

Assessing and Ensuring GOES-R Magnetometer Accuracy

GOES-R Flight Project
Code: 417

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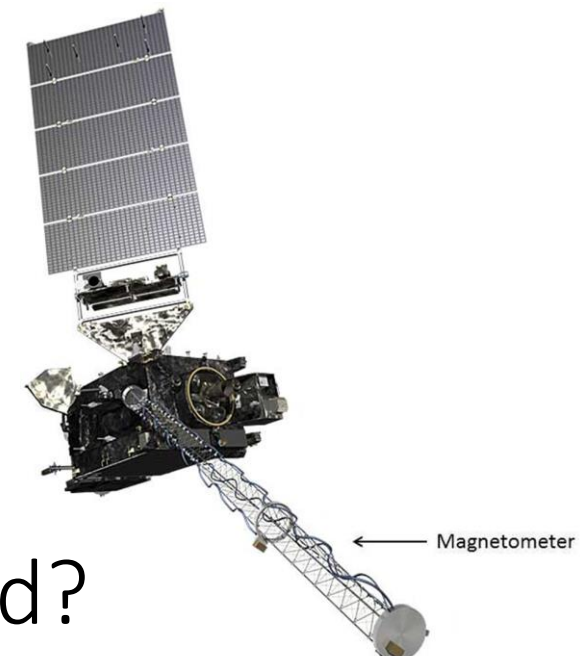
I. Abstract

The GOES-R magnetometer accuracy requirement is 1.7 nanoteslas (nT). During quiet times (100 nT), accuracy is defined as absolute mean plus 3 sigma. During storms (300 nT), accuracy is defined as absolute mean plus 2 sigma. To achieve this, the sensor itself has better than 1 nT accuracy. Because zero offset and scale factor drift over time, it is also necessary to perform annual calibration maneuvers. To predict performance, we used covariance analysis and attempted to corroborate it with simulations. Although not perfect, the two generally agree and show the expected behaviors. With the annual calibration regimen, these predictions suggest that the magnetometers will meet their accuracy requirements.

II. Problem

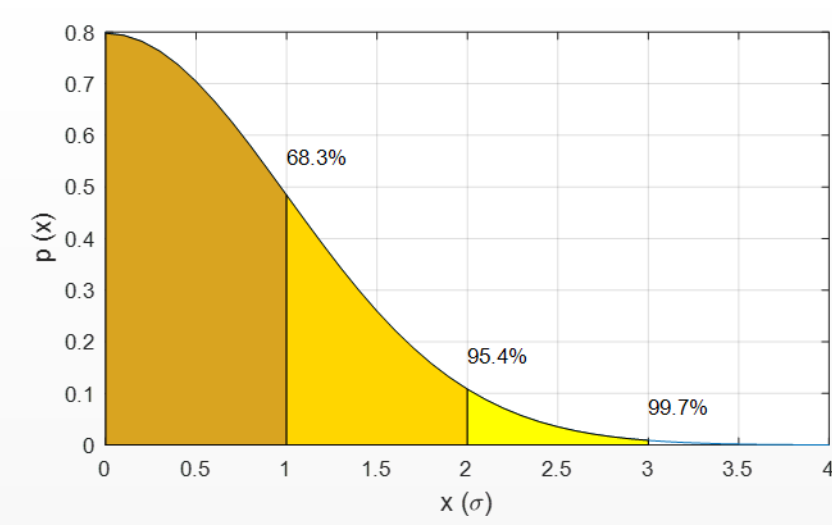
What is the GOES-R magnetometer subsystem?

- Two fluxgate magnetometers
- Mounted on boom 6.5 and 8.5 m from base
- Temperature-controlled sensor and electronics
- Hosted on a magnetically clean spacecraft



