Low-Thrust Transfers from Distant Retrograde Orbits to L_2 Halo Orbits in the Earth-Moon System

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Motivation

- Enable future missions
  - Any mission to a DRO or halo orbit could benefit from the capability to transfer between these orbits
  - Chemical propulsion could be used for these transfers, but at high propellant cost

- Fill gaps in knowledge
  - A variety of transfers using SEP or solar sails have been studied for the Earth-Moon system
  - Most results in literature study a single transfer
  - This is a step toward understanding the wide array of types of transfers available in an N-body force model

Background: CRTBP

- N-body problem still has not been solved analytically
- For three-body case: 18 degrees of freedom, 10 known integrals of motion
- Rely on simplifying assumptions when possible

- Circular Restricted Three Body Problem (CRTBP)
  - "Restricted" three-body problem: mass of the third body (the spacecraft) is negligible compared to the primaries
  - "Circular": the primaries' orbit about their barycenter is perfectly circular

Background: Synodic reference frame

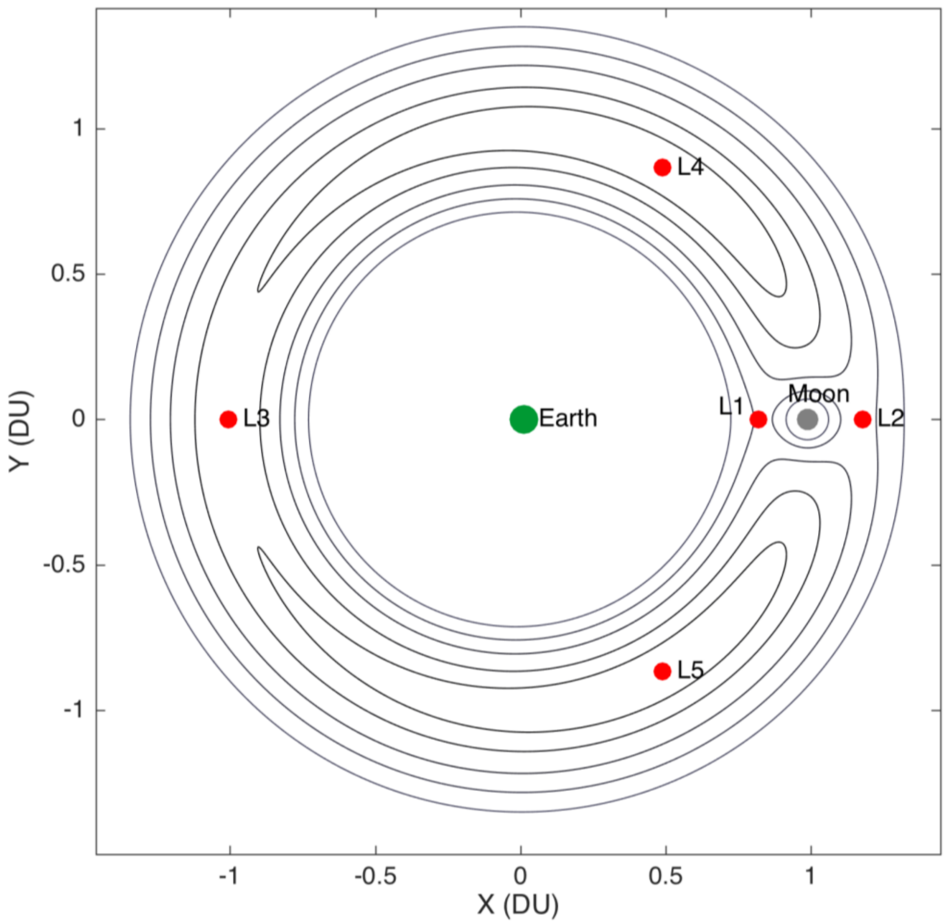
Equations of motion:

$$\ddot{x} = -\left(\frac{(1-\mu)}{r_1^3}(x+\mu) + \frac{\mu}{r_2^3}(x-1+\mu)\right) + 2\dot{y} + x + T_x$$

$$\ddot{y} = -\left(\frac{(1-\mu)}{r_1^3}y + \frac{\mu}{r_2^3}y\right) - 2\dot{x} + y + T_y$$

