Why are we here? Why do I have a job?


  - “During development and operation of the SSME, 27 ground test failures of sufficient severity to be termed “major incident” have occurred.”

  - “Most SSME failures were a result of design deficiencies stemming from inadequate definition of dynamic loads. High cycle fatigue was the most frequent mechanism leading to failure.”
• Introduction to NASA’s new SLS
• Short Review of Basics of Structural Dynamics
• The Critical Role of Structural Dynamics in the Design, Analysis, Testing, and Operation of Rocket Engines:
  – How Rocket Engines Work
  – Turbomachinery
  – Rocket Nozzles
  – Rocket Engine Loads
  – System Hardware and Propellant Feedlines
Travelling To and Through Space

Space Launch System (SLS) – America’s Heavy-lift Rocket

- Provides initial lift capacity of 70 metric tons (t), evolving to 130 t
- Carries the Orion Multi-Purpose Crew Vehicle (MPCV) and significant science payloads
- Supports national and international missions beyond Earth’s orbit, such as near-Earth asteroids and Mars

Test of RS25 Core Stage Engine for Space Launch System
• Free Vibration, Undamped Single Degree of Freedom System

$$\sum F_x = m\ddot{u}$$
$$m\ddot{u} + Ku = 0$$

1) Steady State, simplest, worth remembering:
Assume solution $u = u(t)$ is of form

$$u(t) = A\cos(\omega t)$$
$$\dot{u}(t) = -A\omega \sin(\omega t)$$
$$\ddot{u}(t) = -A\omega^2 \cos(\omega t)$$

Now plug these equalities into eq of motion:
$$m(-A\omega^2 \cos \omega t) + k(A\cos \omega t) = 0$$
$$A\cos \omega t(k - \omega^2 m) = 0$$

For $A\cos \omega t = 0$, $A$ has to $= 0$, i.e., no response ("trivial solution")

Therefore, $k - \omega^2 m = 0$

Solution is Eigenvalue $\lambda = \omega^2 = \frac{k}{m}$  \(\Rightarrow\)  $\omega = \sqrt{\frac{k}{m}}$  \(\text{Rad/sec}\)

Define $\omega \equiv \text{Natural Frequency}$
So, solution for $u = u(t)$ is
$$u(t) = A\cos(\sqrt{\frac{k}{m}} t)$$

where $A$ depends on the initial conditions
Response to Harmonic Excitation

- Ω = Excitation Frequency
- p = Harmonic Excitation Force
- ω = System Natural Frequency = \sqrt{\frac{k}{m}}

\[ \text{eq. of motion: } m\ddot{u} + c\dot{u} + ku = F_o \cos \Omega t \]

steady state response part of the solution is \( \ddot{u}_{ss}(t)) = \ddot{U} \cos(\Omega t + \phi) \)

define static response \( U_{st} \) to force \( F_o \) using \( F_o = k U_{st} \rightarrow U_{st} = \frac{F_o}{k} \)

and define the "Complex Frequency Response"

\[ \bar{H}(\Omega) = \frac{\text{Dynamic Response } \ddot{U} \text{ at frequency } \Omega}{\text{Static Response } \ddot{U}_{st}} \]

\[ \ddot{U}(\Omega) = \bar{H}(\Omega) \ddot{U}_{st} \]

Long Derivation

\[ |\bar{H}(\Omega)| = \frac{1}{\sqrt{(1 - r^2)^2 + (2\zeta r)^2}} \]

where we define the Frequency Ratio \( r = \frac{\Omega}{\omega} \)

Resonance is defined when \( \Omega = \omega \), ie, \( r = 1 \).