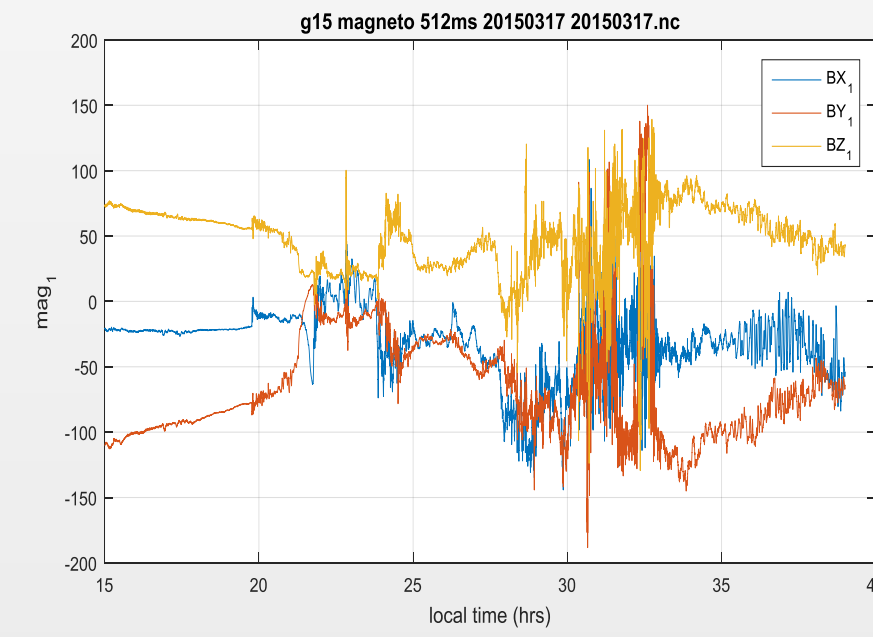
How is performance defined?

- The Error Metric
 - Accuracy (mean error $|\mu|$) and precision (standard deviation σ)
 - Metric $|\mu| + n\sigma$ where n depends on the situation per axis
 - Time span (unspecified but assumed here to be 1 day)
- Requirements
 - Quiet $|\mu| + 3\sigma \leq 1.7 \text{ nT}$
 - Storm $|\mu| + 2\sigma \leq 1.7 \text{ nT}$



What are quiet and storm fields?

- Quiet
 - Based 3-hour average of horizontal field variations around world
 - Field components don't vary much compared to magnitude (100 nT)
- Because it's an average, there can still be locally strong disturbances.



Storm Example

- St. Patrick's Day 2015
- Caused by Coronal Mass Ejection (CME) 12 hours earlier
- Lasted 18 hours

http://www.swpc.noaa.gov/sites/default/files/images/u33/StPatrick%27sDay_Geomagnetic_Storm.pdf

Where does the error come from?

- Zero Offset Random Walk
 - Besides noise and quantization errors, uncertainties in these three calibrations limits performance:
 - 1. Zero offset – initially well-known but executes 0.2 nT/Vyr random walk
 - 2. Misalignments
 - a. launch shock – up to 1.0° inboard and 0.5° relative misalignment
 - b. diurnal – up to 0.15° inboard and 0.10° relative misalignment
 - c. non-orthogonality – constant and known to 0.06° from ground measurement
 - 3. Scale factor – 0.10%/Vyr random walk (initially well-known)

III. Plan

How can we reduce errors?

- Slows make calibrations observable.
- Assuming constant fields, we can solve for zero offsets and misalignments.
- Knowing attitude lets us predict ambient field in body coordinates.

Maneuver Constraints

- For communications, antennas cannot point far from nadir.
- Slew rate and acceleration are limited by reaction wheel torque.



