A photograph of a regolith test bin, likely used for simulating lunar or planetary surface conditions. The bin is filled with a dark, granular material. A person is visible in the background, working with the equipment. An American flag is on the left. Several bright green laser lines are projected across the scene, forming a grid-like pattern. The text is overlaid on the image.

Optical Extinction Measurements of Dust Density in GMRO Regolith Test Bin

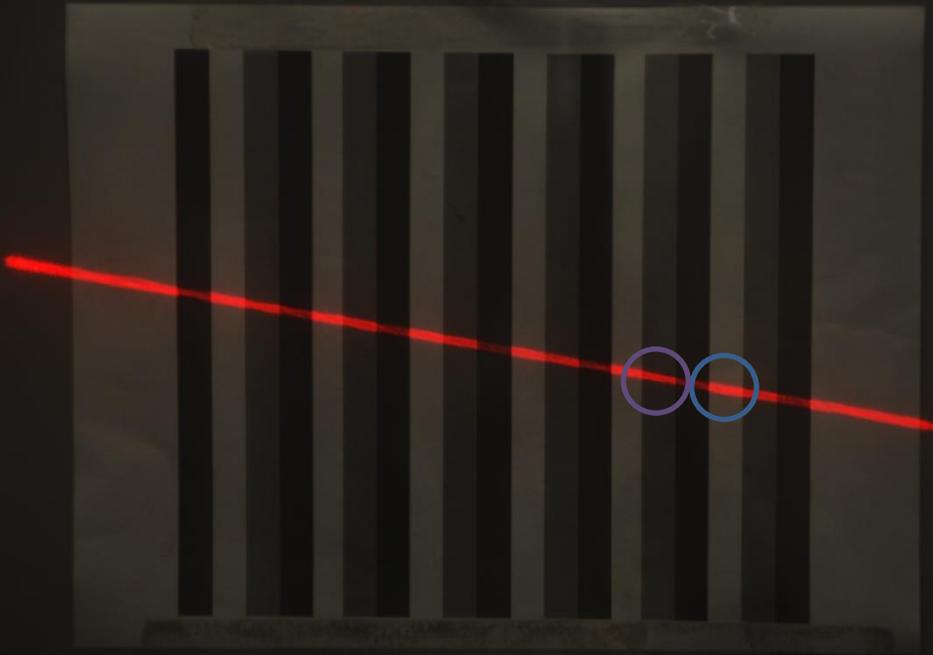
J. Lane, R. Mueller, J. Mantovani, M. Nugent, A. Nick,
J. Schuler, and I. Townsend

*Granular Mechanics and Regolith Operations Lab
Kennedy Space Center, FL 32899*

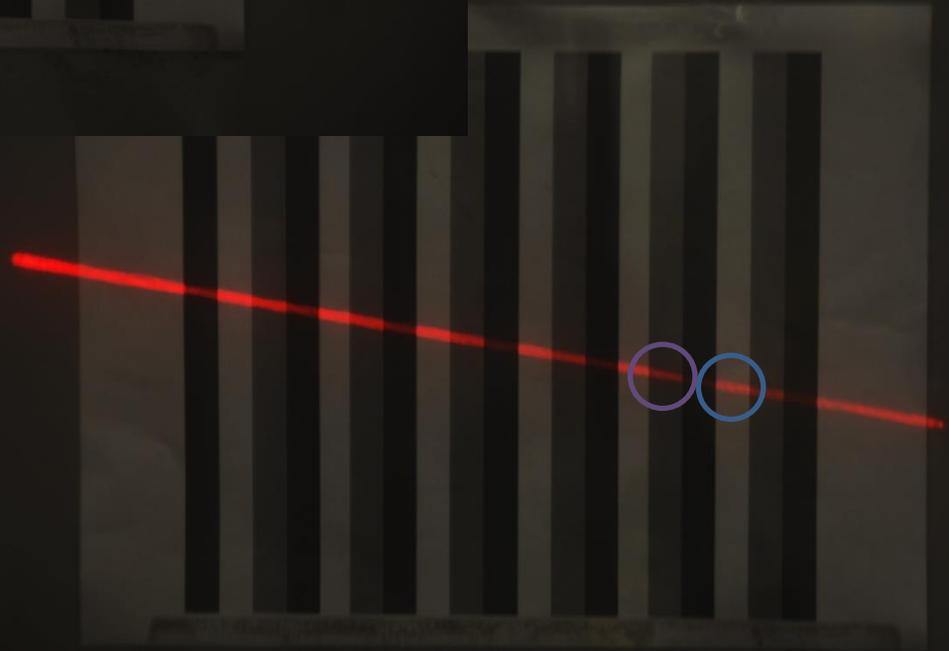
Earth and Space 2016, Orlando, Florida
April 11-15, 2016

Optical Density Using a Target

without dust



with dust



Average particle Size from Optical Density

$$\langle D \rangle = \frac{3\dot{m}L}{2v_p\rho_p \ln((l'_L - l_0)/(l_L - l_0))}$$

$\langle D \rangle$ = average scattering cross-section diameter

L = distance laser travels through dust ≈ 0.5 in = 0.0127 m

\dot{m} = mass transfer rate ≈ 0.0125 kg/s

l_L = luminosity of target with laser without dust

l'_L = luminosity of target with laser with dust

l_0 = luminosity of target without laser (background light only)

ρ_p = particle bulk density ≈ 3100 kg/m³

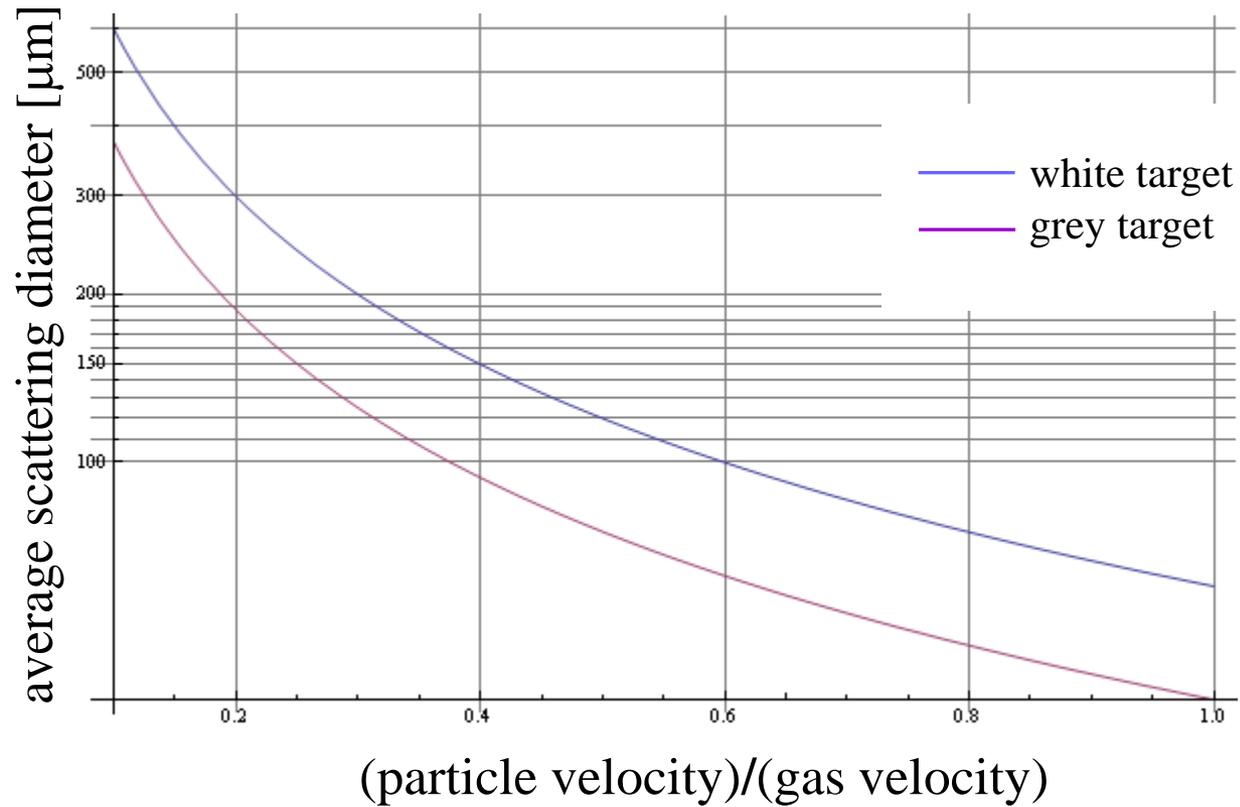
$v_p = e v_g$ = particle velocity m/s (v_g = gas velocity ≈ 17.7 m/s)

$l \equiv 0.299 R + 0.587 G + 0.114 B$, *luminosity* according to PhotoShop

Luminosity Measurements

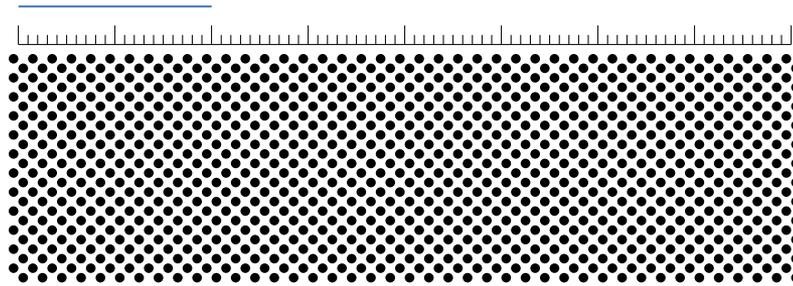
	l_L	l'_L	l_0
white target	97.1	80.5	58.9
gray target	70.9	46.2	29.6

Average particle Size from Optical Density



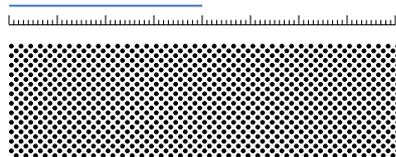
Optical Density 2D Calibration

Test Transparency



$$D = 0.050 \text{ [in]}$$

$$n\sigma = \pi/8$$



$$D = 0.025 \text{ [in]}$$


```

xCalc v2.nb
c = ((pLp - p0) / (pL - p0))
x[pLp, p0, pL] = -Log[c]
x[42.23, 2, 59.76]
Exp[-x[41.9, 2, 59.02]]
x[22.5, 2, 42.61]
Exp[-x[22.5, 2, 42.61]]
x[11.7, 2, 19.4]
Exp[-x[11.7, 2, 19.4]]


$$\frac{-p0 + pLp}{-p0 + pL}$$


$$-\text{Log}\left[\frac{-p0 + pLp}{-p0 + pL}\right]$$


0.361684
0.699754
0.683589
0.504802
0.584344
0.557471
Quit[]
1 - Pi / 8 // N
0.607301
a = 10
L = 1
M = 10 000 000
x = L / M
Exp[-a L] // N
(1 - a x) ^ M // N
10
1
10 000 000


$$\frac{1}{10\,000\,000}$$


0.0000453999
0.0000453997
Limit[(1 - a L / m) ^ m, m -> Infinity]
e^-aL

```

$$c = \frac{p'_L - p_0}{p_L - p_0}$$

White Target: $c = 0.700$

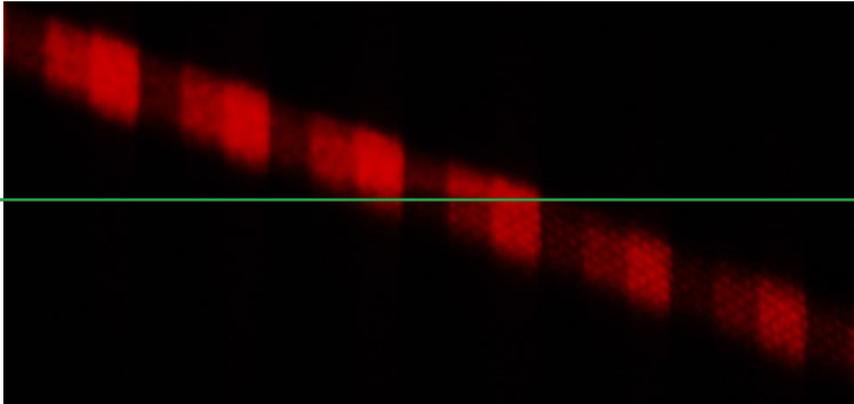
Grey Target: $c = 0.505$

Black Target: $c = 0.557$

$$1 - n\sigma = 1 - \pi/8 = 0.6073$$

2D Test Image

without "dust"

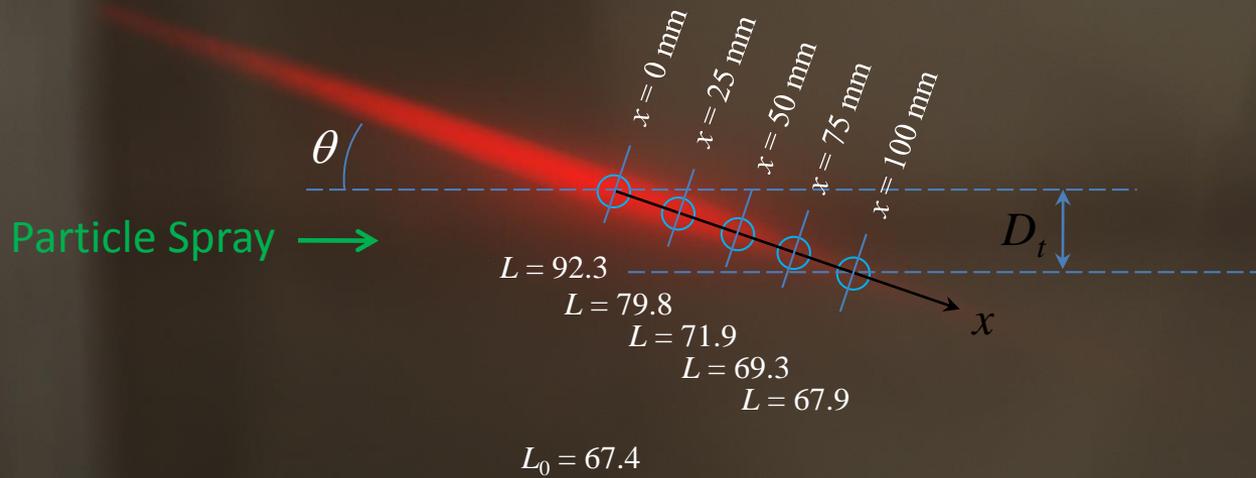


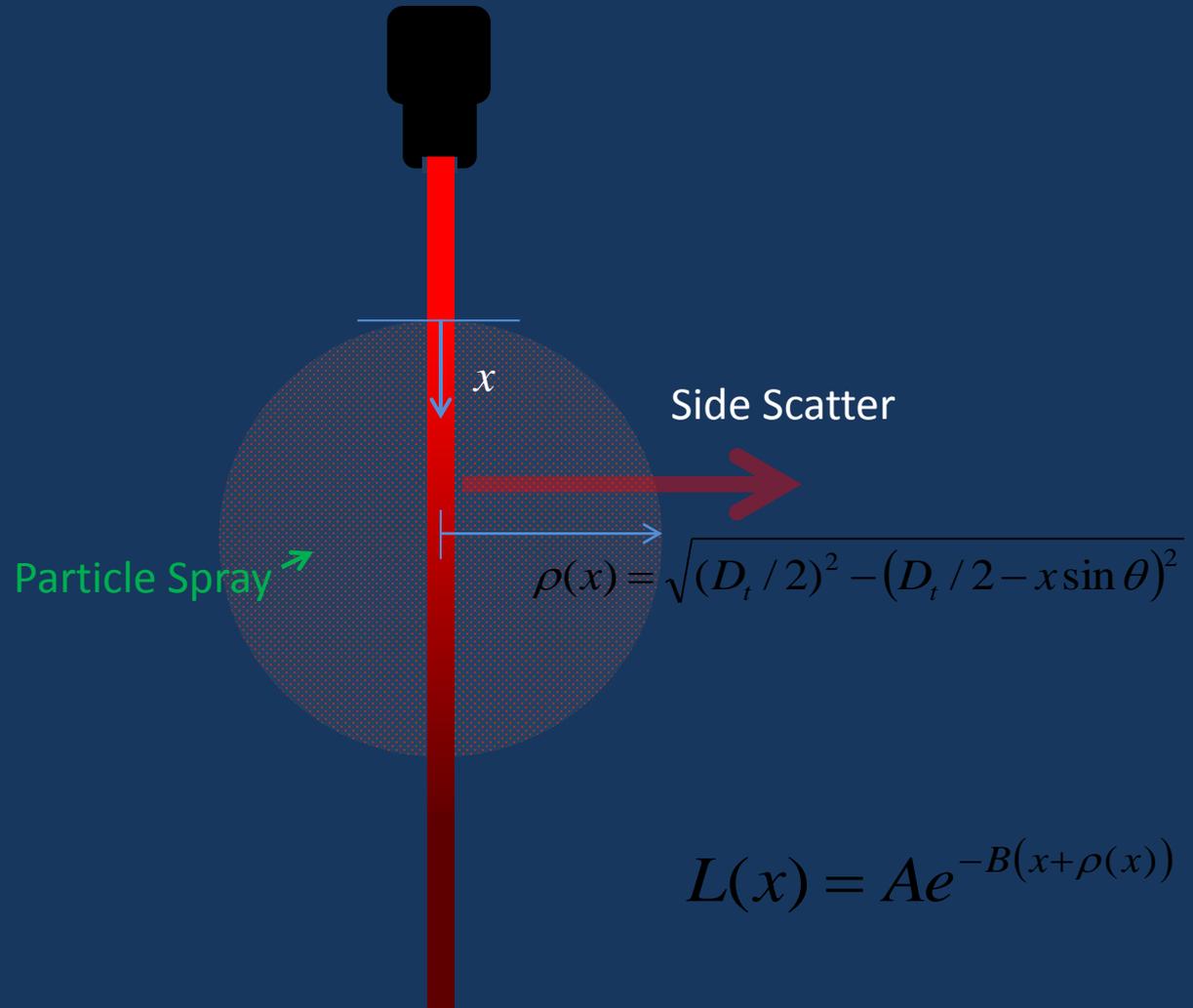
with "dust"



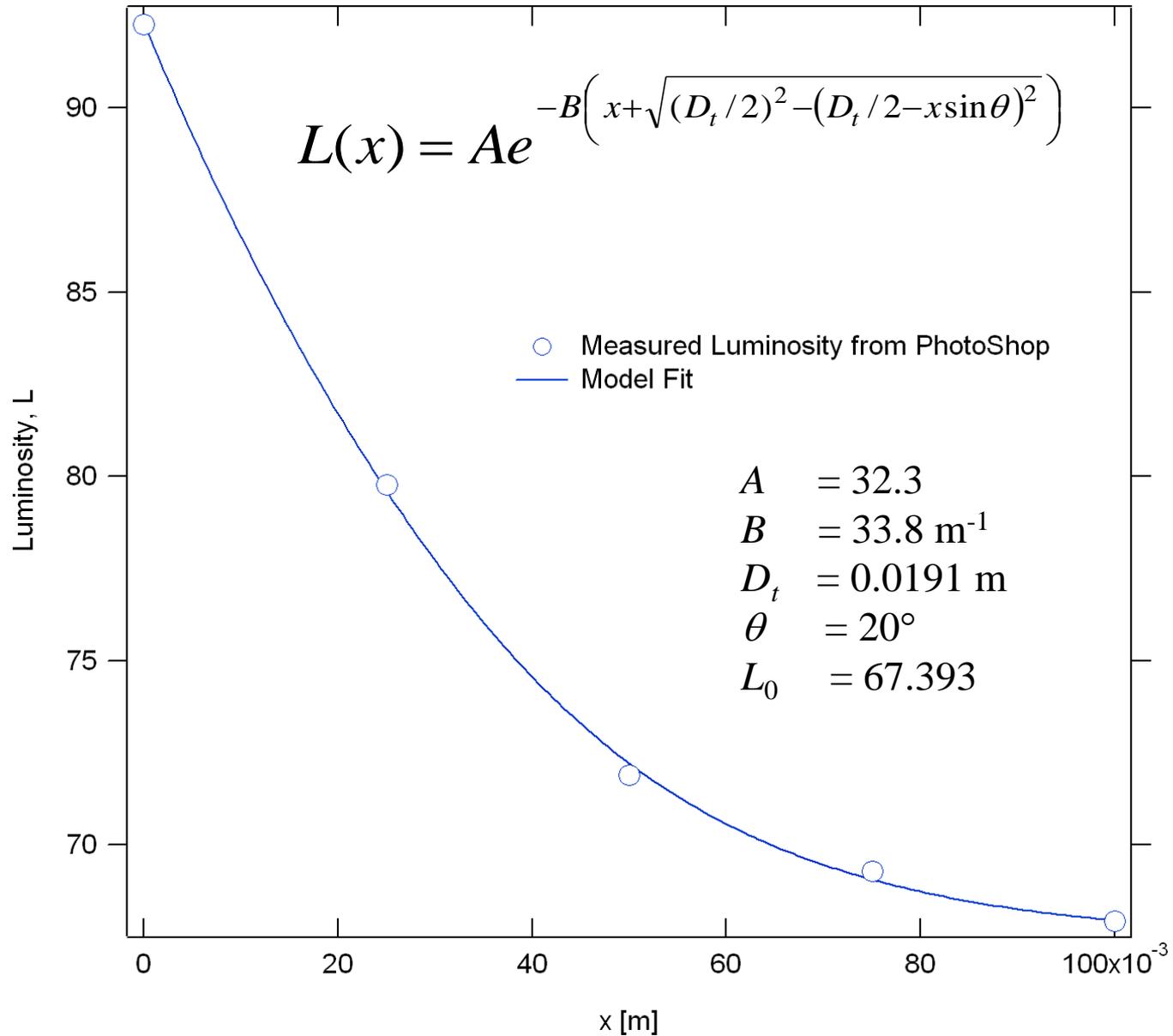
Optical Density Using Side-Scatter

Luminosity Values as a Function of x





Luminosity Values as a Function of x



Average Particle Size from Optical Density

$$\bar{D} = \frac{6\dot{M}}{\pi D_t^2 \xi v_g \rho_p B}$$

\bar{D} = average scattering cross-section diameter of particles

B = from luminosity fit = 33.8 m⁻¹

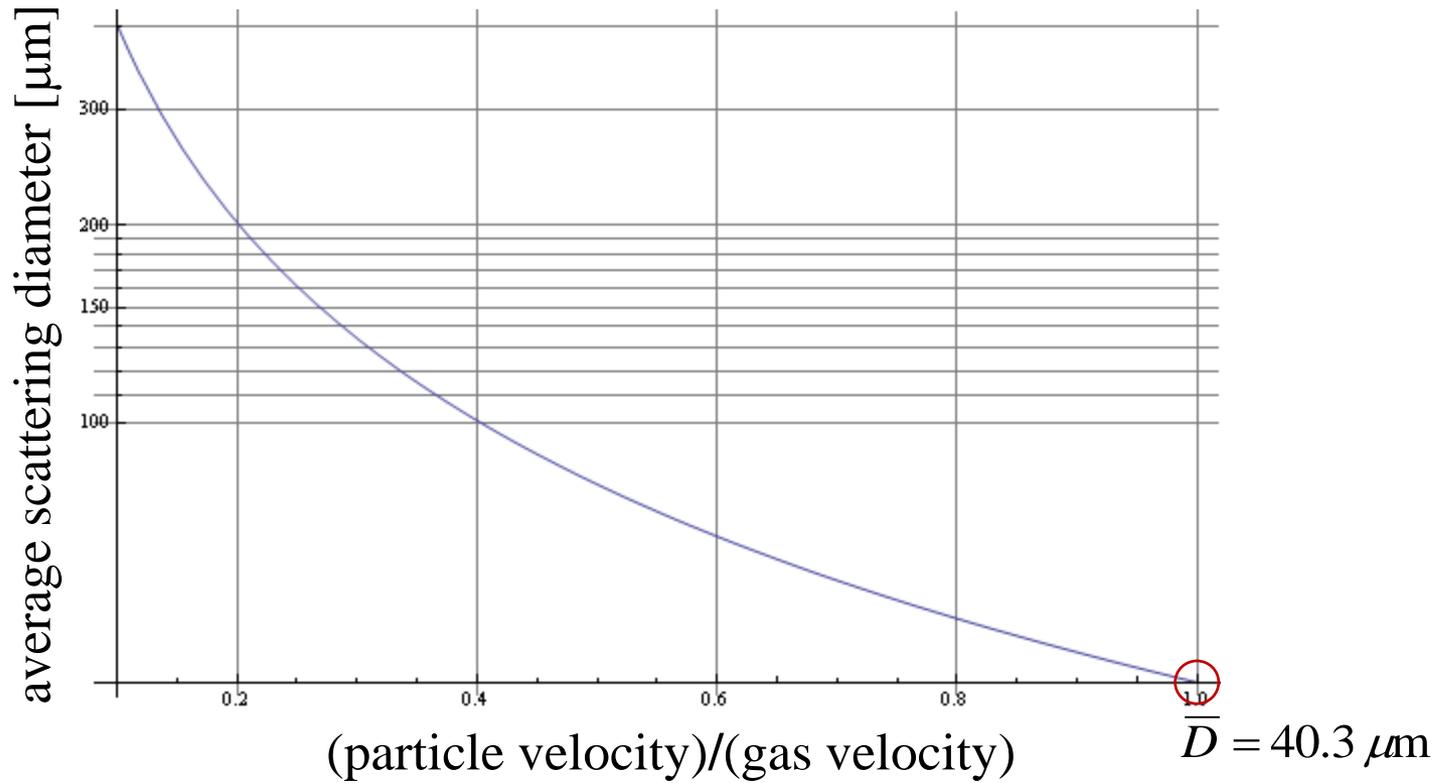
\dot{M} = mass transfer rate ≈ 0.0250 kg/s

