3c). These results are shown in Table A1. There is excellent agreement between these techniques (Ref. 41).

TABLE A1.—COMPARISON OF BEARING FAILURE DISTRIBUTIONS BASED UPON WEIBULL-BASED MONTE CARLO METHOD AND THOSE CALCULATED FROM EQUATIONS (A3a) TO (A3c) FOR 50-mm-BORE DEEP-GROOVE AND ANGULAR-CONTACT BALL BEARINGS (REF. 41)

Ball bearing type	Component	Percent failure	
		Weibull-based Monte Carlo results	Results from Equations (A31) to (A3c)
Deep groove	Inner race	70.1	69.9
	Rolling element	14.8	15.0
	Outer race	15.1	15.0
Angular contact	Inner race	45.4	45.1
	Rolling element	45.2	45.1
	Outer race	9.4	9.7

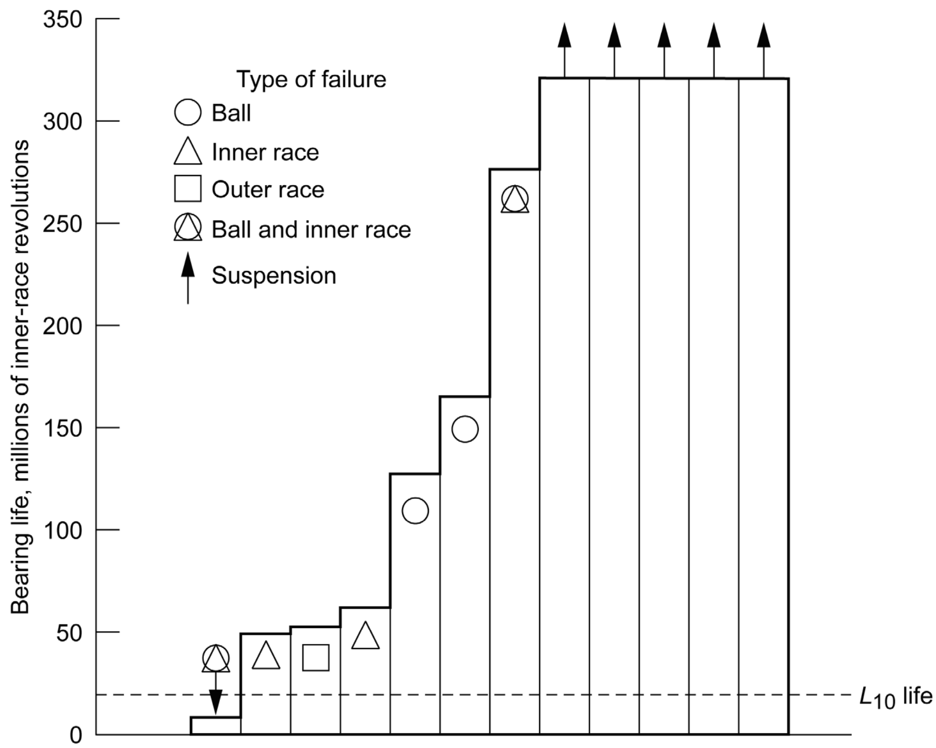

Figure A1.—Rolling-element fatigue lives of AISI 52100 204-size angular-contact ball bearings. Contact angle is 10°; outer-race temperature, 79 °C (175 °F); thrust load, 1108 N (249 lb); inner-ring speed, 10,000 rpm; lubricant, MIL-L-7808; L_{10} life, 20.5×10^6 inner-ring revolutions (34.2 hr); and failure index, 7 out of 12 (Ref. 42).

Figure A1 summarizes rolling-element fatigue life data for ABEC 7 204-size angular-contact ball bearings made from AISI 52100 steel (Ref. 42). The bearings had a free contact angle of 10°. Operating conditions were an inner-ring speed of 10,000 rpm, an outer-ring temperature of 79 °C (175 °F), and a thrust load of 1108 N (249 lb). The thrust load produced maximum Hertz stresses of 3172 MPa (460 ksi) on the inner race and 2613 MPa (379 ksi) on the outer race. From a Weibull analysis of the data, the bearing L_{10} life was 20.5 million inner-race revolutions or approximately 34.2 hr of operation (Ref. 42).

Seven of the twelve bearings failed from rolling-element fatigue. Two of the failed bearings had fatigue spalls on a ball and an inner race. Two bearings had inner-race fatigue spalls. Two bearings had fatigue spalls on a ball, and one bearing had an outer-race fatigue spall. Counting each component that failed as an individual failure independent of the bearing, there were four inner-race failures, four ball failures, and one outer-race failure for a total of nine failed components. Inner-race failures were responsible for 44.4 percent of the failures; ball failures, 44.4 percent; and outer-race failures, 11.2 percent. Using each of these percentages in Equations (A3a) to (A3c) together with the experimental L_{10} life, the lives of the inner and outer races and the ball set were calculated. For purposes of the calculation, the Weibull slope m was assumed to be 1.11, the same as Lundberg and Palmgren (Ref. 3). The resultant component L_{10} lives were 53 million inner-race revolutions (88.3 hr) for both the inner race and ball set and 183.3 million inner-race revolutions (305.5 hr) for the outer race.

