SPACE LAUNCH SYSTEM

Substructure Versus Property-Level Dispersed Modes Calculation

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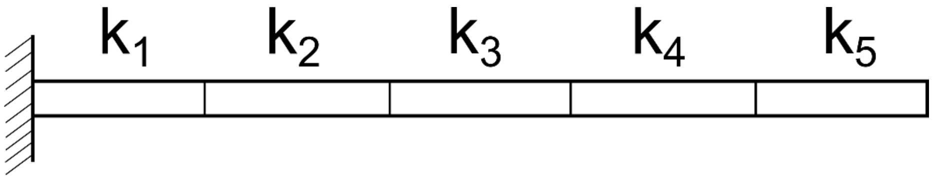
Eric C. Stewart, Ph.D. NASA/MSFC
Jeff A. Peck NASA/MSFC
T. Jason Bush NASA/MSFC
Clay W. Fulcher Jacobs Engineering

Outline

- **Introduction**
  - Need for Dispersed Modes
  - Historical Approach
- **Dispersion Calculations**
  - Substructure vs. Part-level
  - Analytical Sensitivities
- **Frequency Response Function**
- **Examples**
  - Cantilevered Beam
  - TAURUS-T Model
- **Taylor Series side note**
- **Discussion/Conclusions**



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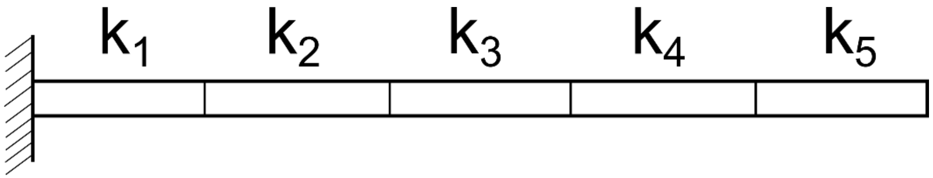
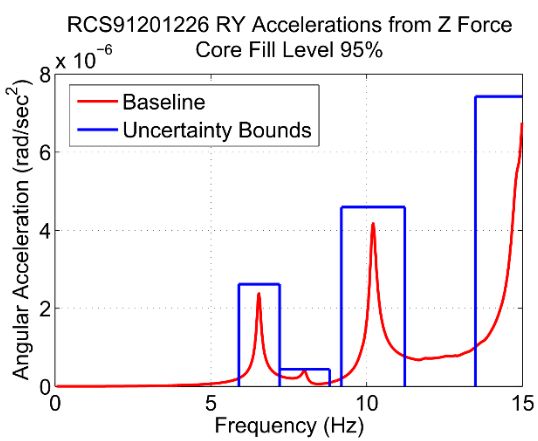
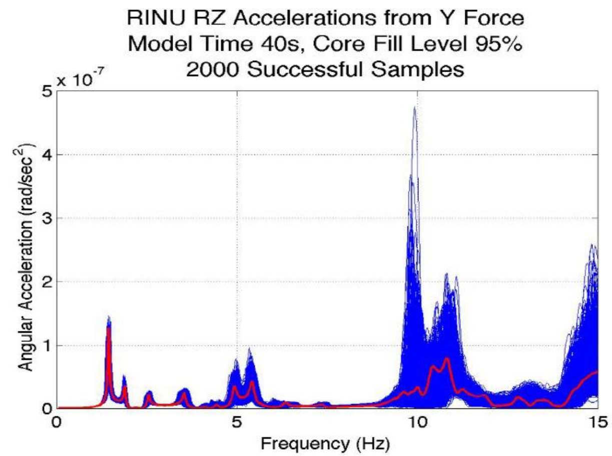


Introduction

- **Guidance, Navigation, and Control (GNC) uses dispersed modes with stability analysis**
  - Calculation of mode with some uncertainty = dispersed modes
  - Used for control system analysis
- **Historical development of dispersions has involved**
  - Overly simplified dispersions
    - 10%-20% frequency dispersions
    - ±100 inches on node dispersions
    - 20%-50% on modal gain amplitudes

    } Anecdotal rules
  - Frequencies & mode shapes dispersed independently
    - Not physics-based or model-based
  - Mode shapes may not be physically realizable
  - Ignores "supermoding"/modal coalescence