<http://www.goes-r.gov/ground/facilities-antennas.html>

Observability

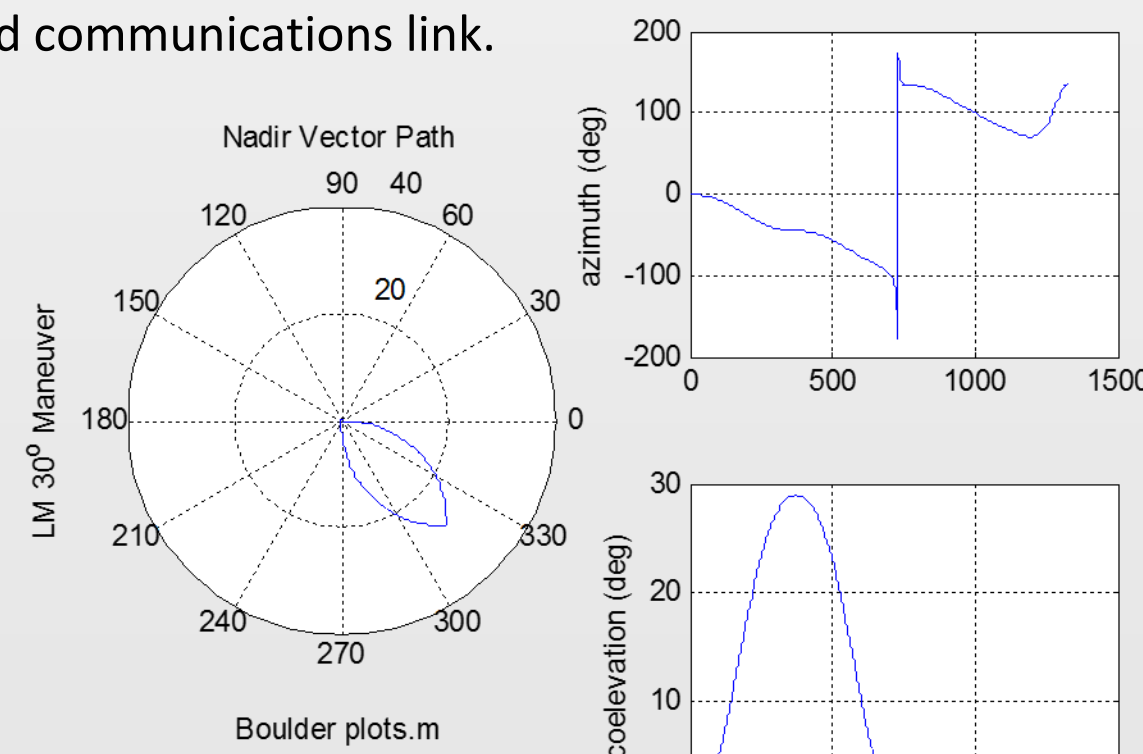
- Rotation makes zero offsets and misalignments on orthogonal axes observable.
- Need rotations about two different axes for full observability.

Unobservability

- Without known field changes, scale factors cannot be calibrated.
- Axis non-orthogonality is virtually unobservable.

What does the calibration maneuver look like?

- The maneuver must keep the z-axis within 30° of nadir to maintain the 40 kps L-band communications link.



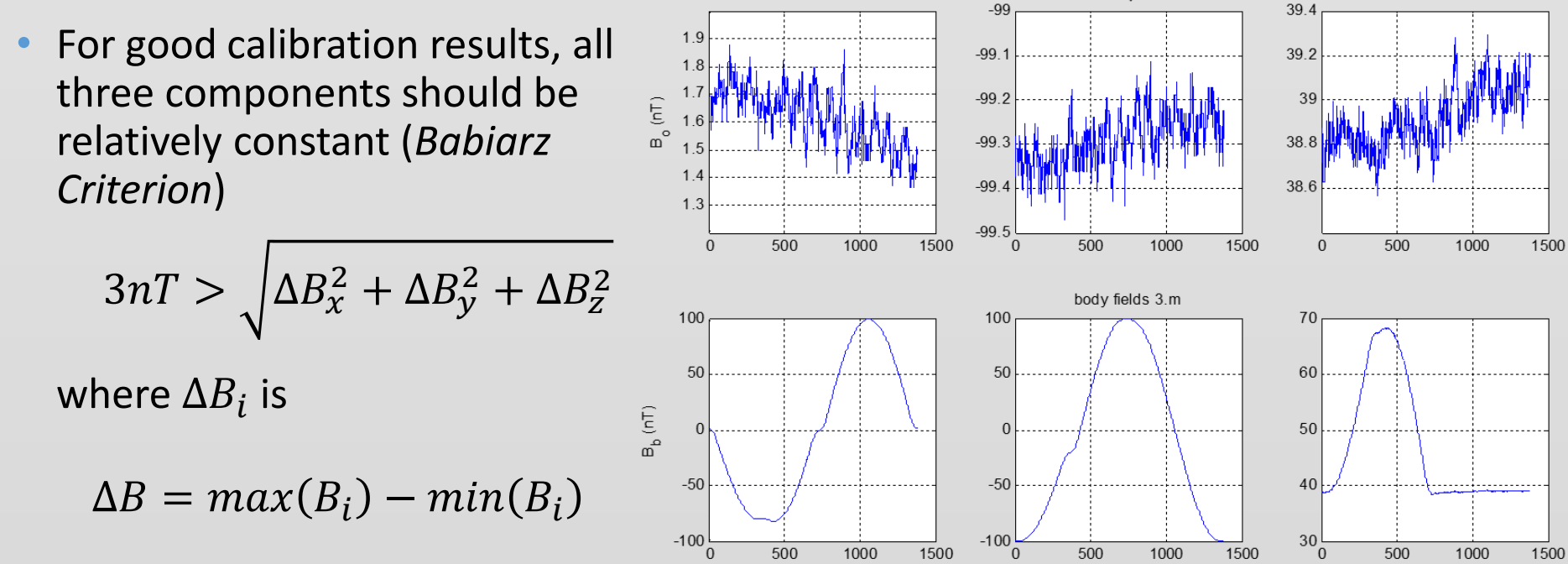
- Slew 94° about axis close to z-axis, 94° about another axis and 179° back to nadir.