$$\ddot{z} = -\left(\frac{(1-\mu)}{r_1^3}z + \frac{\mu}{r_2^3}z\right) + T_z$$

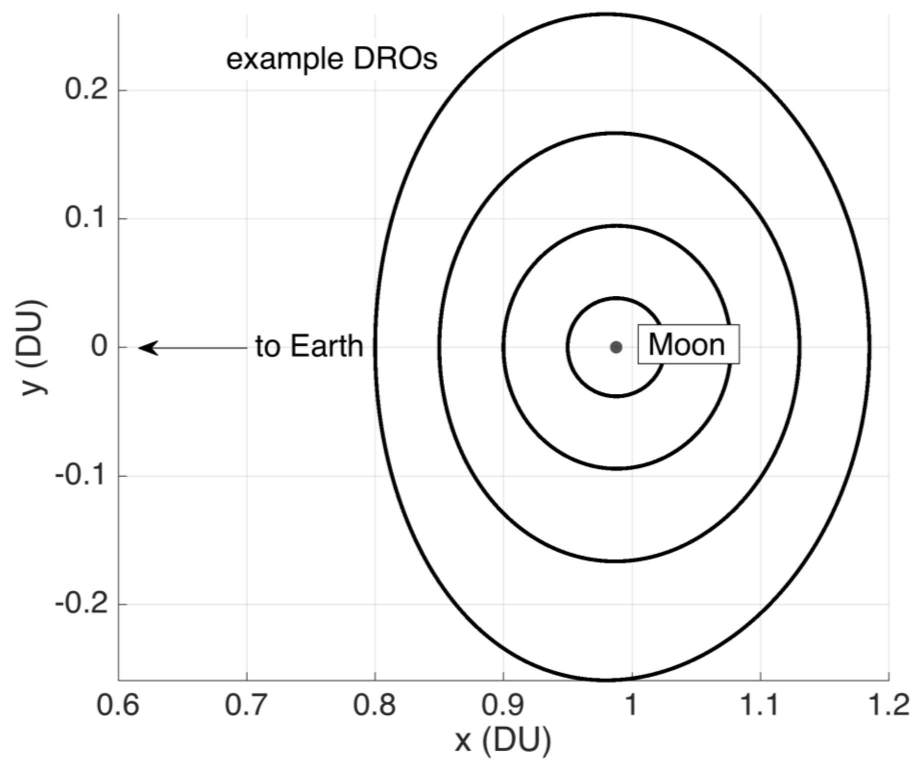
μ = mass ratio
$T_.$ = thrust
r_1 = distance from Earth
r_2 = distance from Moon



Background: DROs

Distant Retrograde Orbit

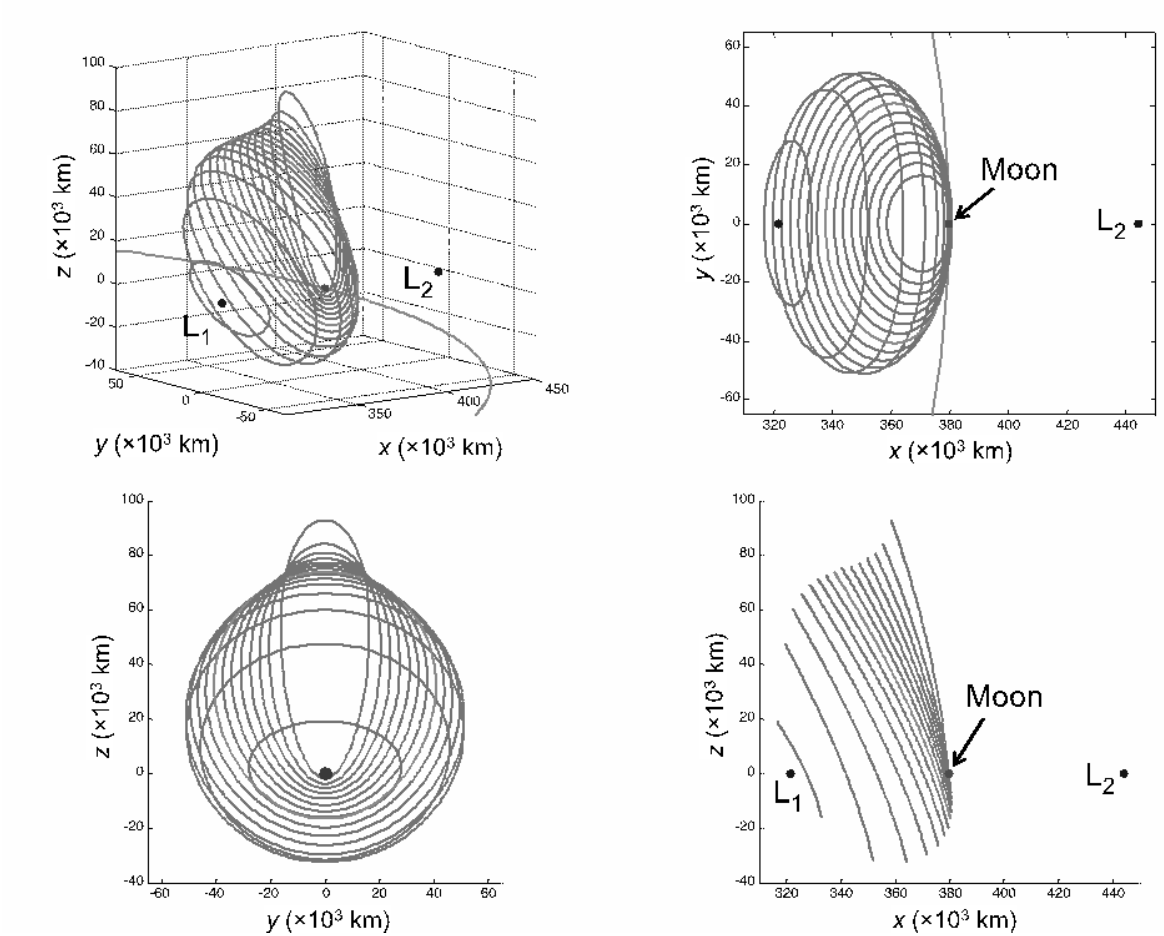
- Highly-perturbed orbit about the Moon
- When viewed in synodic frame, orbit is repeating and retrograde about the Moon
- Situated between libration point orbits and two-body orbits in terms of stability
- Currently being considered as destination orbit for Asteroid Redirect Mission concept



