Hybrid Differential Dynamic Programming with Stochastic Search

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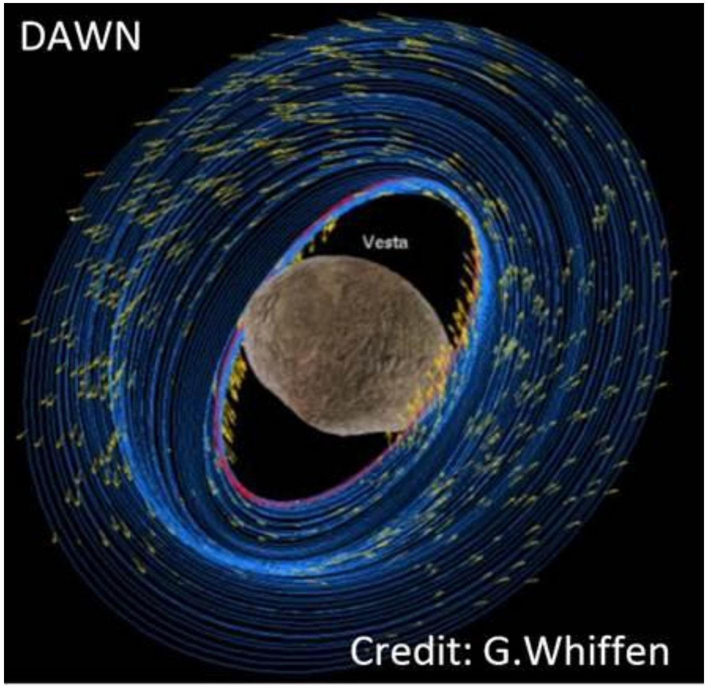
Motivation

Research Objective:

"… to efficiently and robustly optimize high-dimensional spacecraft trajectories" (e.g. low-thrust, many revolutions)



- Large number of decision variables
- Many local optima
- Hard to form an appropriate initial guess

Popular direct methods:
discretize trajectory into *M* stages of *N* decision variables (e.g. thrust vector) and solve the nonlinear programming problem (NLP) of size *M*N*

Differential Dynamic Programming (DDP) instead solves many subproblems, i.e. DDP solves *M* NLPs of size *N*

AND

without the need for 'black box' NLP solvers (SNOPT, IPOPT, fmincon, etc.)

Alphabet Soup

This study implements HDDP to compute spacecraft trajectories and uses MBH as a stochastic search step to find better solutions.

DDP – Differential Dynamic Programming
- a trajectory optimization algorithm

HDDP – Hybrid Differential Dynamic Programming
- a recent variant of DDP by Lantoine and Russell

MBH – monotonic basin hopping
- multi-start algorithm to search many local optima

EMTG – Evolutionary Mission Trajectory Generator
- NASA Goddard open source mission design tool

FBLT – finite burn low-thrust
- two-body equations with continuous low-thrust

ALM – augmented Lagrangian method
- Constrained optimization by adding penalty term to Lagrangian cost function

Dynamic Programming

- Solve a complex problem by breaking it down into smaller subproblems

Differential Dynamic Programming

- Iteratively perform backward sweep on the trajectory to update the control sequence
  - Minimizing quadratic expansion of objective function yields feedback control law

Hybrid Differential Dynamic Programming

- Use state transition matrix (Φ) and state transition tensor to map sensitivities
- NLP techniques
  - Augmented Lagrangian Method
  - Trust-region Methods