D_t = diameter of spray ≈ 0.75 in = 0.0191 m

ρ_p = particle bulk density ≈ 3100 kg/m³

$v_p = \xi v_g$ = particle velocity m/s (v_g = gas velocity ≈ 31.1 m/s)

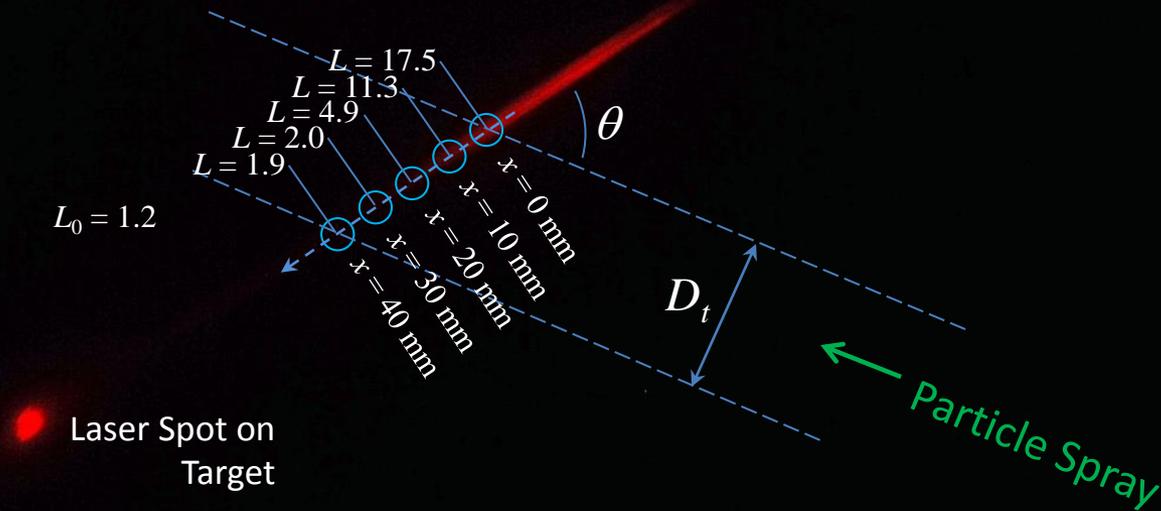
Average Particle Size from Optical Density



Optical Density Using Side-Scatter with Camera at Arbitrary Angle

Luminosity Values as a Function of x

12-16-11 experiment



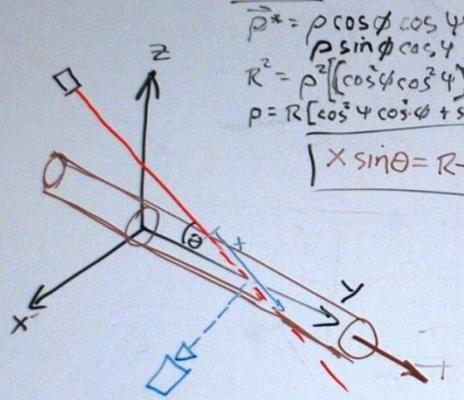
$$\begin{aligned} \phi \neq 0 \\ \psi \neq 0 \\ z = 0 \end{aligned}$$

$$\vec{p}^* = \rho \cos \phi \cos \psi \hat{i} + \rho \sin \phi \cos \psi \hat{j} + \rho \sin \psi \hat{k}$$

$$R^2 = \rho^2 [\cos^2 \psi \cos^2 \phi + \sin^2 \psi]$$

$$\rho = R [\cos^2 \psi \cos^2 \phi + \sin^2 \psi]^{1/2}$$

$$x \sin \theta = R - z$$

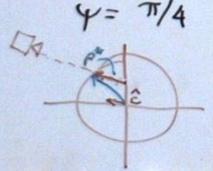


$$x_{max} \sin \theta = 2R$$

$$x_{max} = \frac{2R}{\sin \theta}$$

$$\psi = \pi/2 :$$

$$\rho(x) = x \sin \theta$$



$$\psi = \pi/4$$

$$\rho(z=R) = 0$$

$$\rho(z=0) = R$$

$$\vec{p}^* = \begin{pmatrix} \xi(z) \\ -z/\tan \theta \\ +z \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \rho^* \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \rho^* = R^2$$

$$\rho(z, \phi, \psi) = \xi$$

$$\psi = 0, \phi = 0 :$$

$$\rho^2 + z^2 = R^2$$

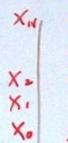
$$\rho = (R^2 - z^2)^{1/2}$$

$$= (R^2 - (x \sin \theta - R)^2)^{1/2}$$

$$\psi = 0 :$$

$$\rho_n' = (R^2 - z_n^2)^{1/2}$$

$$\rho_n = \rho_n' \cos \phi$$



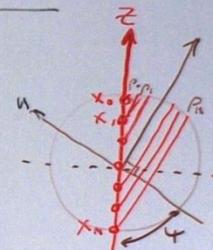
$$\rho(x) = \rho_0(x) \frac{1}{\cos \phi}$$

$$\phi = 0 :$$

$$u = z \cos \psi$$

$$\rho(u) = (R^2 - u^2)^{1/2} - z \sin \psi$$

$$z_n = R - x_n \sin \theta$$



$$\rho(z) = (R^2 - z^2 \cos^2 \psi)^{1/2} - z \sin \psi$$

$$\psi = 0 : \rho = (R^2 - z^2)^{1/2}$$

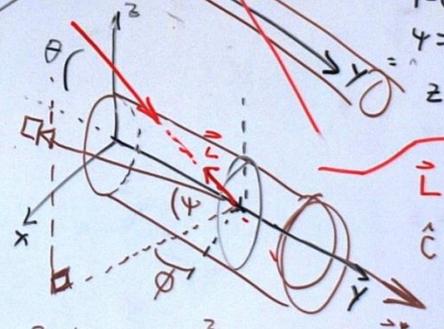
$$\psi = \pi/2 : \rho = R - z$$

$$z = 0 : \rho = R$$

$$\tan \theta = \frac{z}{y}$$

$$\vec{L} = y \hat{j} + z \hat{k} = z \left(\frac{1}{\tan \theta} \hat{j} + \hat{k} \right)$$

$$\hat{C} = \cos \phi \cos \psi \hat{i} + \sin \phi \cos \psi \hat{j} + \sin \psi \hat{k}$$



$$R^2 = (\xi \cos \phi \cos \psi)^2 + (\xi \sin \phi \cos \psi + z)^2$$

$$\vec{p}^* = \xi \cos \phi \cos \psi \hat{i} + (\xi \sin \phi \cos \psi + z) \hat{k}$$

```

ps[ξ_] = {ξ Cos[φ] Cos[ψ], ξ Sin[φ] Cos[ψ] - z / Tan[θ], ξ Sin[ψ] + z};
MatrixForm[ps[ξ]]
xzP = {{1, 0, 0}, {0, 0, 0}, {0, 0, 1}};
MatrixForm[xzP]
Solve[R^2 == (xzP.ps[ξ]).(xzP.ps[ξ]), ξ]
{ρ1[x_, φ_, ψ_], ρ2[x_, φ_, ψ_]} = ξ /. %;

FullSimplify[ρ1[z, φ, ψ], {R > 0 == 0, Im[z] == 0, Im[φ] == 0, Im[ψ] == 0}]
FullSimplify[ρ2[z, φ, ψ], {R > 0 == 0, Im[z] == 0, Im[φ] == 0, Im[ψ] == 0}]

FullSimplify[ρ2[z, 0, 0], {R > 0, Im[z] == 0, Im[φ] == 0, Im[ψ] == 0}]
FullSimplify[ρ2[0, 0, ψ], {R > 0 == 0, Im[z] == 0, Im[φ] == 0, Im[ψ] == 0}]
FullSimplify[ρ2[z, 0, ψ], {R > 0 == 0, Im[z] == 0, Im[φ] == 0, Im[ψ] == 0}]
FullSimplify[ρ2[z, 0, Pi/2], {R > 0 == 0, Im[z] == 0, Im[φ] == 0, Im[ψ] == 0}]
FullSimplify[ρ2[z, φ, 0], {R > 0 == 0, Im[z] == 0, Im[φ] == 0, Im[ψ] == 0}]

```

$$\begin{pmatrix} \xi \cos[\phi] \cos[\psi] \\ -z \cot[\theta] + \xi \cos[\psi] \sin[\phi] \\ z + \xi \sin[\psi] \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\left\{ \xi \rightarrow \frac{-16 z \sin[\psi] - \sqrt{-4 (8 R^2 - 8 z^2) (-6 - 2 \cos[2 \phi] - \cos[2 \phi - 2 \psi] + 2 \cos[2 \psi] - \cos[2 \phi + 2 \psi]) + 256 z^2 \sin[\psi]^2}}{2 (6 + 2 \cos[2 \phi] + \cos[2 \phi - 2 \psi] - 2 \cos[2 \psi] + \cos[2 \phi + 2 \psi])} \right\},$$

$$\left\{ \xi \rightarrow \frac{-16 z \sin[\psi] + \sqrt{-4 (8 R^2 - 8 z^2) (-6 - 2 \cos[2 \phi] - \cos[2 \phi - 2 \psi] + 2 \cos[2 \psi] - \cos[2 \phi + 2 \psi]) + 256 z^2 \sin[\psi]^2}}{2 (6 + 2 \cos[2 \phi] + \cos[2 \phi - 2 \psi] - 2 \cos[2 \psi] + \cos[2 \phi + 2 \psi])} \right\}$$

$$\frac{2 \left(2 z \sin[\psi] + \sqrt{(R - z) (R + z) (3 + \cos[2 \phi] - 2 \cos[2 \psi] \sin[\phi]^2) + 4 z^2 \sin[\psi]^2} \right)}{3 + \cos[2 \phi] - 2 \cos[2 \psi] \sin[\phi]^2}$$

$$\rho(x, \phi, \psi) = \frac{2 \left(-2 z \sin[\psi] + \sqrt{(R - z) (R + z) (3 + \cos[2 \phi] - 2 \cos[2 \psi] \sin[\phi]^2) + 4 z^2 \sin[\psi]^2} \right)}{3 + \cos[2 \phi] - 2 \cos[2 \psi] \sin[\phi]^2}$$

$$\sqrt{R^2 - z^2}$$

R

$$-z \sin[\psi] + \sqrt{R^2 - z^2 + z^2 \sin[\psi]^2}$$

R - z

$$\sqrt{R^2 - z^2} \operatorname{Abs}[\cos[\phi]] \operatorname{Sec}[\phi]^2$$

Quit[]

Particle
Spray

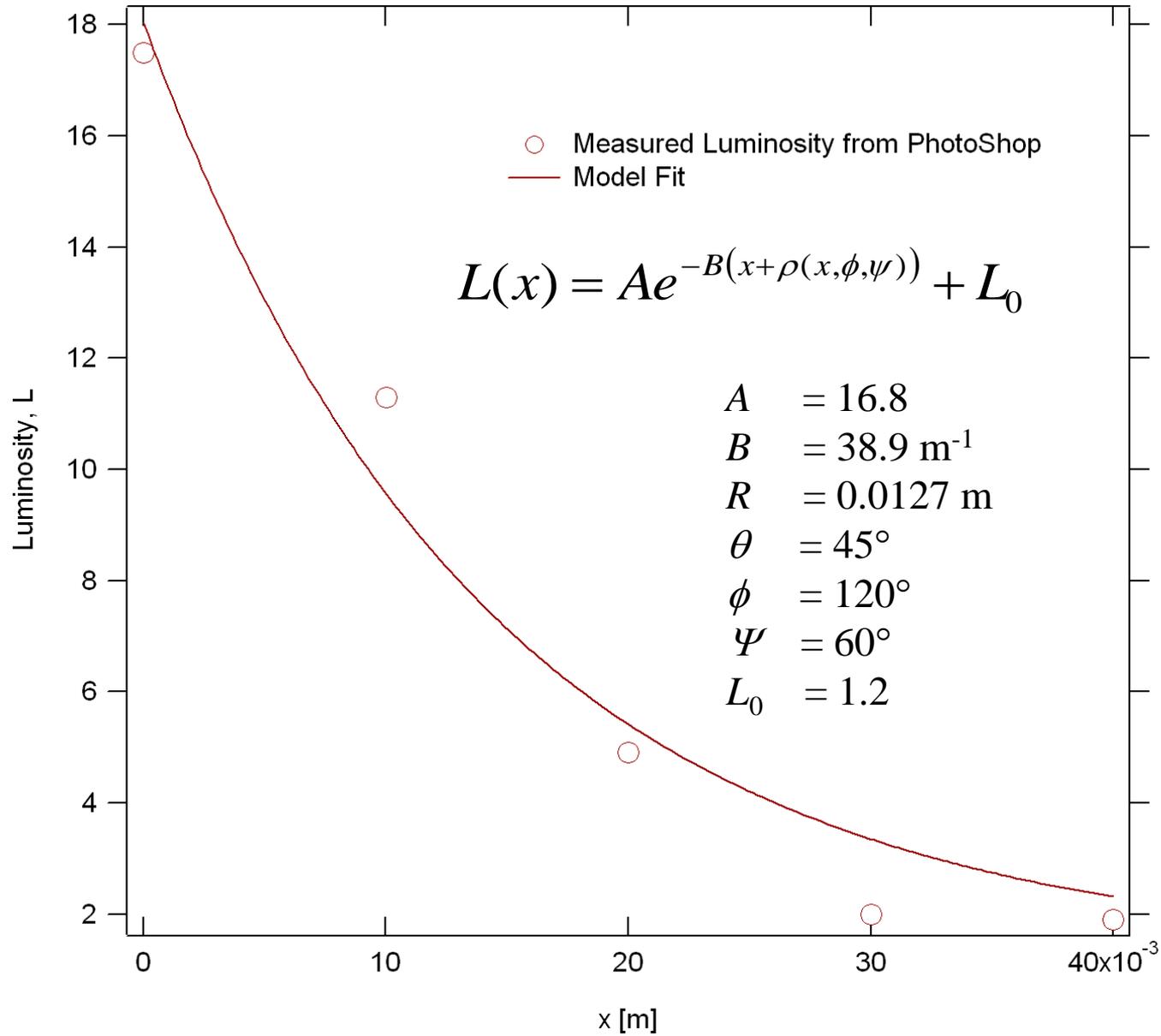
side scatter
towards
camera

$$\rho(z, \phi, \theta) = \frac{2 \left(-2z \sin \psi + \left((R-z)(R+z) \left(3 + \cos 2\phi - 2 \cos 2\psi \sin^2 \phi \right) + 4z^2 \sin^2 \psi \right)^{1/2} \right)}{3 + \cos 2\phi - 2 \cos 2\psi \sin^2 \phi}$$

$$z = R - x \sin \theta$$

$$L(x) = A e^{-B(x + \rho(x, \phi, \psi))}$$

Luminosity Values as a Function of x



Average Particle Size from Optical Density

$$\bar{D} = \frac{6\dot{M}}{\pi D_t^2 \xi v_g \rho_p B}$$

\bar{D} = average scattering cross-section diameter of particles

B = from luminosity fit = 38.9 m⁻¹

\dot{M} = mass transfer rate \approx 0.0370 kg/s

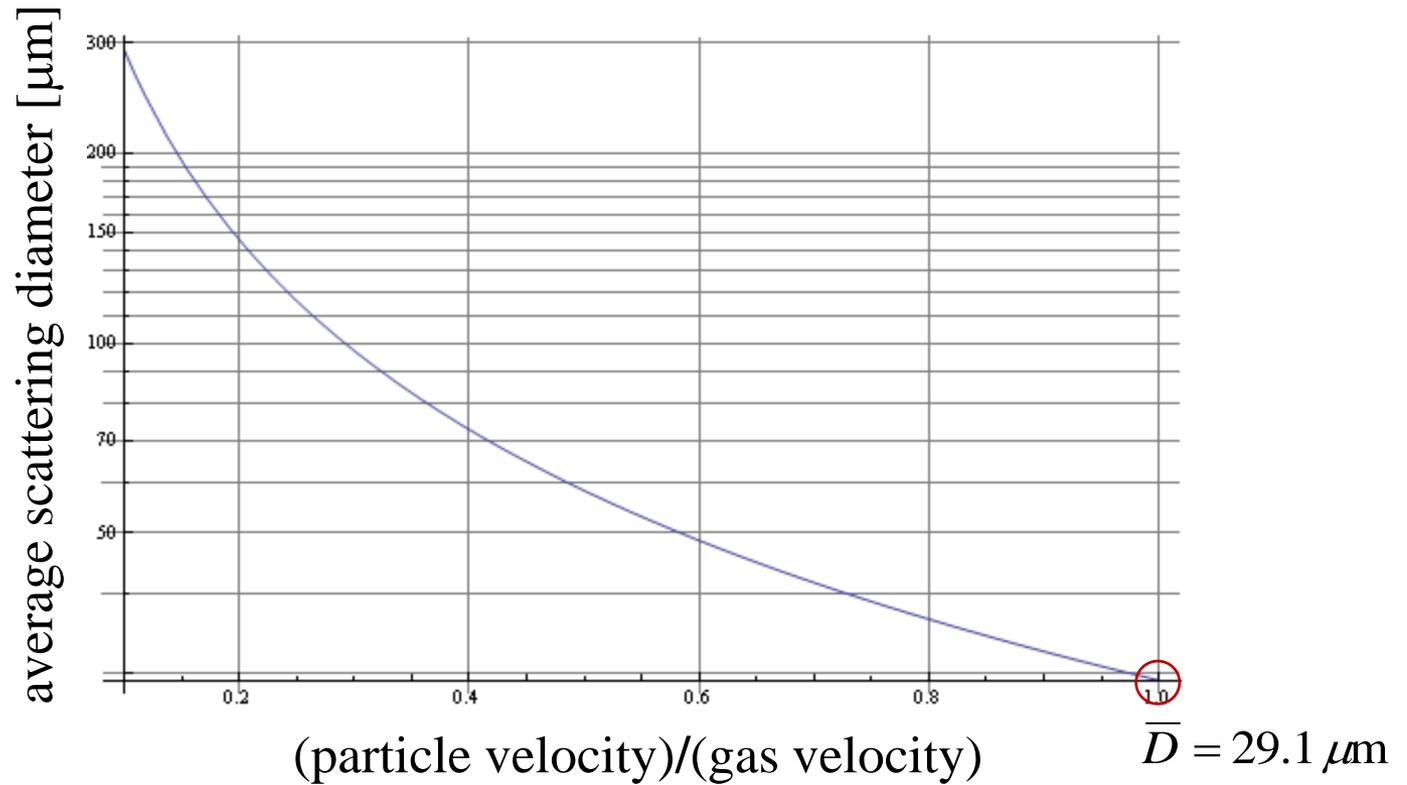
D_t = diameter of spray \approx 1.0 in = 0.0254 m

