Aerodynamic Shape Optimization with Goal-Oriented Error Estimation and Control

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Motivation

• Challenges of simulation-based design
  - High CFD expertise in mesh generation
    ‣ Long setup time
    ‣ High cost due to repeated flow solves on fine meshes or high uncertainty due to inappropriate meshes

• Success of error estimation and mesh adaptation in goal-oriented simulations
Objectives

Adaptive discretization of aerodynamic shape optimization problems

Accuracy
- Improve design confidence
- Direct control over objective function discretization error

Automation
- Reduce level of CFD expertise
  - Eliminate the need to handcraft a mesh appropriate for all candidate designs
  - Shorten problem setup time

Progress toward improved efficiency
- Reduce cost by systematically increasing depth of refinement as designs improve
  - Progressive optimization strategy
Problem Formulation

\[
\min_{X} \quad J(X, Q) \\
\text{subject to} \\
R(X, Q) = 0 \quad \forall X \in \Omega
\]

\[\frac{dJ}{dX} \rightarrow 0\]

• Gradient-based optimization

• Steady Euler equations

Spatial Discretization: \( J_H(X, Q_H), R_H(X, Q_H) \)

• Second-order finite-volume method

• Cartesian mesh with embedded boundaries

✓ Complex geometry
✓ Automation
✓ \( h \)-refinement

Cut cells
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Discretization Error

• Leverage adjoint method
  ▪ Error estimates via the method of adjoint weighted residuals
  ▪ Objective function gradient via the discrete adjoint method
Dual Role of Adjoints

Gradients

\[ J_H = f(X, Q_H) \]

\[ \text{e.g. } C_D + (C_L - C_L^*)^2 \]

Error Estimates

\[ e = |J_h - J_H| \]
Dual Role of Adjoint

Gradients

\[ J_H = f(X, Q_H) \]

\[ \frac{dJ}{dX} = \frac{\partial J}{\partial X} + \frac{\partial J}{\partial Q} \frac{dQ}{dX} \]

\[ 0 = \frac{\partial R}{\partial X} + \frac{\partial R}{\partial Q} \frac{dQ}{dX} \]

\[ \psi = \frac{\partial J}{\partial Q} \]

\[ \frac{dJ}{dX} = \frac{\partial J}{\partial X} - \psi^T \frac{\partial R}{\partial X} \]

Error Estimates

\[ e = |J_h - J_H| \]

\[ J_h \approx J_h(Q_H) + \frac{\partial J(Q_H)}{\partial Q} \Delta Q \]

\[ 0 \approx R_h(Q_H) + \frac{\partial R(Q_H)}{\partial Q} \Delta Q \]

\[ J_h \approx J_h(Q_H) - \psi^T R_h(Q_H) \]
Error Estimation Details

\[ J_h(Q_h) \approx J_h(Q_H) - \psi_h^T R_h(Q_H) \]

\[ J_c = J_h(Q_H) - \psi_H^T R_h(Q_H) \]

Error Estimate

\[ \mathcal{E} = C |J_c - J_H| \]

Refinement Indicator

\[ \eta_H = \left( \tilde{\psi}_h - \psi_H \right)^T R_h(Q_H) \]

\[ \eta = \sum_{i=1}^{N} |\eta_i| \]
Verification: Supersonic Vortex

\[ J = \int_{B_{r_0}} p \, dl \]

Uniform Refinement

True Error

Error Estimate

\[ \mathcal{E} = C \left| J_C - J_H \right| \]

- No limiter, \( O(h^2) \)
- Effectivity close to 1
Verification: Supersonic Vortex

\[ J = \int_{B_{r_0}} p \, dl \]

\[ J = J_H + J_C - J_H \]

Refinement Indicator

\[ \eta_H = \left| \left( \tilde{\psi}_h - \psi_H \right)^T R_h(Q_H) \right| \]

- Sharp estimate of remaining error
- Localization very conservative
Optimization with Mesh Adaptation

- Integration into existing, fixed mesh, optimization framework
  - Build sequence of adapted meshes
  - Pass values of objective and gradient from finest mesh to optimizer
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- Integration into existing, fixed mesh, optimization framework
  - Build sequence of adapted meshes
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- In each design iteration, perform fixed (user specified) number of adaptations
  - Fixed depth strategy
  - Robust and precise control over computational resources
Optimization with Mesh Adaptation