Introduction

Three methods to calculate dispersions

- Top-down: tweak the mode frequencies and shapes as per the historical methodology (10%-20%)
- Bottom-up: apply uncertainty factors to the properties of the individual finite elements in the model **(Property-Level dispersions)**
  - May no be possible if models are very large or using superelements
- Middle ground: apply uncertainty factors to the stiffness and mass matrices describing groups of elements **(Substructure dispersions)**
  - Great if already using reduced substructures
- Taylor series approximations
  - Builds on property-level or substructure dispersions

Current Presentation

- Compare property-level and substructure dispersions
  - Beam
  - TAURUS-T
- Analytical Sensitivities – In Work

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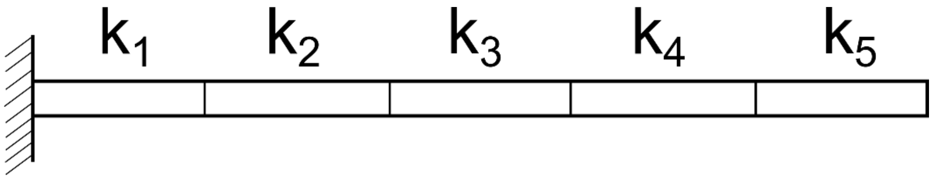
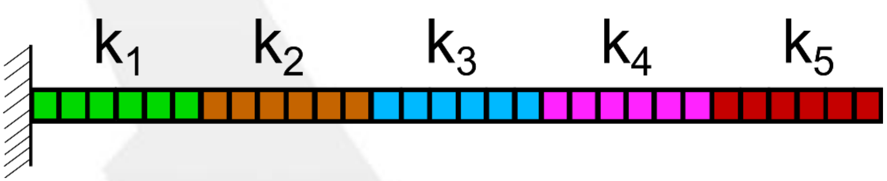
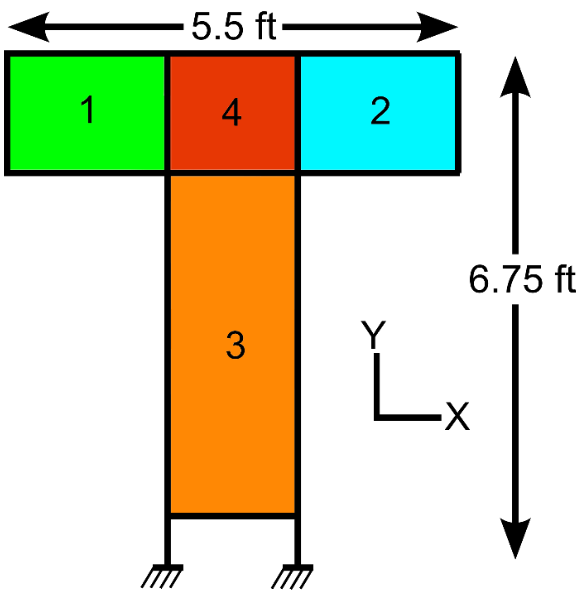


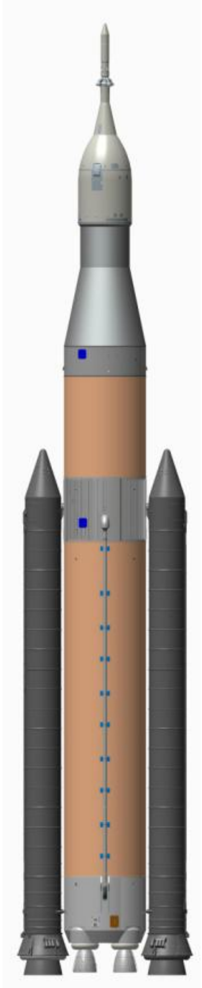
Dispersion Calculations: Substructure

- **Group together elements and treat as a single substructure**
  - Apply the model uncertainty to the stiffness and mass matrices of each substructure
  - Uncertainty factors (μ, ν) must be large enough to envelope potential uncertainties in the model
  - Beam – Young's modulus and density
  - TAURUS-T – Young's modulus, density, spring rates
  - Integrated vehicle – mass and stiffness matrices of elements
    - Core, boosters, LVSA, MPCV, etc