ρ_p = particle bulk density \approx 3100 kg/m³

$v_p = \xi v_g$ = particle velocity m/s (v_g = gas velocity \approx 31.1 m/s)*

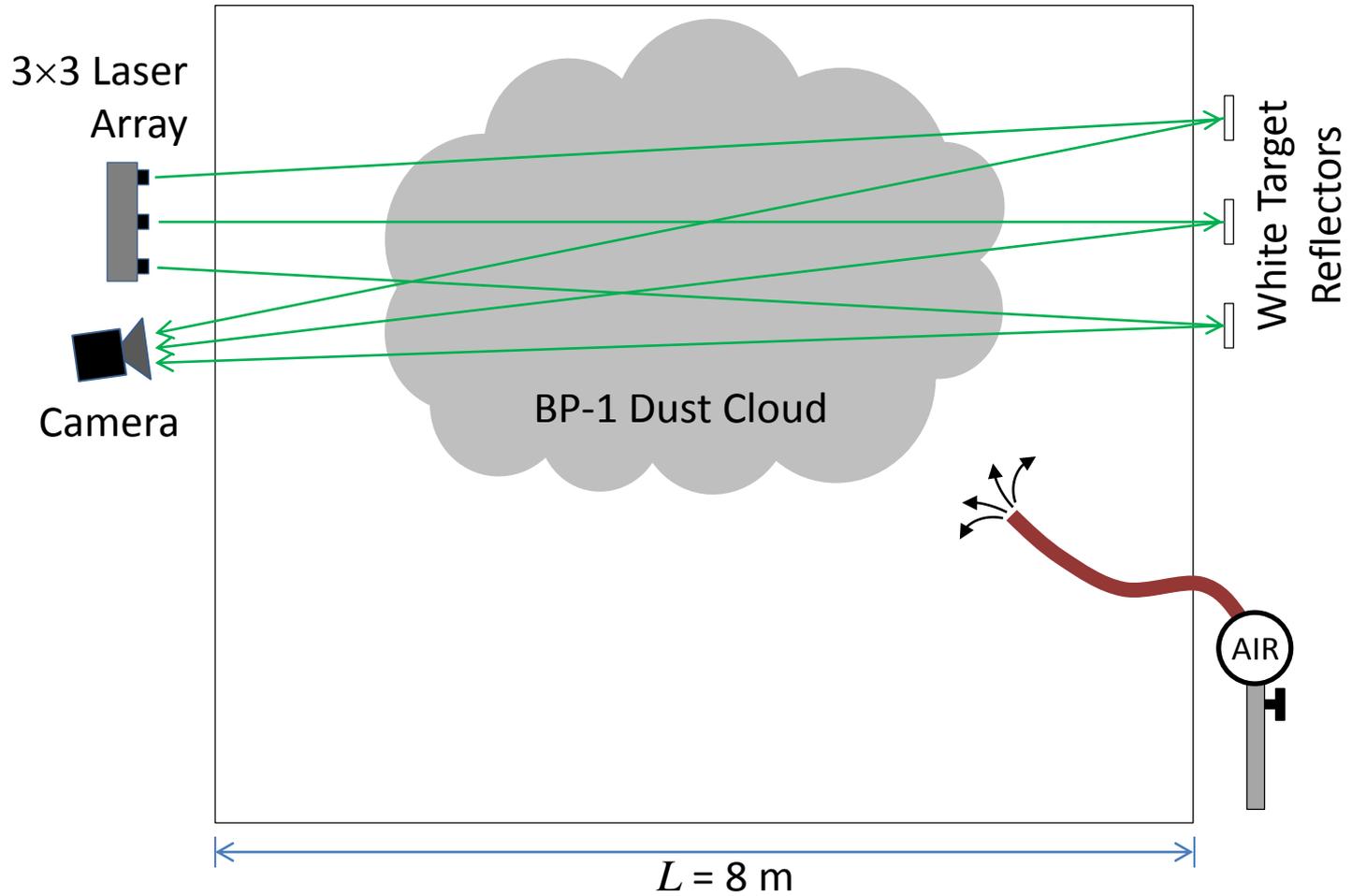
* gas flow = 4.7 CFM through a 3/8 inch pipe

Average Particle Size from Optical Density

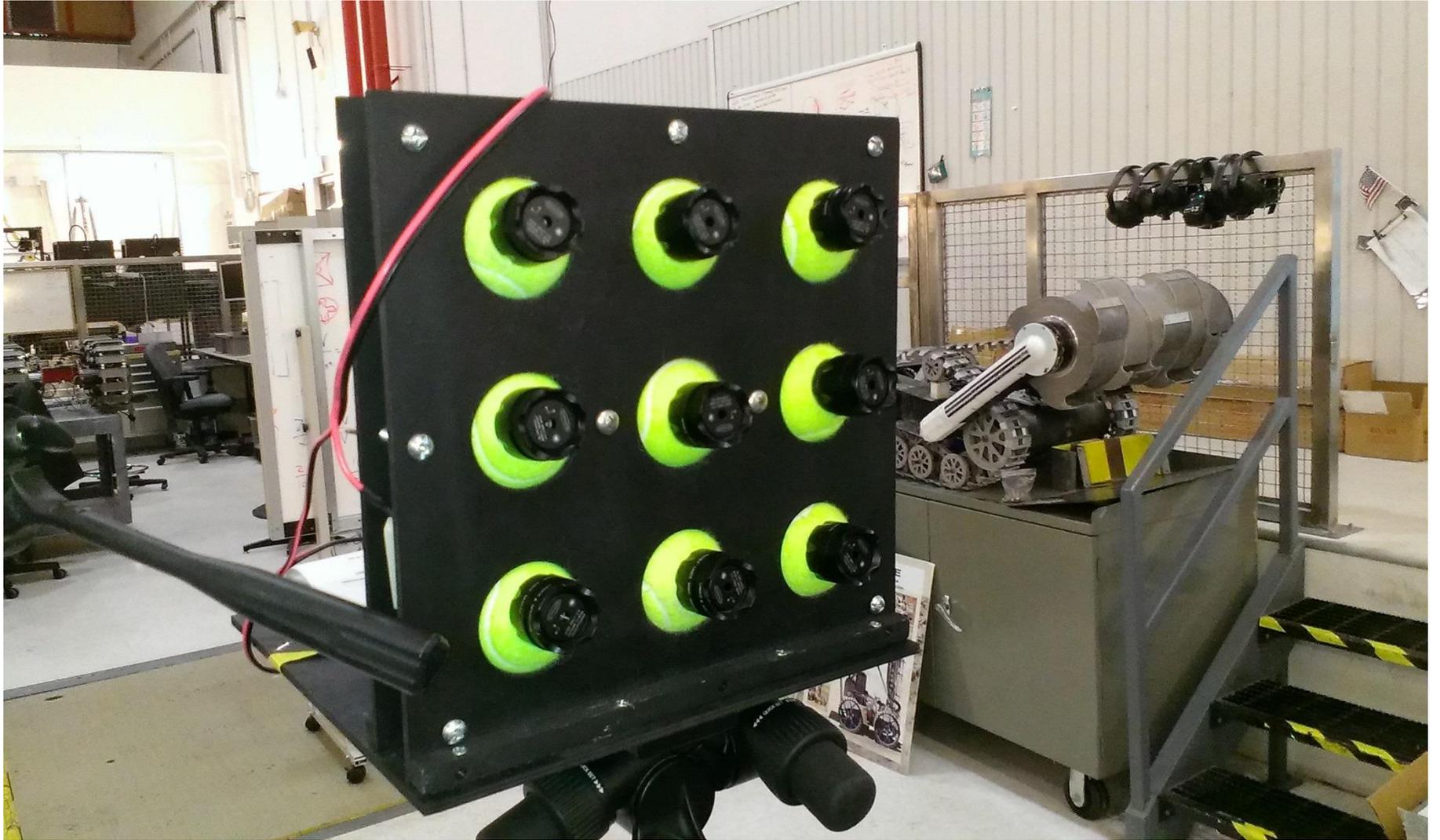


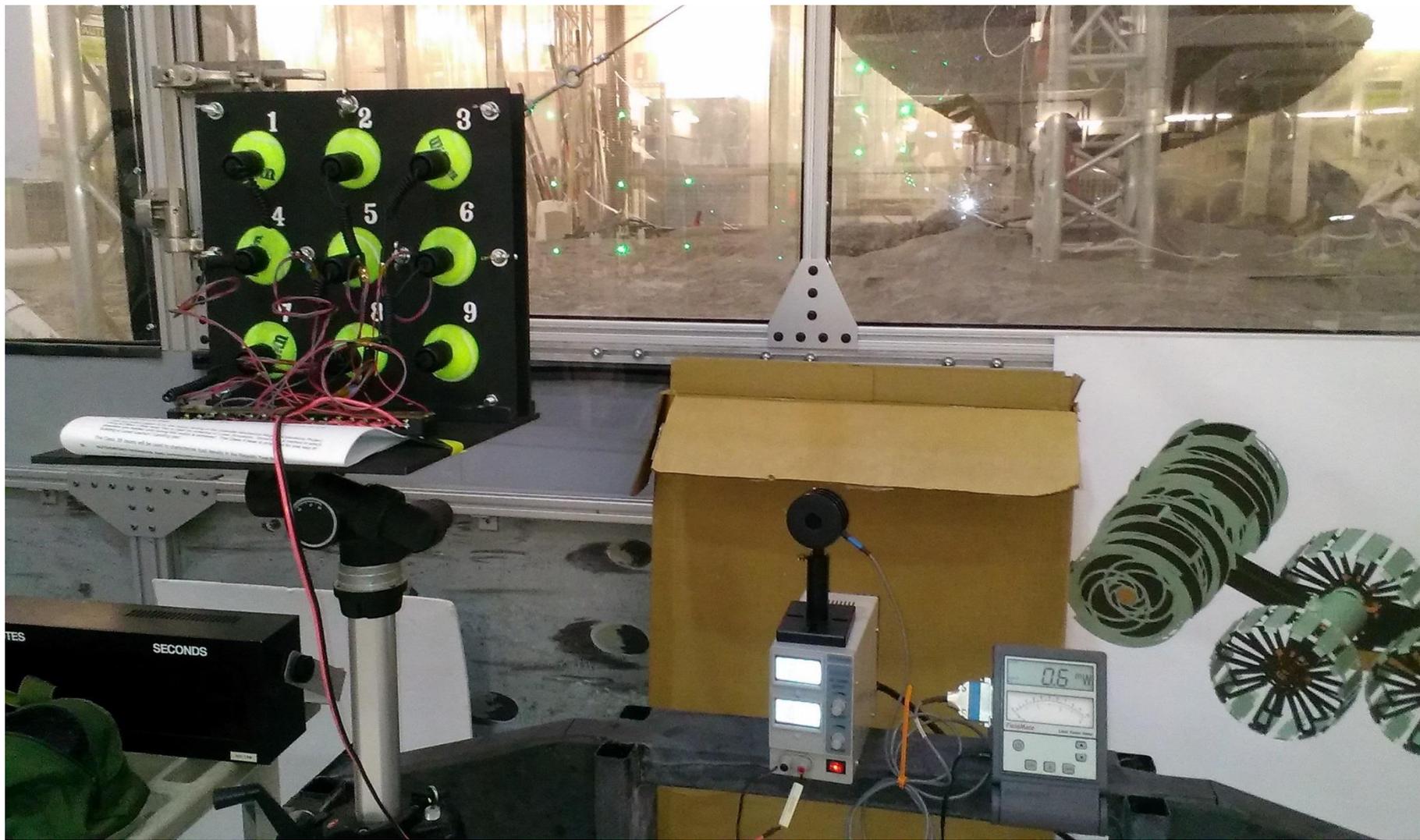
Regolith Test Bin (RTB)
Swampworks,
Kennedy Space Center, FL

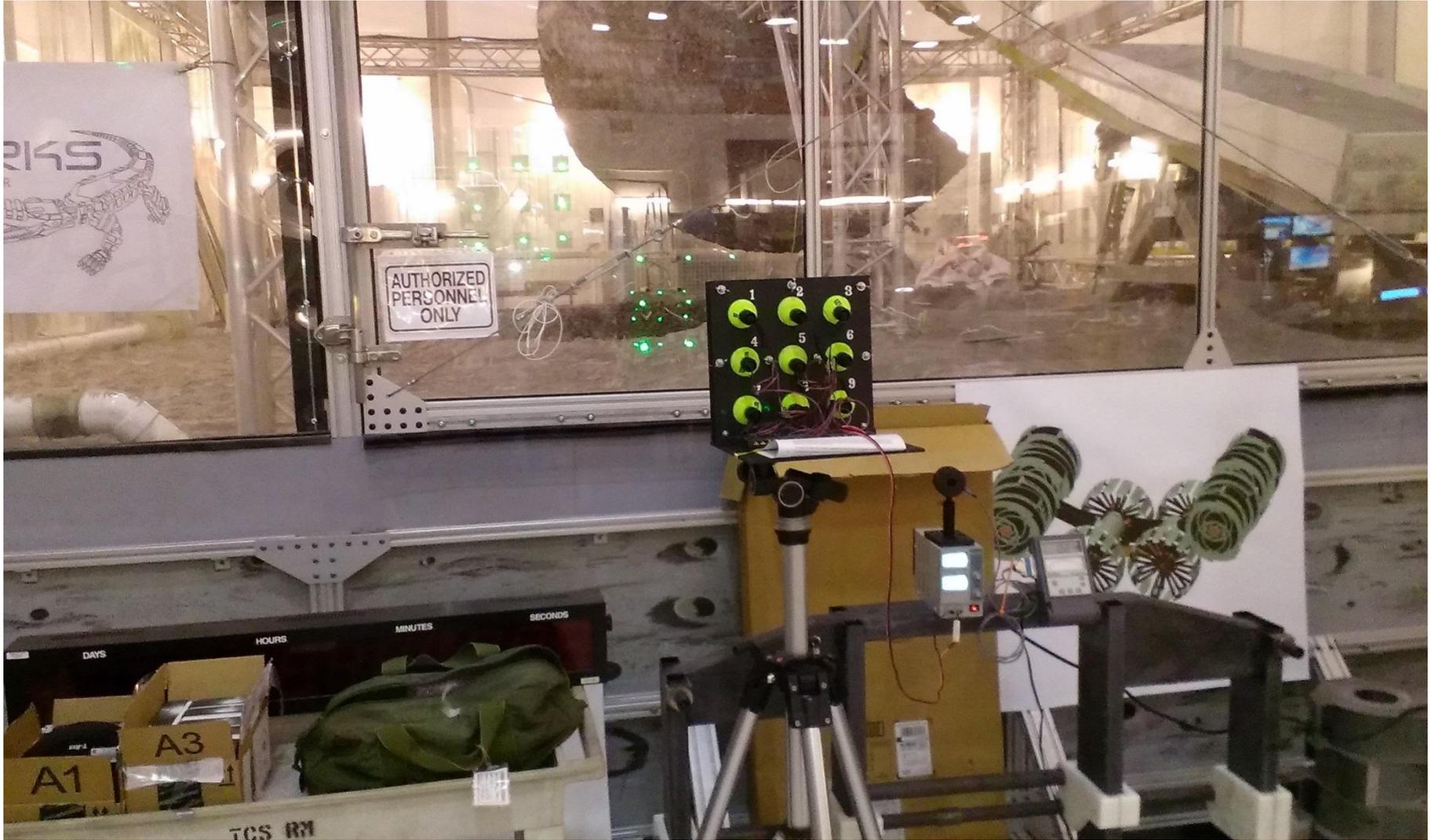
Planetary Regolith Test Bed













Intensity of Reflected Beam:

$$I(x) = I_0 e^{-\pi Q_e \alpha x / 4}$$

Extinction Coefficient:

$$\alpha = -\frac{4}{\pi Q_e x} \ln \left(\frac{I(x)}{I_0} \right)$$

Q_e = Extinction Efficiency (≈ 2)

x = Total Optical Distance

= $2L$ (where L is bin width ≈ 7.9 m)

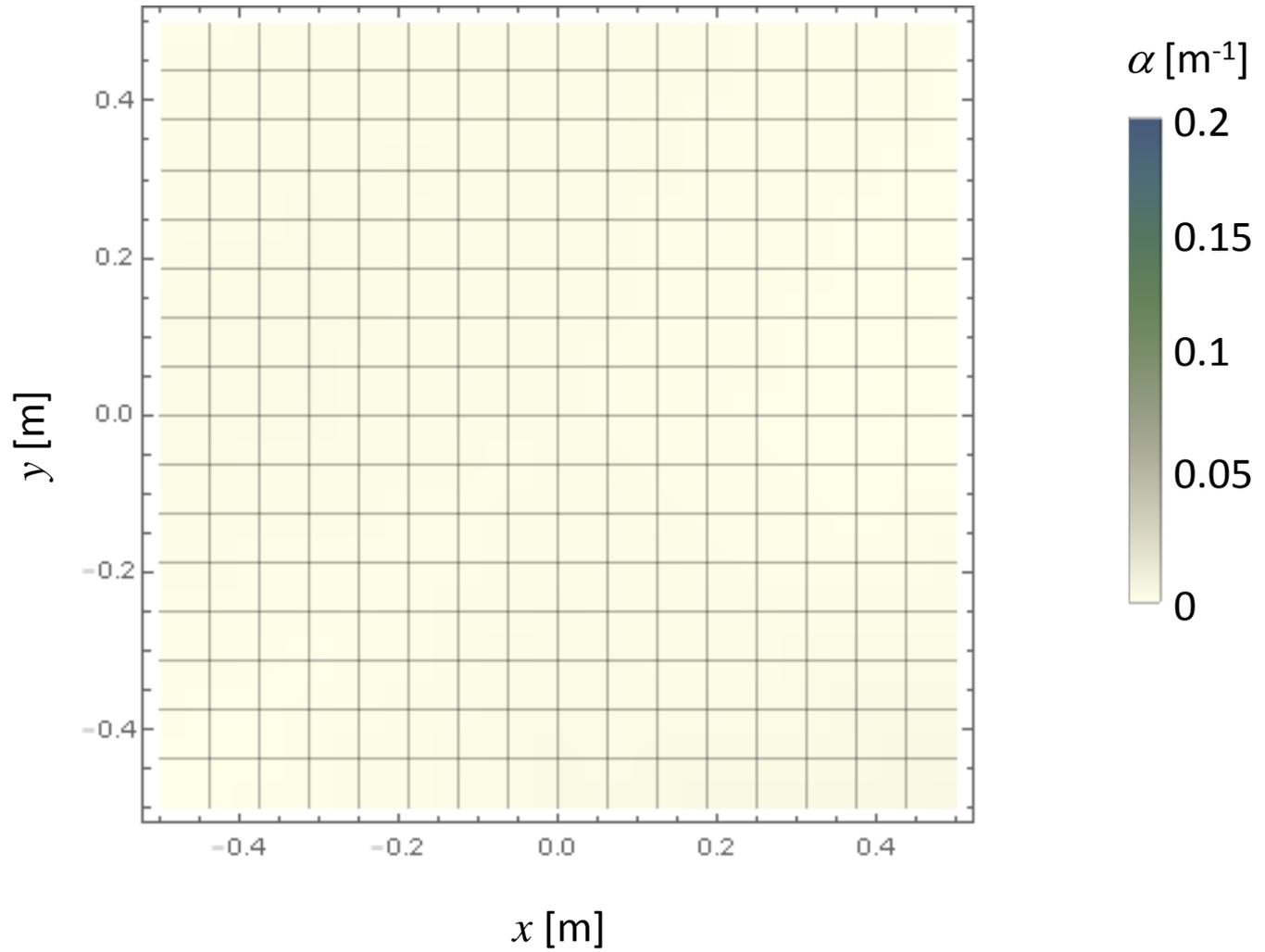
$$\alpha = -\frac{4}{\pi Q_e x} \ln \left(\frac{I(x)}{I_0} \right) \text{ [m}^{-1}\text{]}$$

$$\alpha = \int_0^{\infty} D^2 N(D) dD$$

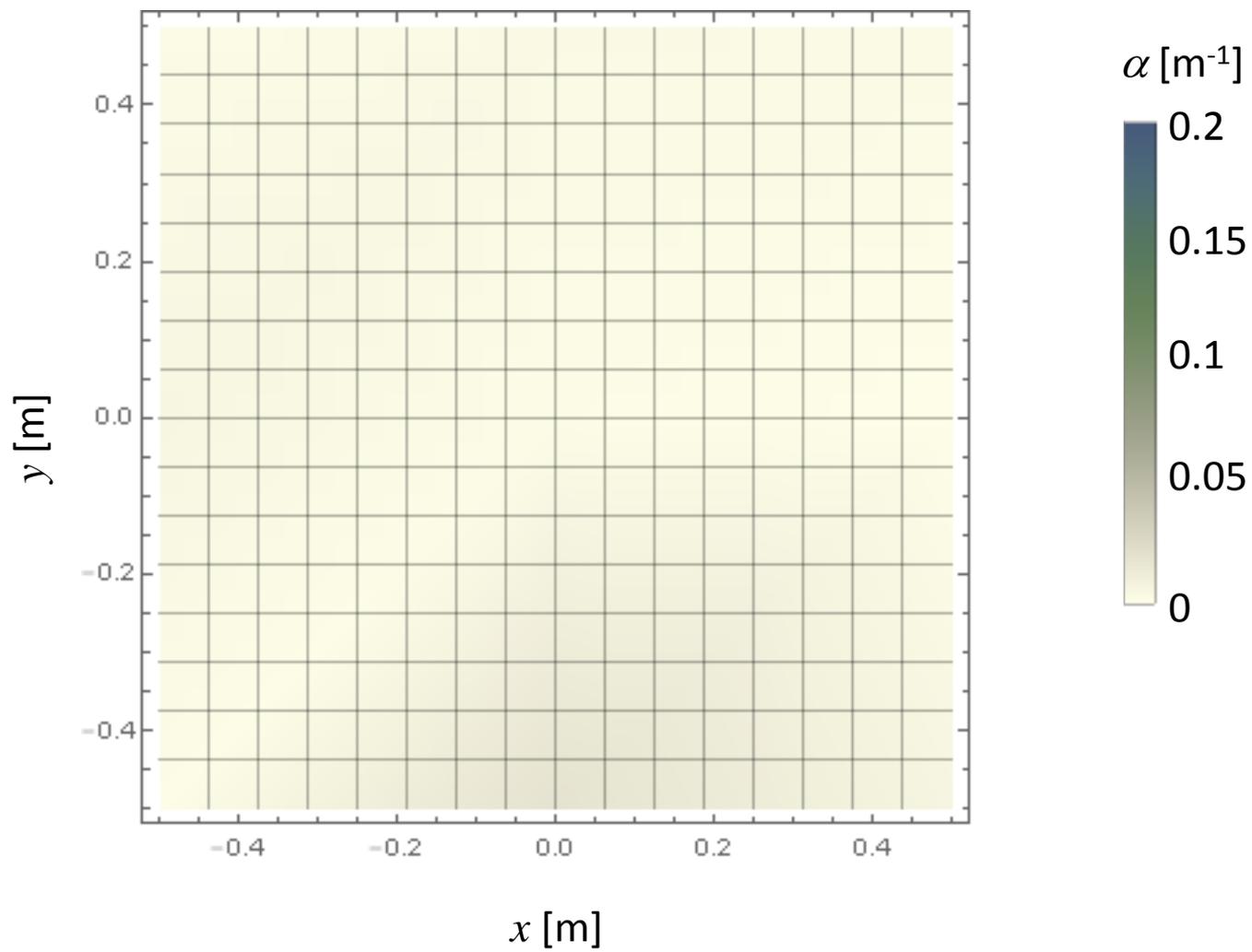
D = Particle Diameter [m]

$N(D)$ = Particle Size Distribution [$\text{m}^{-3} \text{m}^{-1}$]

