

Learning L/M specificity  
for  
ganglion cells

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# Talk Plan

Review the cone-indiscriminate wiring model for ganglion cells and describe its theoretical weaknesses.

Show how associative learning in the retina could generate cone-specific ganglion cells.

Discuss some implications.

# Cone Array Model

Spatially random array of cones with a proportion  $p_L$  of L cones and  $p_M = 1 - p_L$  of M cones.

$c_{Li}$  will denote the signal from an L cone  
and

$c_{Mi}$  is the signal from an M cone.

# Ganglion Cell Model

$g_L$  , an L-center ganglion cell output is

$$g_L = c_{L0} - (\text{sum } w_{Li} c_{Li} + \text{sum } w_{Mi} c_{Mi})$$

$w$ 's are greater than or equal to 0

$$W_T = W_L + W_M$$

$$= \text{sum } w_{Li} c_{Li} + \text{sum } w_{Mi} c_{Mi}$$

is less than or equal to 1.

A balanced cell output to a uniform stimulus is color pure.

$$g_L = c_{L0} - (\sum w_{Li} c_{Li} + \sum w_{Mi} c_{Mi})$$

$$g_L = L - (\sum w_{Li} L + \sum w_{Mi} M)$$

$$= L(1 - W_L) - M W_M$$

$$= L(1 - W_T + W_M) - M W_M$$

$$= W_M (L - M) \text{ if } W_T = 1.$$

If wiring is indiscriminate,  $E[W_M] = p_M$

# Cone Noise Effects

Suppose the cones add a constant independent noise with mean zero and variance  $n^2$  to the signal.

If the weights of  $N$  surround cones are equal and there are  $NM$   $M$  cones, the signal to noise ratio in the case above is

$$s/n = ((L-M) NM/N) / (n \sqrt{1+1/N})$$

# Cone-specific Case

If we delete the connections from the same type of cone, and there is at least one cone of the opposite type, the signal to noise ratio is that when  $N = NM$

$$s/n = (L-M) / (n \sqrt{1+1/NM})$$

The ratio of the signal-to-noise ratio for the indiscriminate case to that of the cone specific case is

$$NM(NM+1)/(N(N+1))$$

# Loss Ratio Table

$$N = 6, pL = 2/3$$

ratio	2/42	6/42	12/42	20/42	30/42	42/42
dB	-26	-17	-11	-6	-3	0
Prob	0.18	0.25	0.22	0.16	0.10	0.03

Prob all surrounds same as center = 0.06

Average loss = -13 dB



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# Cone Images for Training

The cones are assumed to be presented with a sequence of training images that provide each of the cone types in each position a series of values.

The average behavior of the learning process depends on certain average properties of the images.

# Cone Images for Training

For all L and M cones we assume the average over images of the input squared is the same.

$$E[L_i^2] = E[M_i^2] = \sigma^2.$$

For any two cones of opposite type we assume that the average of their cross product is the same,

$$E[L_i M_j] = r_{LM} \sigma^2$$

# Spatial Correlations

The correlations of cones of the same type at different distances in realistic images are a function of the distance. We assume that the distances are similar enough that the average correlation between a surround cell and the center of the same type

$$E[L_j L_k] = E[M_j M_k] = r_s \sigma^2$$

# Learning rule

$$\Delta w_i = -a g c_i$$

then the constraint is enforced that  
 $W_L + W_M = W_T$ .

For a gL cell, the learning rule will result in  $W_L$  going to zero if  $a$  is small enough to average out the random variations and  
 $dL_{Mij} = E[w_{Li}] - E[w_{Mj}] < 0$

# Learning Condition

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$$\begin{aligned} dLM_{ij} &= E[w_{Li}] - E[w_{Mj}] < 0 \\ &= -a (\sigma^2 (1-W_T+W_M)(r_D-r_{LM})) \\ &\quad - n (w_{Li} - w_{Mj}) \end{aligned}$$

Need  $\sigma^2 \gg n$ , the image power dominates the cone noise and

$r_D \gg r_{LM}$ , the spatial correlation stronger than the correlation between L and M

# Correlations

If the stimuli are uniform,  $r_D = 1$  and  $r_D - r_{LM}$  is positive, but possibly small.

Need  $\sigma^2 \gg n$ , so the image power dominates the cone noise and

$r_D \gg r_{LM}$ , the spatial correlation stronger than the correlation between L and M



# Discussion

Associative learning is a promising mechanism for weeding out useless connections in the retina.

This learning process essentially does a local principle components analysis. Buchsbaum and Gottschalk had proposed a global PCA to obtain the transformation of the cone signals that would best code information for the optic nerve.

# Discussion

We have previously proposed two cortical learning processes, an associative one and a translation invariant one.

The former mainly reduced the gain variability from the cone-indiscriminant ganglion cell model.

The translation invariant learning mechanism seems to be still necessary to make the actual L/M distinction.