For nearly all rolling-element bearings the number of inner-race failures is greater than those of the outer race. Accordingly, from Equations (A3a) and (A3c), the life of the outer race will be greater than that of the inner race. Zaretsky (Ref. 1) noted that for radially loaded bearings (ball or roller), the percentage of failures of the rolling-element set was generally equal to and/or less than that of the outer race. For thrust-loaded ball or roller bearings, Zaretsky (Ref. 1) further noted that the percent for the rolling-element set was equal to or less than that for the inner race but more than for the outer race. In order to account for material and processing variations, Zaretsky developed what is now referred to as Zaretsky's Rule (Ref. 1):

For radially loaded ball and roller bearings, the life of the rolling-element set is equal to or greater than the life of the outer race. Let the life of the rolling-element set (as a system) be equal to that of the outer race.

From Equation (A2)

$$\left(\frac{1}{L_{10}}\right)^m = \left(\frac{1}{L_{IR}}\right)^m + 2\left(\frac{1}{L_{OR}}\right)^m \quad \text{(A4)}$$

where $L_{RE} = L_{OR}$.

For thrust-loaded ball and roller bearings, the life of the rolling-element set is equal to or greater than the life of the inner race but less than that of the outer race. Let the life of the rolling-element set (as a system) be equal to that of the inner race.

From Equation (A2),

$$\left(\frac{1}{L_{10}}\right)^m = 2\left(\frac{1}{L_{IR}}\right)^m + \left(\frac{1}{L_{OR}}\right)^m \quad \text{(A5)}$$

where $L_{RE} = L_{IR}$.

Examples of using Equations (A4) and (A5) are given in References 15 and 16. As previously stated, the resulting values for L_{IR} and L_{OR} from these equations are not the same as those from Equation (A1). They will be higher.

Appendix B—On Converting Life Results From the Lundberg-Palmgren Life Equation to the Zaretsky Life Equation

Zaretsky et al. (Ref. 17) modified the Lundberg-Palmgren life equation (Refs. 3 and 4) by making three changes: (1) replacing the orthogonal shear stress with the maximum shear stress; (2) eliminating the dependence on the Weibull slope in the first term (which involves the shear stress τ); and (3) removing the term involving the depth to critical shear stress.

If we solve for the life L_Z from the Zaretsky Equation (3) in terms of the Lundberg-Palmgren life L_{LP}, Equation (1), we have

$$L_Z = k\left(L_{LP}\right)\left(\frac{\tau_o}{\tau_{\max}}\right)^c \left(\frac{V_{LP}}{V_Z}\right)^{1/m} \left(\frac{1}{z_o}\right)^{h/m} \tag{B1}$$

Where k is a constant that incorporates the dimension of $z^{h//m}$, where z is the depth to the critical shear stress and $h/m = 2.071$. The value of k is unknown. It will take a series of life tests to establish the value of this constant. In this article, we assume that k is unity when z is expressed in millimeters. The value of k will vary with different units of z. For example, for z in inches and with line contact, the constant becomes $k = (1/25.4)^{2.071} = 0.001232$.

For roller bearings with line contact, the maximum orthogonal shear stress $\tau_o = 0.25S_{\max}$, while the maximum shear stress $\tau_{\max} = 0.300S_{\max}$, where $S_{\max}$ is the maximum Hertz stress. Therefore $\tau_o/\tau_{\max} = 0.25/0.300 = 0.833$. Likewise, for ball bearings with typical conformity of 0.52, $\tau_o = 0.249S_{\max}$ and $\tau_{\max} = 0.317S_{\max}$ therefore $\tau_o/\tau_{\max} = 0.249/0.317 = 0.785$.

The Lundberg-Palmgren life equation is semi-empirical. The exponents for the various terms were chosen to fit the experimental data available at the time. In their 1952 paper Lundberg and Palmgren (Ref. 4) show the Weibull slope for 10 roller bearing tests, with 30 bearings in each test. The test bearings included tapered, cylindrical, and spherical roller bearings. The exponents varied from 0.7 to 1.4.

Lundberg and Palmgren (Ref. 4) adjusted their exponents in order to have an integer value for p, the load-life exponent, where $p = (c\text{-}h+1)/(2m)$. For roller bearings with line contact, they chose $m = 9/8$, which makes $p = 4$.

For the term involving the critical shearing stress in the LP life equation (Eq. (1); Lundberg and Palmgren (Refs. 3 and 4), chose exponent $c = 10.33$ for either ball bearings or roller bearings, that is, $L{\sim}1/\tau^{c/m}$. For roller bearings with line contact, $c/m = 10.33/1.125 = 9.182$, and for ball bearings with point contact, $c/m = 9.306$.