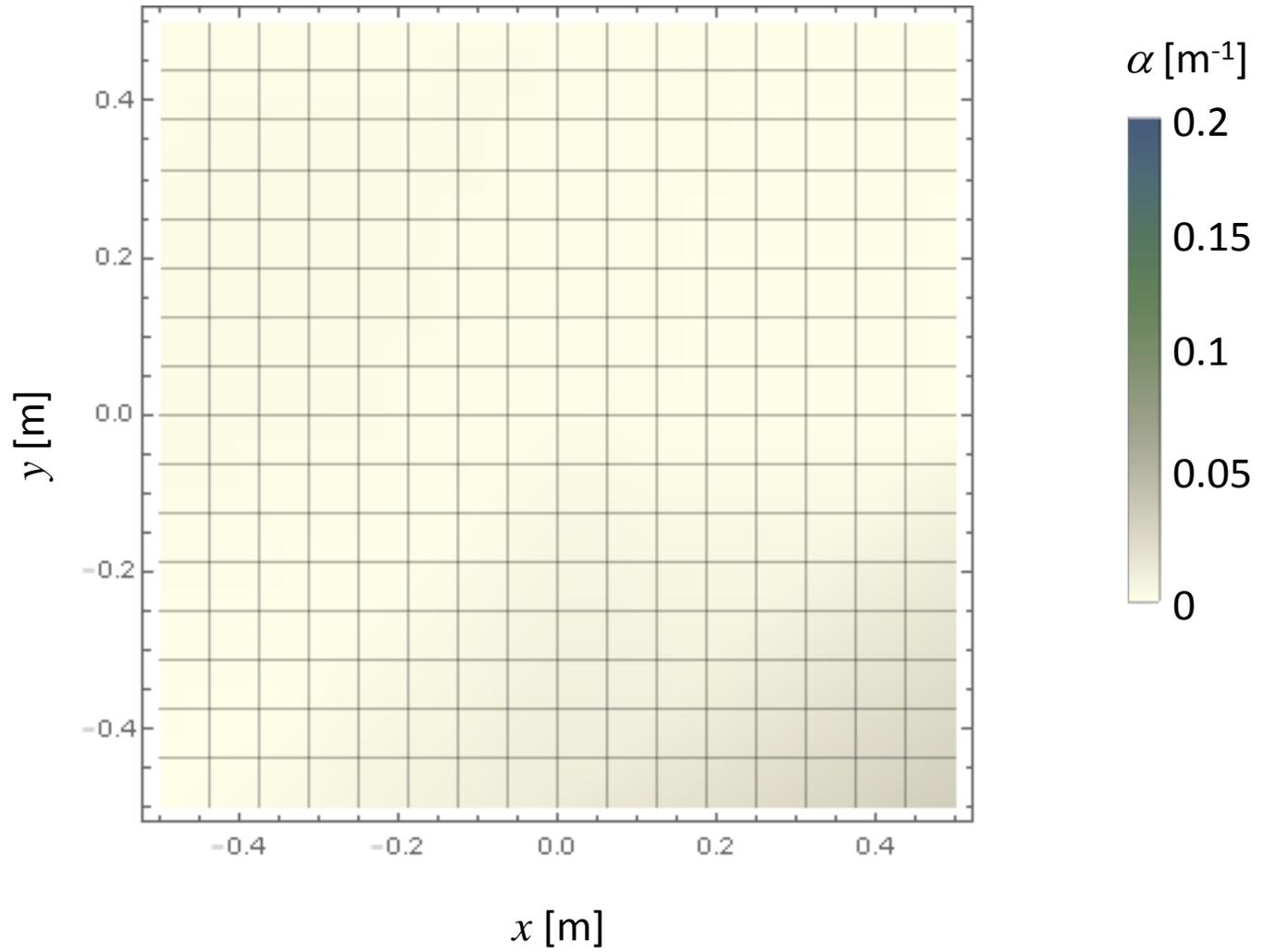
$t = 10$ [s]



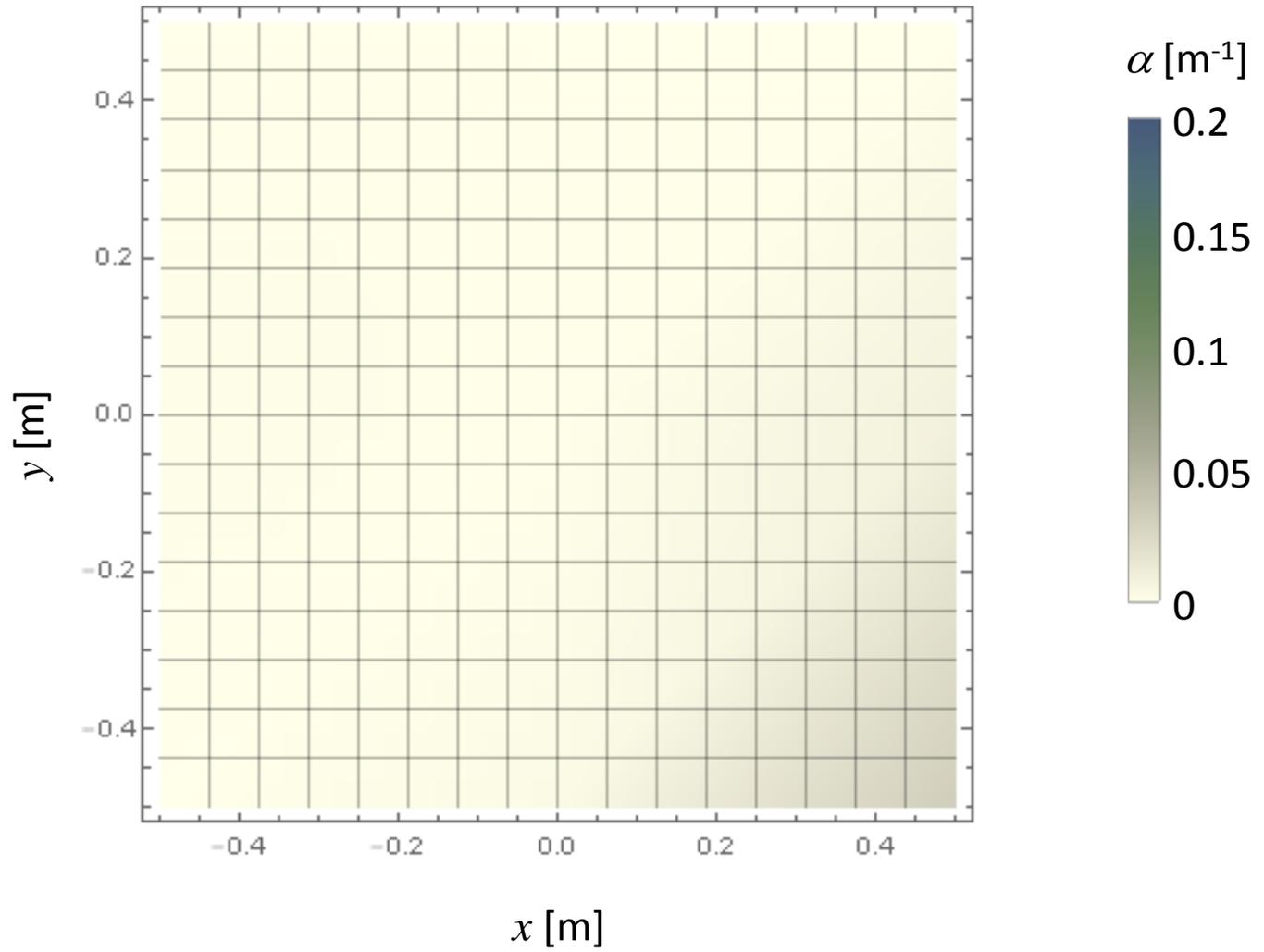
$t = 12$ [s]



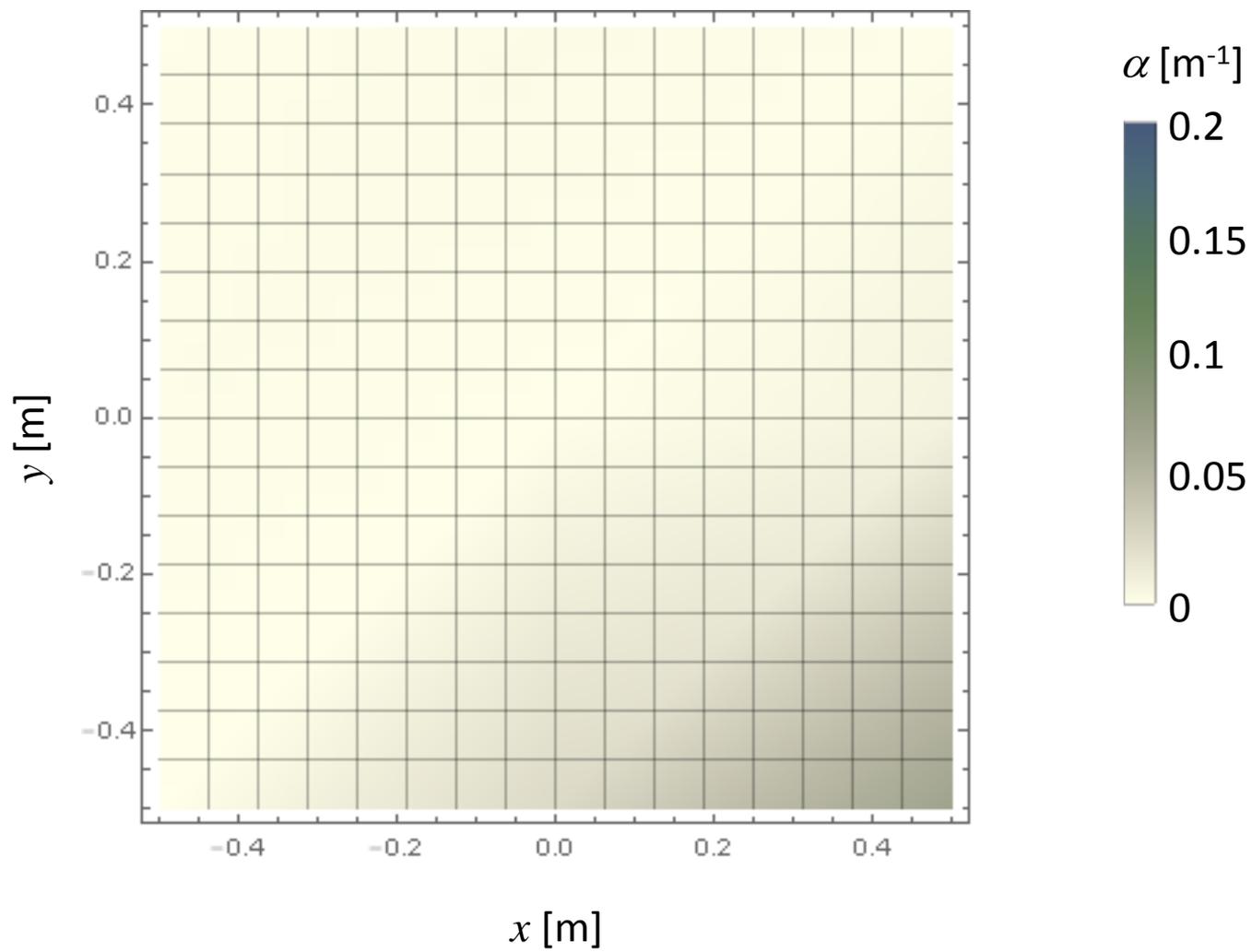
$t = 14$ [s]



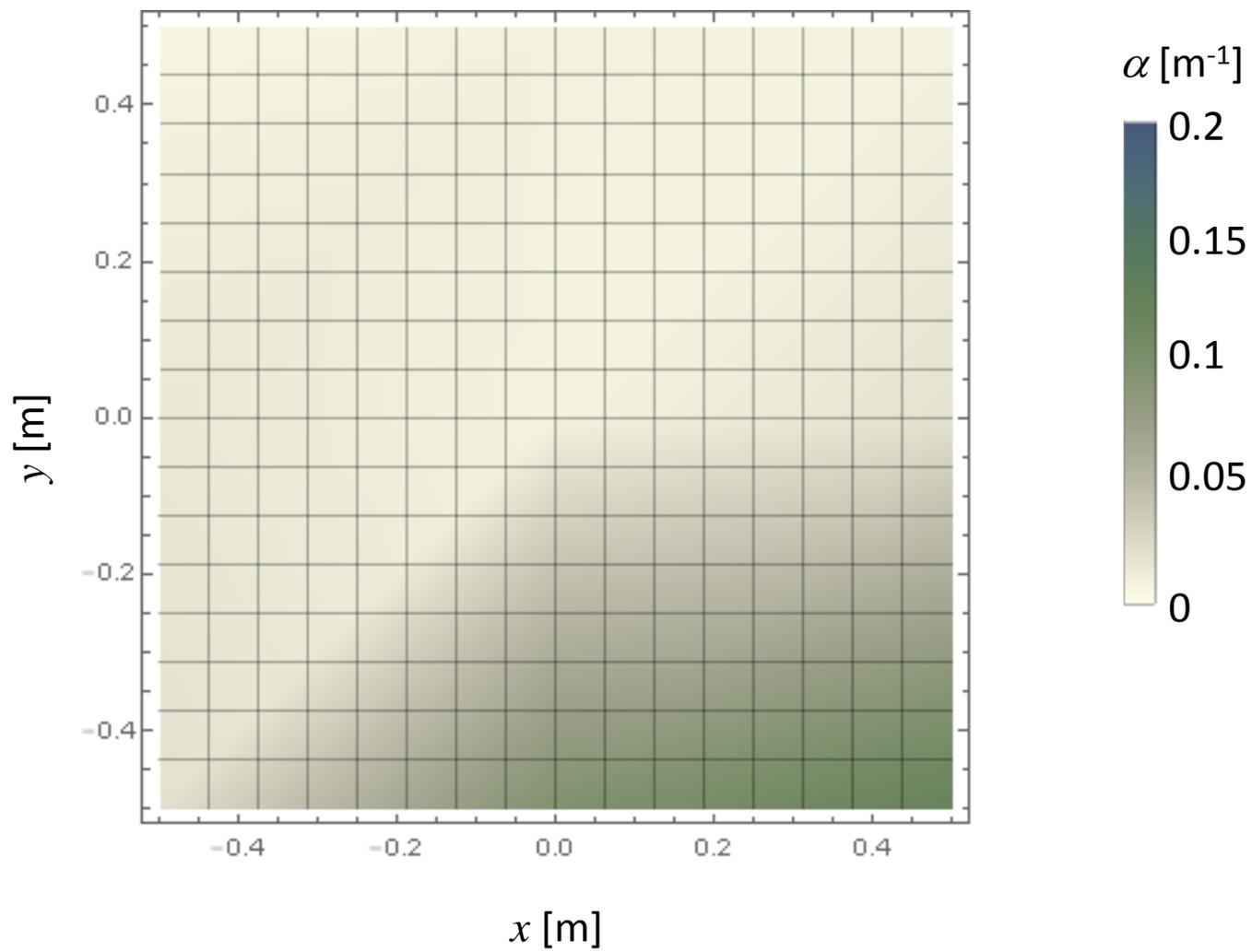
$t = 16$ [s]



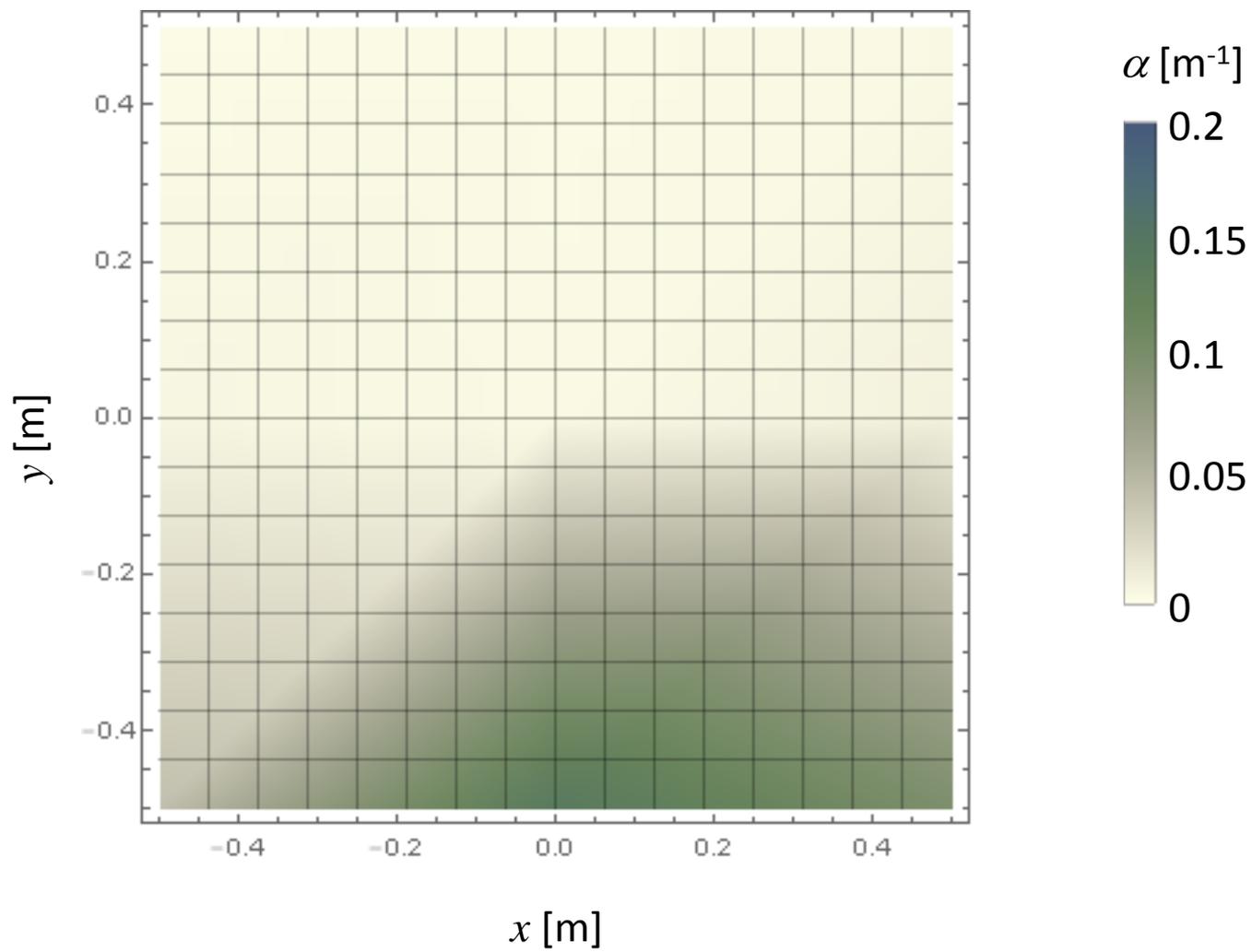
$t = 18$ [s]



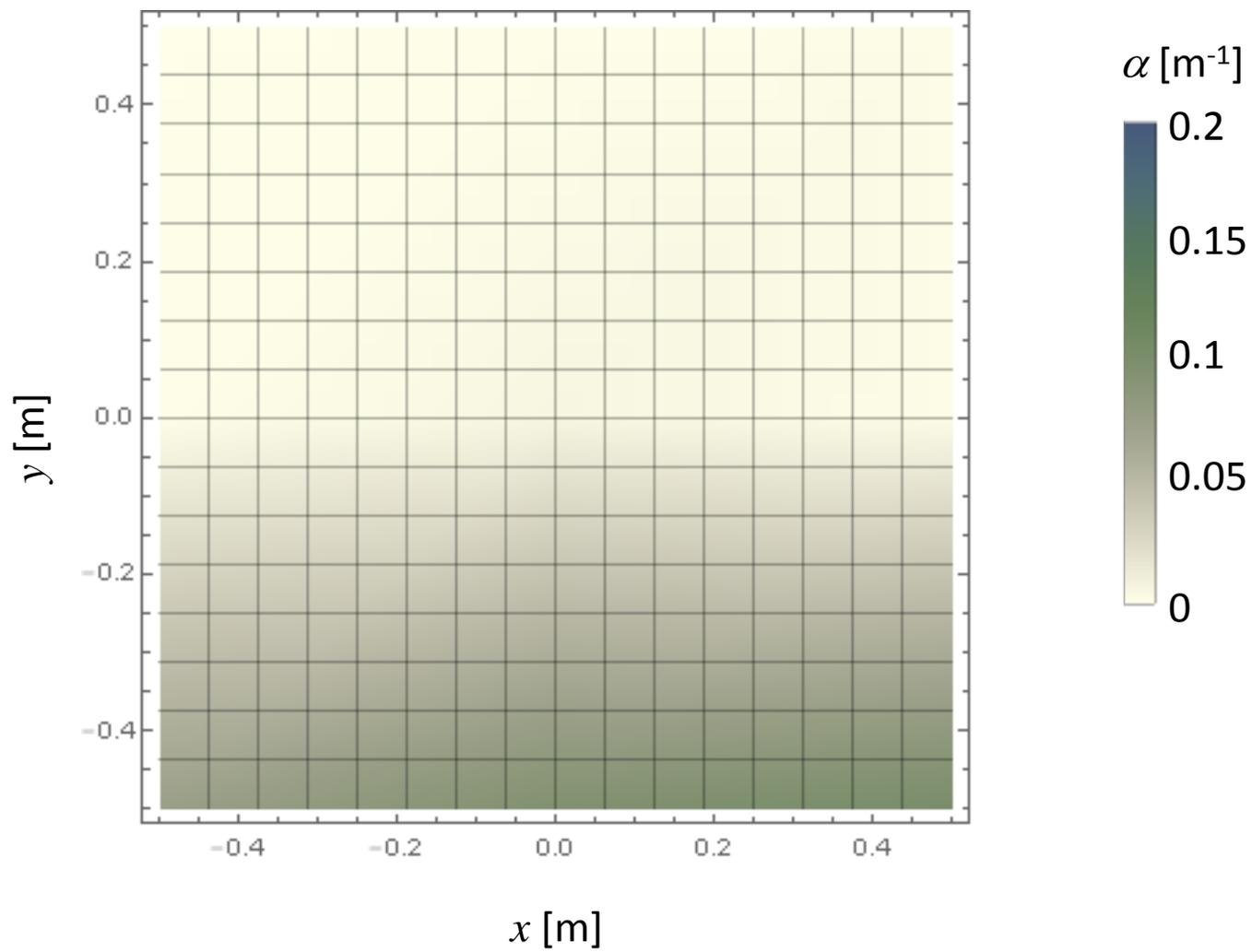
$t = 20$ [s]