$$K = \sum_{i=1}^{N_{SE}} \mu_i k_i \qquad M = \sum_{i=1}^{N_{SE}} \nu_i m_i$$

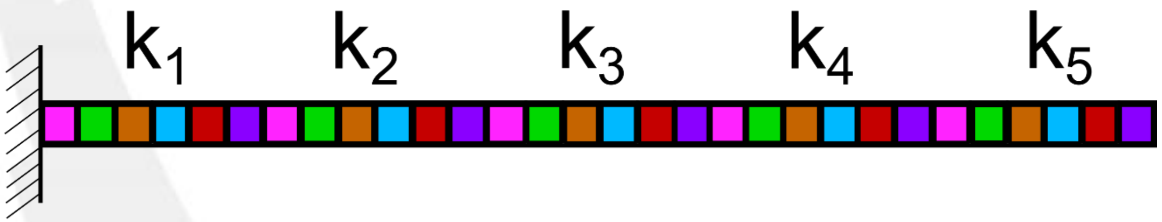


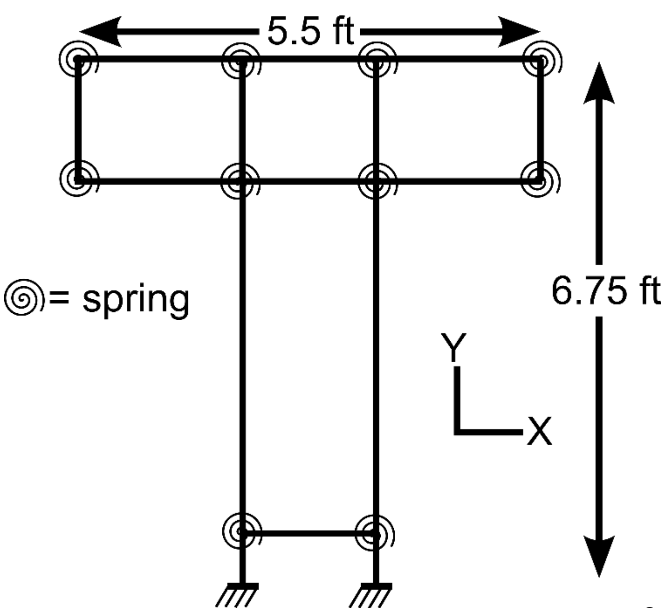




Dispersion Calculations: Property-Level

- **Treat all finite elements independently**
  - Apply the model uncertainty to stiffness and mass matrices of each element
  - May use uncertainty factors that reflect unknowns due to manufacturing or material tolerances
    - Will likely be smaller than prescribed using substructure uncertainty
  - Beam – Young's modulus and density
  - TAURUS-T – Young's modulus, density, spring rates, bar element dimensions
  - Integrated vehicle – material stiffnesses, density, bar dimensions, beam dimensions, shell thicknesses, etc.
    - Core, boosters, LVSA, MPCV, etc





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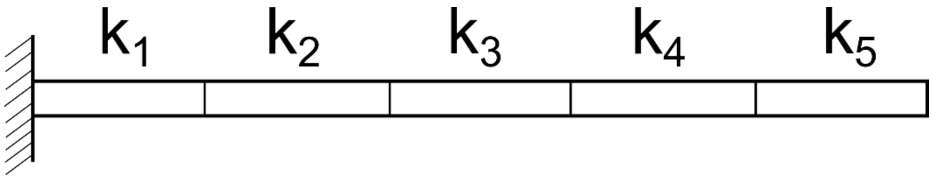