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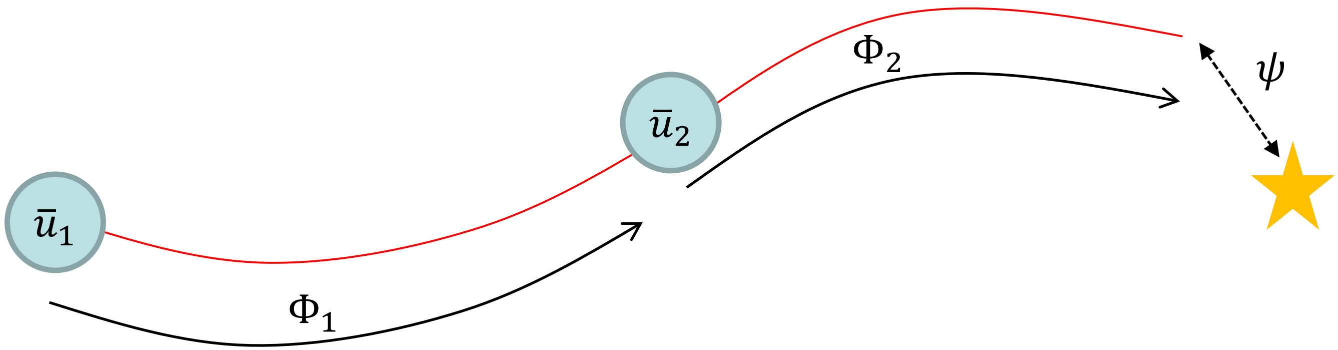
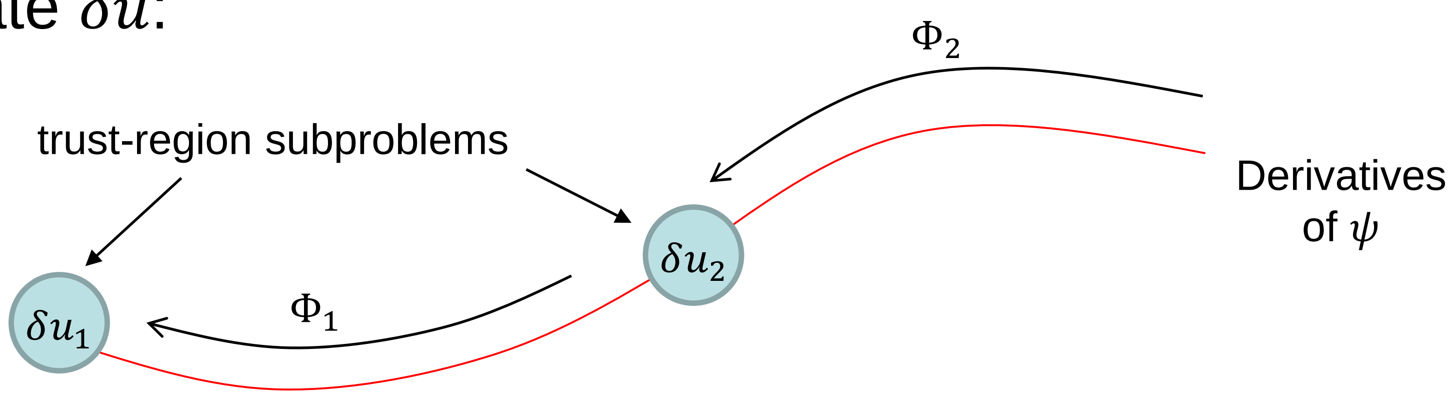
HDDP Iteration

Forward pass on nominal control sequence $\bar{u}$:



Backward sweep recursively solves subproblems for control update δu:



Augmented Lagrangian Method

Terminal constraints are enforced by added penalty to cost function:

$$J = h + \lambda^T \psi + \sigma \, \psi^T \psi$$

h: original objective

λ: Lagrange multipliers

ψ: constraint violations

σ: penalty parameter

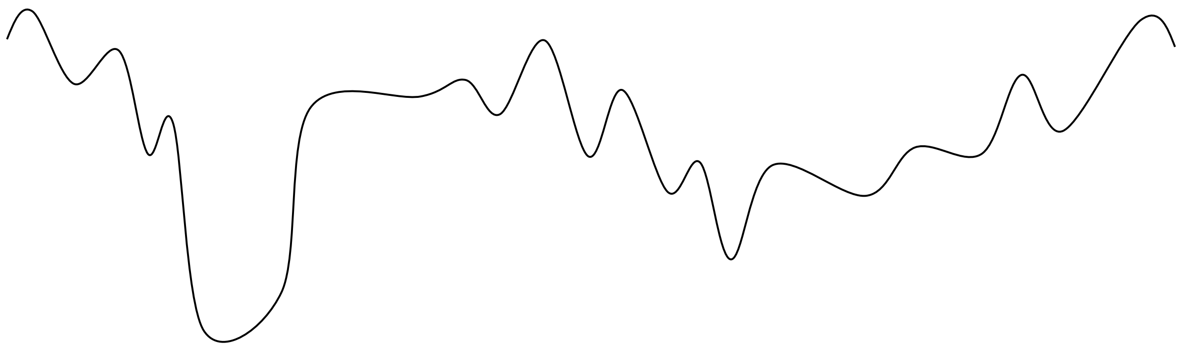
An initial σ must be sufficiently large, and continually increased to drive $\psi \to 0$.