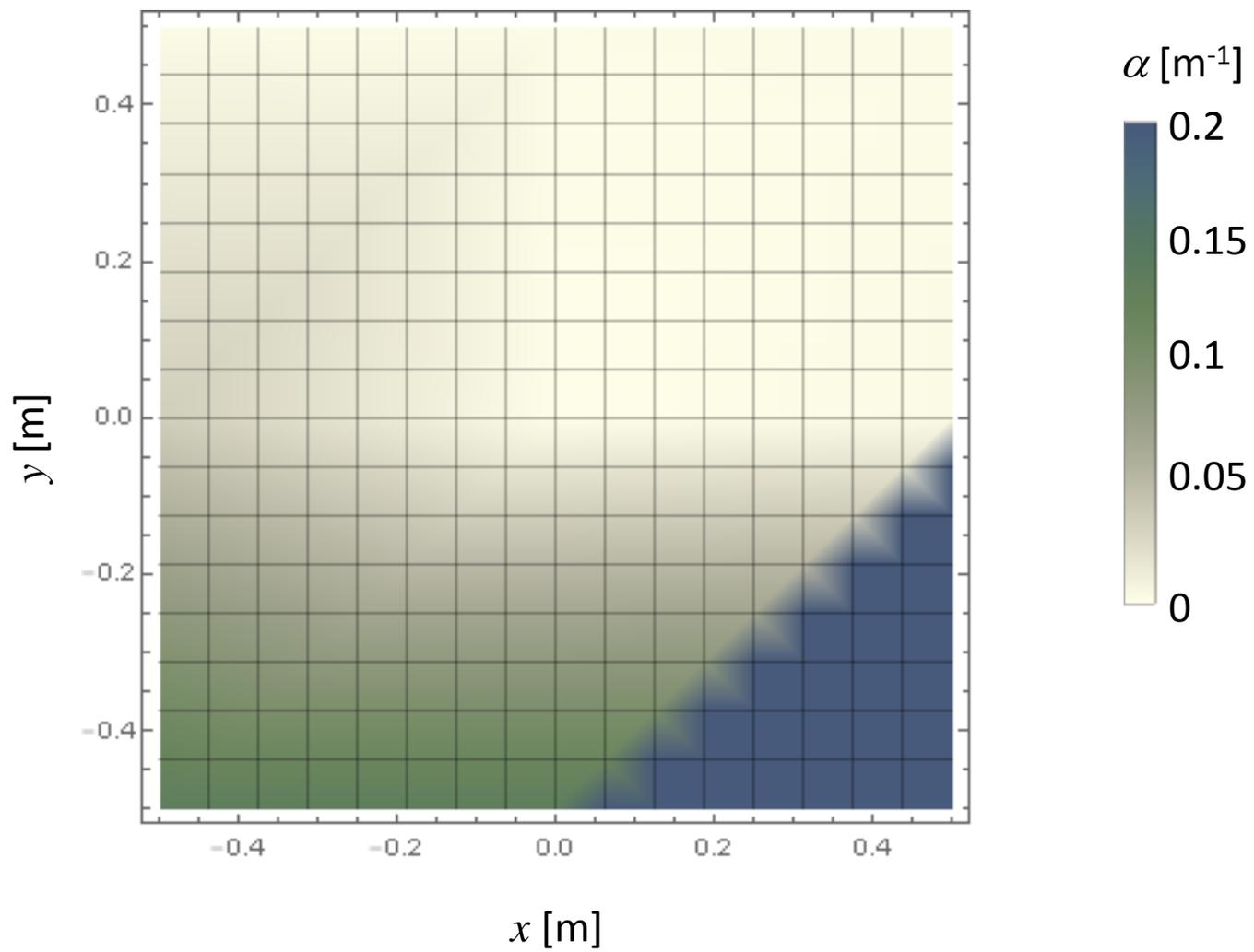
$t = 22$ [s]



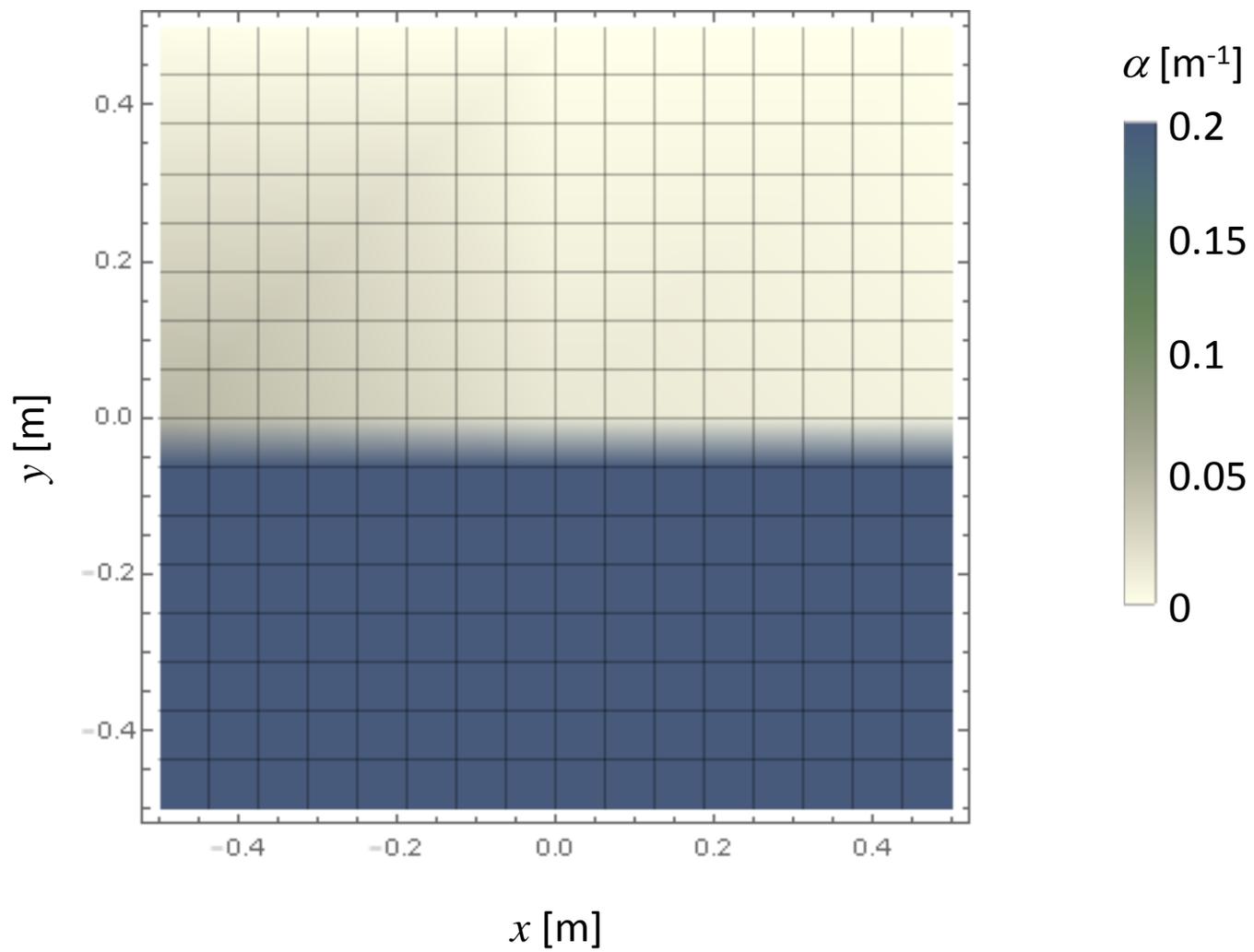
$t = 24$ [s]



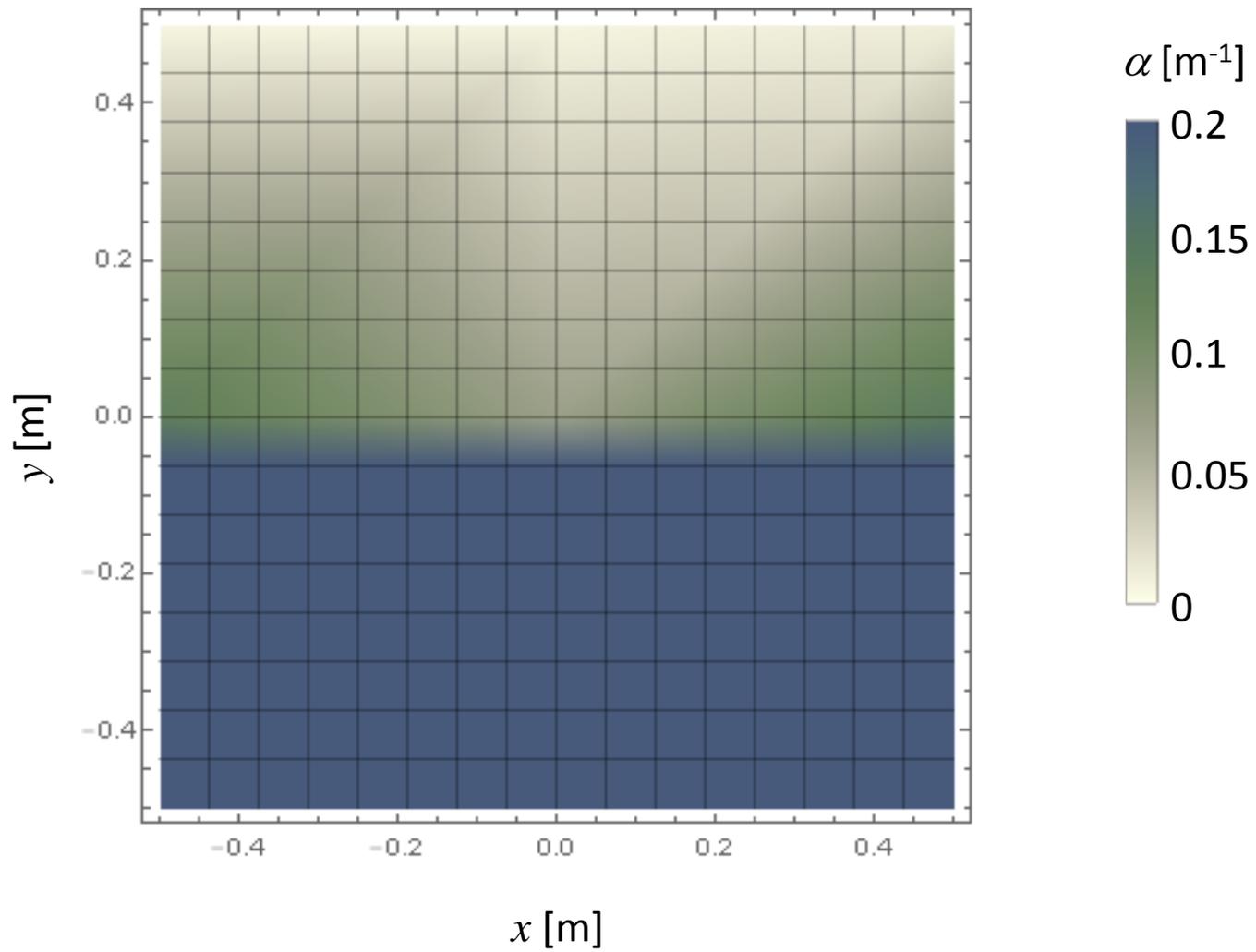
$t = 26$ [s]



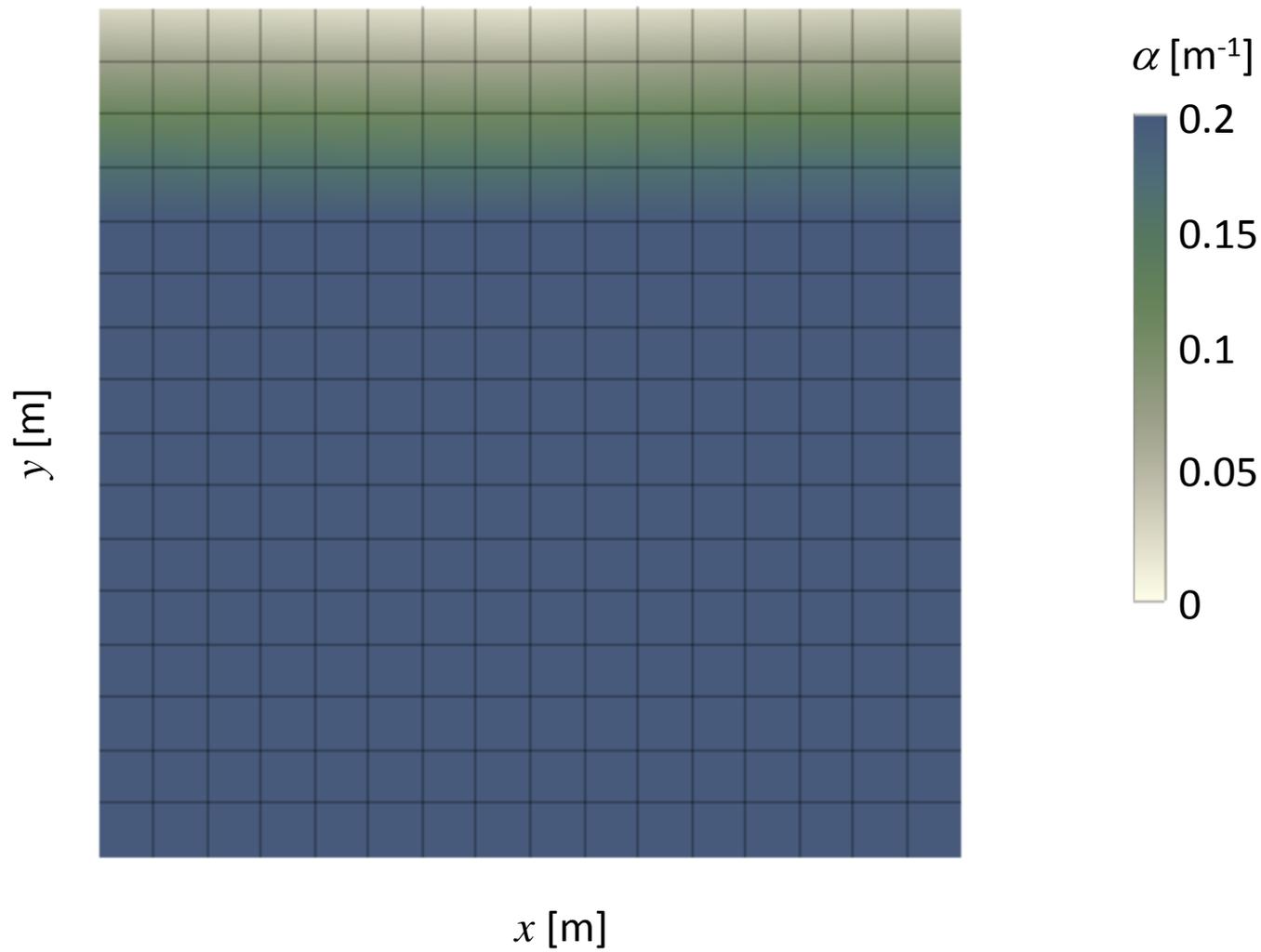
$t = 28$ [s]



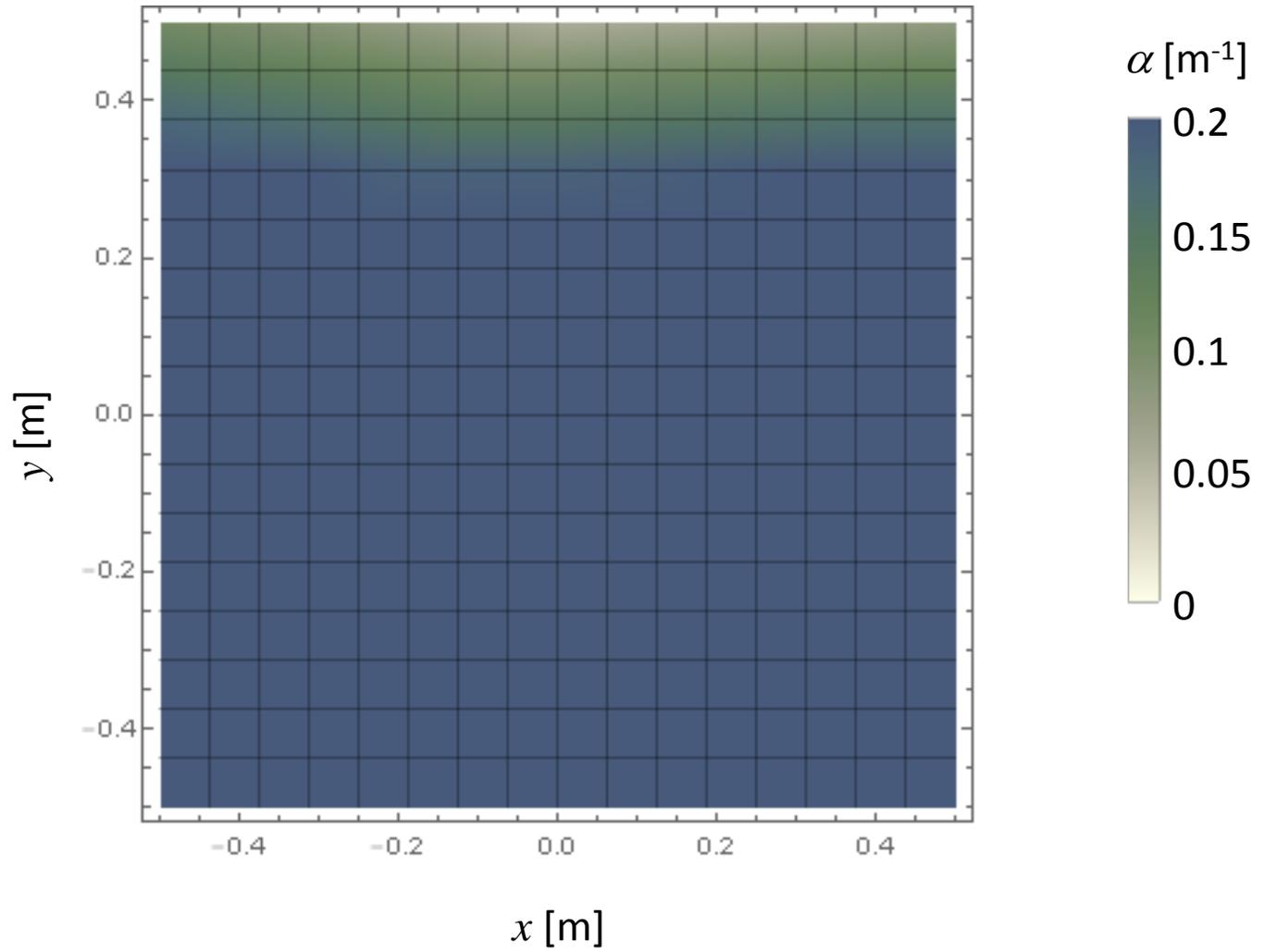
$t = 30$ [s]



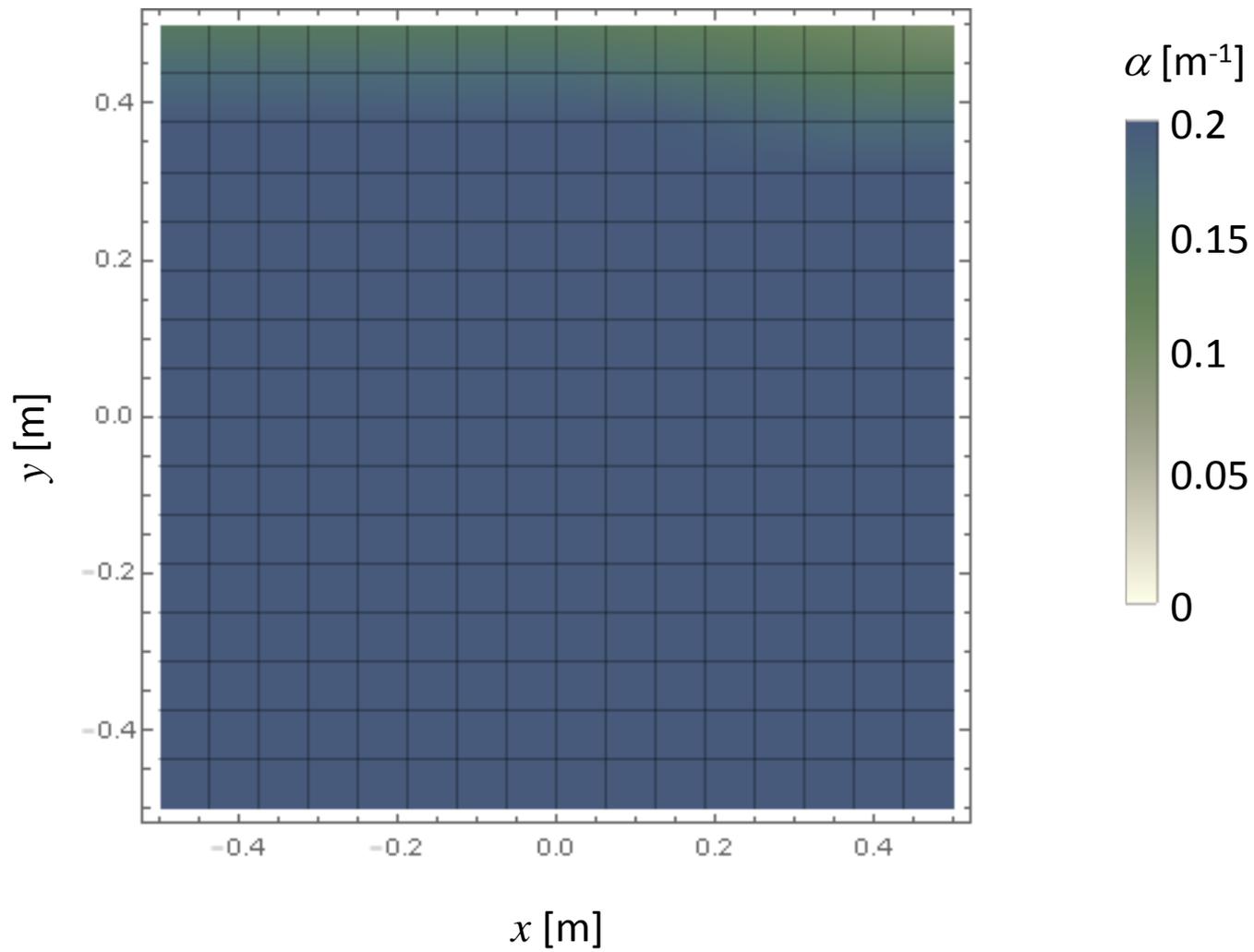
$t = 32$ [s]



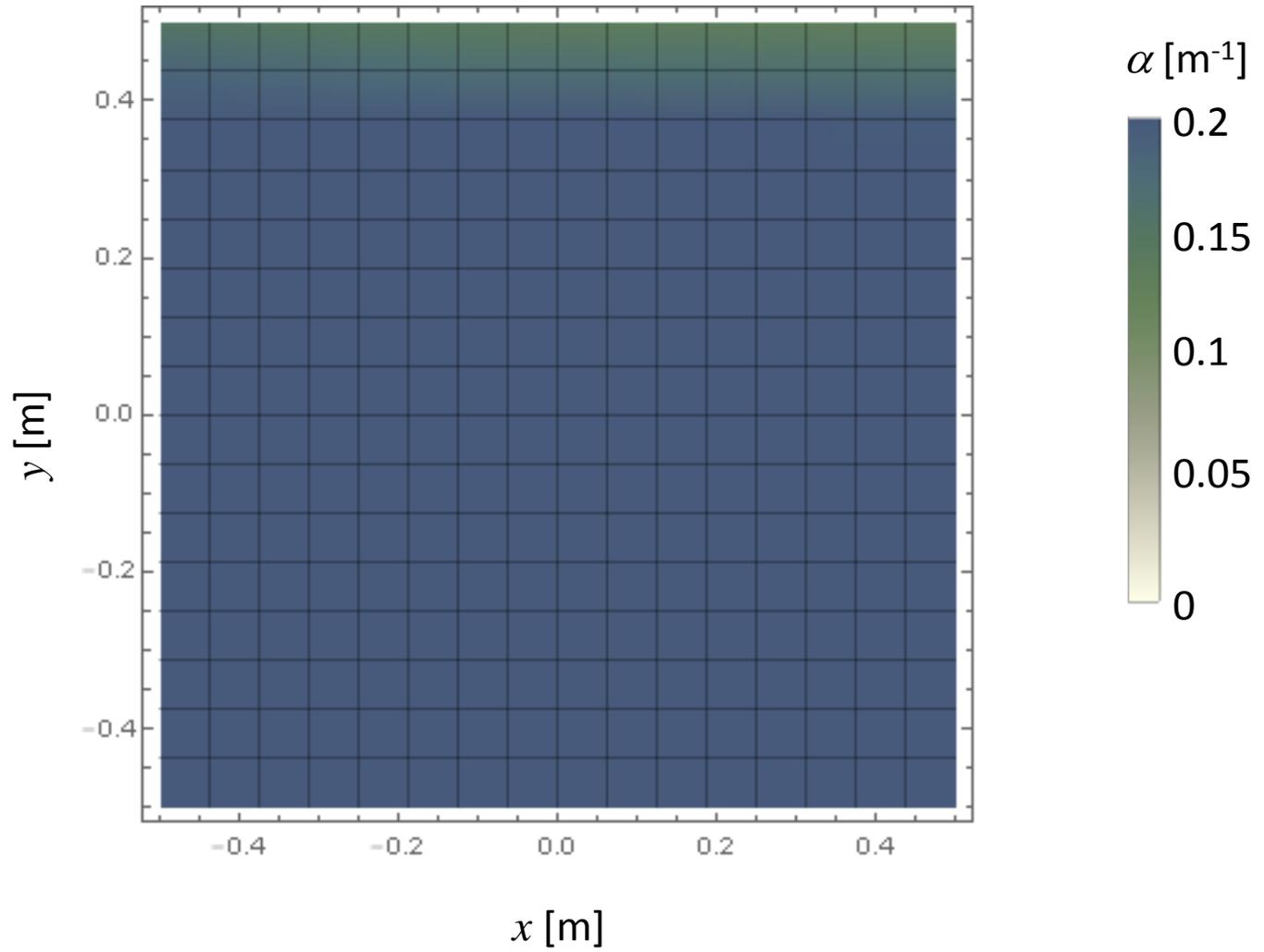
$t = 44$ [s]