At \( r = 1 \), \[ |\bar{H}(\Omega)| = \frac{1}{2\zeta} \equiv \text{Quality Factor } Q \]
P=2 lb * cos(Ωt), m=1 lb-sec^2/in

c=0.6 lb-sec/in, k=9 lb/in

\[ \omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{9}{1}} = 3 \]

\[ \zeta = \frac{c}{2\sqrt{km}} = \frac{0.6}{2\sqrt{(9)(1)}} = 0.1 \]

\[ U_{\text{static}} = \frac{F_o}{k} = \frac{2}{9} = 0.222 \]

At resonance, \( |U| = Q \cdot U_{\text{static}} = \)

\[ |U| = \frac{1}{2\zeta} \cdot (0.222) = 1.111 \]

For example, at \( \Omega = 2.8 \), \( r = \frac{\Omega}{\omega} = \frac{2.8}{3} = 0.9333 \), so

\[ |\bar{H}(\Omega)| = \frac{1}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}} = \frac{1}{\sqrt{(1-0.93333^2)^2 + (2 \cdot 1 \cdot 0.93333)^2}} = 4.408 \]
Modal Analysis of Multiple Discrete DOF Systems

Solutions for Undamped, Free Vibration of MDOF Systems with N dof's.

\[ [M] \ddot{u} + [K]u = 0 \]

Assume \( m \) solutions (≡ eigenvectors ≡ modes) of form

\[ \{u\}_m = \{\phi\}_m e^{i(\omega_m t + \alpha_m)} \]

\( m = 1, \ldots, M \), where \( M \leq N \)

Continuous \hspace{1cm} Discrete MDOF

\[ w(x) = U_m(x) \iff \{\phi\}_m \]

\[ \phi_{21} \rightarrow \bar{u}_{2m}(t) = \phi_{2m} e^{i(\omega_1 t + \alpha_1)} \]

\[ \phi_{41} \rightarrow \phi_{61} \]
Other Spatial Solutions are other Mode Shapes

Clamped-Free Boundary Conditions

Mode 1 at $f_1$ Hz

Mode 2 at $f_2$ Hz

Mode 3 at $f_3$ Hz
Frequency and Transient Response Analysis uses Concept of Modal Superposition using Generalized (or Principal Coordinates).

![Math equation: \[ \begin{bmatrix} M \end{bmatrix}\ddot{u} + [C]\dot{u} + [K]u = \{ P(t) \} \]

- **Mode Superposition Method** – transforms to set of uncoupled, SDOF equations that we can solve using SDOF methods.
- First obtain \( [\Phi]_{mass} \). Then introduce coordinate transformation:

\[
\{ u \} = N [\Phi] \{ \eta \}_M \quad \text{where} \quad [\Phi] = [\phi_1 \phi_2 \phi_3 \ldots \phi_M]
\]

\[
[M][\Phi]\dot{\eta} + [C][\Phi]\dot{\eta} + [K][\Phi]\eta = \{ P(t) \}
\]

- Generalized (or Modal) Force - dot product of each mode with excitation force vector - means response directly proportional to similarity of spatial shape of each mode with spatial shape of the force (Orthogonality).
for the SDOF equation of motion,
\[ m\ddot{u} + c\dot{u} + ku = p \rightarrow \ddot{u} + 2\zeta\omega\dot{u} + \omega^2 u = F_0e^{i\Omega t} \]

\[
|U(\Omega)| = \frac{F_0}{k} \sqrt{\frac{1}{1 - \left(\frac{\Omega}{\omega}\right)^2 + \left(2\zeta\frac{\Omega}{\omega}\right)^2}}
\]

So we get the same equations in \( \eta \):

\[
\dot{\eta}_m + 2\zeta_m\omega m\dot{\eta}_m + \lambda_m\eta_m = \{\phi\}^T_m \{P(t)\}
\]

\[
|\eta_m(t)| = \frac{\{\phi\}^T_m \{F\}}{\lambda_m} \sqrt{\frac{1}{1 - \left(\frac{\Omega}{\omega_m}\right)^2 + \left(2\zeta_m\frac{\Omega}{\omega_m}\right)^2}}
\]
Random Vibrations

- For structures undergoing random vibration (vibration whose magnitude can only be characterized statistically), random vibration analysis gives the statistical characterization of the response.

\[ PSD(\omega) = \frac{1}{\pi} \int_{0}^{\infty} f(\tau)e^{i\omega\tau} d\tau \]

- 1 sample out of a population

- Perform a Fourier Transform of the excitation to generate a Power Spectral Density (PSD).
• Envelope population of PSD’s to generate Random Excitation specification.
• Apply Excitation as series of frequency response analyses, generates response.
• Response will also be in frequency domain, and can be converted to a PSD.