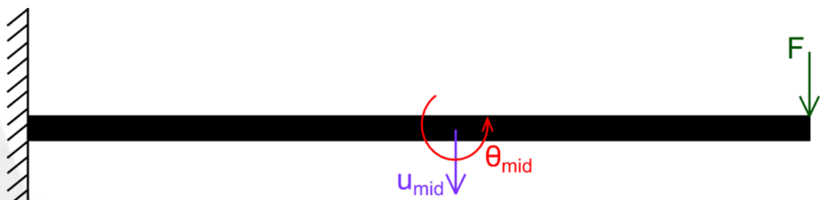
Evaluation of Dispersions: FRF

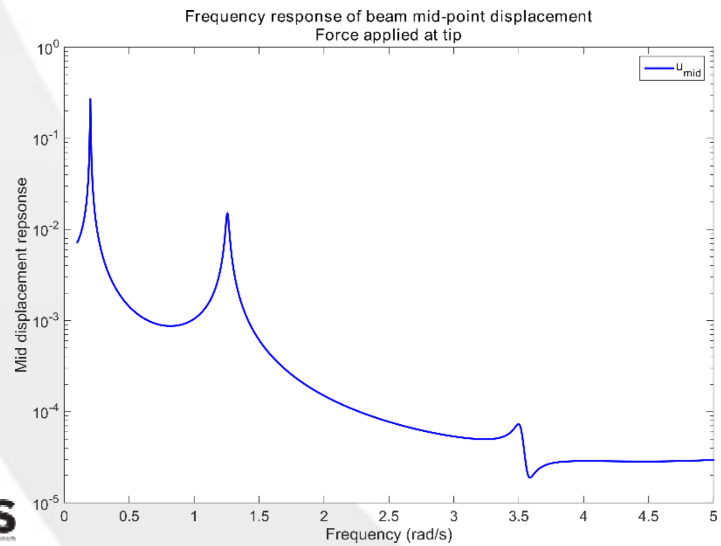
- **Equation of motion**

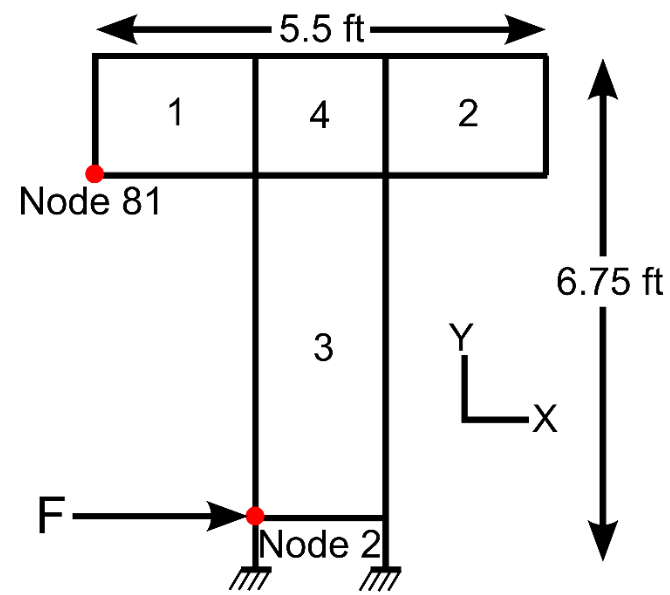
$$[I]\{\ddot{\eta}\} + [2\zeta\omega_r]\{\dot{\eta}\} + [\omega_r^2]\{\eta\} = [\Phi]^T\{F_0\}$$

- **Transfer function between force at degree of freedom *j* and output at degree of freedom *i***

$$\overline{H}_{ij}(t) \equiv \overline{H}_{u_i/p_j}(\Omega) = \sum_{r=1}^{N} \frac{\phi_{ir}\phi_{jr}^T}{\omega_i^2} \frac{1}{\left(1 - \left(\frac{\Omega}{\omega_r}\right)^2\right) + i\left(2\zeta_i\left(\frac{\Omega}{\omega_r}\right)\right)}$$







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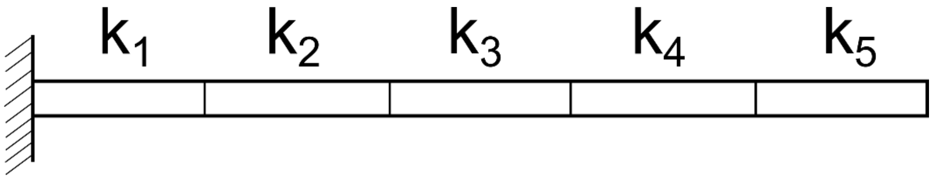
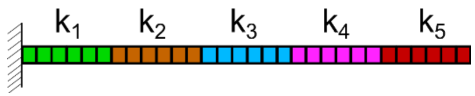
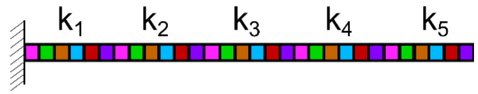
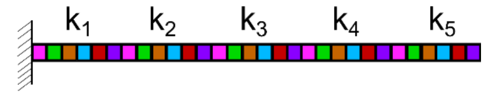
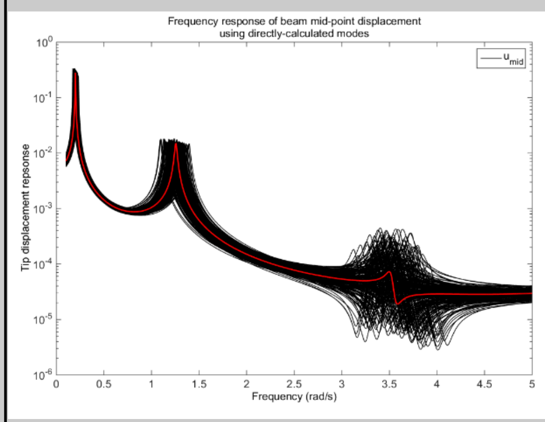
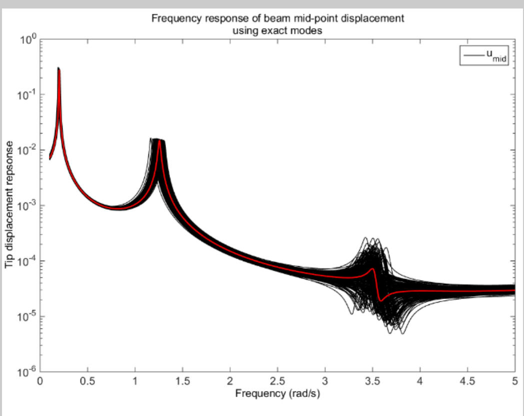
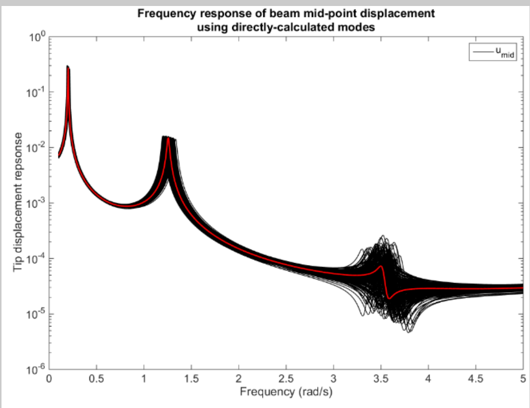