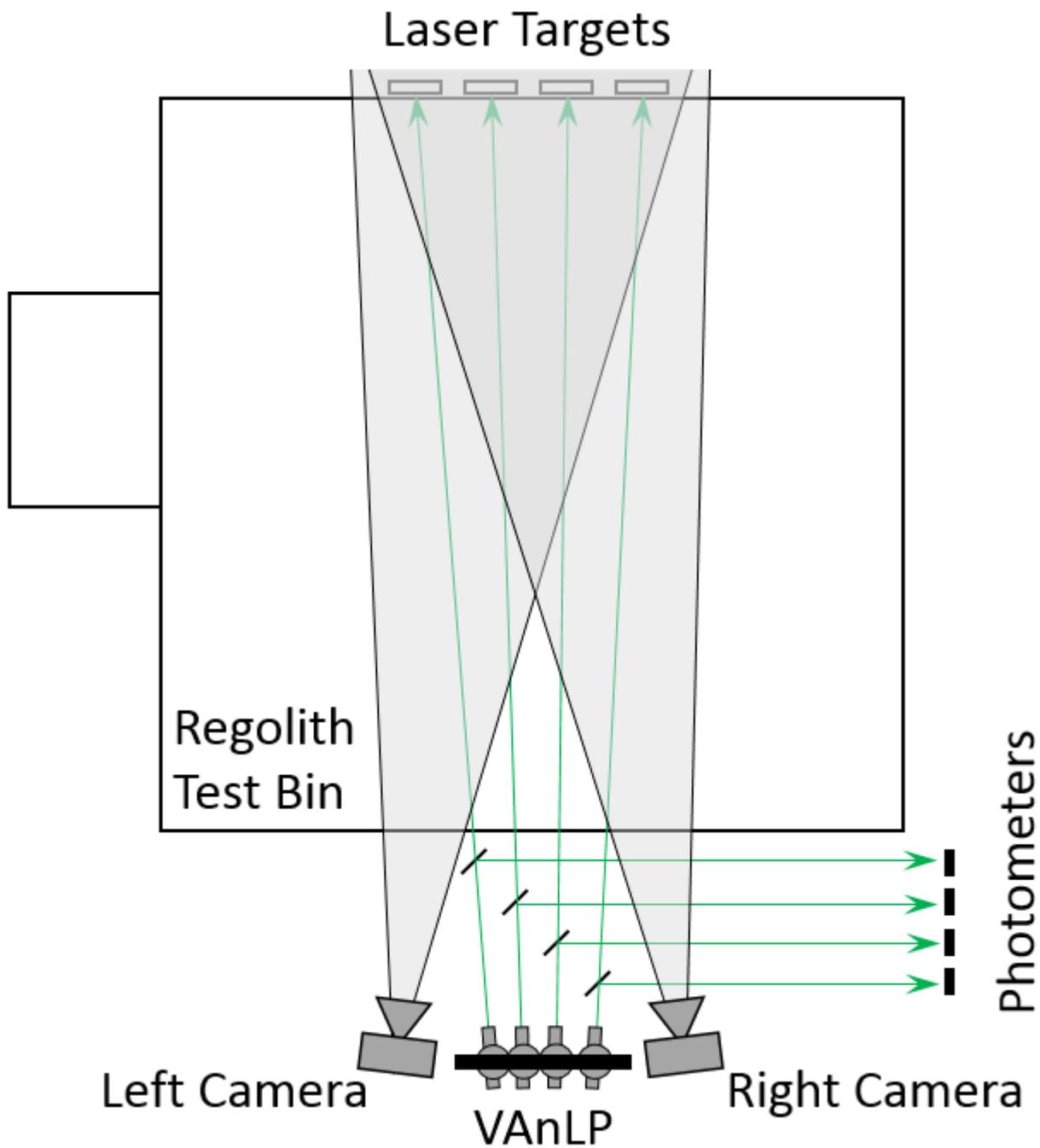


$t = 45$ [s]



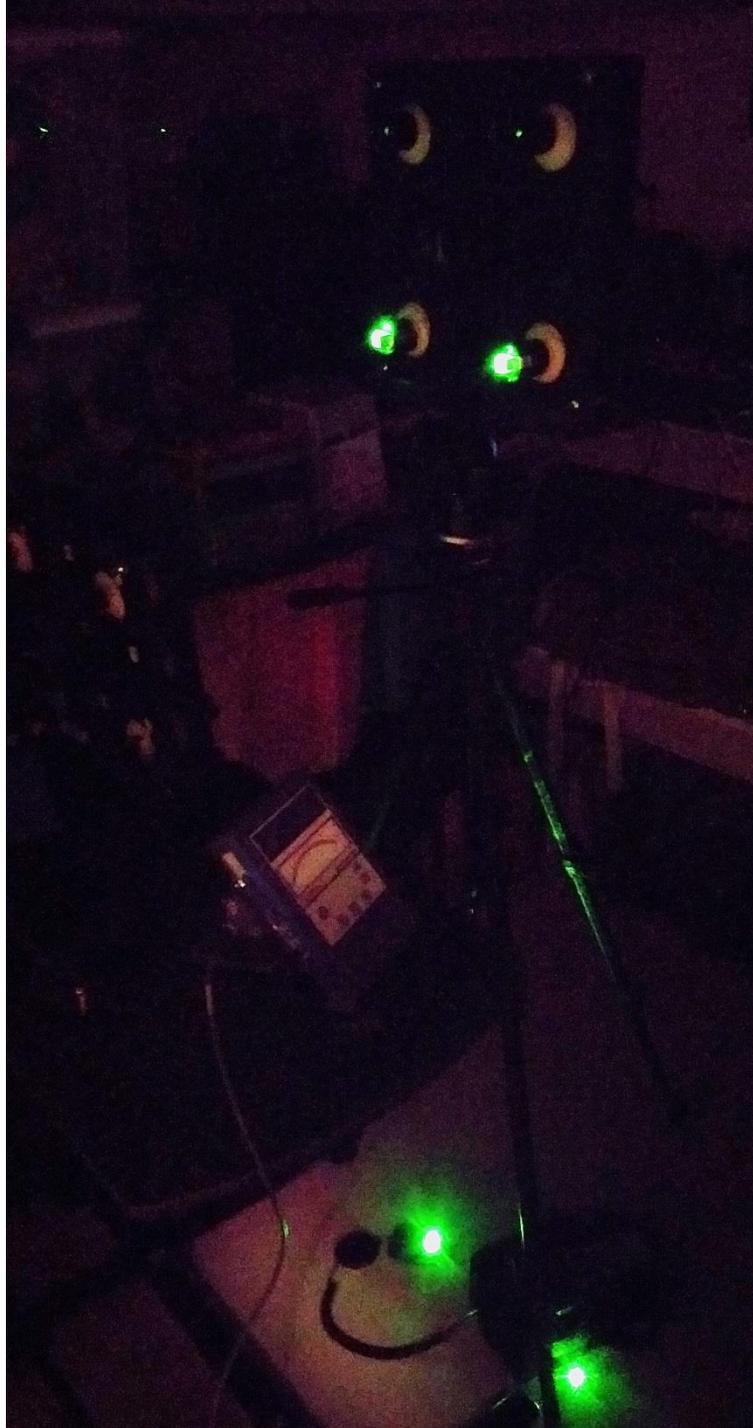
$t = 46$ [s]



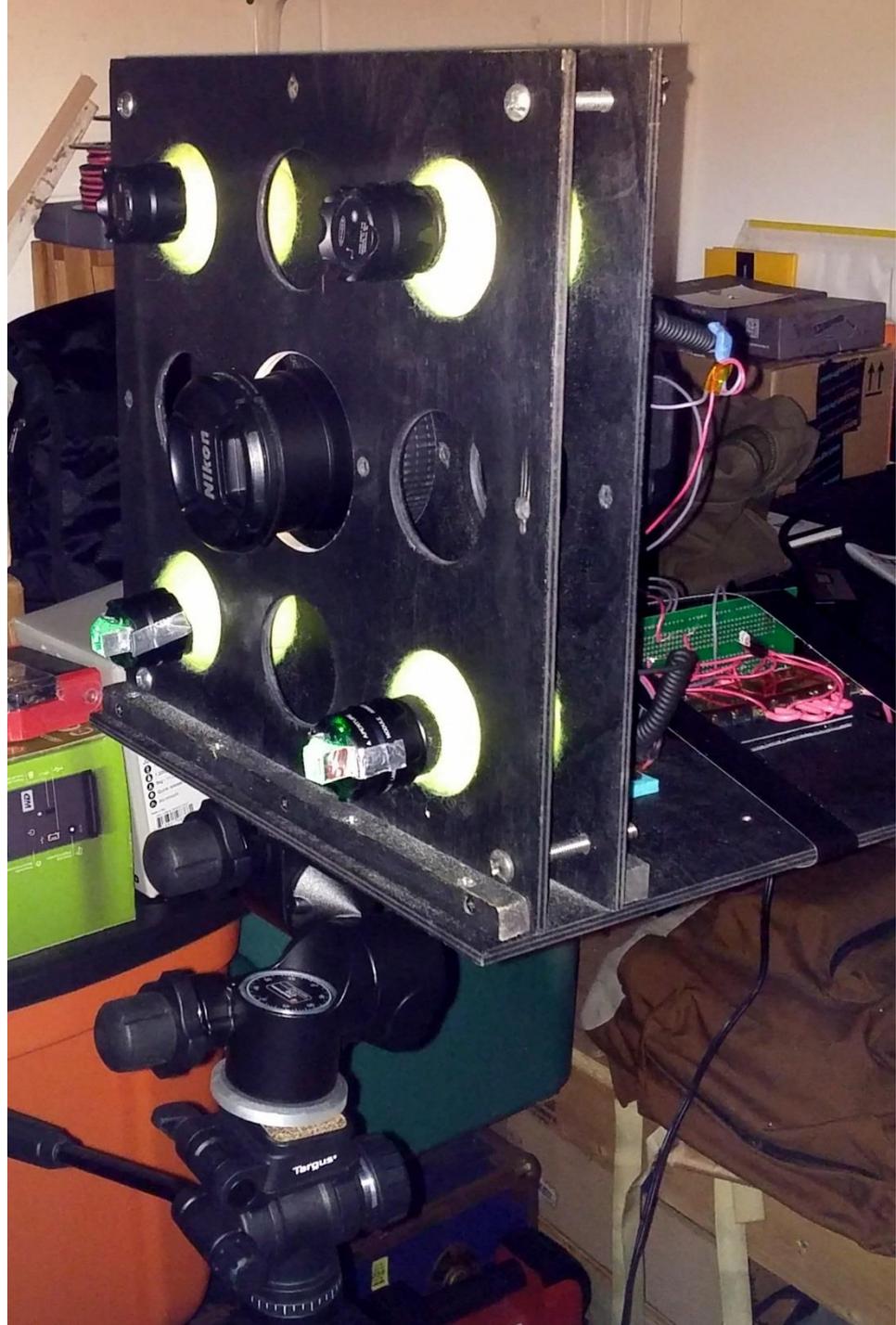


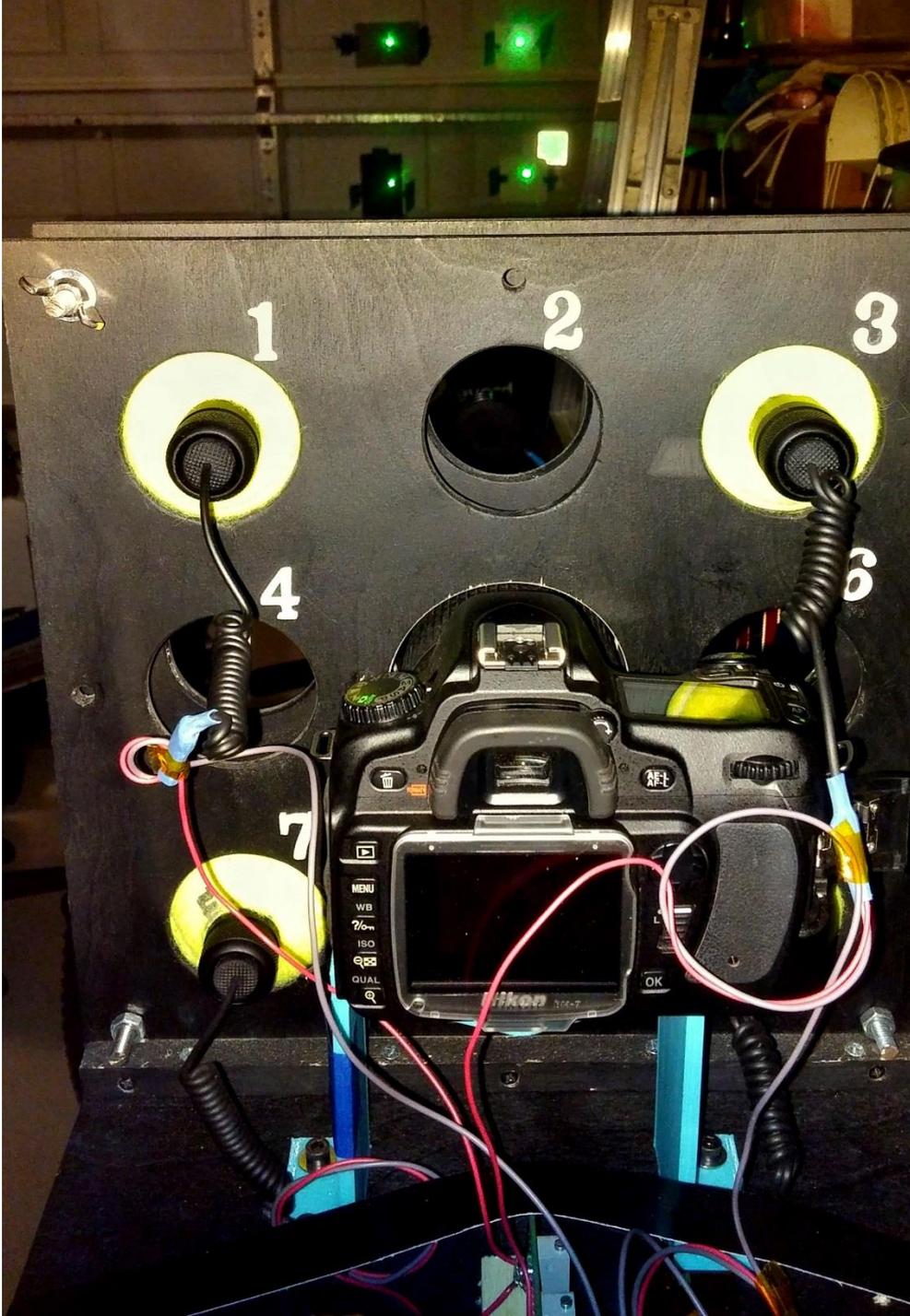
Variable angle eight laser pointer plus camera (VA8LP+C).