When he modified the life relation, removing the dependence on the Weibull slope, Zaretsky adjusted the exponent c so that it is equal to the published values of the Lundberg-Palmgren quotient c/m. But then Zaretsky will not vary the exponent if the Weibull exponent is changed from the nominal value. Therefore, for roller bearings, the term $(\tau_o/\tau_{\max})^c = (0.833)^{9.182} = 0.1875$ and for ball bearings, $(\tau_o/\tau_{\max})^c = (0.785)^{9.306} = 0.1057$.

The stressed volume in either life equation is the product of the circumference of the rolling-element running track, times the width of the contact, times the depth to the critical shear stress. In changing from the orthogonal to maximum shear stress, the only parameter for the stressed volume that changes is the depth to the critical shear stress.

The maximum shear stress occurs at greater depth than the maximum orthogonal shear stress; therefore, $z_{\max}$ is greater than z_o. For cylindrical roller bearings, $z_o = 0.5b$, while $z_{\max} = 0.786b$, where b = the half-width of the Hertzian contact stress zone and the Weibull slope $m = 1.125$. Therefore, the term $(V_{LP}/V_Z)^{1/m} = (z_o/z_{\max})^{1/m} = (0.50/0.786)^{1/1.125} = 0.6689$.

For ball bearings with typical conformity, $f = 0.52$, $z_o = 0.49b$ while $z_{\max} = 0.767b$. Thus, $(V_{LP}/V_Z)^{1/m} = (0.49/0.767)^{1/1.11} = 0.6679$.

To remove the dependence on the depth to the critical shear stress for the Zaretsky life equation, the LP life is divided by $z_o^{h/m}$, where for line contact, the exponent $h/m = 2.33/1.125 = 2.071$ and for point contact, $h/m = 2.33/1.11 = 2.099$.

For roller bearings with line contact, Equation (B1) simplifies to Equation (B2)

$$L_Z = k\left(L_{LP}\right)\left(0.1875\right)\left(0.6689\right)\left(\frac{1}{0.5b}\right)^{2.071} = k\left(L_{LP}\right)\left(0.1254\right)\left(\frac{1}{0.5b}\right)^{2.071} \tag{B2}$$

and for ball bearings with point contact, Equation (B1) simplifies to Equation (B3).

$$L_Z = k\left(L_{LP}\right)\left(0.1056\right)\left(0.6689\right)\left(\frac{1}{0.49b}\right)^{2.0991} = k\left(L_{LP}\right)\left(0.07054\right)\left(\frac{1}{0.49b}\right)^{2.0991} \tag{B3}$$

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TABLE 1.—GEAR FATIGUE LIFE DATA FOR 1710 MPa (248 ksi) MAXIMUM HERTZ STRESS

Material	Compressive residual stress, MPa (ksi)	Gear fatigue L_{10} life million cycles	Weibull slope	Material life factor, F_M	90 percent limits on true L_{10} life[a] million cycles
VIM-VAR AISI M-50[b]	0	60	3.8	1.0	39 to 77
VIM-VAR AISI 9310[c]	200 (29)	48	1.3	0.8	20 to 94
VIM-VAR M50 NiL (AMS 6278)[c]	400 (58)	217	2.3	3.6	117 to 688

[a] From Johnson (Ref. 23)
[b] From Townsend et al. (Ref. 22)
[c] From Townsend and Bamberger (Ref. 14)

TABLE 2.—BEARING PROPERTIES

Bearing description	Bore, mm	Outside diameter, mm	Number of elements	Element diameter, *d*		Roller length	
				mm	in.	mm	in.
210-size cylindrical roller bearing	50	90	10	13	0.5118	13	0.5118
210-size angular-contact ball bearing	50	90	14	12.7	½	n/a	n/a

TABLE 3.—RELATIVE L_{10} LIVES FOR LUNDBERG-PALMGREN AND ZARETSKY MODELS AT MAXIMUM HERTZ STRESS S_{max} = 1710 MPa

Steel	Roller bearing				Angular-contact ball bearing			
	Lundberg-Palmgren model		Zaretsky model		Lundberg-Palmgren model		Zaretsky model	
	No fit	m6 fit	No fit	m6 fit	No fit	m6 fit	No fit	m6 fit
AISI M-50	1	0.52	21	11	1	0.67	15	10
AISI 9310	0.86	0.37	19	8	0.82	0.49	12	7
M50 NiL (entire bearing)	4.18	1.29	94	30	3.75	1.87	56	28
M50 NiL (inner race only)	2.3	1.06	42	22	1.48	1.08	21	16

TABLE 4.—LOAD- AND STRESS-LIFE EXPONENTS FOR CYLINDRICAL ROLLER BEARING WITH RADIAL LOAD

Steel material	Lundberg-Palmgren life equation				Zaretsky life equation			
	No fit		m6 shaft fit		No fit		m6 shaft fit	
	Stress-life exp., *n*	Load-life exp., *p*	Stress-life exp., *n*	Load-life exp., *p*	Stress-life exp., *n*	Load-life exp., *p*	Stress-life exp., *n*	Load-life exp., *p*
AISI M-50	8.2	3.6	7.6	3.3	10.3	4.5	9.7	4.2
AISI 9310	10.3	4.5	9.3	4.1	12.4	5.4	11.4	5.0
M50 NiL	13.8	6.0	11.9	5.2	15.9	6.9	14.0	6.1
M50 NiL (IR only)	10.9	4.7	11.0	4.8	12.5	5.4	12.8	5.6