- In each design iteration:
  - Start with same initial mesh
  - Adapt until prescribed refinement level is attained
- May be inefficient
Progressive Optimization

- Increase mesh refinement in each optimization subproblem
  - Converge a sequence of improving discretizations

\[ X \rightarrow X^* \text{ as } \mathcal{E} \rightarrow 0 \]
Progressive Optimization

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\[ J \]

\[ h \rightarrow 0 \]

\[ X^* \]

\[ J^* \]
Progressive Optimization

• Increase mesh refinement in each optimization subproblem
  - Converge a sequence of improving discretizations

• Stopping Criterion
  1. Gradient or KKT norms, or stall
  2. Specified number of search directions
  3. Diminishing changes in objective function
  4. Ratio of design improvement to error: refine when
     \[ J_{i-1} - J_i < \epsilon \]
Results

**Sonic-Boom Mitigation Inverse Design**

Optimize aircraft shape by prescribing quieter near-field signals

\[ J = \frac{1}{p_\infty^2} \int (p - p_{\text{target}})^2 dS \]

1. Pressure-signature analysis
2. Shape optimization on a fixed mesh
3. Progressive optimization
Pressure Signature of Delta-Wing Body

Determine pressure signature 3.6 body-lengths below the model

Freestream Conditions:
- $M_\infty = 1.68$
- $C_L = 0.15$
Mesh and Solution

Initial Mesh: 879 cells

12 Adaptations: 4.5M cells

Isobars

Sensor

Refinement Indicator:
\[ \eta_H = \left| \left( \tilde{\psi}_h - \psi_H \right)^T R_h(Q_H) \right| \]

Near-field on symmetry plane
Pressure Signature

- Initial mesh, 879 cells
- 10 adaptations, 590k cells
- 12 adaptations, 4.5M cells
- 13 adaptations, 12M cells
- Experiment

Graph showing pressure signature with distance along sensor on the x-axis and $\Delta p/p_\infty$ on the y-axis.
Error Convergence

\[ J = \frac{1}{p^2} \int (p - p_\infty)^2 dS \]

- Error bars represent level of discretization error

\[ \mathcal{E} = 2 |J_c - J_H| \]

- Remaining error term is small and is \( O(h^2) \)
- Error indicator is \( O(h) \) (due to localization)
Inverse Design on Fixed Meshes

Approach: use adaptation to guide construction of a fixed mesh for shape optimization runs

$M_\infty = 1.6^\circ$

$\alpha = 0.612$

$h/L = 2.0$

9.3 M Cells

Full aircraft configuration: 180 design variables
Optimization Targets

\[ J = \frac{1}{p_\infty^2} \int (p - p_{\text{target}})^2 \, dS \]

On-track, \( \Phi = 0^\circ \)

Off-track, \( \Phi = 15^\circ \)
Optimization Results

- 50 design iterations (SNOPT)
- Ground noise 76.7 PLdB, 9.6 dB reduction in perceived loudness

![Graph showing optimization results with distance along sensor (ft) on the x-axis and Δp/p_{inf} on the y-axis. The graph compares Final Design, Optimization Target, and Baseline Signal.]

- On-track, Φ = 0°
- Off-track, Φ = 15°

Free Polar Graph Paper from http://incompetech.com/graphpaper/polar/
Optimization with Adaptation

Model Problem Setup

- Prescribe a target signature from a known shape
- 10 design variables that control body radius
- $M_\infty = 1.5$ and $\alpha = 0^\circ$
Optimization with Adaptation

Model Problem Setup

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Consider two cases

1. **Fixed-depth strategy**: 7 refinements in each design iteration
2. **Progressive optimization**: Increment from 4 to 7 refinements (allow designs to advance as far as possible on each level)

7 Adaptations, ~650k cells
Optimization with Adaptation

Fixed-Depth Strategy
7 Refinements

Progressive Optimization

Progressive optimization is about a factor of two faster than fixed-depth strategy.
Summary and Outlook

- Progress toward a gradient-based optimization framework with capability to perform adaptive meshing in each design iteration
  - Promising approach to enhance accuracy, efficiency and automation of simulation-based design

- Future work
  - Use of error estimates to limit oversolving
  - Transfer of Hessian matrix as the design moves from mesh to mesh
  - Dynamic error control and mesh re-use
Questions

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Cart3D website and publications:
http://people.nasa.gov/aftosmis/cart3d/