Tuning σ and its update factor κ_σ can be tedious.

Stochastic Search

HDDP is a gradient based method that will converge to a solution nearby an initial guess.

- motivates a stochastic search across many local optima



MBH has previously been implemented for controlled random search in spacecraft trajectory design.

- ESA Advanced Concept Team and EMTG
  - ❖ applied to ‘black box’ NLP solvers

HDDP now fills the role of the NLP solver.

Monotonic Basin Hopping

1. HDDP computes nominal solution from initial guess
2. Until MBH stopping criteria (compute time or N_{hop}):
   a. Introduce perturbations to decision variables

      random draw from Pareto distribution

   b. If $rand(0,1) < \rho_{time-hop}$

      Shift time variables forward or backward 1 synodic period

   c. Reinitialize HDDP with perturbed solution as initial guess
   d. Accept iterate if:

      - feasible and cost improves
      - infeasible and violations reduced

Computed Examples

- Example Earth-Mars rendezvous transfer from Lantoine and Russell is used for validation and test case for variable time of flight with MBH.

Spacecraft	
m_0	1000.0 kg
T_{max}	.5 N
I_{sp}	2000 s

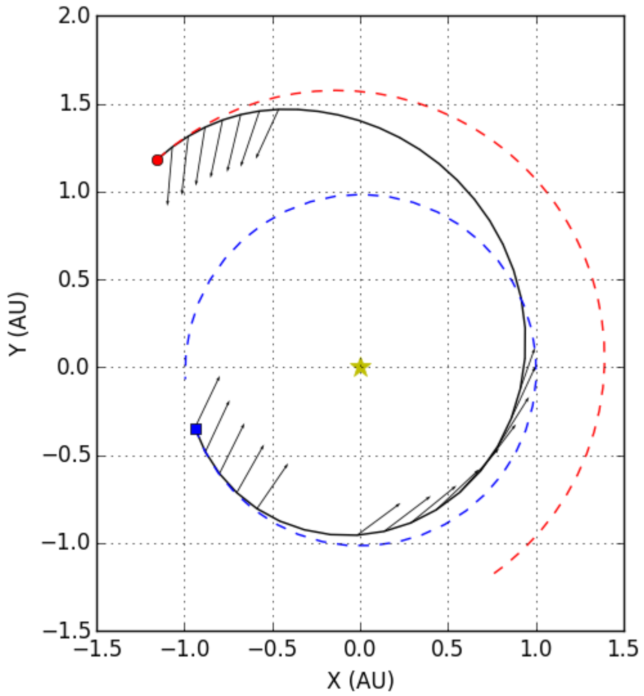
Mission	
t_0	April 12, 2007
TOF	348.79 days

- Objective is to maximize final mass with penalty on position and velocity errors at Mars arrival