- Time 22 minutes. Quicker would be better so field has less time to change.

- Maneuver will be done during Post-Launch Test and may be repeated if the ambient field changes much.

And the maneuver observations?

- The top plots show ambient magnetic field in the orbital frame.



- The bottom plots show ambient magnetic field in the body frame.

How can we predict performance?

Generate Fields

- Quiet – GOES-12 NCEI 'quiet' data
- Storm – distributed through 300 nT sphere

Estimate Those Fields

- Estimate fields and calibrations.
- Generate observations with remaining calibration errors.

Compute Metrics

- Covariance – 2σ or 3σ averaged over sampled fields
- Simulation – $|\mu| + 2\sigma$ or 3σ averaged over sampled fields and mission trials

Analysis or Simulation?

- Covariance analysis paints a clear picture and is quick to run.
- Simulation is more flexible and can be easier to understand.

IV. Implementation

Why not covariance *and* simulation? Good idea!

Covariance Equations

To get a realistic error estimate, one needs to include the effect of what is solved for and what is known with limited accuracy.

The total covariance P is the sum of the solve-for covariance P_x and "consider" covariance P_c

$$P = P_x + P_c$$

The least squares normal matrix gives the solve-for covariance due to observation noise only P_x

$$P_x = (H_x^T W H_x)^{-1}$$

Here, W is the observation weighting matrix. H_x is the partial derivative of the observation \hat{z} with respect to the solve-for parameter vector \hat{x}

$$H_x = \frac{\partial \hat{z}}{\partial \hat{x}}$$

The consider covariance P_c is

$$P_c = (P_x H_x^T W H_y) P_{y0} (P_x H_x^T W H_y)^T$$

Here P_{y0} is the *a priori* consider covariance, *i.e.* what we know but with finite accuracy. H_y is the observation partial derivative matrix with respect to the consider parameters, e.g. the calibrations not solved-for