TABLE 5.—LOAD- AND STRESS-LIFE EXPONENTS FOR ANGULAR-CONTACT BALL BEARING WITH THRUST LOAD

Steel material	Lundberg-Palmgren life equation				Zaretsky life equation			
	No fit		m6 shaft fit		No fit		m6 shaft fit	
	Stress-life exp., *n*	Load-life exp., *p*	Stress-life exp., *n*	Load-life exp., *p*	Stress-life exp., *n*	Load-life exp., *p*	Stress-life exp., *n*	Load-life exp., *p*
AISI M-50	9.0	2.8	8.6	2.7	11.1	3.4	10.7	3.3
AISI 9310	10.9	3.4	10.3	3.2	13.0	4.0	12.4	3.8
M50 NiL	14.0	4.3	12.8	4.0	16.1	5.0	14.9	4.6
M50 NiL (IR only)	10.1	3.1	10.5	3.2	12.1	3.7	12.5	3.9

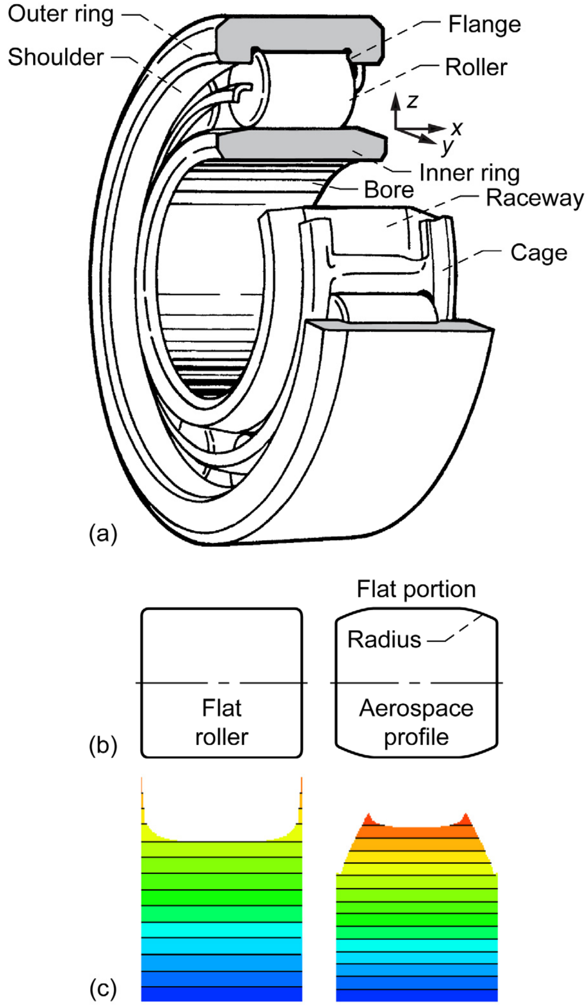

Figure 1.—Cylindrical roller bearing. (a) Schematic. (b) Flat and aerospace roller profiles. (c) Typical stress distribution across rollers.