• The area under the PSD curve is defined as the “mean square ($\Phi^2$)”. 
• Root of the Mean Square (RMS) of entire response PSD equals 1 standard deviation of response for a Gaussian distribution.
By general agreement, the design value for a random response is generally a value that exceeds the response 99.865% of the time.

This value is 3 sigma for a normal distribution. So we simply multiply the RMS by 3 and use that as our design value.
How a Rocket Engine Works, and why it needs Structural Dynamic Analysis

- Liquid Fuel (LH2, Kerosene) and Oxidizer (LO2) are stored in fuel tanks at a few atmospheres.

- Turbines, driven by hot gas created by mini-combustors, tied with shaft to pump, suck in propellants, increases their pressures to thousands of psi, producing substantial harmonic forces at specific frequencies.

- High pressure propellants sent to Combustion Chamber, which ignites mixture with injectors, produces large forces in a wide band of frequencies, most of which are random.

- Hot gas directed to converging/diverging nozzle to give flow very high velocity for thrust.

- Both the random and the harmonic loads propagate through every component on the engine and last throughout engine operation.

\[ \text{Thrust} = F = \dot{m} V_e + (p_e - p_{atm}) A_e \]

\[ V = \text{Velocity} \]
\[ \dot{m} = \text{mass flow rate} \]
\[ p = \text{pressure} \]
Turbine Components (vanes, stators, blades) experience large harmonic excitations from up & downstream components, and multiples of these counts.
Crack found during ground-test program can stop engine development
If crack propagates, it could liberate a piece

- At very high rotational speeds could be catastrophic (i.e., engine will explode)
- Can cause large unbalance in rotor shaft, driving it unstable, causing engine failure.
First obtain speed range of operation from performance group.

- For Rocket Engines, there are generally several “nominal” operating speeds dependent upon phase of mission (e.g., reduce thrust during “Max Q”).
- However, since flow is the controlling parameter, actual rotational speeds are uncertain (especially during design phase).
- For new LPS engine being built at MSFC, assuming possible variation +/-5% about each of two operating speeds.

In addition, speed generally isn’t constant, but instead “dithers”.†

<table>
<thead>
<tr>
<th>Rated Power Level</th>
<th>70%</th>
<th>100%</th>
</tr>
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<tbody>
<tr>
<td>Low Range</td>
<td>20759.4</td>
<td>26125</td>
</tr>
<tr>
<td>Nominal</td>
<td>21852</td>
<td>27500</td>
</tr>
<tr>
<td>High Range</td>
<td>22944.6</td>
<td>28875</td>
</tr>
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</table>

†Implementation of Speed Variation in the Structural Dynamic Assessment of Turbomachinery Flow Path Components
Andrew M. Brown, R. Benjamin Davis and Michael K. DeHaye
Sinusoidal excitation Frequency = N * j * d
- Speed N (RPM)
- d = Number of flow distortions arising from adjacent upstream and downstream blade and vane counts
- “harmonics” j = 1, 2, 3
Now Structure: Create FEM of component, Modal Analysis

Example: Turbine Blades

Mode 12 at 36850 Hz

Mode 13 at 38519 Hz

Modal Animations very useful for identifying problem modes, optimal damper locations
Create “Campbell Diagram”

- Simplest Version of Campbell Diagram is just a glorified Resonance Chart.
Cyclic Symmetry and Matching of Nodal Diameter of Modes with Excitation Necessary Condition for Resonance

- For structures with repeating sectors, “Cyclic Symmetry” mathematical transformations enable generation of mode shape of entire structure at huge computational savings.
- These structures exhibit “Nodal Diameter” type modes.
- For disks and disk dominated modes, 5ND Traveling Wave will only excite a 5ND mode.

\[ p(\theta) = P_0 \sin 3\theta \]

5ND travelling wave Mode of Bladed-Disc

- On the other hand, 3ND excitation (perhaps from pump diffusers) will not excite a 5ND structural mode.
- Max ND=# sectors/2 for even
  (# sectors-1)/2 for odd
• Sampling by discrete number of points on structure of pressure oscillation results in spatial Nodal Diameter excitation at the difference of the two counts.