Background: Halo orbits

Halo orbit

- When viewed in synodic reference frame, traces a “halo”



Examples of orbits about L1

Background: Collocation

- Collocation: direct optimization method, transcribes optimal control problem to NLP problem
- Analogy: Runge-Kutta implicit integration for orbit propagation
- Solution described by a set of discrete nodes, or collocation points
- Can classify methods as “global” or “local”
  - Global: a continuous, high-order polynomial used for the entire time history. Differential defect constraints are difference between function derivative and dynamics
  - Local: a low-order polynomial is used to relate a few adjacent collocation points. Differential defect constraints are difference between local polynomial and dynamics

Background: Collocation

- Global method used: Pseudospectral collocation on Legendre-Gauss-Lobatto nodes

- Approximation:

$$x(\tau) \approx \sum_{k=0}^{N} x(\tau_k)\mathcal{L}_k(\tau)$$

$\mathcal{L}_k$: Lagrange basis polynomials
τ : transformed time s.t. $\tau \in [-1,1]$

$$\mathcal{L}_k(\tau) = \frac{1}{N(N+1)L_N(\tau_k)} \frac{(\tau^2-1)\dot{L}_N(\tau)}{\tau-\tau_k}$$

$$L_N(\tau) = \frac{1}{2^N N!} \frac{d^N}{d\tau^N} (\tau^2 - 1)^N$$

L_N : Legendre polynomials of order *N*

Global method used: Pseudospectral collocation on Legendre-Gauss-Lobatto nodes

- Derivative of the state vector analytically approximated as

$$\dot{x}(\tau_k) \approx \sum_{i=0}^{N} D_{ki} x^N(\tau_i)$$

Differential defect constraints: difference between approximation & differential equations

$$D_{ki} = \begin{cases} -\frac{L_N(\tau_k)}{L_N(\tau_i)} \frac{1}{\tau_k - \tau_i}, & k \neq i \\ \frac{N(N+1)}{4}, & k = i = 0 \\ -\frac{N(N+1)}{4}, & k = i = N \\ 0, & else \end{cases}$$

D_k : Analytical differentiation matrix

Local method used: Hermite-Simpson

Defect constraints:

$$\zeta(\tau_k) = x(\tau_{k+1}) - x(\tau_k) - \frac{h_k}{6}\left(f_k + 4\bar{f}_{k+1} + f_{k+1}\right)$$

Where

$$\bar{f}_{k+1} = f\left[\bar{x}_{k+1}, \bar{u}_{k+1}, p, \tau_k + \frac{h_k}{2}\right]$$

$$\bar{x}_{k+1} = \frac{1}{2}\left(x(\tau_k) + x(\tau_{k+1})\right) + \frac{h_k}{8}(f_k - f_{k+1})$$

Background: optimal control problem definition

The optimal control problem is defined as follows:

Minimize the performance index

$$J = \varphi\left[x\left(t_f\right), p, t_f\right] + \int_{t_0}^{t_f} L[x(t), u(t), p, t]dt$$

$$t \in \left[t_0, t_f\right]$$

Subject to differential constraints

$$\dot{x}(t) = f[x(t), u(t), p, t]$$

Path constraints

$$h_L \leq h[x(t), u(t), p, t] \leq h_U$$

Event constraints

$$e_L \leq e\left[x(t_0), u(t_0), x\left(t_f\right), u\left(t_f\right), p, t_0, t_f\right] \leq e_U$$

x state
u control
p parameters
t time

Bound constraints

$$u_L \le u(t) \le u_U$$
$$x_L \le x(t) \le x_U$$
$$p_L \le p \le p_U$$
$$t_{0,L} \le t_0 \le t_{0,U}$$
$$t_{f,L} \le t_f \le t_{f,U}$$