$$h = -m_f$$

$$\psi = \begin{bmatrix} r_f - r_M \\ v_f - v_M \end{bmatrix}$$

Fixed TOF Validation



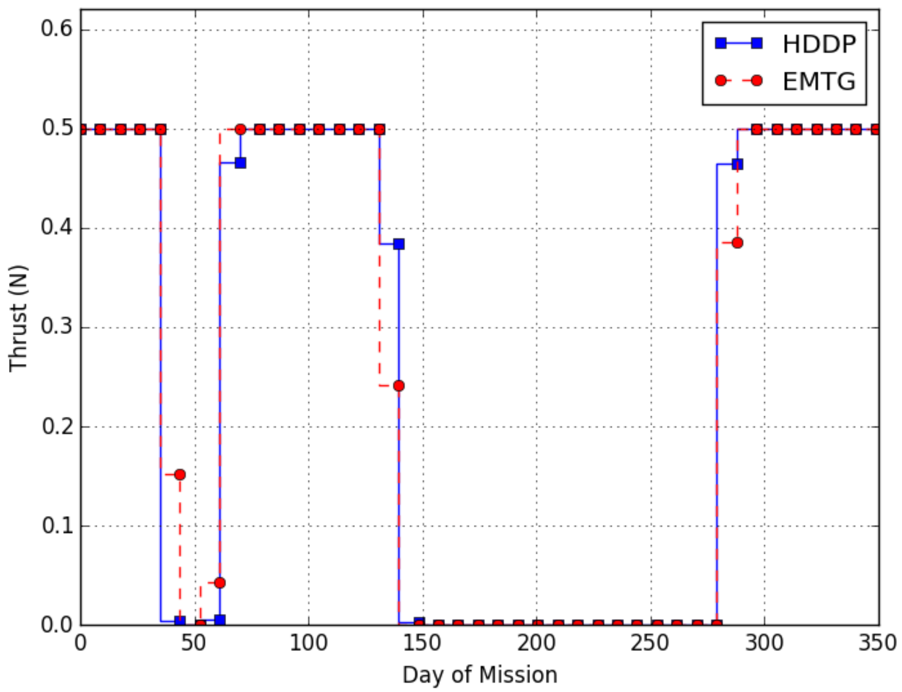


Table 1: Comparison of Spacecraft Final Mass

HDDP *standard*	$m_f = 598.66$ kg
HDDP – FBLT	$m_f = 603.29$ kg
EMTG – FBLT	$m_f = 603.45$ kg

Table 2: Comparison of Lagrange multipliers

HDDP *standard*	$\lambda = [0.5095, -1.2700, -0.2665, 0.1178, 2.0701, 0.13404]^T$
HDDP – FBLT	$\lambda = [1.0793, -2.3127, -0.5920, -0.1125, 2.9337, 0.0463]^T$

MBH can be used to cover for a poorly tuned HDDP

Table 3: Survey of ALM tuning and improvements with MBH

σ_0	κ_s	Iterations	m_f (kg)	$m_f, N_{hop} = 30$
10	1.1	123	failed	603.29*
	1.5	541	failed	603.04
	2.0	643	601.87	602.80
100	1.1	231	failed	603.07
	1.5	181	602.93	603.17
	2.0	892	601.67	602.04
1000	1.1	352	failed	602.95
	1.5	1119	602.50	603.14
	2.0	940	601.75	602.21

Variable Time

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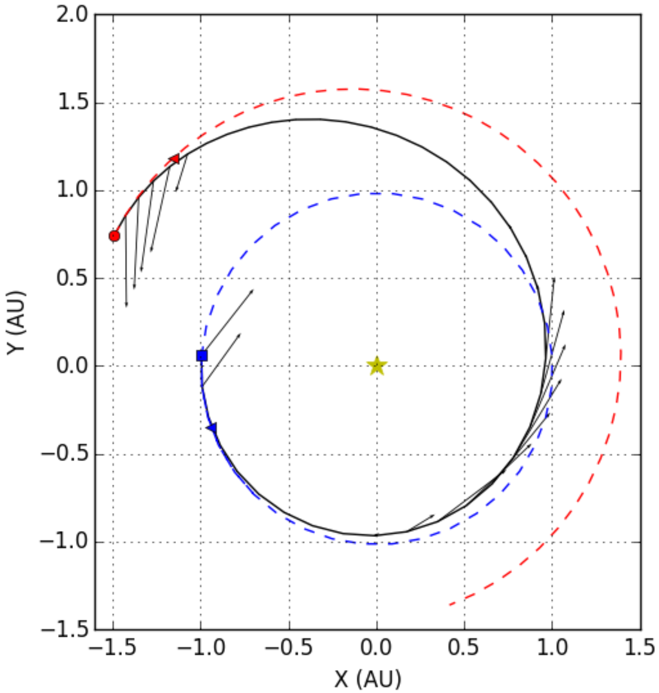
δt_0 and δt_f are now decision variables