• E.g., a 74 wave number pressure field (coming from 2x37 vanes), exciting 69 blades results in a Nodal Diameter mode of 69-74=-5, where sign indicates direction of traveling 5ND wave (plot courtesy Anton Gagne).

“Blade/Vane” Interaction Causes Different ND Excitation
Tyler-Sofrin Blade-Vane Interaction Charts

<table>
<thead>
<tr>
<th>Upstream Nozzle Multiples</th>
<th>37</th>
<th>74</th>
<th>111</th>
<th>148</th>
</tr>
</thead>
<tbody>
<tr>
<td>Downstream Stator Multiples</td>
<td>57</td>
<td>114</td>
<td>171</td>
<td>228</td>
</tr>
<tr>
<td>Blade multiples</td>
<td>69</td>
<td>32</td>
<td>-5</td>
<td>N/A</td>
</tr>
<tr>
<td></td>
<td>138</td>
<td>N/A</td>
<td>27</td>
<td>-10</td>
</tr>
<tr>
<td></td>
<td>207</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
</tbody>
</table>

- 74N excites 5ND mode at 40,167 hz
- 4 revolution CFD analysis such that primary temporal Fourier Component $F_0 e^{i \Omega t}$ has that frequency.
Modal Analysis has Multiple Uses

• Redesign Configuration to move excitations ranges away from natural frequencies
• Redesign component to move resonances out of operating range.
• Put in enough damping to significantly reduce response
• Use as first step in “Forced Response Analysis” (applying forces and calculating structural response).
Modal analysis of original design indicated resonance with primary mode by primary forcing function.

- Since excitation simultaneously from upstream and downstream blades, critical to change design to avoid resonance.
- Extensive optimization effort performed to either move natural frequency out of range and/or change count of turbine blades to move excitation.

**Stator Airfoil Thickness Changes**

- Initial (R02)
- R03b2
- R03b2_t2
- R03b2_t4
- R03b2_t5
- R03b2_t6

_Courtesy D. O’Neal_
Range of +/- 5% on natural frequencies to account for modeling uncertainty.
Can Also Use Modal Analysis in Failure Investigations

- Examination of **Modal Stress** Plots provides link to location of observed cracking.

\[
\begin{align*}
\phi_m &= \begin{bmatrix} \phi_1 \\ \phi_2 \\ \vdots \\ \phi_N \end{bmatrix}^m \\
\sigma_m &= \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \vdots \\ \sigma_N \end{bmatrix}^m
\end{align*}
\]

SSME HPFTP 1st Stage Turbine Blade

Crack Origination Site at Inside of tip core
If Forced Response Analysis Required, need Forcing Function on Turbine Components from CFD

• For “Frequency Response” Analysis, apply Fourier coefficients coming from CFD such that excitation frequencies match Campbell crossovers.
Forced Response Analysis in Failure Investigations

- SSME HPFTP 1\textsuperscript{st} Stage Impeller.

Mode shape

Crack location 1\textsuperscript{st} splitter

Frequency Response Analysis
Damping

• Damping is critical parameter for forced response prediction, so “whirligig” test program used to obtain data.
• Whirligig is mechanically-driven rotor with bladed-disk excited by pressurized orifice plate to simulate blade excitation.

• SDOF Curve fit technique applied to selected top-responding blades to derive damping from response.
Injectors in Main Combustion Chamber

- Test of SSME in 1981 failed due to burn through of 149 liquid oxidizer Injectors, caused by high-cycle fatigue cracking.
- Failure investigation showed design insufficient to withstand huge random load caused by combustion and flow induced vibration from hot gas (flow by cylinders causing vortex shedding).
Nozzle is a major portion of the overall dynamics of the engine, frequently the structural backbone if components mounted onto it.

Accurate assessment of Nozzle response critical for evaluating both HCF and Ultimate.

Nozzle material complex
- Tube-wall construction filled with liquid hydrogen
- Graphite phenolic composite, Young’s Modulus can be highly temperature dependent
- Exotic high temperature metals, still close to melting.