Beam Dispersions







Dispersion Type	Substructure	Part Level	Part Level
Variations	±20%	±10%	±20%
Mode 1 %change	29%	11.7%	16.3%
Mode 2 %change	24%	11.6%	14.7%
Mode 3 %change	22%	9.3%	14.3%
FRF			

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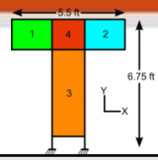
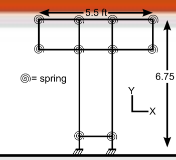
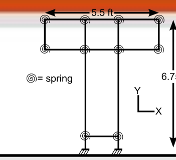
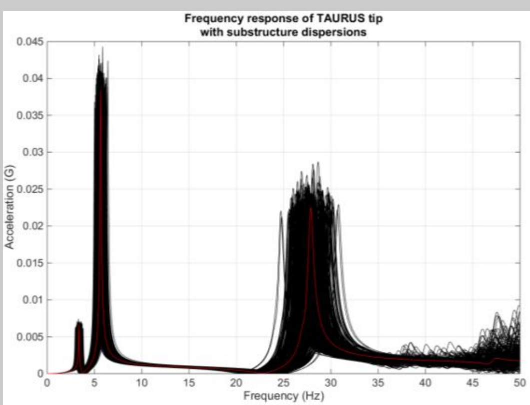
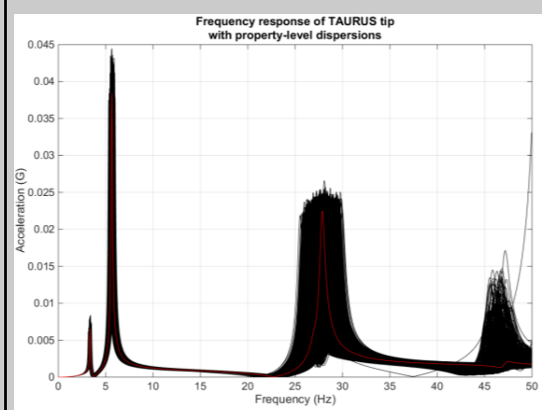
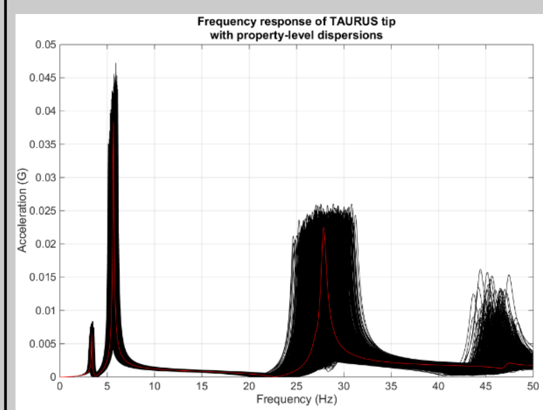
TAURUS-T Dispersions