x state
u control
p parameters
t time

and

$$t_f - t_0 \ge 0$$

Initial guesses are given for $x, u, p,$ and t

Problem implementation

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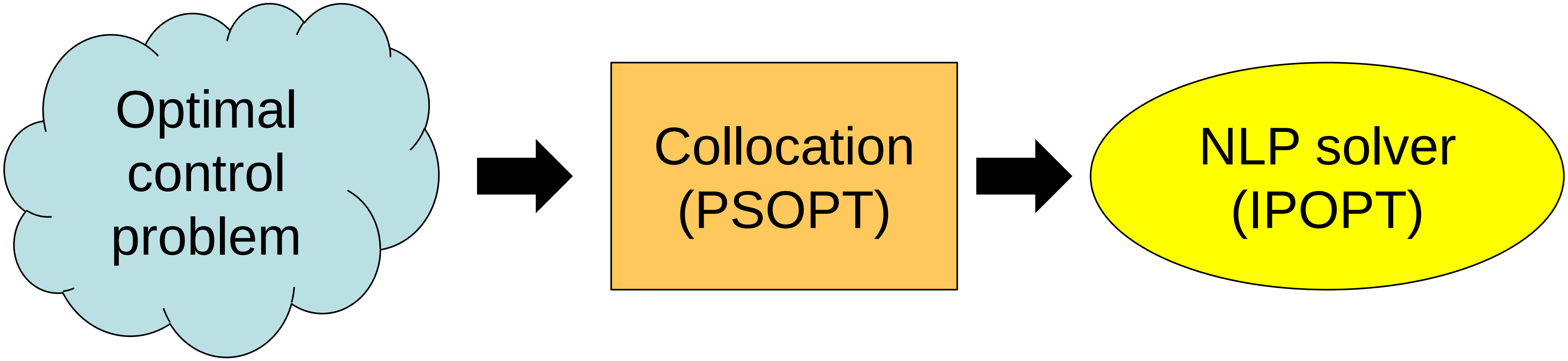
- PSOPT (PseudoSpectral OPTimal control) used for implementation of collocation
  - Open-source software, uses collocation to transcribe optimal control problem to NLP problem
  - NLP problem then solved by IPOPT (Interior Point OPTimizer)



All 3 of these boxes must be well implemented
This research focuses on the 1st box: defining the optimal control problem

Initial guess generation

- Low-thrust transfers in N-body force fields have many local minima
- Collocation yields a locally optimal solution
- Established tools exist for optimizing a transfer when there is a good initial guess available
- Systematic, well-informed choice of a trajectory requires knowledge of the relationship between the initial guess and the solutions it can yield.
- Generating an initial guess is perhaps the least-understood aspect of the problem
- This research used initial guesses that stayed in the vicinity of the Moon, with varying #’s of revolutions

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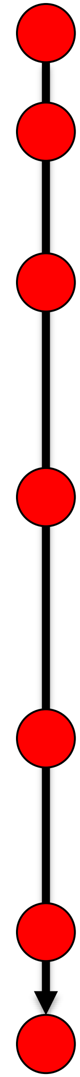
Initial guess generation

Propagated states in DRO

Propagated states in L_2 halo orbit

Concatenate list of states in each orbit



Interpolate to nodes

Initial guess generation

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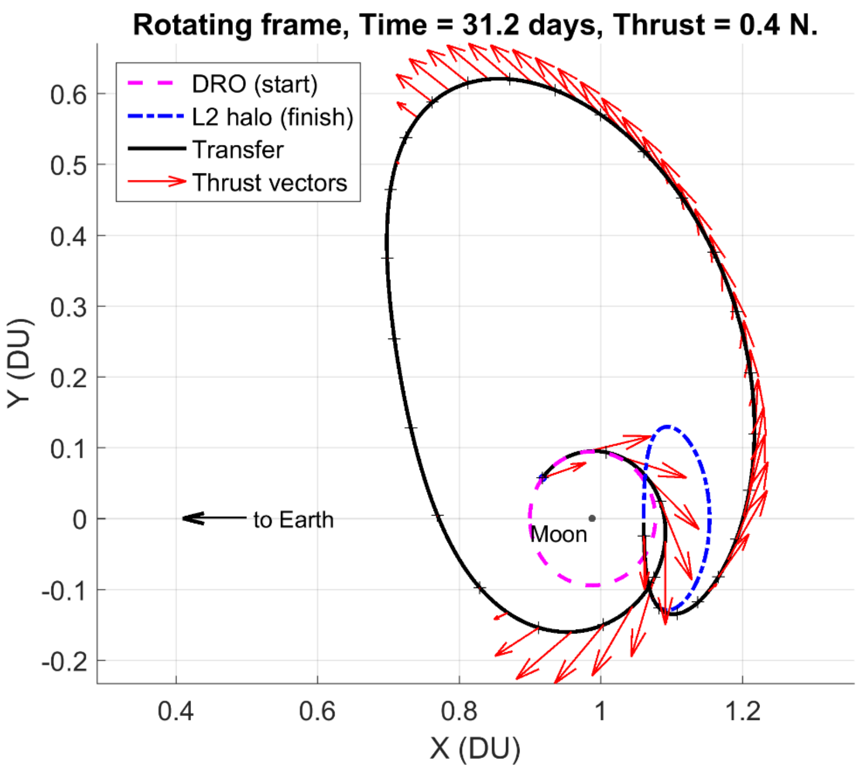
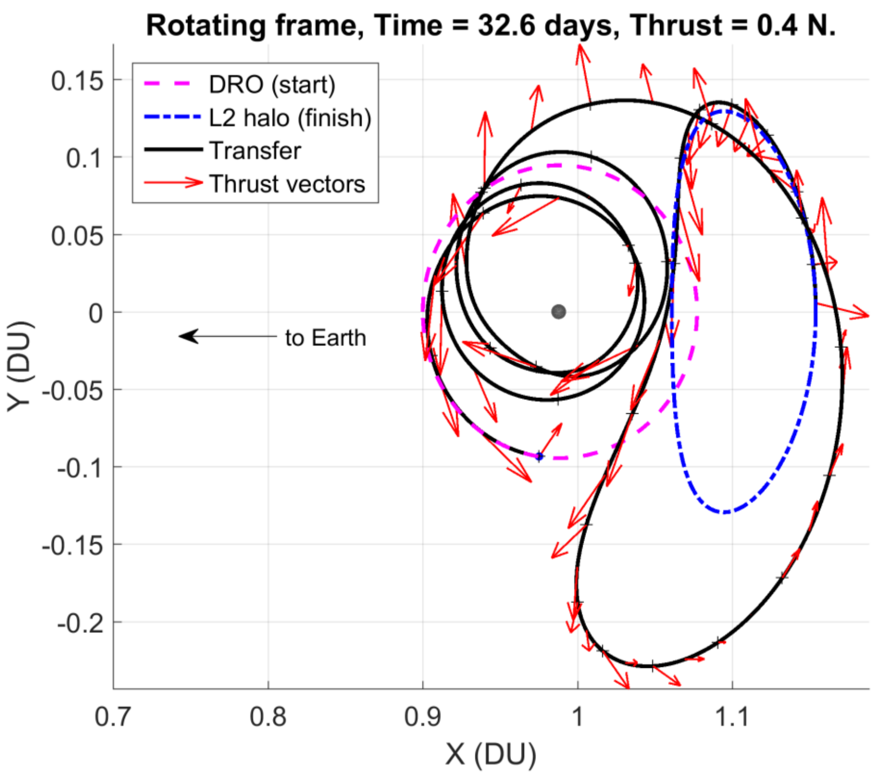
- Initial guess has large discontinuity in the middle
- Shapes the converged solution:
  - # of revolutions in DRO
  - # of revolutions in halo orbit