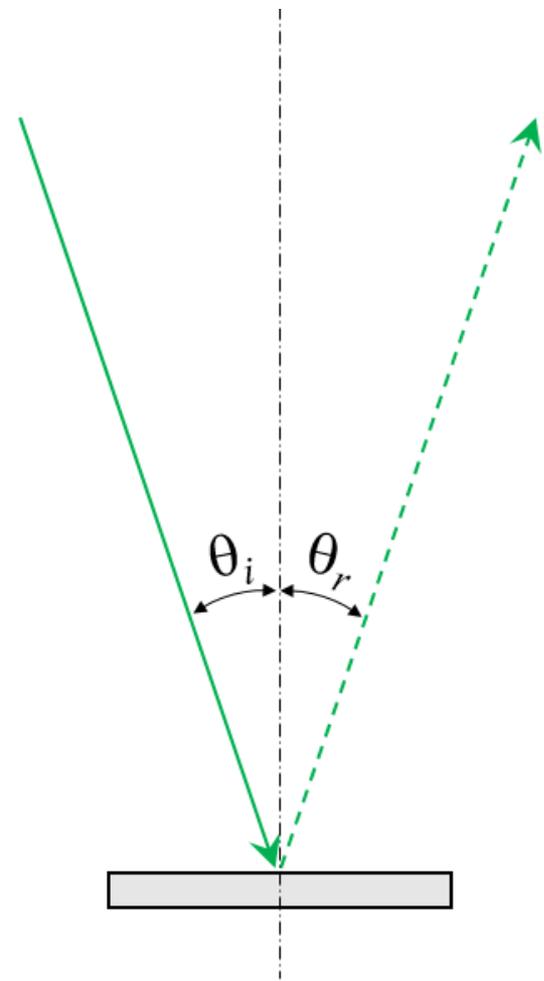






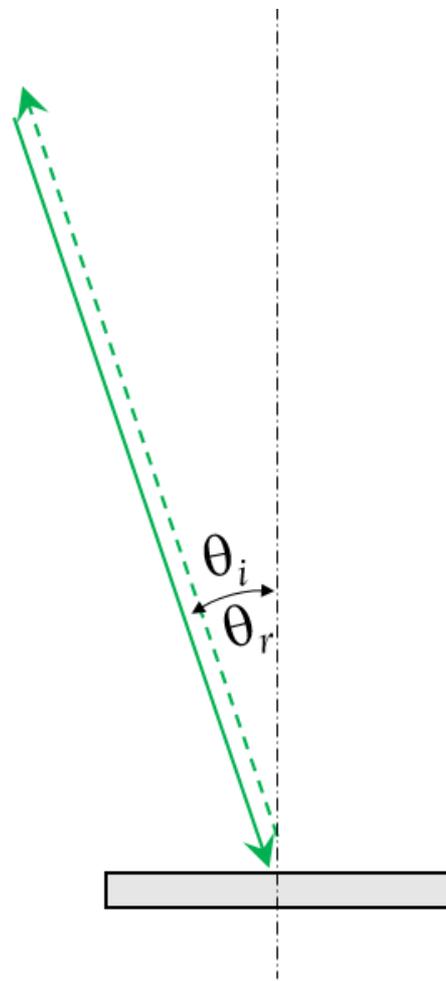






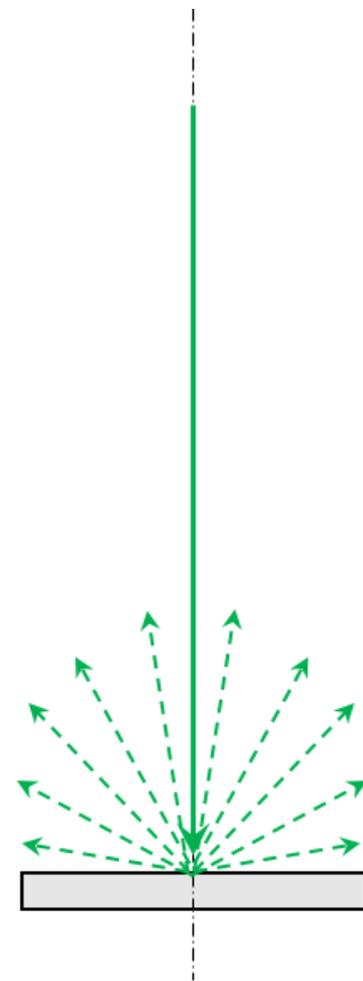
Specular Reflection

$$\theta_r = -\theta_i$$



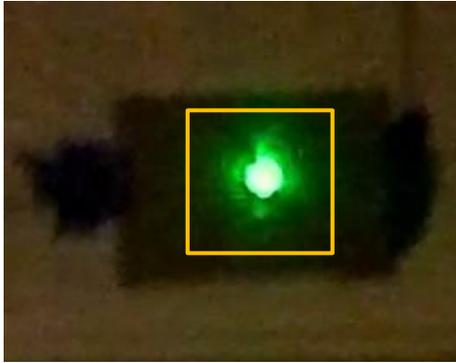
Retro Reflection

$$\theta_r = \theta_i$$

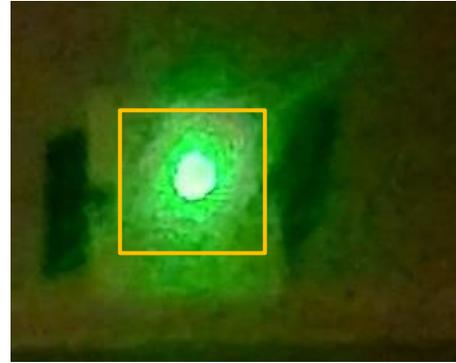


Diffuse Reflection

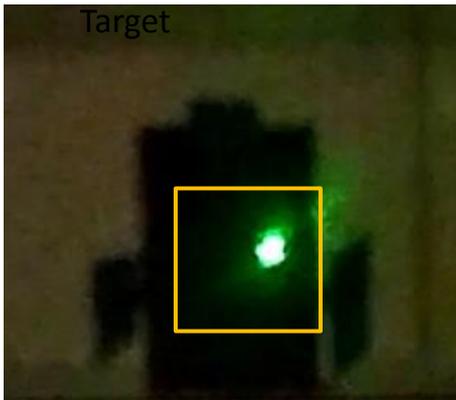
$$\theta_r = 0 \text{ to } \pi$$



Diffuse (Gray) Target

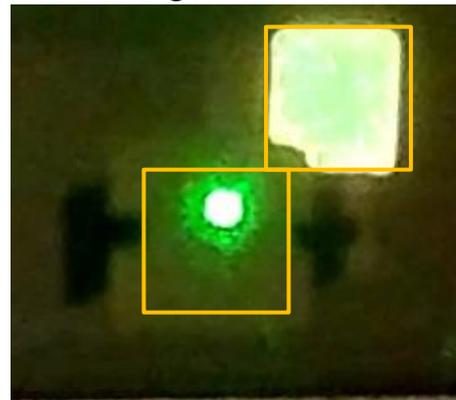


Diffuse (White) Target

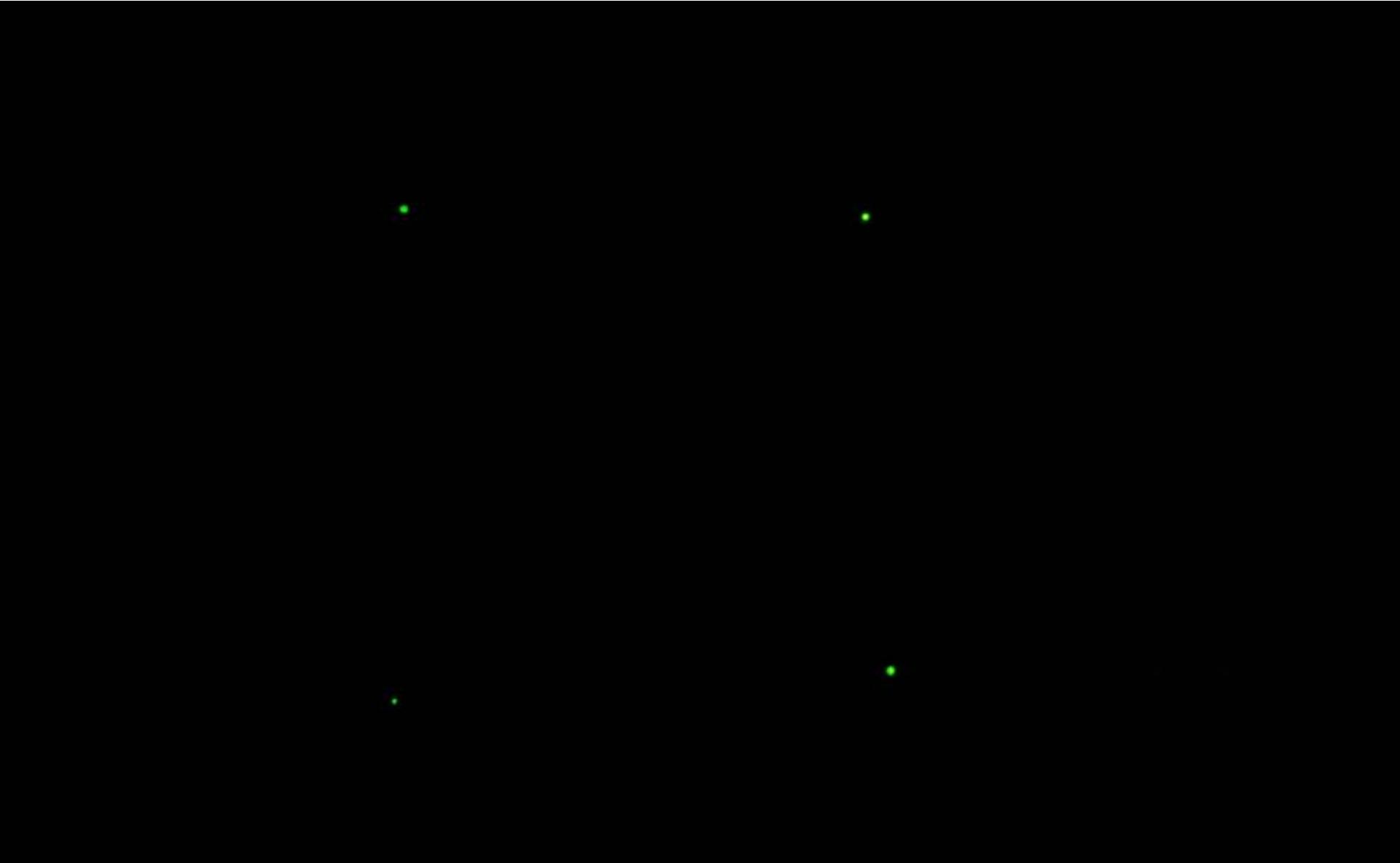


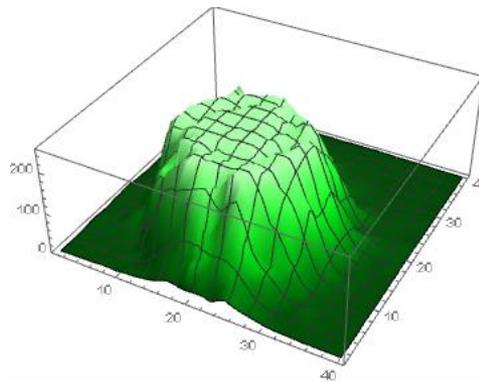
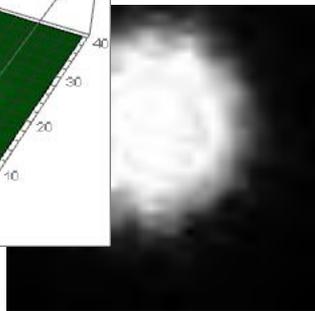
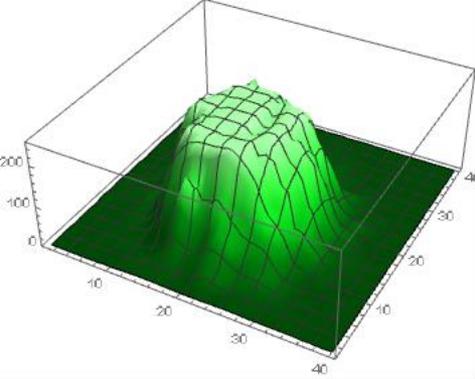
Diffuse (Black)
Target

Diffuse (White)
Target Retro Target



Camera image, DSC_0037.jpg corresponding to above Figures c and d:
ISO = 100, shutter speed = 1/320 s, and aperture = $f/11$.





DSC_0037.JPG

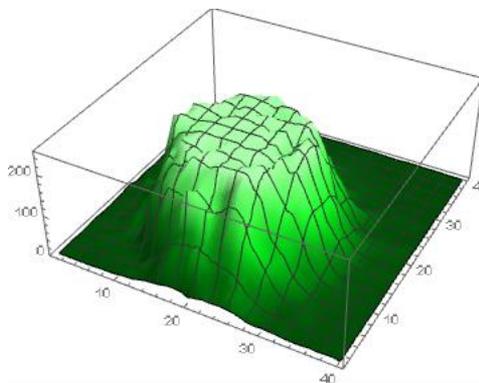
No Filter

Max I = 255.

Avg I = 218.847

Avg D = 23.4529

Sat Cnt = 87



DSC_0038.JPG

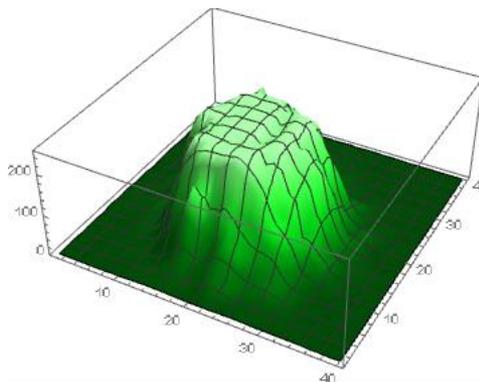
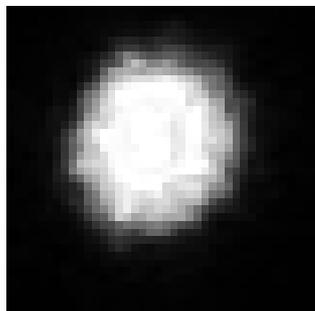
No Filter

Max I = 255.

Avg I = 220.741

Avg D = 23.1524

Sat Cnt = 76



DSC_0039.JPG

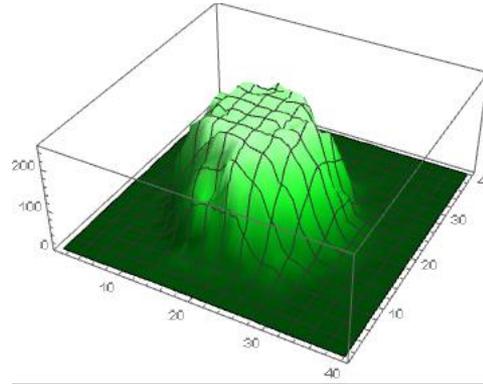
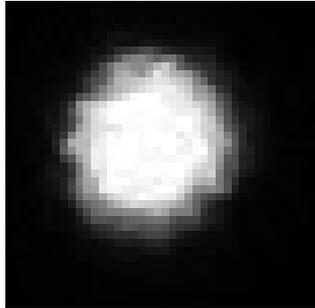
YG Filter

Max I = 255.

Avg I = 217.703

Avg D = 20.7143

Sat Cnt = 63



DSC_0040.JPG

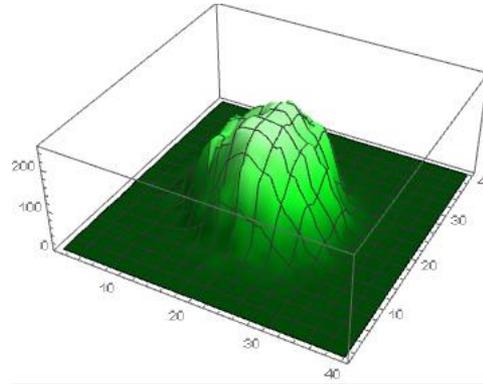
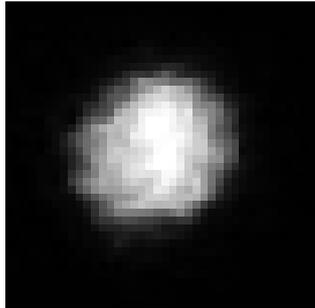
YG Filter

Max I = 255.

Avg I = 216.244

Avg D = 20.0267

Sat Cnt = 42



DSC_0041.JPG

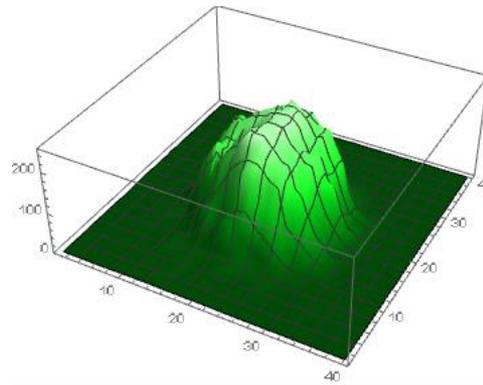
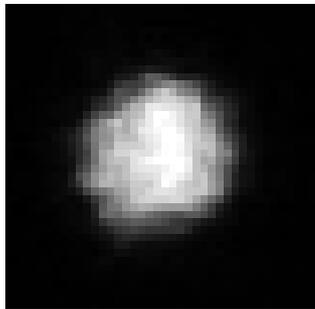
2×YG Filter

Max I = 255.

Avg I = 197.824

Avg D = 16.7746

Sat Cnt = 16



DSC_0042.JPG

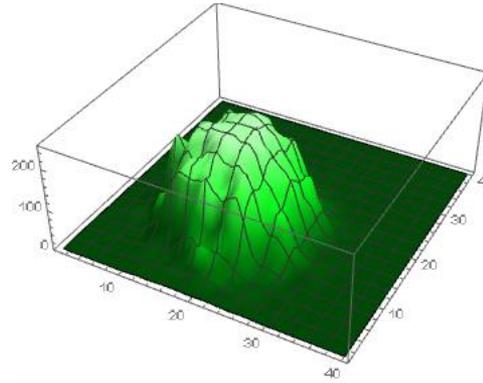
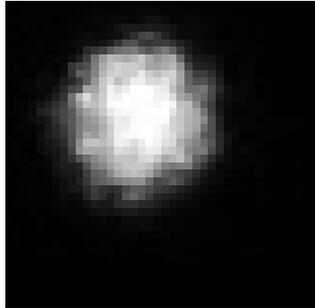
2×YG Filter

Max I = 255.

Avg I = 198.391

Avg D = 16.5453

Sat Cnt = 14



DSC_0043.JPG

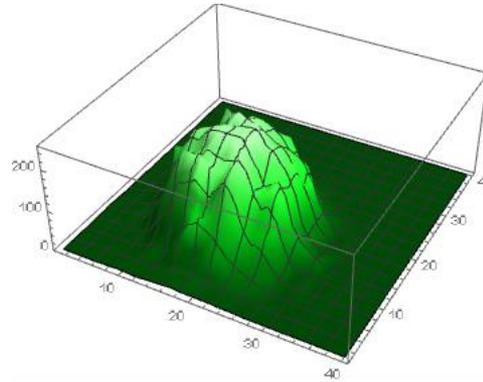
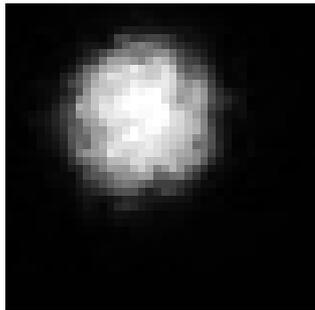
SG1 Filter

Max I = 255.

Avg I = 200.598

Avg D = 16.5068

Sat Cnt = 12



DSC_0044.JPG

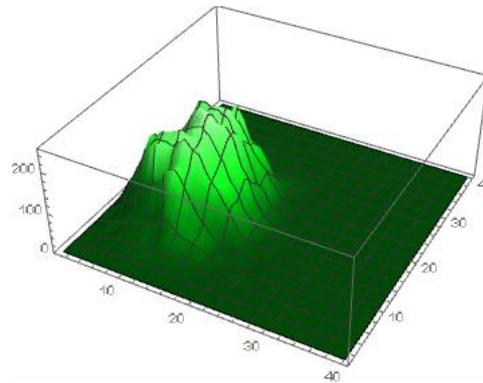
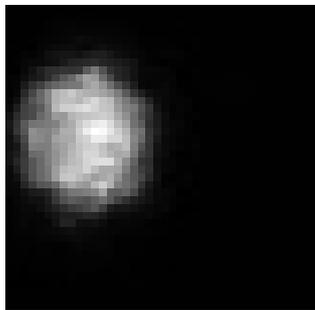
SG1 Filter

Max I = 255.

Avg I = 198.065

Avg D = 16.5837

Sat Cnt = 12



DSC_0045.JPG

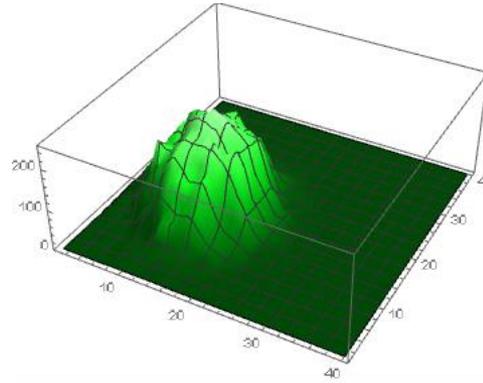
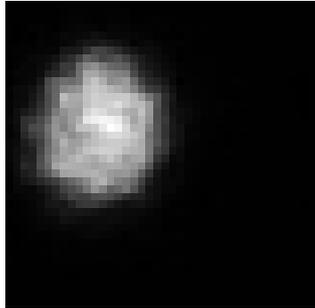
SG2 Filter

Max I = 255.

Avg I = 180.729

Avg D = 13.3512

Sat Cnt = 1



DSC_0046.JPG

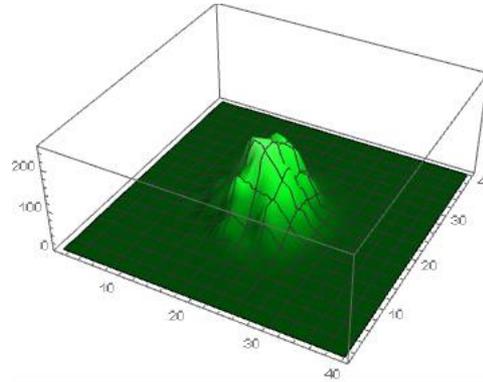
SG2 Filter

Max I = 254.

Avg I = 176.553

Avg D = 14.3175

Sat Cnt = 0



DSC_0047.JPG

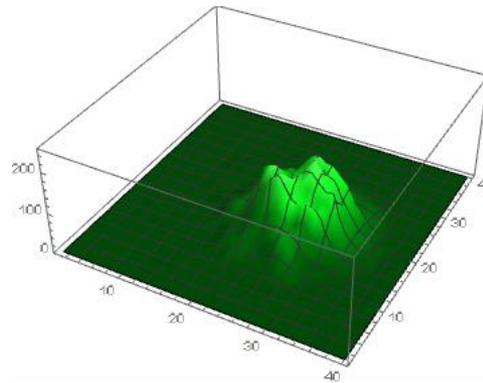
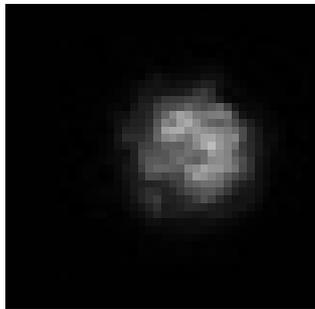
SG2+YG Filter

Max I = 197.

Avg I = 136.62

Avg D = 9.50789

Sat Cnt = 0



DSC_0048.JPG

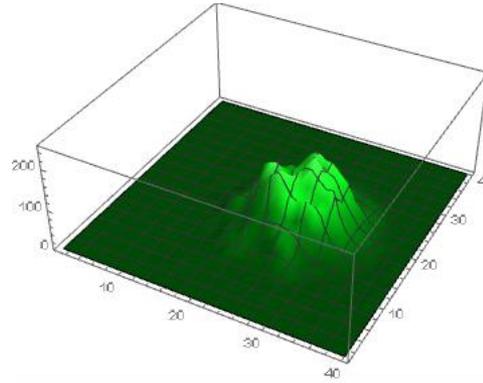
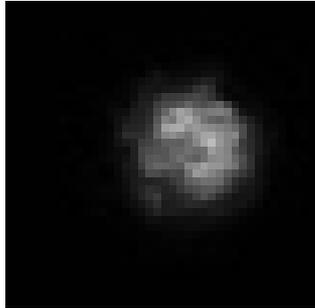
SG2+YG Filter

Max I = 188.

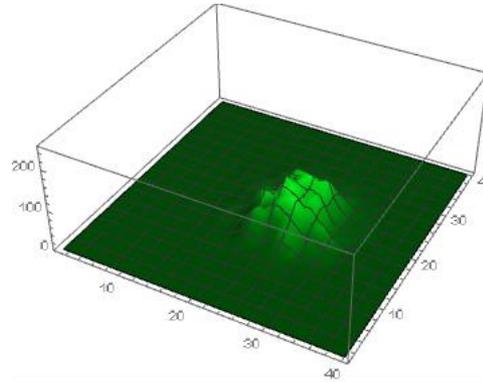
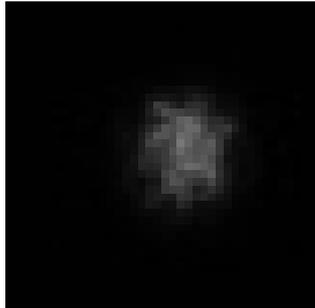
Avg I = 128.167

Avg D = 10.3418

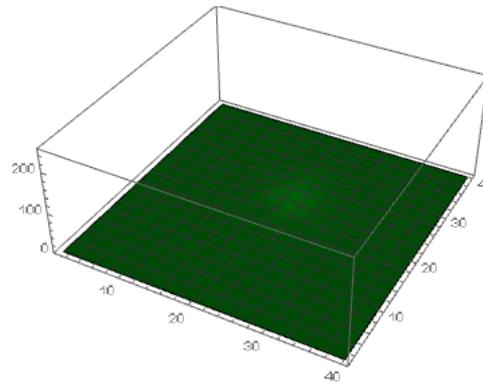
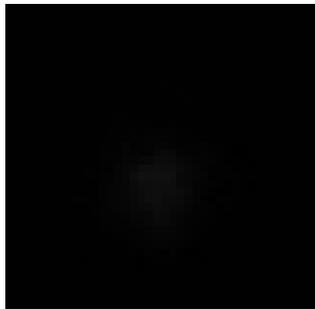
Sat Cnt = 0



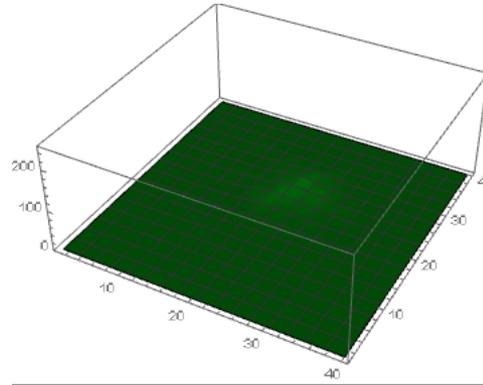
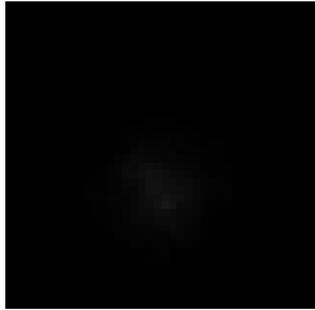
DSC_0049.JPG
SG2+2×YG Filter
Max I = 130.
Avg I = 86.6522
Avg D = 7.65304
Sat Cnt = 0



DSC_0050.JPG
SG2+2×YG Filter
Max I = 119.
Avg I = 82.1475
Avg D = 8.81292
Sat Cnt = 0



DSC_0051.JPG
SG1+SG2+2×YG
Filter
Max I = 16.
Avg I = 11.8864
Avg D = 7.48482
Sat Cnt = 0



DSC_0052.JPG

SG1+SG2+2×YG
Filter

Max I = 22.