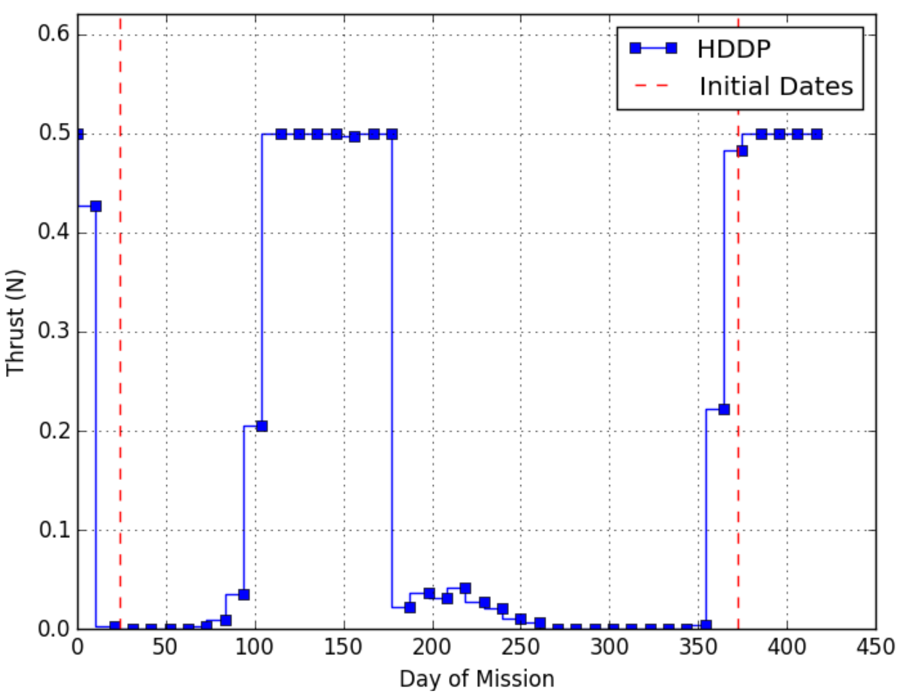


Table 4: Variable Time of Flight Results

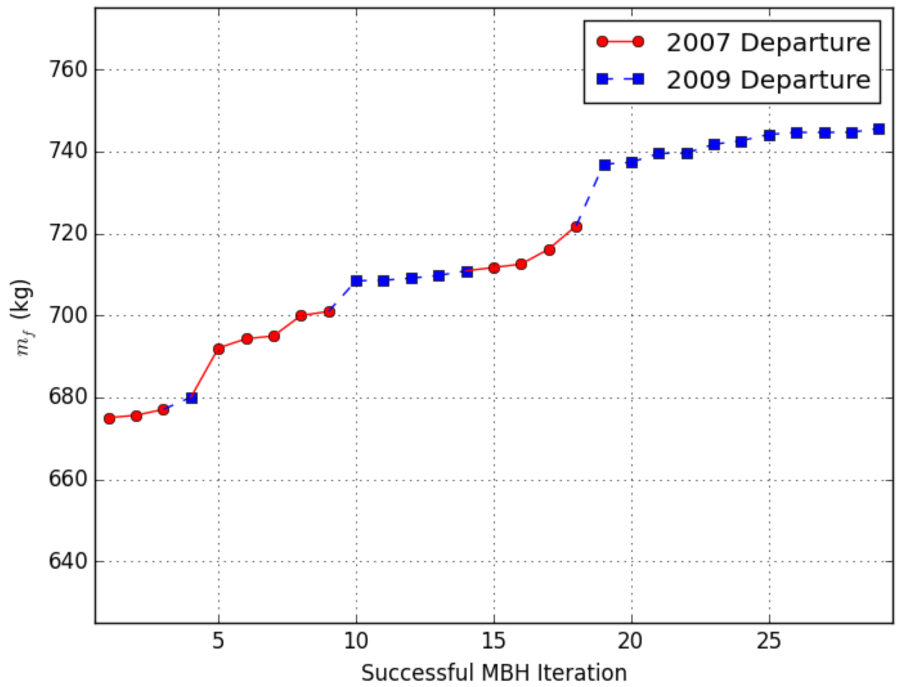
δt_0	-23.89 days
δt_f	43.78 days
m_f	674.95 kg