Dispersion Type	Substructure	Part Level	Part Level
Variations	±10% E, spring rates, ρ	±5% on dim1 & dim2 50%-200% on springs E,ρ: Gaussian w/ σ=0.5%	±10% Spring rates, E, ρ, beam dim1 & dim2
Mode 1 %change	25%	11%	16%
Mode 2 %change	25%	11%	17%
Mode 3 %change	22%	16%	23%
FRF			

Design Sensitivities

- **Use the eigenvalue sensitivities to show why substructure dispersions are more conservative**

$$\frac{d\lambda_i}{dX_j} = \phi_i^T \left(\frac{dK}{dX_j} - \lambda_i \frac{dM}{dX_j} \right) \phi_i$$

	k1	k2	k3	k4	k5	k6	k7	k8	k9	k10
λ1	0.1273	0.1156	0.1045	0.0940	0.0840	0.0747	0.0659	0.0577	0.0501	0.0432
λ2	0.1132	0.0775	0.0487	0.0268	0.0116	0.0030	0.0002	0.0026	0.0091	0.0185
λ3	0.1015	0.0501	0.0173	0.0022	0.0022	0.0131	0.0292	0.0448	0.0551	0.0572
λ4	0.0905	0.0289	0.0028	0.0060	0.0261	0.0475	0.0575	0.0509	0.0321	0.0116

Note: numbers shown are the absolute values of the sensitivities

- **Substructure dispersions have the cumulative effect of the parts**
- **Part-level dispersions: some element stiffness values within a substructure go up while some go down**

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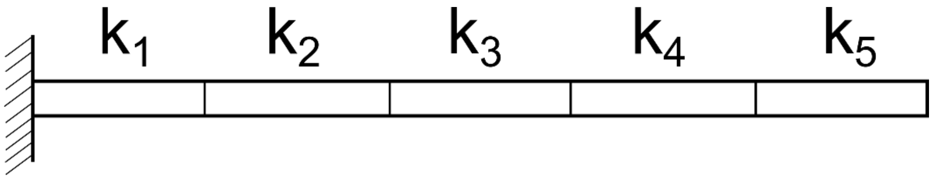

Taylor Series Approximations

- One cost-reduction method is to approximate modes with Taylor series

$$\lambda_D = \lambda_B + \sum \frac{\partial \lambda_B}{\partial X_i} dX_i + \frac{1}{2}\frac{\partial \lambda_B}{\partial X_i}(dX_i)^2$$

$$\phi_D = \Phi_B + \sum \frac{\partial \Phi_B}{\partial X_i} dX_i + \frac{1}{2}\frac{\partial \Phi_B}{\partial X_i}(dX_i)^2$$

- The first and second derivative of the eigenvalues and eigenvectors are easily calculated
- A pseudo-inverse method used to get eigenvector sensitivities

$$\frac{d\lambda_i}{dX_j} = \phi_i^T \left(\frac{dK}{dX_j} - \lambda_i \frac{dM}{dX_j}\right)\phi_i$$

$$(K - \lambda_i M)\frac{d\phi_i}{dX_j} = \left(\frac{dK}{dX_j} - \lambda_i \frac{dM}{dX_j} - \frac{d\lambda_i}{dX_j} M\right)\phi_i$$

- The approximation only for beam (TAURUS results within month)