Upper Stage Engine Designs can be unusual to allow for optimization during ascent
- RL10B has extension that stows until deployed; undeployed configuration has very active modes that were challenge to prove ok during ascent.

Ref: Impact of Dynamics on the Design of the RL-10B-2 Extendible Carbon-carbon Exit Cone, Mary Baker et.al, 1998
“Side Loads” in Rocket Nozzles is Major Fluid/Structural Dynamic Interaction Issue

- Start-up, shut-down, or sea-level testing of high-altitude engines, ambient pressure higher than internal nozzle wall pressures.
- During transient, pressure differential moves axially down nozzle.
- At critical $p_{\text{wall}}/p_{\text{ambient}}$, flow separates from wall ("Free Shock Separation"), allows ambient air at much higher pressure to rush in & induce huge SL.
- Primary Nozzle Failure Mode for most Rocket Engines is Buckling due to Side Loads
- Caused failures of both nozzle actuating systems (Japanese H4 engine), sections of nozzle itself (SSME).
- Existing “Skewed Plane” Side Load calculation method assumes separation at two different axial stations, integrates the resultant $\Delta P^*dA$ loads.
- Method calibrated to maximum and minimum possible separation locations to be intentionally conservative.
Also postulated that FSS initiates aeroelastic coupling with 2\textsuperscript{ND} mode.

FASTRAC engine designed to operate in overexpanded condition during ground test, so this could be a problem.

Test/analysis program initiated with goal of obtaining physics-based, predictable value.

Strain-gauge measurements taken on nozzle during hot-fire test

Flow separation clearly identified at steady-state operation.

- Video, Pressure and strain-gage data from thin-wall nozzle show self-excited vibration loop tying structural 2ND mode and flow separation.

- Some methodologies developed by industry have used this phenomena to calculate forcing function.
Follow-on Testing to Measure Magnitude of Side Loads

- Simple “skewed plane” method still favored though, but attempt made to quantify actual value
- Simply measuring forces using pressure transducers impractical.
- “Side Load” measurement setup based on Frey, et.al., 2000, consisting of very stiff nozzle (“lumped mass”) attached to flexible “strain tube”.
- Accelerometers and pressures measured in nozzle, strains measured on strain tube.
- Hypothesis: system is SDOF, measure accel response, back-calculate forcing function FRF. (“Easier said than done” – many problem with static indeterminancy)

\[
[PSD_{AccelResponse}(\Omega)] = [H^*(\Omega)][PSD_{ForceInput}(\Omega)][H(\Omega)]^T
\]
- Instead decided to calibrate strains with series of static loads applied in transverse directions, then measure strains during test and back out loads.

Frey, M, et. al, Subscale Nozzle Testing at the P6.2 Test Stand, 36\textsuperscript{th} AIAA Joint Propulsion Conference, 17-19 July 2000, Huntsville, Alabama
• However, response strains were dominated by resonant frequency, so entire forcing function at other bandwidths essentially “filtered” out by system structural dynamics.
• We realized that instead of using entire response bandwidth, if we filter our resonant portion, we’re left with the quasi-static response, which still covers most of the excitation bandwidth.

Therefore calculate side load by measuring response strain that has dynamically amplified portion filtered out, and back out the force using calibration.

• Able to use this methodology to come up with improved estimates of magnitude of side load, and in particular to show 20% load reduction using Truncated Ideal Contour instead of traditional thrust optimized contour (called “Ruf-Brown Knockdown factor” 😊!!)
Random (combustion sources), harmonic loads (turbopumps) propagate through every component on engine, so Engine System Model required to generate major component interface loads and stresses. Initial magnitudes impractical to quantify.

However, can measure the engine dynamic environment at key locations in the engine near primary vibration sources.

For a new engine, data from “similar” previous engine designs is scaled to define an initial engine vibration environment using “Barrett Criteria.”

\[ G_n(f) \frac{g^2}{Hz} = G_r(f) \frac{N_n T V W_n}{N_r T V W_n} \]
• Accelerometer measurements taken during hot-fire testing of engine used to update or create environments.
• Specification created by “enveloping” this data, generally up to 2000 hz.
• For engines with multiple sources of excitation (thrust chamber, turbomachinery), different excitation criteria used for each “zone”.