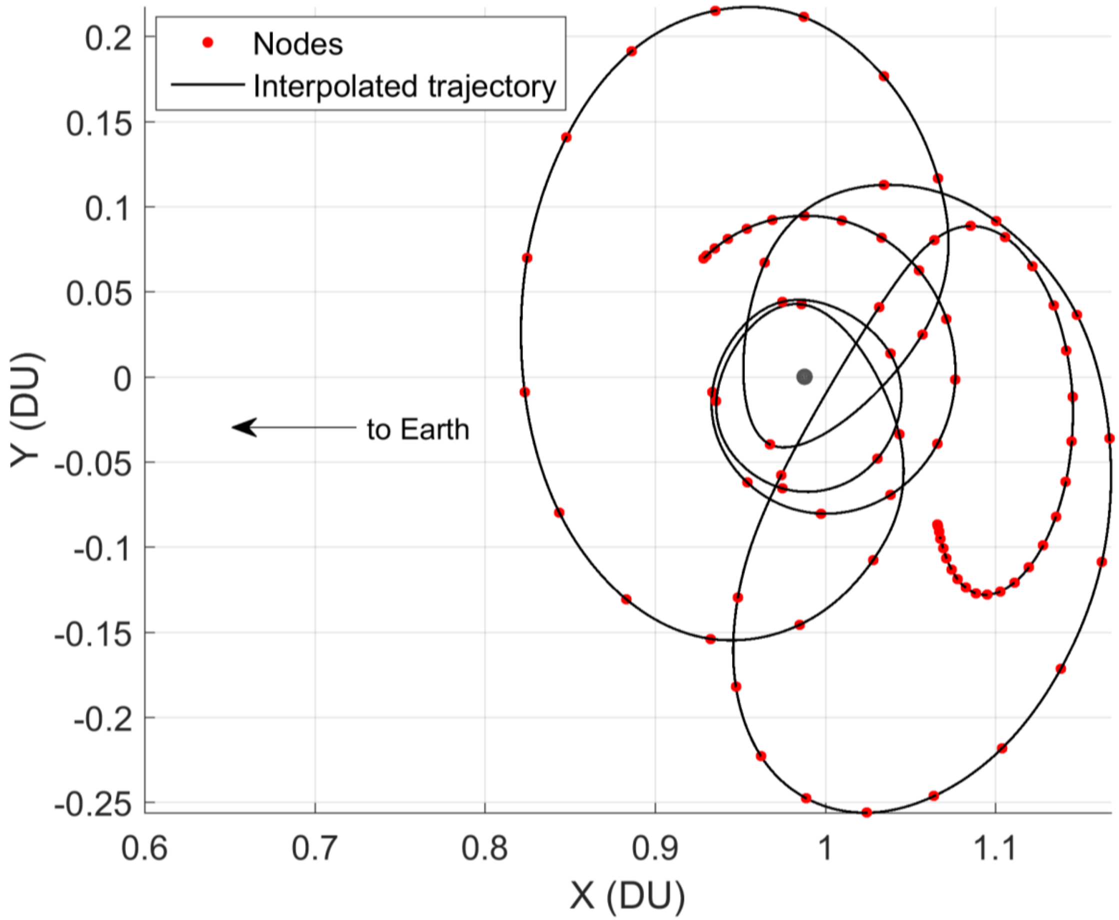


Lunar flybys

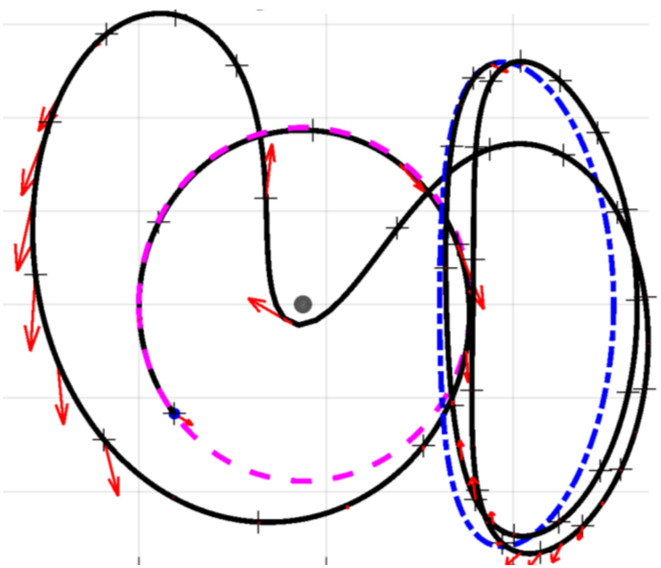
Close lunar flybys save propellant, but are very sensitive

- Harder to converge solutions
  - More nodes required to represent quickly-changing dynamics accurately
  - Approximation methods may have trouble representing the transfers
  - Automatic mesh refinement necessary (in PSOPT, only available with Hermite-Simpson)
- More dangerous for operations
  - Errors in state execution or in maneuver execution are magnified after the flyby
  - Risks could be mitigated by enforcing a coasting period before the flyby (to obtain an accurate OD solution)
- To avoid these challenges, a “keep-out” zone was used. Spacecraft not allowed closer than ~9 lunar radii to the Moon

Lunar flybys



Close approach to moon, poorly represented by single-phase pseudospectral method



Close approach to moon, well-represented by Hermite-Simpson method automatic mesh refinement