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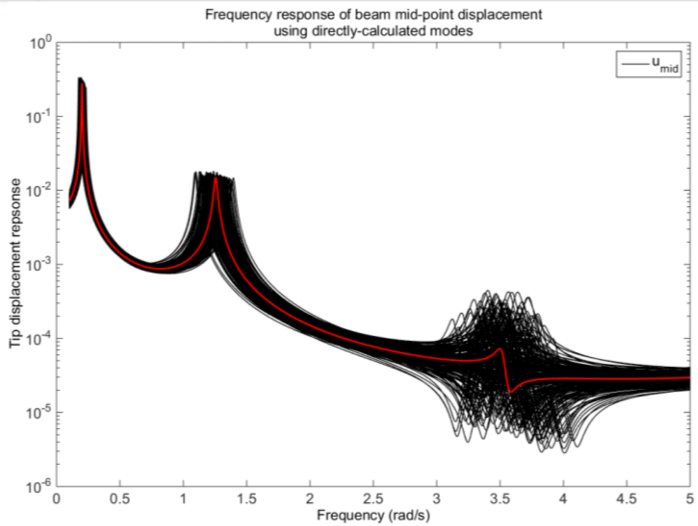
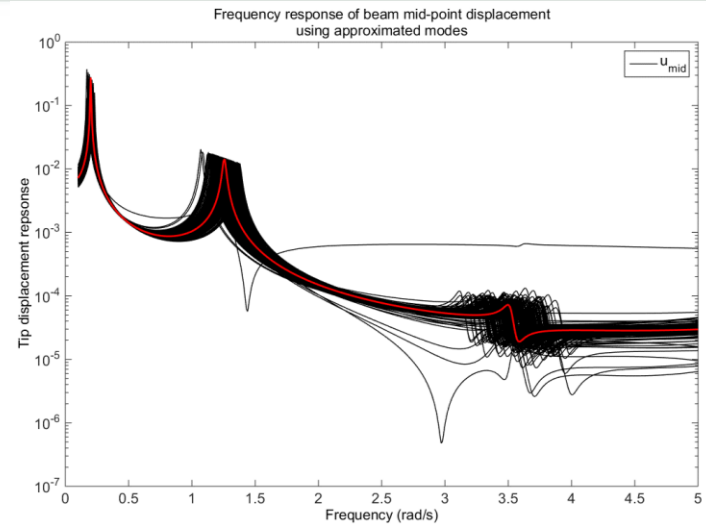
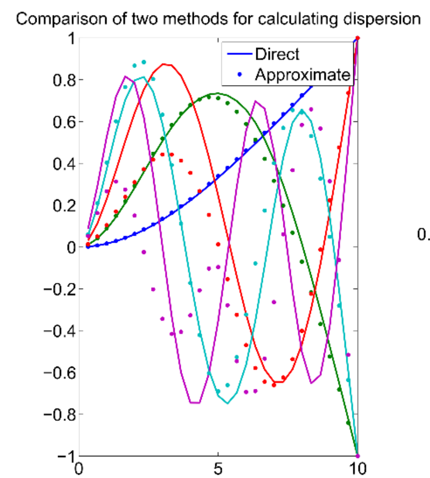
Taylor Series Approximations

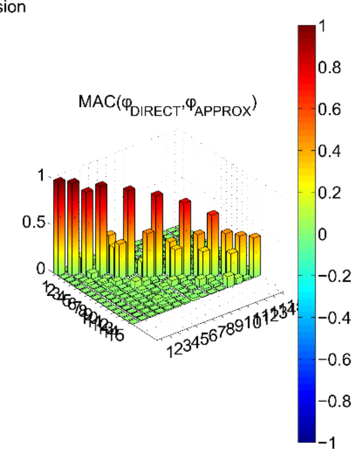




- **Taylor series approximation of FRF response**
  - Good for first two modes, poor for higher order modes
  - Gains at the peaks are linear with respect to frequency, not so for exact FRF

- **Compare exact and approximate modes with modal assurance criteria**
  - With the ±10% dispersion values, the approximation breaks down





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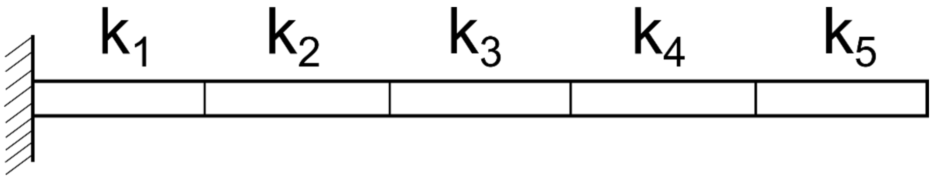


Discussion/ Conclusions

- **Two(-ish) methods of calculating modal dispersions**

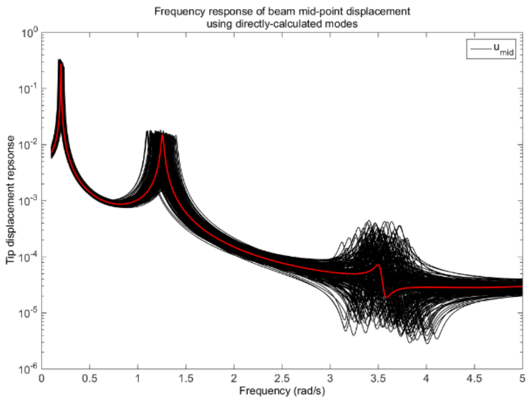
- **Substructure dispersions**
  - Group together elements that are spatially close
  - Apply uncertainty factors to substructure stiffness and mass matrices
  - Developed to be more model-realistic than 100 inch method
  - Requires large uncertainty values to get to traditional levels of uncertainty
  - Can be performed on reduced or full finite element models