$$H_y = \frac{\partial \hat{z}}{\partial \hat{y}}$$

Simulation Processing

Loop over missions

- Pick field value to use (one set of values for whole mission)
- Randomize static parameters
- Update calibration annually
- Loop over months
 - Randomize diurnal parameters (misalignments)
 - Propagate drift parameters (zero offset and scale factor)
 - Accumulate errors and squared errors
- Compute mean error and standard deviations
- Save results
- Compute performance metrics and plot

```
1 loop through monthly evaluations
2 for i=1:nMonths
3     % single-frame ambient field standard deviations
4     Pcov = zeros(3); Pcov = zeros(3); Pcov = zeros(3);
5     % evaluation standard deviations
6     sdcov = sdcov + diurnal + random;
7     sdcov = sdcov + diurnal + random;
8     % loop through evaluation data points and compute average covariances
9     for j=1:nDataPoints
10        % cov = avg_cov(Bx, Bx, alpha, sdcov, sdcov);
11        % cov = avg_cov(Bx, By, alpha, sdcov, sdcov);
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228       % cov = avg_cov(Bx, Bz, alpha, sdcov, sdcov);
229       % cov = avg_cov(By, Bx, alpha, sdcov, sdcov);
230       % cov = avg_cov(By, By, alpha, sdcov, sdcov);
231       % cov = avg_cov(By, Bz, alpha, sdcov, sdcov);
232       % cov = avg_cov(Bz, Bx, alpha, sdcov, sdcov);
233       % cov = avg_cov(Bz, By, alpha, sdcov, sdcov);
234       % cov = avg_cov(Bz, Bz, alpha, sdcov, sdcov);
235       % cov = avg_cov(Bx, Bx, alpha, sdcov, sdcov);
236       % cov = avg_cov(Bx, By, alpha, sdcov, sdcov);
237       % cov = avg_cov(Bx, Bz, alpha, sdcov, sdcov);
238       % cov = avg_cov(By, Bx, alpha, sdcov, sdcov);
239       % cov = avg_cov(By, By, alpha, sdcov, sdcov);
240       % cov = avg_cov(By, Bz, alpha, sdcov, sdcov);
241       % cov = avg_cov(Bz, Bx, alpha, sdcov, sdcov);
242       % cov = avg_cov(Bz, By, alpha, sdcov, sdcov);
243       % cov = avg_cov(Bz, Bz, alpha, sdcov, sdcov);
244       % cov = avg_cov(Bx, Bx, alpha, sdcov, sdcov);
245       % cov = avg_cov(Bx, By, alpha, sdcov, sdcov);
246       % cov = avg_cov(Bx, Bz, alpha, sdcov, sdcov);
247       % cov = avg_cov(By, Bx, alpha, sdcov, sdcov);
248       % cov = avg_cov(By, By, alpha, sdcov, sdcov);
249       % cov = avg_cov(By, Bz, alpha, sdcov, sdcov);
250       % cov = avg_cov(Bz, Bx, alpha, sdcov, sdcov);
251       % cov = avg_cov(Bz, By, alpha, sdcov, sdcov);
252       % cov = avg_cov(Bz, Bz, alpha, sdcov, sdcov);
253       % cov = avg_cov(Bx, Bx, alpha, sdcov, sdcov);
254       % cov = avg_cov(Bx, By, alpha, sdcov, sdcov);
255       % cov = avg_cov(Bx, Bz, alpha, sdcov, sdcov);
256       % cov = avg_cov(By, Bx, alpha, sdcov, sdcov);
257       % cov = avg_cov(By, By, alpha, sdcov, sdcov);
258       % cov = avg_cov(By, Bz, alpha, sdcov, sdcov);
259       % cov = avg_cov(Bz, Bx, alpha, sdcov, sdcov);
260       % cov = avg_cov(Bz, By, alpha, sdcov, sdcov);
261       % cov = avg_cov(Bz, Bz, alpha, sdcov, sdcov);
262       % cov = avg_cov(Bx, Bx, alpha, sdcov, sdcov);
263       % cov = avg_cov(Bx, By, alpha, sdcov, sdcov);
264       % cov = avg_cov(Bx, Bz, alpha, sdcov, sdcov);
265       % cov = avg_cov(By, Bx, alpha, sdcov, sdcov);
266       % cov = avg_cov(By, By, alpha, sdcov, sdcov);
267       % cov = avg_cov(By, Bz, alpha, sdcov, sdcov);