MBH is now applied to the variable time problem in HDDP

- Perturb time variables only
- 20% chance of hopping between 2007 and 2009 opportunities
- Stopping criteria: $N_{hop} = 100$
- 1st order state transition matrix only to quickly generate results
- Reset σ with each hop

Table 5: Results for $N_{hop} = 100$

δt_0	836.85 days
δt_f	894.95 days
m_f	745.57 kg



HDDP + MBH

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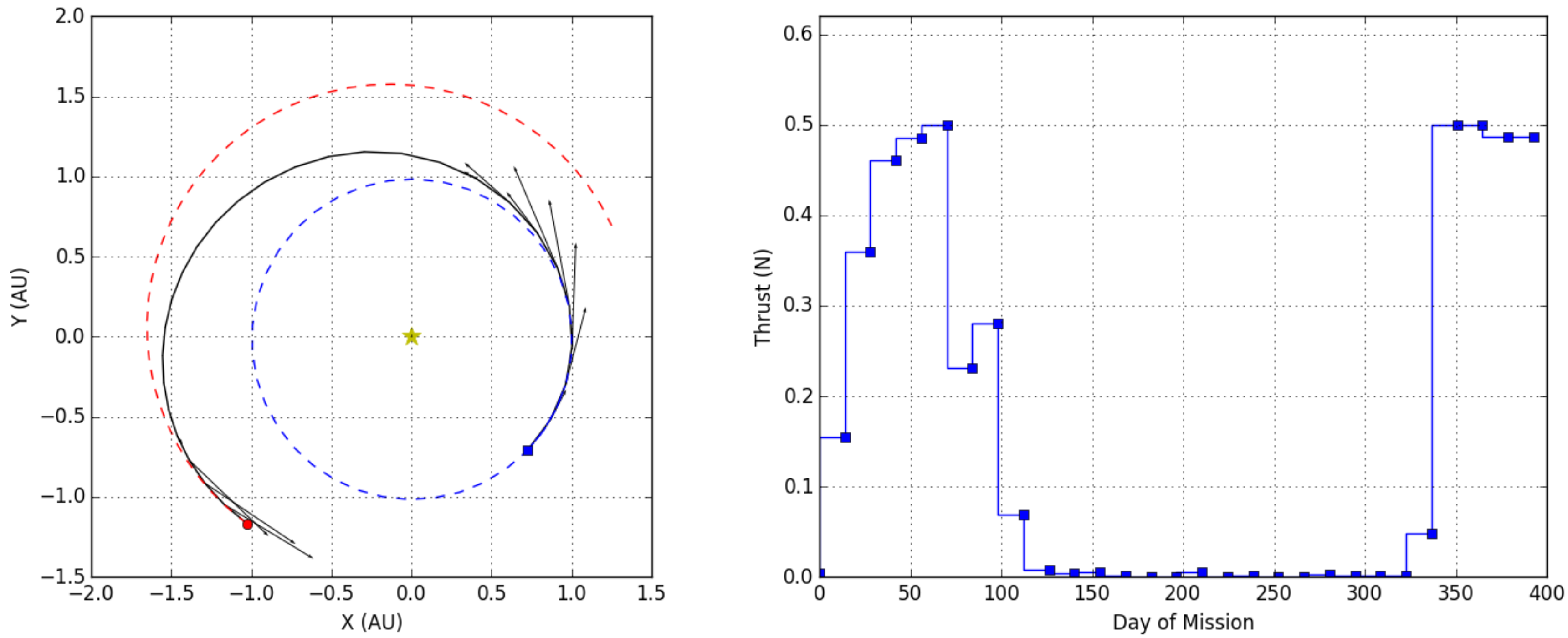

- Transfer in next synodic period
- Departure/arrival in different phases of Earth/Mars orbit
- Reduced to a 2-burn solution
- Compare 745.57 kg final mass to 674.95 kg before hopping

Summary

- Implementation based on HDDP by Lantoine and Russell validated with EMTG
- Successfully introduced time variables to an Earth-Mars rendezvous example
- Employed MBH as stochastic search step with HDDP computing the spacecraft trajectory
- MBH shown to guide HDDP through large steps in time variables and across synodic period
- Additional benefit found in MBH helping HDDP overcome poor tuning of algorithm parameters

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