<table>
<thead>
<tr>
<th>Frequency</th>
<th>R5 Ax</th>
<th>R5 Rad</th>
<th>R5 Tan</th>
</tr>
</thead>
<tbody>
<tr>
<td>270</td>
<td>0.2</td>
<td>0.2</td>
<td>0.4</td>
</tr>
<tr>
<td>500</td>
<td>1.0</td>
<td>0.7</td>
<td>1.0</td>
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<tr>
<td>600</td>
<td>4.5</td>
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<td>1.8</td>
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<tr>
<td>1000</td>
<td>1.3</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td>1200</td>
<td>2.5</td>
<td>0.8</td>
<td>0.9</td>
</tr>
<tr>
<td>2000</td>
<td>4.6</td>
<td>4.4</td>
<td>1.1</td>
</tr>
</tbody>
</table>

- Harmonic excitation obtained by taking peaks from overall data signal, then calculating the RMS of the sine using the PSD magnitudes of the peak & adjacent bins.

\[
g_{rms} = \sqrt{bw(A_1 + A_2 + A_3)}
\]

\[
g_{peak} = \frac{g_{rms}}{.707}
\]
Calculating System Dynamic Loads

• Requirement of calculation is that engine response matches the measured (enveloped) accelerations

• Several ways this can be done
  - System “Direct” Approach (used in Fastrac, RS–68)
    Directly apply an enforced acceleration at the points where environments are defined.
  - System Equivalent Applied Force Methods (“Industry Approach”)
    Determine a set of applied forces that will reproduce the measured environment.
  - Component Approach (“ESF Approach”)
    Calculate loads on a component basis by fixing both ends and exciting structure with random load

• Most methods used to date result in loads which are almost always over-conservative.
Direct Approach using Dynamic Only Portion

- Apply engine acceleration environments directly to the model.
- Constrain nodes to have a given random acceleration PSD

\[ X_f = \text{Free DOF} \]
\[ X_s = \text{Support DOF where accelerations are applied} \]

\[
\begin{bmatrix}
M_{ff} & M_{fs} \\
M_{sf} & M_{ss}
\end{bmatrix}
\begin{Bmatrix}
\ddot{X}_f \\
\ddot{X}_s
\end{Bmatrix}
+ \begin{bmatrix}
C_{ff} & C_{fs} \\
C_{sf} & C_{ss}
\end{bmatrix}
\begin{Bmatrix}
\dot{X}_f \\
\dot{X}_s
\end{Bmatrix}
+ \begin{bmatrix}
K_{ff} & K_{fs} \\
K_{sf} & K_{ss}
\end{bmatrix}
\begin{Bmatrix}
X_f \\
X_s
\end{Bmatrix}
= \begin{Bmatrix}
0 \\
F_s
\end{Bmatrix}
\]

- \(X_f\) can be further partitioned into dynamic portion \(X_{fd}\) and quasi-static portion \(X_{fs}\).
  - \(X_{fs}\) is static response to base acceleration at 0 hz. It only exists because of assumption that base drive accelerations are uncorrelated, when in fact, the connecting structure forces some level of correlation.
  - \(X_{fs}(t)\) is calculated by ignoring the mass and damping terms in 1st row:

\[
K_{ff} X_{fs}(t) = -K_{fs} X_s(t)
\]

\[
X_{fs}(t) = -K_{ff}^{-1} K_{fs} X_s(t) = K_I X_s(t)
\]

Fastrac Results
Pseudo-Static and Dynamic Components

- NASTRAN Finite Element Code random analysis methods used to calculate PSD’s of quasi-static portion and total response
- Difference of two is the remaining “dynamic” portion that gives true response.