268       % cov = avg_cov(Bz, Bx, alpha, sdcov, sdcov);
269       % cov = avg_cov(Bz, By, alpha, sdcov, sdcov);
270       % cov = avg_cov(Bz, Bz, alpha, sdcov, sdcov);
271       % cov = avg_cov(Bx, Bx, alpha, sdcov, sdcov);
272       % cov = avg_cov(Bx, By, alpha, sdcov, sdcov);
273       % cov = avg_cov(Bx, Bz, alpha, sdcov, sdcov);
274       % cov = avg_cov(By, Bx, alpha, sdcov, sdcov);
275       % cov = avg_cov(By, By, alpha, sdcov, sdcov);
276       % cov = avg_cov(By, Bz, alpha, sdcov, sdcov);
277       % cov = avg_cov(Bz, Bx, alpha, sdcov, sdcov);
278       % cov = avg_cov(Bz, By, alpha, sdcov, sdcov);
279       % cov = avg_cov(Bz, Bz, alpha, sdcov, sdcov);
280       % cov = avg_cov(Bx, Bx, alpha, sdcov, sdcov);
281       % cov = avg_cov(Bx, By, alpha, sdcov, sdcov);
282       % cov = avg_cov(Bx, Bz, alpha, sdcov, sdcov);
283       % cov = avg_cov(By, Bx, alpha, sdcov, sdcov);
284       % cov = avg_cov(By, By, alpha, sdcov, sdcov);
285       % cov = avg_cov(By, Bz, alpha, sdcov, sdcov);
286       % cov = avg_cov(Bz, Bx, alpha, sdcov, sdcov);
287       % cov = avg_cov(Bz, By, alpha, sdcov, sdcov);
288       % cov = avg_cov(Bz, Bz, alpha, sdcov, sdcov);
289       % cov = avg_cov(Bx, Bx, alpha, sdcov, sdcov);
290       % cov = avg_cov(Bx, By, alpha, sdcov, sdcov);
291       % cov = avg_cov(Bx, Bz, alpha, sdcov, sdcov);
292       % cov = avg_cov(By, Bx, alpha, sdcov, sdcov);
293       % cov = avg_cov(By, By, alpha, sdcov, sdcov);
294       % cov = avg_cov(By, Bz, alpha, sdcov, sdcov);
295       % cov = avg_cov(Bz, Bx, alpha, sdcov, sdcov);
296       % cov = avg_cov(Bz, By, alpha, sdcov, sdcov);
297       % cov = avg_cov(Bz, Bz, alpha, sdcov, sdcov);
298       % cov = avg_cov(Bx, Bx, alpha, sdcov, sdcov);
299       % cov = avg_cov(Bx, By, alpha, sdcov, sdcov);
300       % cov = avg_cov(Bx, Bz, alpha, sdcov, sdcov);
301       % cov = avg_cov(By, Bx, alpha, sdcov, sdcov);
302       % cov = avg_cov(By, By, alpha, sdcov, sdcov);
303       % cov = avg_cov(By, Bz, alpha, sdcov, sdcov);
304       % cov = avg_cov(Bz, Bx, alpha, sdcov, sdcov);
305       % cov = avg_cov(Bz, By, alpha, sdcov, sdcov);
306       % cov = avg_cov(Bz, Bz, alpha, sdcov, sdcov);
307       % cov = avg_cov(Bx, Bx, alpha, sdcov, sdcov);
308       % cov = avg_cov(Bx, By, alpha, sdcov, sdcov);
309       % cov = avg_cov(Bx, Bz, alpha, sdcov, sdcov);
310       % cov = avg_cov(By, Bx, alpha, sdcov, sdcov);
311       % cov = avg_cov(By, By, alpha, sdcov, sdcov);
312       % cov = avg_cov(By, Bz, alpha, sdcov, sdcov);
313       % cov = avg_cov(Bz, Bx, alpha, sdcov, sdcov);
314       % cov = avg_cov(Bz, By, alpha, sdcov, sdcov);
315       % cov = avg_cov(Bz, Bz, alpha, sdcov, sdcov);
316       % cov = avg_cov(Bx, Bx, alpha, sdcov, sdcov);
317       % cov = avg_cov(Bx, By, alpha, sdcov, sdcov);
318       % cov = avg_cov(Bx, Bz, alpha, sdcov, sdcov);
319       % cov = avg_cov(By, Bx, alpha, sdcov, sdcov);
320       % cov = avg_cov(By, By, alpha, sdcov, sdcov);
321       % cov = avg_cov(By, Bz, alpha, sdcov, sdcov);
322       % cov = avg_cov(Bz, Bx, alpha, sdcov, sdcov);
323       % cov = avg_cov(Bz, By, alpha, sdcov, sdcov);
324       % cov = avg_cov(Bz, Bz, alpha, sdcov, sdcov);
325       % cov = avg_cov(Bx, Bx, alpha, sdcov, sdcov);
326       %
```