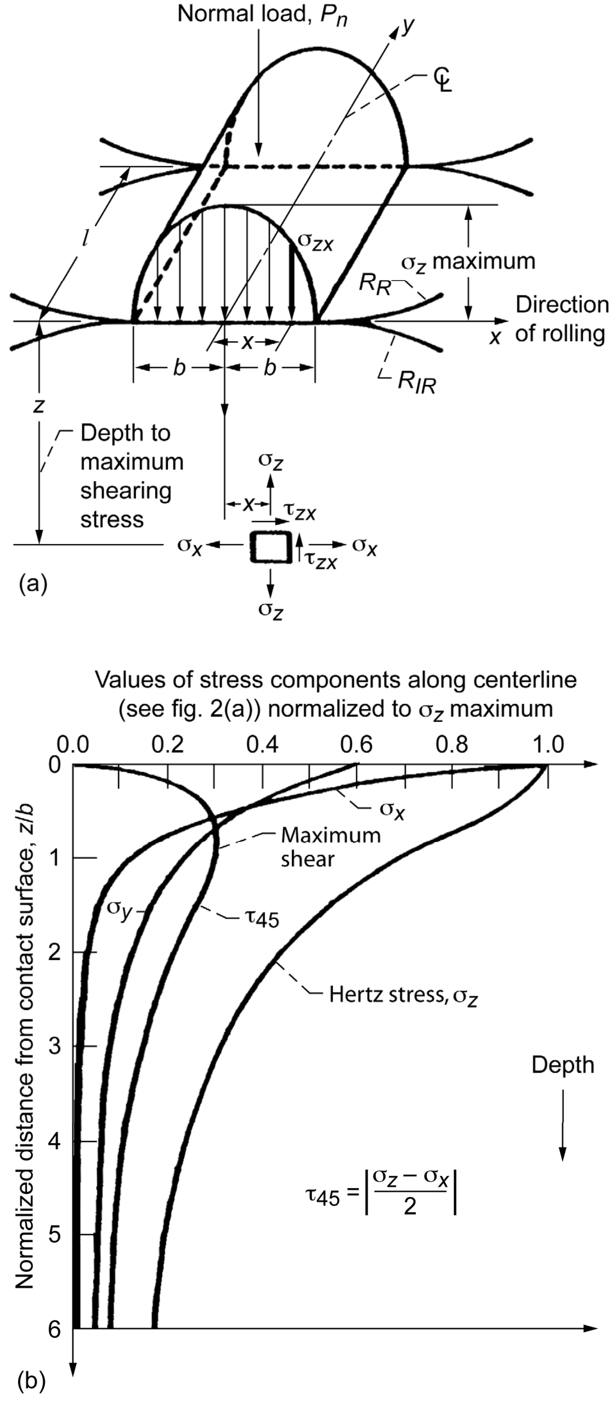

Figure 2.—Subsurface stress field under line contact for a frictionless contact. (a) Hertz stress distribution for roller on raceway showing principal stresses at depth *z* below surface. (b) Distribution of principal and shearing stress as a function of depth *z* below surface.

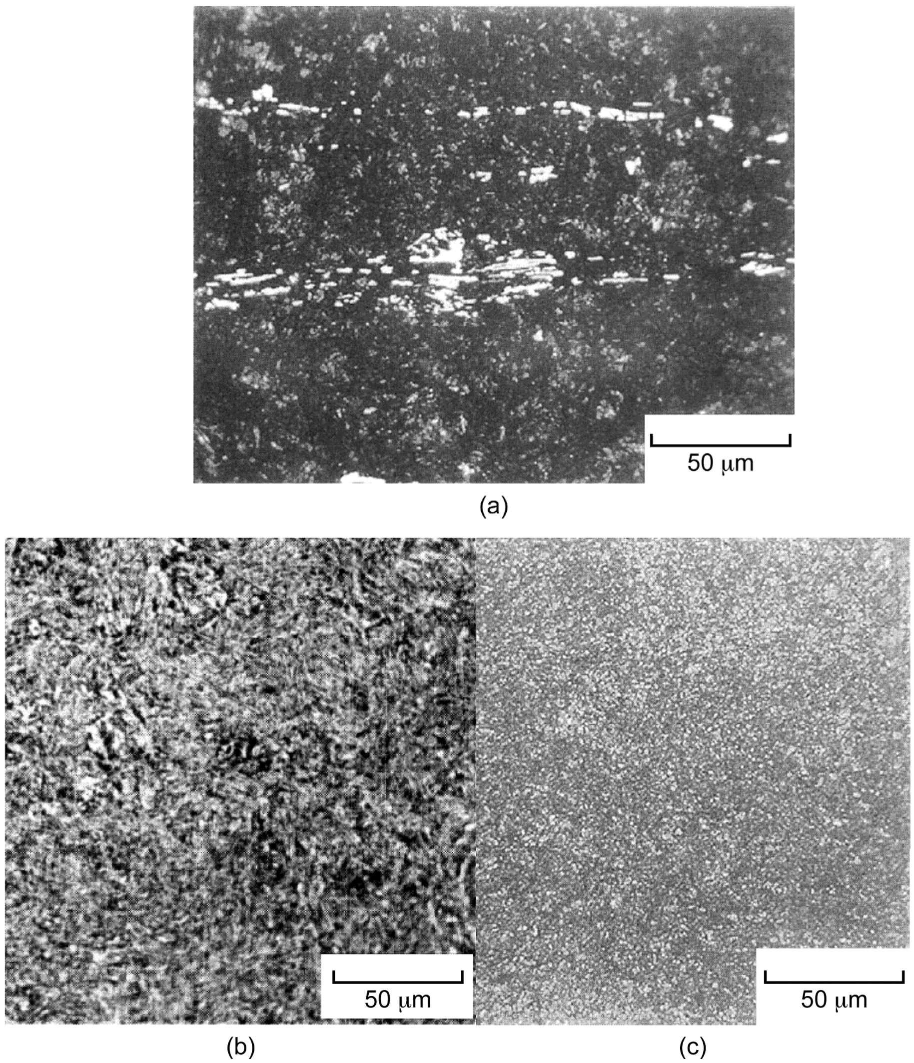

Figure 3.—Comparison of microstructures of three steels in bearing materials database discussed in this article. (a) AISI M-50 steel showing carbide banding. (b) AISI 9310 steel. (c) M50 NiL (AMS 6278).

