ASW Covariance Introduction and Formation

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OD Parameters Generated by ASW Solutions

- **Solved for: State parameters**
  - Six parameters needed to determine 3-d state fully
  - Cartesian: three position and three velocity parameters in orthogonal system
  - Element: six orbital elements that describe the geometry of the orbit
- **Solved for: Non-conservative force parameters**
  - Ballistic coefficient (C_DA/m); describes vulnerability of spacecraft state to atmospheric drag
  - Solar radiation pressure (SRP) coefficient (C_RA/m); describes vulnerability of spacecraft state to visible light momentum from sun
- **Considered: ballistic coefficient and SRP consider parameter**
  - Not solved for but "considered" as part of the solution
  - Derived from information outside of the OD itself
- **Covariance matrix includes variances/covaraínces for all solved-for parameters, with potential alteration by consider parameters**

Covariance Matrix Construction: Symbolic Example

- Three estimated parameters (a, b, and c)
- Variances of each along diagonal
- Off-diagonal terms the product of two standard deviations and the correlation coefficient (ρ); matrix is symmetric

	a	**b**	**c**	…
a	σ_a^2	$\rho_{ab}\sigma_a\sigma_b$	$\rho_{ac}\sigma_a\sigma_c$	…
b	$\rho_{ab}\sigma_a\sigma_b$	σ_b^2	$\rho_{bc}\sigma_a\sigma_c$	…
c	$\rho_{ac}\sigma_a\sigma_c$	$\rho_{bc}\sigma_a\sigma_c$	σ_c^2	…
…	…	…	…	…

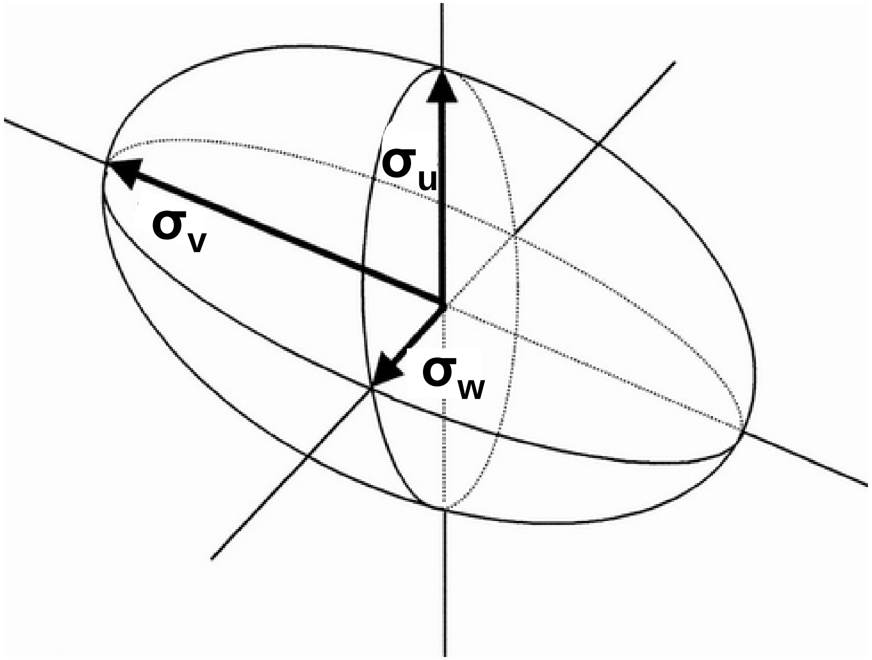
Example Covariance from CDM

- **8 x 8 matrix typical of most ASW updates**
  - Some orbit regimes not suited to solution for both drag and SRP; these covariances 7 x 7
- **Mix of different units often creates poorly conditioned matrices**
  - Condition number of matrix at right is 9.8E+11—terrible!
- **Often better numerically (and more intuitive) to separate matrix into sections**
- **First 3 x 3 portion (amber) is *position* covariance—often considered separately**