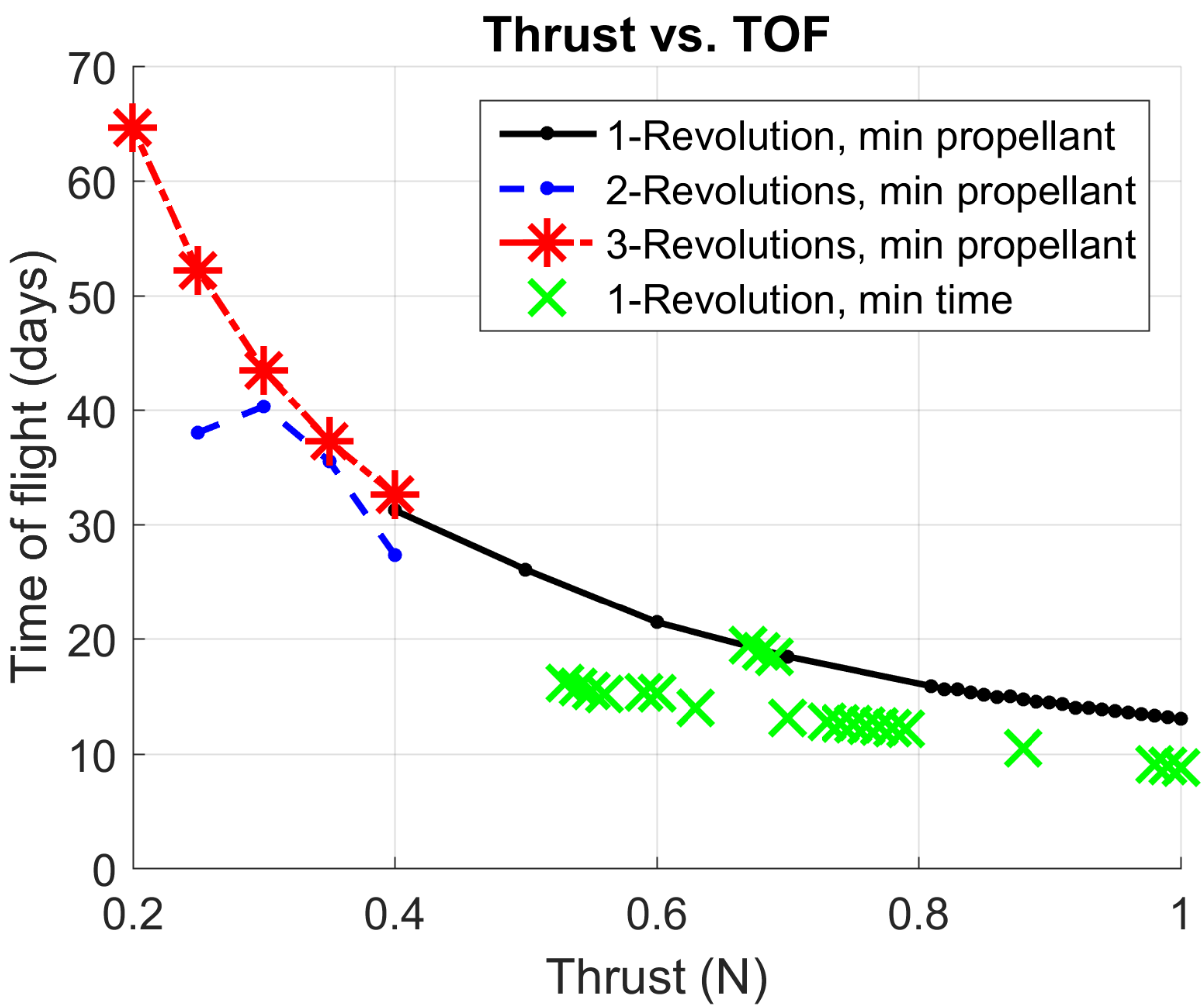
Strategy to find families of transfers

1. Generate an initial guess
2. Using the pseudospectral method, run the problem with zero cost function. This allows the optimizer to quickly find a feasible (but not optimal) transfer.
3. Using the Hermite-Simpson method, set the objective function to maximize the final mass, and run the optimizer.
4. Decrease the maximum thrust limit slightly. Using the Hermite-Simpson method again and the solution from step (3) as the initial guess, run the optimizer.
5. Repeat step (4) until the problem no longer converges.

Results: Families of transfers

- Four families examined:
  - 1-revolution, minimum time
  - 1-revolution, minimum propellant
  - 2-revolution, minimum propellant
  - 3-revolution, minimum propellant
- For 1-revolution: started at 1-Newton thrust, then used that solution as the new initial guess, with thrust slightly reduced
  - Repeat until solution no longer converges (0.4 N)
  - Then, use different initial guess (2-rev, 3-rev)
- For 2-revolution & 3-revolution: started at 0.4 N