- **Part-level dispersions**
  - Apply uncertainty factors to element dimensions and material properties
  - Realistic uncertainty values applied
    - Manufacturing tolerances
    - Material quality control
  - Provides most physically realistic modal dispersions
  - Uses the full finite element model, thus costly
  - Can provide an estimate of the model uncertainty
  - Least conservative

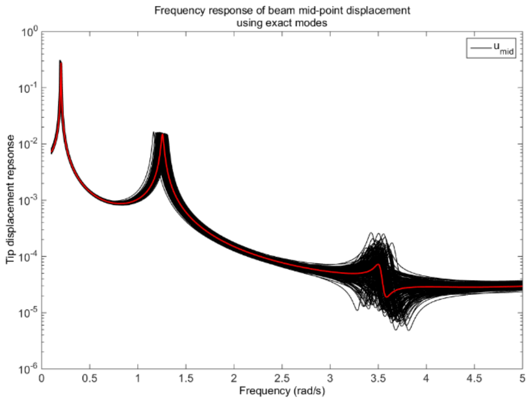
- **Taylor series dispersions**
  - Potential cost savings
  - Quickly lose accuracy

Beam Dispersions

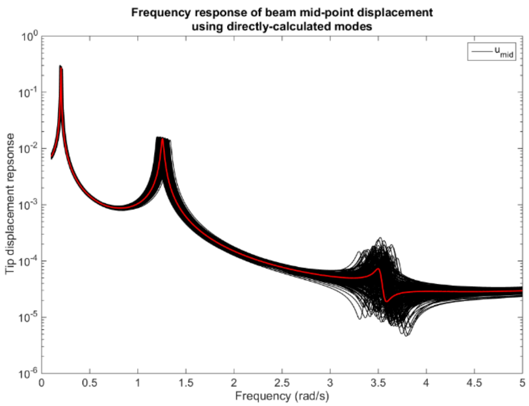
Substructure
- Each mass and stiffness allowed to vary ±20%
- First three frequencies vary 29%, 24%, and 22%



Part level – 10%
- Properties (E, ρ) varied ±10%
- Modes vary by 11.7%, 11.6%, and 9.3%



Part level – 20%
- Properties varied ±20%
- Modes vary by 16.3%, 14.7%, and 14.3%



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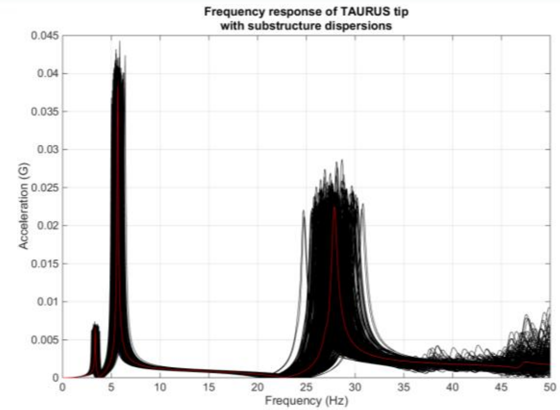
TAURUS-T Dispersions

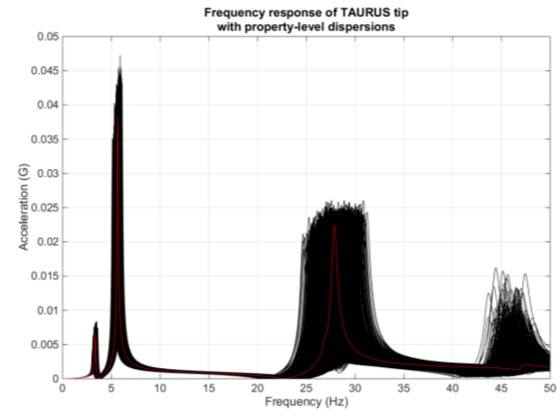
Substructure

- Stiffness (E, spring rates) and mass varied ±10%
- First three peaks vary 25%, 25%, 22%



Part Level

- Vary spring rates, beam dimensions, Young's modulus, and density ±10%
- First three peaks vary 16%, 17%, and 23%



Part Level

- Cross-sectional dimensions varied 5%
- Spring rates varied 50%-200%
- Young's modulus and density varied with Gaussian distribution with σ=0.5%*nominal
- First three peaks vary 11%, 11%, 16%