	U (m)	V (m)	W (m)	Udot (m/s)	Vdot (m/s)	Wdot (m/s)	B (m2/kg)	AGOM (m2/kg)
U	6.84E+01	-2.73E+02	6.38E+00	2.76E-01	-7.14E-02	8.75E-03	-3.83E-02	-3.83E-02
V	-2.73E+02	1.10E+05	3.23E+01	-1.17E+02	-8.99E-02	2.51E-02	-1.28E-01	-1.28E-01
W	6.38E+00	3.23E+01	4.47E+00	-3.26E-02	-6.83E-03	1.81E-03	-3.73E-03	-3.73E-03
Udot	2.76E-01	-1.17E+02	-3.26E-02	1.24E-01	1.10E-04	-2.47E-05	1.46E-04	1.46E-04
Vdot	-7.14E-02	-8.99E-02	-6.83E-03	1.10E-04	7.57E-05	-9.39E-06	4.10E-05	4.10E-05
Wdot	8.75E-03	2.51E-02	1.81E-03	-2.47E-05	-9.39E-06	2.06E-05	-4.39E-06	-4.39E-06
B	-5.07E-03	1.30E+00	4.34E-05	-1.38E-03	7.97E-07	7.26E-07	1.64E-05	-6.28E-07
AGOM	-3.83E-02	-1.28E-01	-3.73E-03	1.46E-04	4.10E-05	-4.39E-06	-6.28E-07	2.31E-05

Position Covariance Ellipse

- **Position covariance defines an "error ellipsoid"**
  - Placed at predicted satellite position
  - Square root of variance in each direction defines each semi-major axis (UVW system used here)
  - Off-diagonal terms rotate the ellipse from the nominal position shown
- **Ellipse of a certain "sigma" value contains a given percentage of the expected data points**
  - 1-σ: 19.9%
  - 2-σ: 73.9%
  - 3-σ: 97.1%
  - Note how much lower these are than the univariate normal percentage points



Batch Epoch Covariance Generation (1 of 2)

- **Batch least-squares update (ASW method) uses the following minimization equation**
  - $dx = (A^TWA)^{-1}A^TWb$
    - dx is the vector of corrections to the state estimate
    - A is the time-enabled partial derivative matrix, used to map the residuals into state-space
    - W is the "weighting" matrix that provides relative weights of observation quality (usually 1/σ, where σ is the standard deviation generated by the sensor calibration process)
    - b is the vector of residuals (observations – predictions from existing state estimate)
- **Covariance is the collected term $(A^TWA)^{-1}$**
  - A the product of two partial derivative matrices:
    - $A = \frac{\partial(obs)}{\partial X_0} = \frac{\partial(obs)}{\partial X}\frac{\partial X}{\partial X_0}$
    - First term: partial derivatives of observations with respect to state at obs time
    - Second term: partial derivatives of state at obs time with respect to epoch state

Batch Epoch Covariance Generation (2 of 2)

- **Formulated this way, this covariance matrix is called an *a priori* covariance**
  - A does not contain actual residuals, only transformational partial derivatives
  - So $(A^TWA)^{-1}$ is a function only of the amount of tracking, times of tracks, and sensor calibration relative weights among those tracks
    - Not a function of the actual residuals from the correction
- **Limitations of *a priori* covariance**
  - Does not account well for unmodeled errors, such as transient atmospheric density prediction errors
    - Because not examining actual fit residuals
  - W-matrix only as good as sensor calibration process
    - Principal weakness of present process, but expected to be improved eventually with JSpOC Mission System (JMS) upgrades

ASW Covariance Propagation

- **Covariance in VCM is virginal (unaltered) covariance**
- **When propagating VCM covariance, the propagator**
  - Scales the covariance by the weighted RMS if it is greater than unity
    - C* = C * WRMS ^ 2
    - Was an early attempt to improve covariance realism; not clear this is still a good idea
  - Applies the consider parameter to the ballistic coefficient variance
    - C*(7,7) = C*(7,7) + Cpd ^ 2
    - More later on how this value is determined
  - May apply a consider parameter to the solar radiation pressure variance
    - C*(9,9) = C*(9,9) + Cps ^ 2
    - Presently not used (Cps set to 0)
  - Propagates the altered covariance using linearized dynamics
    - Phi * C** * Phi^T
  - Converts propagated matrix from equinoctial to Cartesian coordinates

Altered Covariance Positions

- **Ballistic coefficient consider parameter (DCP) applied to ballistic coefficient variance (orange)**
- **If used, solar radiation pressure consider parameter applied to solar radiation pressure variance (purple)**