Most of low-frequency response for this component is due to pseudo-static
New “Dynamic” Methodology Significantly Reduces Loads

Comparison of RMS Values for Max Load Component

- Method tested by performing multi-axis shaker test of Fastrac engine, enabling measurement of response and excitation.
## Typical Fastrac Engine Load Set

<table>
<thead>
<tr>
<th></th>
<th>Shear 1 (lbs)</th>
<th>Shear 2 (lbs)</th>
<th>Axial (lbs)</th>
<th>Bending 1 (in-lbs)</th>
<th>Bending 2 (in-lbs)</th>
<th>Torque (in-lbs)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Glue Bracket 3 (GB-3)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
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<tr>
<td>Sine X</td>
<td>97</td>
<td>7</td>
<td>0</td>
<td>3</td>
<td>78</td>
<td>72</td>
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<tr>
<td>Sine Y</td>
<td>91</td>
<td>7</td>
<td>0</td>
<td>3</td>
<td>98</td>
<td>70</td>
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<tr>
<td>Sine Z</td>
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<td>0</td>
<td>2</td>
<td>78</td>
<td>52</td>
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<tr>
<td><strong>Sine Peak (RSS)</strong></td>
<td>178</td>
<td>11</td>
<td>0</td>
<td>5</td>
<td>148</td>
<td>113</td>
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<td><strong>Random Peak (RSS)</strong></td>
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<table>
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<tr>
<th></th>
<th>Shear 1 (lbs)</th>
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<th>Axial (lbs)</th>
<th>Bending 1 (in-lbs)</th>
<th>Bending 2 (in-lbs)</th>
<th>Torque (in-lbs)</th>
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<td><strong>Stringer Bracket 3 (Lower Support)</strong></td>
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<td>SB-6</td>
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Structural Dynamic Analysis Required for all Components Near Engine.

- In 2002 Cracks found in Orbiter Main Propulsion System Feedline Flowliner

*Courtesy C. Larsen, NASA Engineering Safety Center*
St. Dynamics Tasks in Failure Investigation of Cracked Flowliners

- Assess loads and environments on flowliner
  - Difficult to characterize highly dynamic, cavitating, cryogenic flow environment
  - Analyze hot fire test data (flow induced environments)
  - Develop loading spectra (X lbs at Y hz for Z sec) for fracture analysis.

- Assess Dynamic Response
  - Finite Element Models created, modal analysis
  - Identify relevant modes for each flight condition
  - Assess strain transfer factors (test measured locations at mid-ligament to crack initiation / field stress)
Huge NASA-wide team assembled. Structural Dynamic team played key role:

Flowchart of Analyses

External Constraints:
- Certification failure modes
- Orbiter flight data
- BTA/GTA test data
- LPTP/SSME operations

Flight Rationale

Residual Fatigue Life

Fatigue Loading Spectrum

Fracture Mechanics Analysis

Combined Loads Time Histories

Static Loads: Temp & Residual

Flow-Induced Dynamic Loads

 Ops/Launch Mechanical Loads

Structural Dynamics Role

Forcing Functions

Fluid / Structure Interaction

Flowliner Dynamic Response

Feedline Flow Physics

Huge NASA-wide team assembled. Structural Dynamic team played key role:
Flowliner Dynamic Analysis Results

• Dynamic analysis determined source of cracking was several modes excited by downstream inducer blade count and cavitation.
• Tested flowliner dynamic response to validate models.
• Performed fracture analysis and computed expected service life based upon observed crack sizes. Solution was improved and more frequent inspections.
Modal Testing Critical for Validating Models

- Use Instrumented Hammer to do quick impact onto structure, which contains broadband frequency content.
- Response measured using an accelerometer or vibrometer.
- Fourier Transform of response/excitation (FRF) generated, imaginary part at each location gives magnitude of mode shape.
- Compare test & analytical mode shapes, update if necessary.

Huge complication for pump-side components in LOX is that fluid-added mass reduces natural frequencies by 20-40%.

- Operating temperature and spin also affect frequencies.
• Structural Dynamics is one of the Critical Disciplines for the successful Design, Development, Testing of Rocket Engines.

• It is applied from the smallest component (turbine blades), all the way to the entire engine and propellant feedlines.

• Successful application of Structural Dynamics requires extensive knowledge of Fourier Techniques, Linear Algebra, Random Variables, Finite Element Modeling, and essentials of SDOF and MDOF vibration theory.

• Working knowledge of Fluid Dynamics and Data Analysis also extremely useful.

• It all pays off when you get to see a successful engine firing!