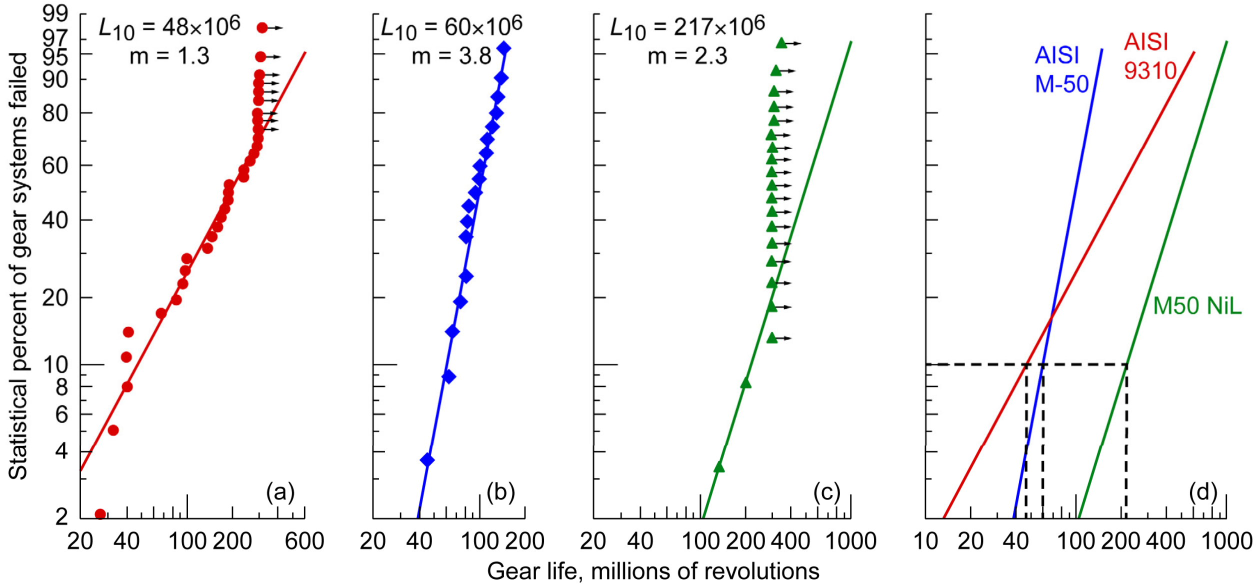

Figure 4.—Gear fatigue data from test gears made from three bearing steels. (a) AISI 9310 (Ref. 14). (b) AISI M-50 (Ref. 22). (c) M50 NiL (AMS 6278 (Ref. 14). (d) Summary.

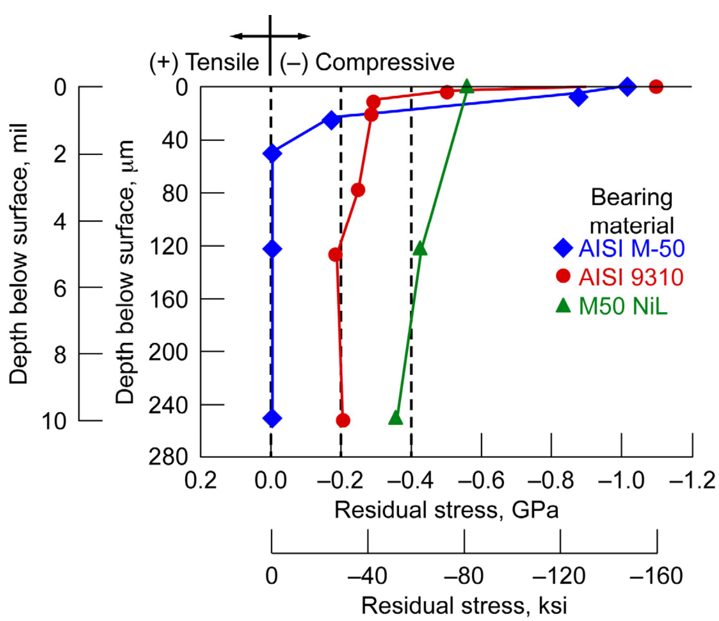

Figure 5.—Residual stress in bearing steels used for gear fatigue tests (Ref. 1). Residual stresses were measured by x-ray diffraction.