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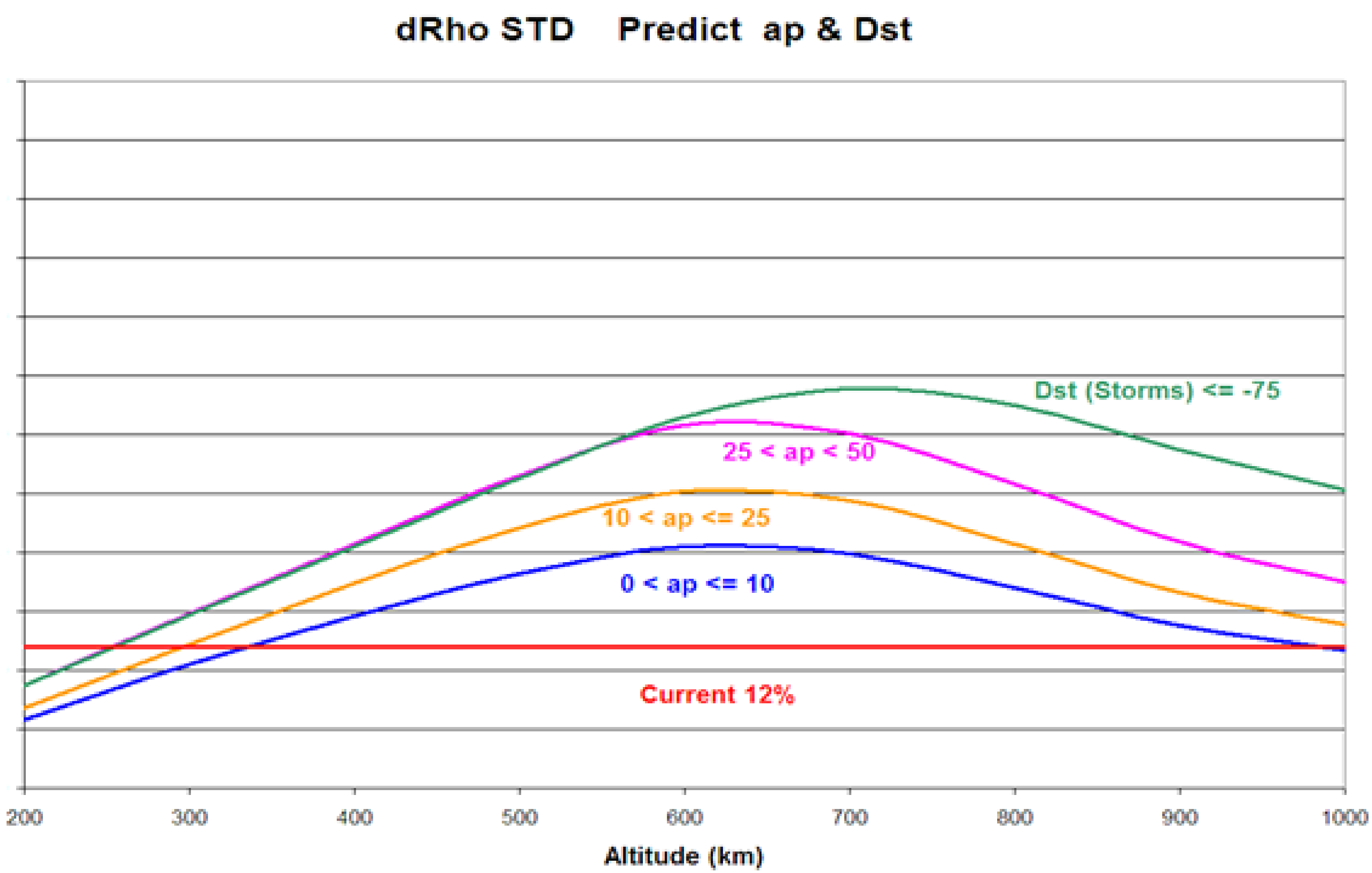
AGOM	-3.83E-02	-1.28E-01	-3.73E-03	1.46E-04	4.10E-05	-4.39E-06	-6.28E-07	2.31E-05