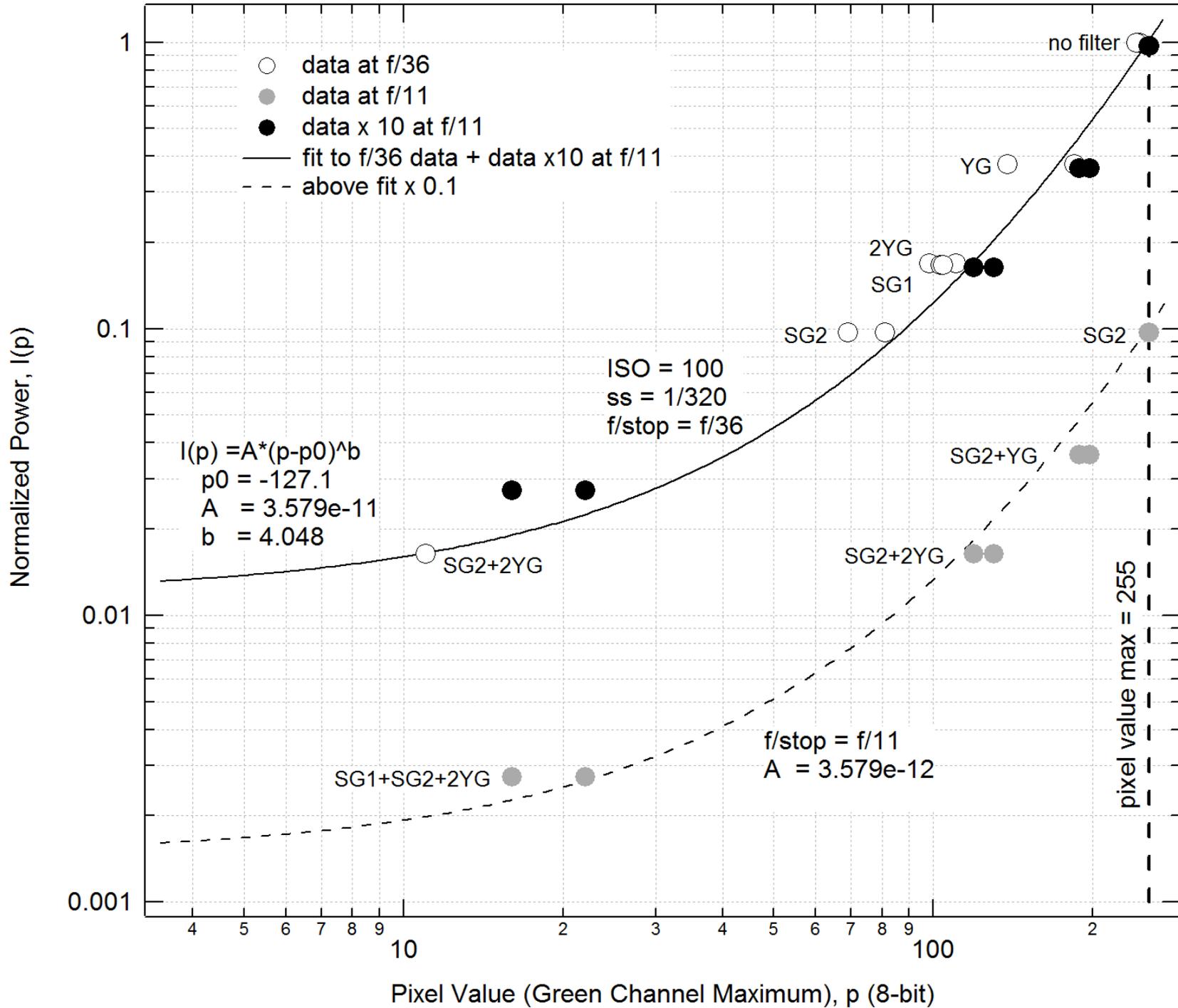
Avg I = 14.8148

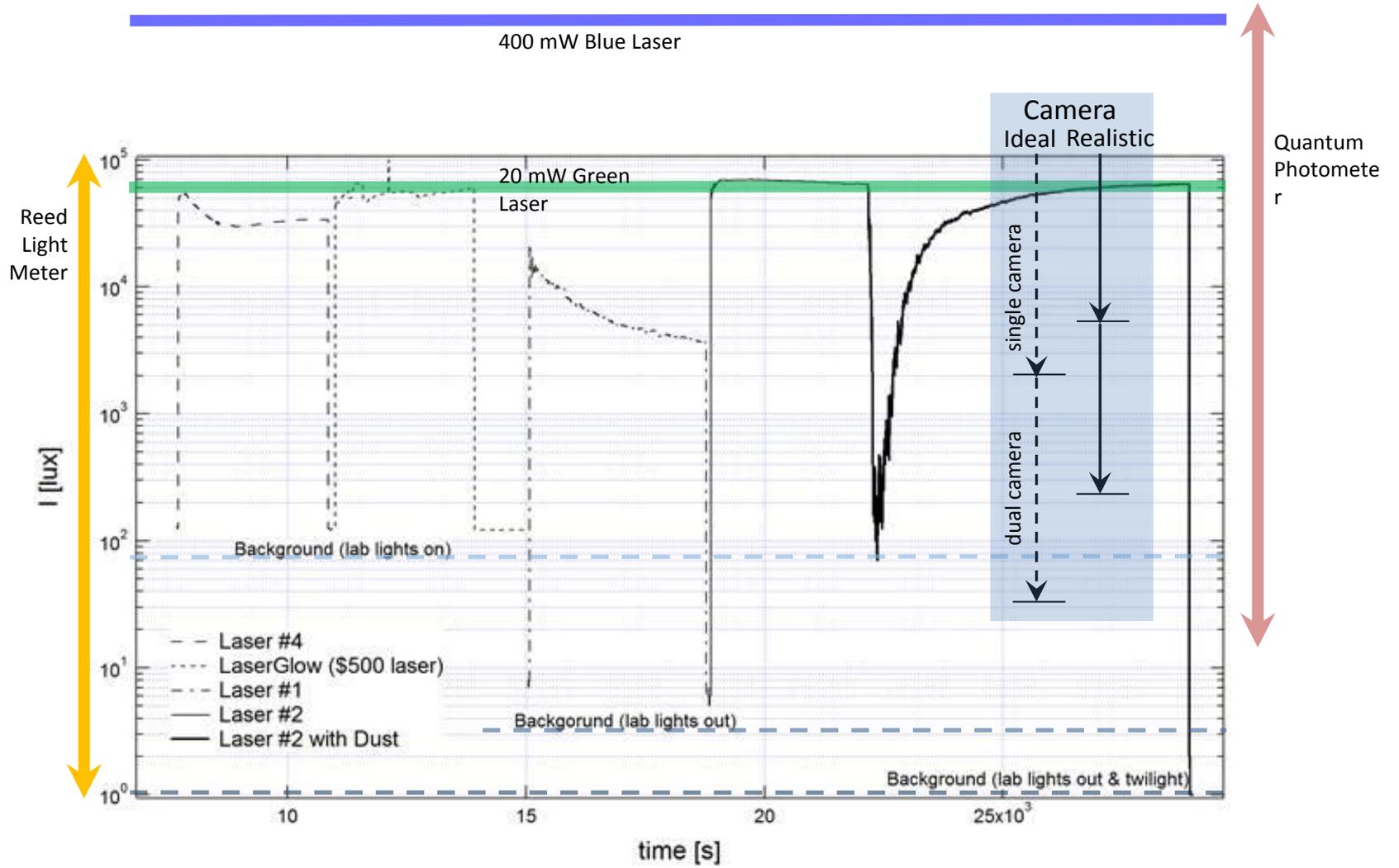
Avg D = 5.86323

Sat Cnt = 0

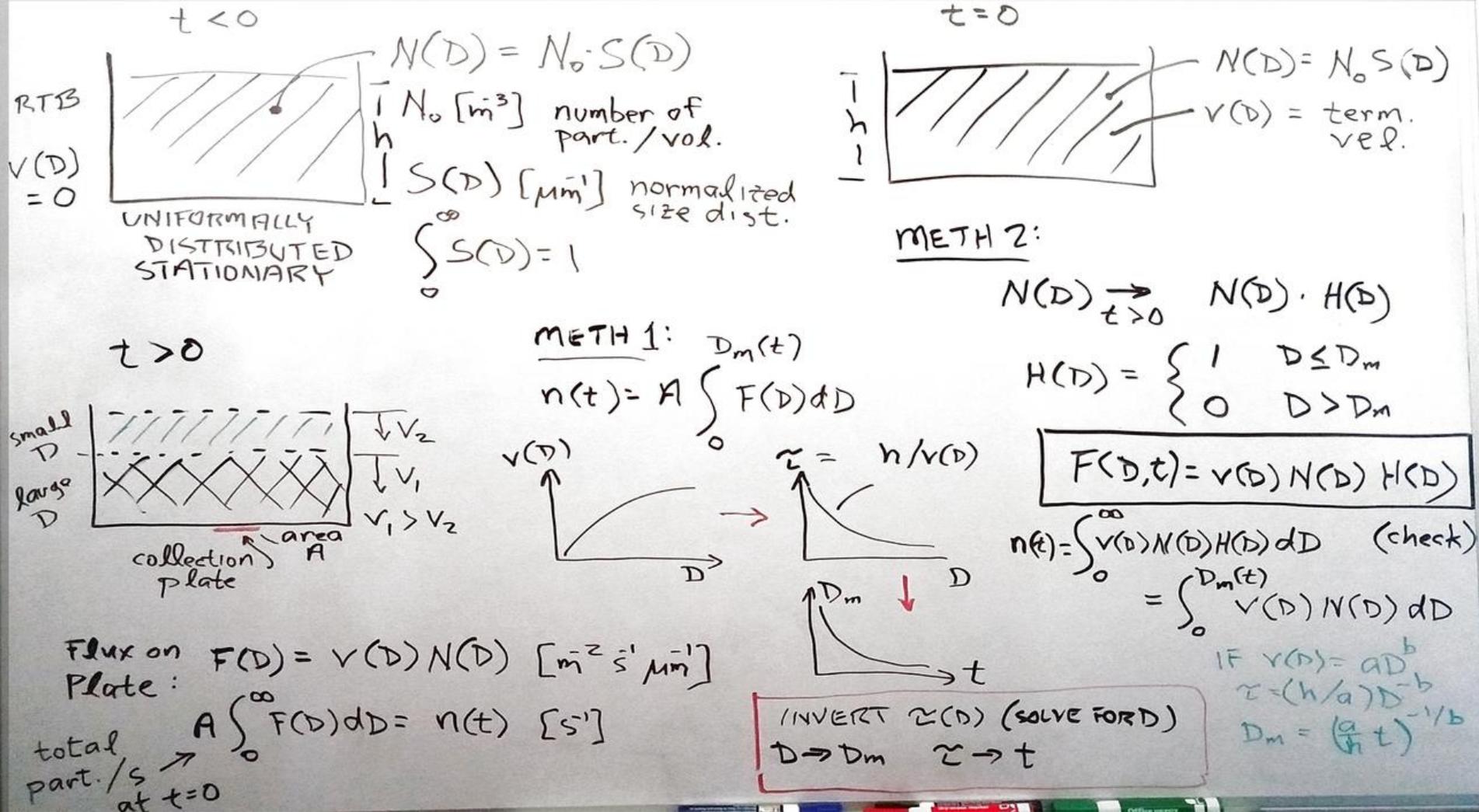
Image processing results for $f/11$ test.

Image File	Max Intensity	Saturated Pixel Count	Saturated Pixel Fraction		Spot Diameter [pixels]	Image Width [pixels]	Image Height [pixels]	spot x coord	spot y coord
DSC_0037.JPG	255.	87	0.201389	218.847	23.4529	3872	2592	2465	2058
DSC_0038.JPG	255.	76	0.180523	220.741	23.1524	3872	2592	2465	2059
DSC_0039.JPG	255.	63	0.186944	217.703	20.7143	3872	2592	2456	2056
DSC_0040.JPG	255.	42	0.133333	216.244	20.0267	3872	2592	2457	2058
DSC_0041.JPG	255.	16	0.0723982	197.824	16.7746	3872	2592	2470	2072
DSC_0042.JPG	255.	14	0.0651163	198.391	16.5453	3872	2592	2470	2074
DSC_0043.JPG	255.	12	0.0560748	200.598	16.5068	3872	2592	2451	2050
DSC_0044.JPG	255.	12	0.0555556	198.065	16.5837	3872	2592	2452	2049
DSC_0045.JPG	255.	1	0.00714286	180.729	13.3512	3872	2592	2443	2047
DSC_0046.JPG	254.	0	0.	176.553	14.3175	3872	2592	2445	2049
DSC_0047.JPG	197.	0	0.	136.62	9.50789	3872	2592	2491	2063
DSC_0048.JPG	188.	0	0.	128.167	10.3418	3872	2592	2493	2072
DSC_0049.JPG	130.	0	0.	86.6522	7.65304	3872	2592	2494	2074
DSC_0050.JPG	119.	0	0.	82.1475	8.81292	3872	2592	2494	2081
DSC_0051.JPG	16.	0	0.	11.8864	7.48482	3872	2592	2484	2098
DSC_0052.JPG	22.	0	0.	14.8148	5.86323	3872	2592	2485	2098



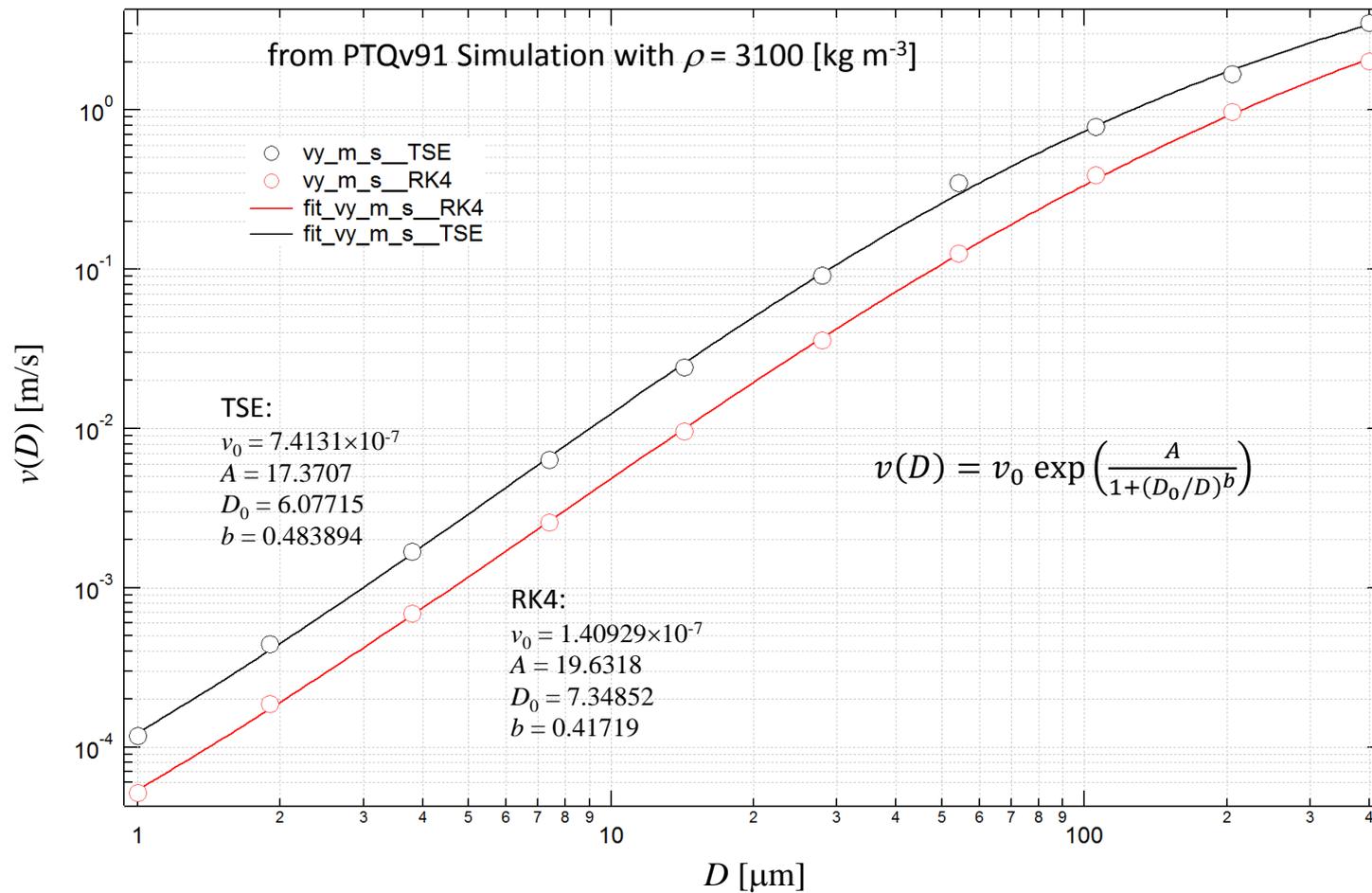


Simulation of Particle Settling In Regolith Test Bin



These results are generated by using the particle trajectory simulation code (PTQv91), with some minor modifications. The plot shows predicted terminal velocity of simulant particles versus diameter. Note that 1 μm particles take 10000 s (~ 3 h) to settle 1 m according to this simulation. That would mean that 1 μm particles thrown up to the ceiling (4 m) could take 12 h to totally settle. On the other end of the spectrum, The 100 μm particles fall one meter in 2 s (8 s from top of the bin).

Dust Particle Terminal Velocity in Regolith Bin



LASER Side Scatter Model And Measurements

The following model is a 4-parameter model composed of three sample points along the laser beam, as viewed by an oblique angle from the camera. The contour graphs show the solution of the 4 parameters - the first is the initial intensity value [W/m²], and the last three are optical extinction coefficients in [1/m]. These values agree well with the laser spot measurements.

$$e_{min} = 0.0154047 \quad P = \begin{pmatrix} 974\,000. \\ 0.01006 \\ 0.02032 \\ 0.0822 \end{pmatrix}$$

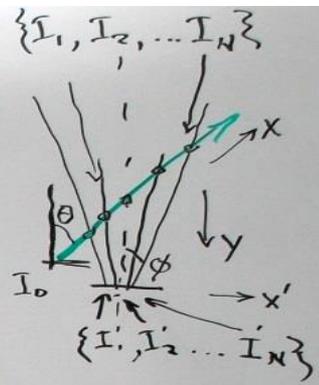
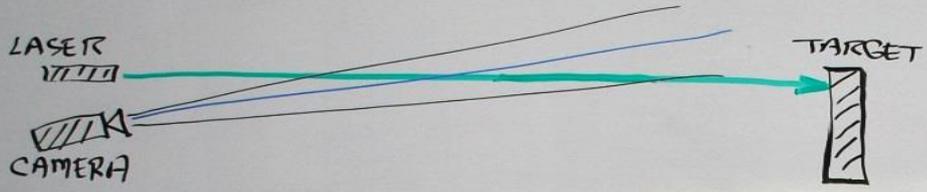
Advantages of this technique as compared to the spot measurement are:

- 1) Provides discrete data along the beam path.
- 2) Does not require an initial calibration measurement of the no dust case.

Disadvantages:

- 1) More calculations and processing is required.
- 2) The results are very dependent on optical alignment (camera relative to laser).

$\text{Exp}[-2\mu \text{imk}/L]$ $P' = (1-x)P_0 + P x$



$\theta \rightarrow 0 \Rightarrow y \rightarrow x$ FOR $N=3$:

$$I' = \begin{pmatrix} I_1 e^{-\alpha_1 \Delta x_1} (1 - e^{-\alpha_1 b / \cos \theta}) F_0 / x_1^2 \\ I_2 e^{-\alpha_1 \Delta x_1 - \alpha_2 \Delta x_2} (1 - e^{-\alpha_2 b / \cos \theta}) F_0 / x_2^2 \\ I_3 e^{-\alpha_1 \Delta x_1 - \alpha_2 \Delta x_2 - \alpha_3 \Delta x_3} (1 - e^{-\alpha_3 b / \cos \theta}) F_0 / x_3^2 \end{pmatrix}$$

$$= I_0 F_0 \begin{pmatrix} (e^{-2\alpha_1 \Delta x_1}) (1 - e^{-\alpha_1 b / \cos \theta}) x_1^{-2} \\ (e^{-2\alpha_1 \Delta x_1 - 2\alpha_2 \Delta x_2}) (1 - e^{-\alpha_2 b / \cos \theta}) x_2^{-2} \\ (e^{-2\alpha_1 \Delta x_1 - 2\alpha_2 \Delta x_2 - 2\alpha_3 \Delta x_3}) (1 - e^{-\alpha_3 b / \cos \theta}) x_3^{-2} \end{pmatrix}$$

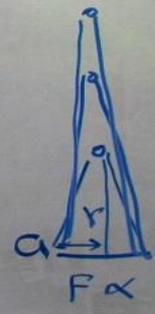
$$I'(x') = I_0 F_0 e^{-2 \int \alpha(x) dx} (1 - e^{-\alpha(x) b / \cos \theta}) x^{-2}, \quad x \gg a$$

$$S(\alpha) = \frac{F_0}{y^2} (1 - e^{-\alpha(x) b / \cos \theta})$$

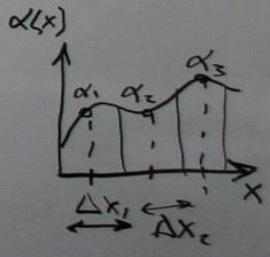
$$I(x) = I_0 e^{-\int \alpha(x) dx}$$

$$I'(x') = I(x) e^{-\int \alpha(y) dy} S(\alpha)$$

$$I = \begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix} = I_0 \begin{pmatrix} e^{-\alpha_1 \Delta x_1} \\ e^{-\alpha_1 \Delta x_1 - \alpha_2 \Delta x_2} \\ e^{-\alpha_1 \Delta x_1 - \alpha_2 \Delta x_2 - \alpha_3 \Delta x_3} \end{pmatrix}$$



$$F_0 \approx 4\pi a^2 ?$$



$$\frac{m_x d}{2f} = -\tan \phi$$

LASER LINE:

$$Y(x') = \frac{1}{\sin \theta} (x' - x_0)$$

CAMERA RAY:

$$Y_k(x') = \frac{1}{\sin \phi_k} x'$$

$$\phi_0 = -\frac{1}{2}\phi$$

$$\phi_{N/2} = 0$$

$$\phi_N = \frac{1}{2}\phi$$

$$\phi_k = \frac{\phi_0}{N} \left(k - \frac{N}{2}\right)$$

$$Y(x') = Y_k(x')$$

$$\frac{1}{\sin \theta} (x' - x_0) = \frac{x'}{\sin \phi_k}$$

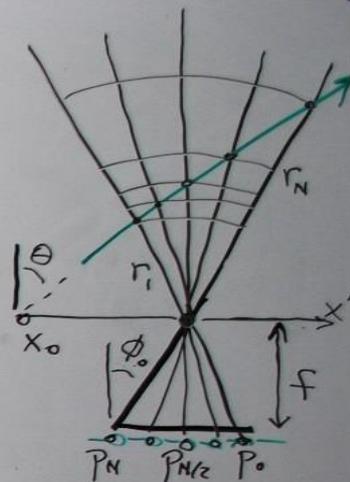
$$\left(\frac{\sin \phi_k}{\sin \theta} - 1\right) x' = \frac{\sin \phi_k}{\sin \theta} x_0$$

$$x'_k = \left(\frac{\sin \phi_k}{\sin \phi_k - \sin \theta}\right) x_0 \equiv \lambda_k x_0$$

$$Y_k = \left(\frac{1}{\sin \phi_k - \sin \theta}\right) x_0 = \Psi_k x_0$$

$$\begin{aligned} \Delta x_k &= \left[(x'_k - x'_{k-1})^2 + (Y_k - Y_{k-1})^2 \right]^{1/2} \\ &= \left[(\lambda_k - \lambda_{k-1})^2 + (\Psi_k - \Psi_{k-1})^2 \right]^{1/2} x_0 \end{aligned}$$

$$\begin{aligned} x_n &= \left[(x'_n - x_0)^2 + Y_n^2 \right]^{1/2} \\ &= \left[(\lambda_n - 1)^2 + \Psi_n^2 \right]^{1/2} x_0 \end{aligned}$$



$m_x \times m_y$
Image array

Questions