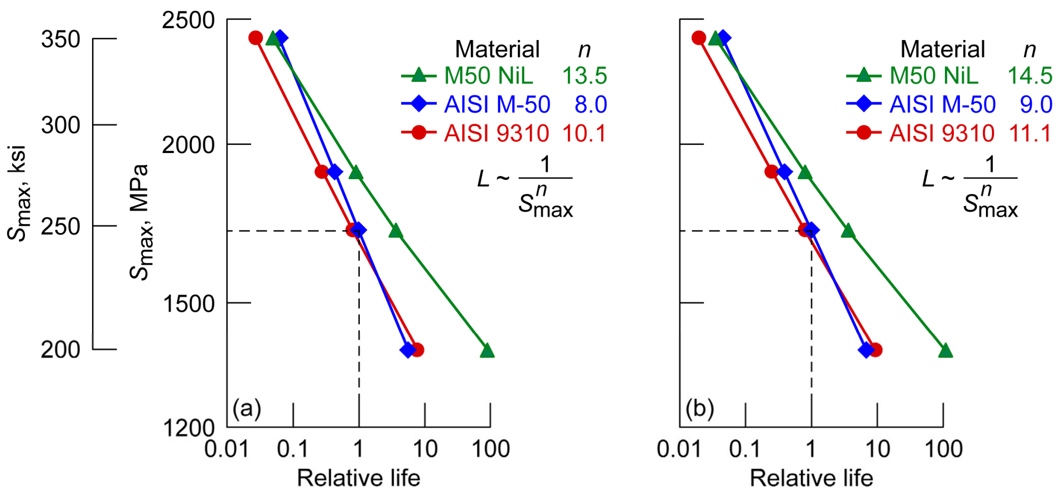

Figure 6.—Effect of material and residual stress on stress-life relationship for single race. (a) Roller bearing inner race. (b) Ball bearing inner race.

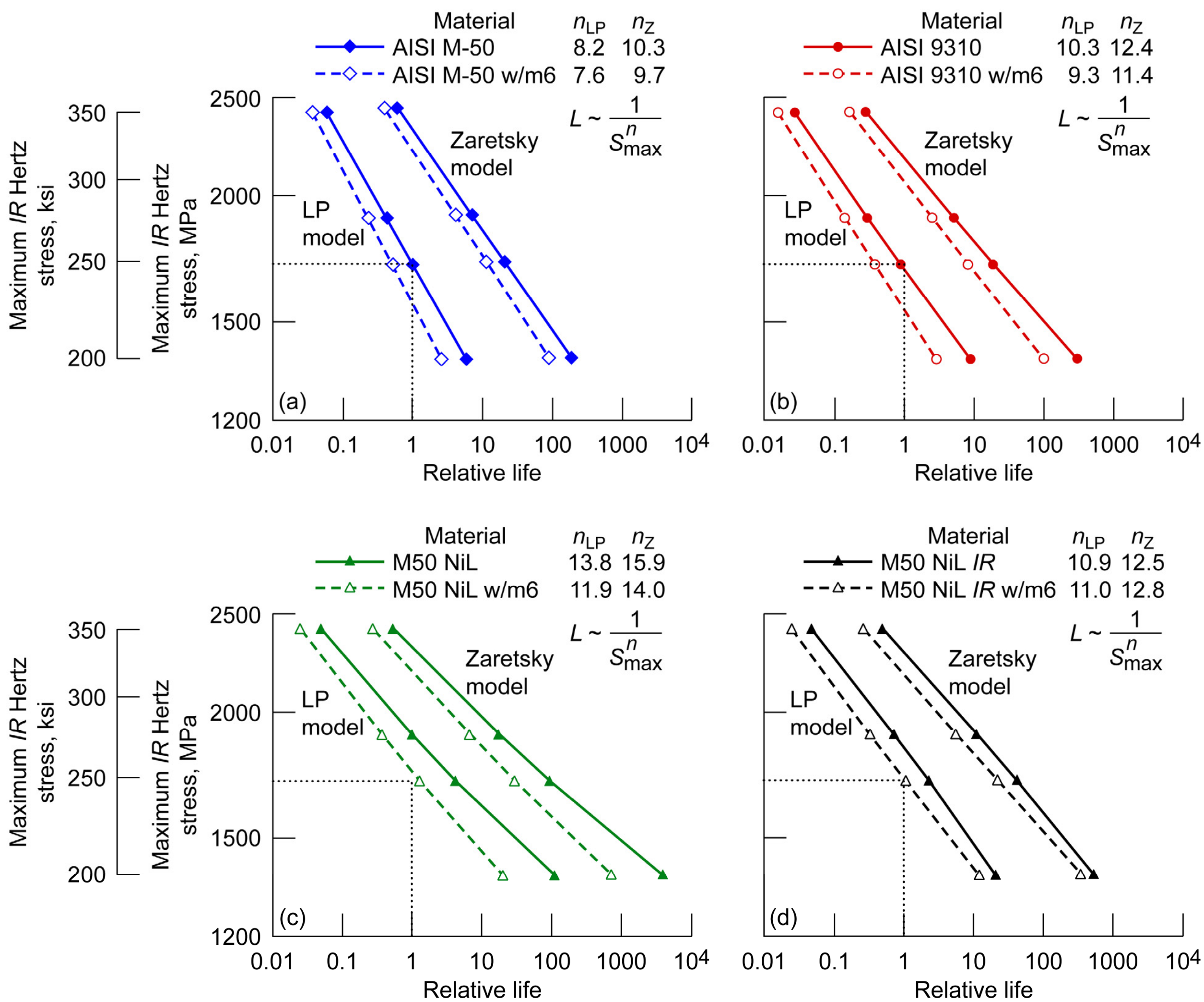

Figure 7.—Relative- and stress-life relationship for 210-size cylindrical roller bearings with aerospace roller profile made from different steels for both the Lundberg-Palmgren (LP) and Zaretsky (Z) life models. (a) AISI M-50. (b) AISI 9310. (c) M50 NiL. (d) M50 Nil inner race, AISI M-50 outer race, and rolling elements. Results are shown with and without m6 interference fit. Reference life is for AISI M-50 material without fit. The dotted lines show the reference life of 1.0 with the reference value of maximum Hertz stress of 1710 MPa (248 ksi). The life equals 1.0 at reference stress only for the LP model for bearings made from AISI M-50.