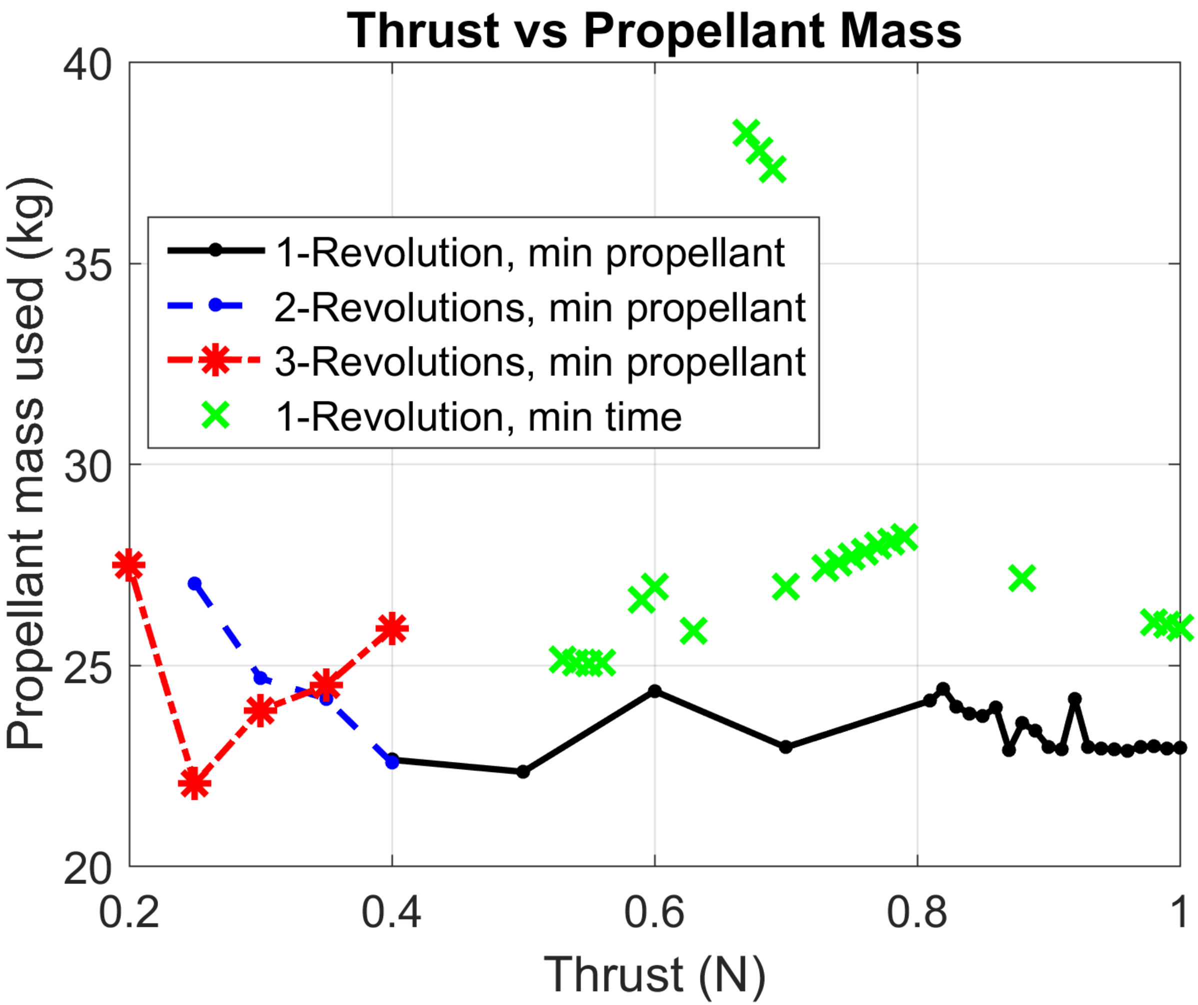
Results: Families of transfers



Not all cases converged completely – easy to get stuck in local optima

Lower thrust generally requires greater time of flight

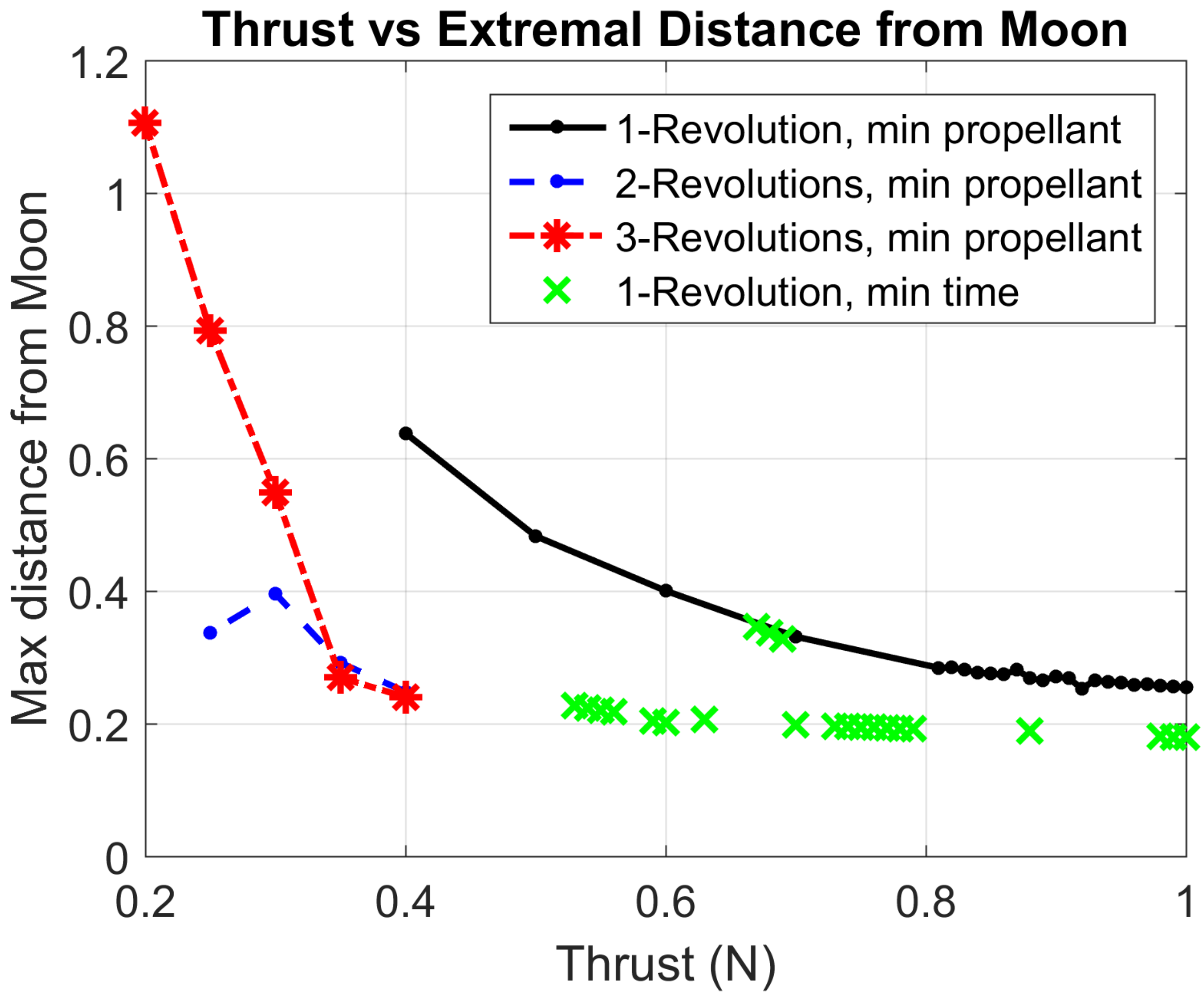
Results: Families of transfers



Most solutions require 23-28 kg propellant (of 1500 kg initial mass)

Adding a lunar flyby reduces propellant to as low as 18 kg, but these were hard to find systematically

Results: Families of transfers



Within a family, lower thrust generally requires traveling further from the Moon

Initial guesses kept solutions near the Moon – when thrust was reduced too much, the solution failed to converge after 3,000 iterations

Conclusion

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Summary

- A variety of transfer trajectories exist from DRO to L_2 halo orbit
- Collocation methods are capable of optimizing transfers in N-body force model
- When a good initial guess is not available, it is possible to use a poor one
- Shape of initial guess strongly influences shape of converged solution

Future Work

- Explore different types of initial guesses
- Use other implementations of collocation-based optimal control
- Use higher-fidelity dynamics
- Extend to other transfers in Earth-Moon system

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