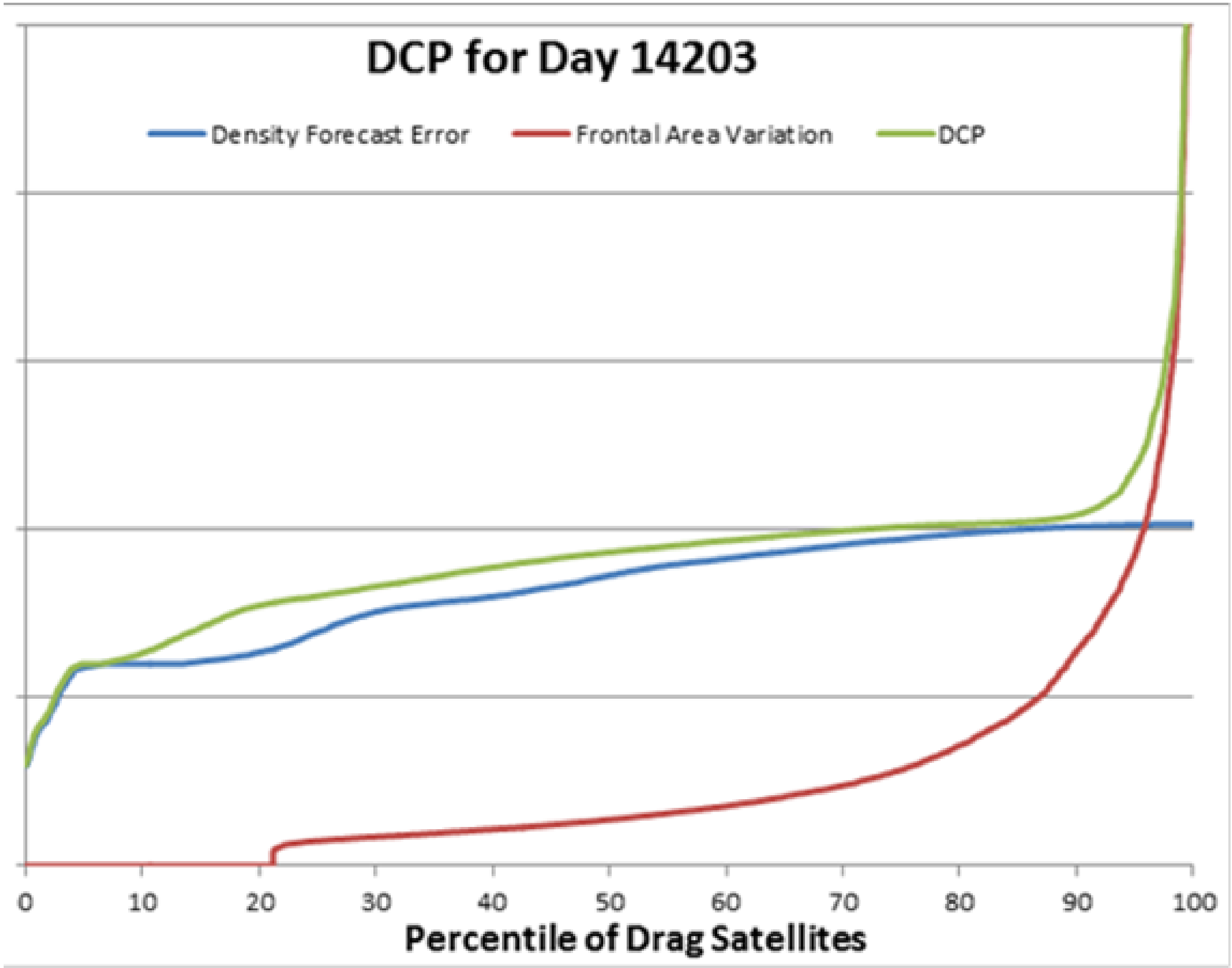
Dynamic Consider Parameter (DCP)

- **Specifies global error in the atmospheric density forecast**
  - Parameterizes percent RMS error in terms of
    - Satellite height (perigee altitude)
    - Geomagnetic activity (a_p and Dst)
  - Density forecast error combination of solar/geomagnetic indices and DCA
    - Directly compared numerous forecast densities to actual density
    - Discretized heights from 200 km to 1000 km averaged over lat / lon
  - Sample high, low, and medium solar cycles (2001, 2005, 2013)
    - Found most variation parameterizable via a_p conditions (versus F_{10})
    - Functions optimized for 3-day predictions—this is the tuning point!
- **Determines satellite-specific frontal area variation in prediction**
  - Quantifies ballistic coefficient RMS error through satellite histories
    - Looks back in time up to a year in most cases (upfront preprocessing)
    - Ascertains error / target for 3-day predictions (accounting for time-lags)
- **Combine the two uncertainty components to obtain DCP value**
  - Additive in variance sense as the root sum of squares

B Consider Parameter Values



DCP Components RMS Uncertainty



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