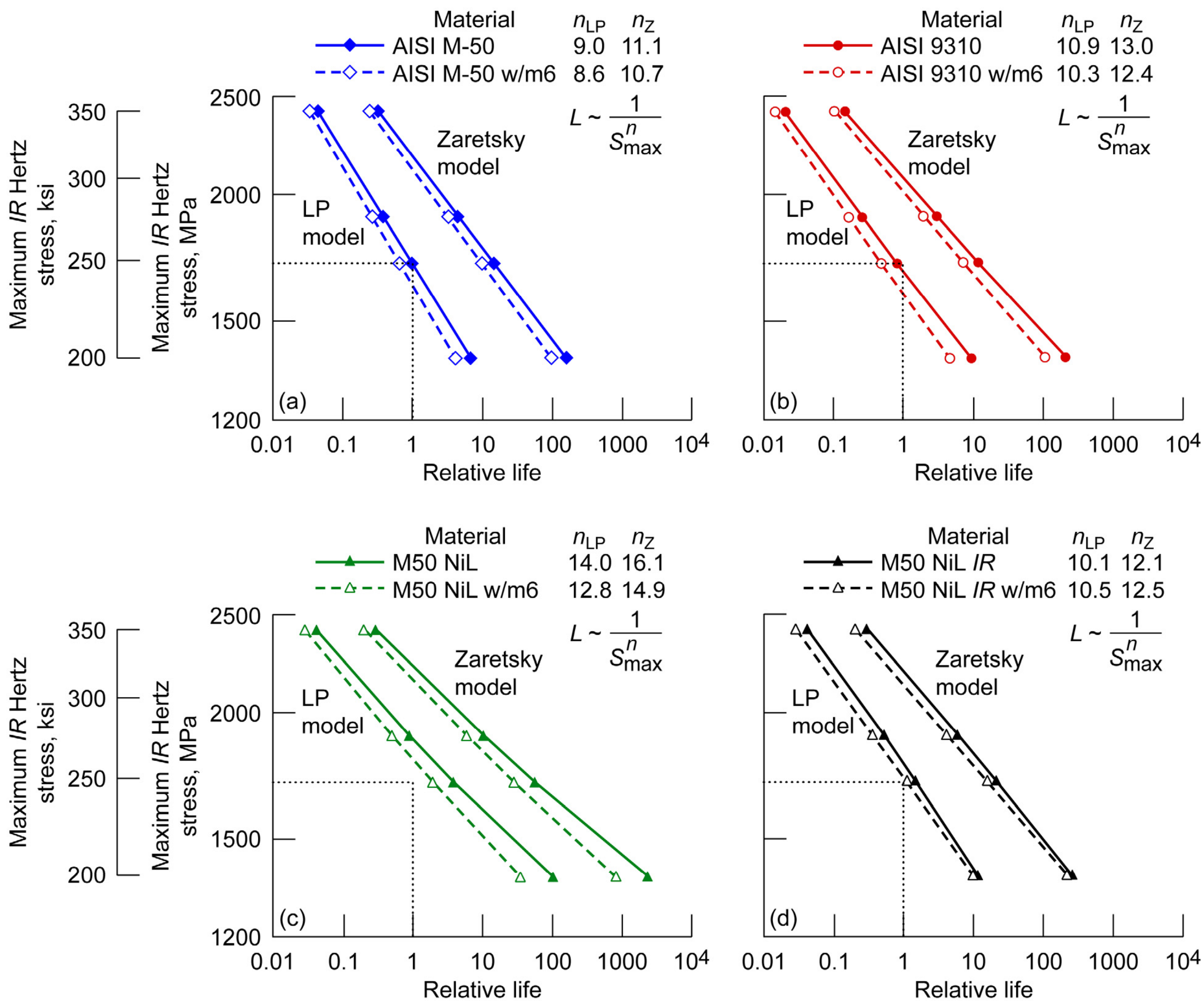

Figure 8.—Relative- and stress-life relationship for 210-size angular-contact ball bearings made from different steels for both the Lundberg-Palmgren (LP) and Zaretsky (Z) life models. (a) AISI M-50. (b) AISI 9310. (c) M50 NiL. (d) M50 Nil inner race, AISI M-50 outer race, and rolling elements. Results are shown with and without m6 interference fit. Reference life is for AISI M-50 material without fit.