

High Resolution Image Reconstruction from Projection of Low Resolution Images Differing in Sub-pixel Shifts

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What is Super Resolution (SR)?

- A High Resolution (HR) image computed from several observed low-resolution (LR) images that differ in sub-pixel shifts and rotation is called Super Resolution (SR).
- SR Increases high frequency components and removing the degradations caused by the imaging process of the low resolution camera.

Why Super Resolution?

- Higher spatial resolution gives more details
- Spatial Resolutions cannot be increased indefinitely by hardware
- The limitations of spatial resolution come from
 - Diffraction of optical system
 - Signal to noise ratio of the sensor system
 - Spacecraft orbits in space-borne imaging

Approaches to Computing SR

- Frequency Domain Approach
- Spatial Domain Approaches
 - Non-Iterative approach: Interpolation and Restoration
 - Statistical Approaches:
 - Maximum A posterior (MAP)
 - Maximum Likelihood (MLE)
- Wavelet-Based construction
- Set Theoretic Methods

Spatial Domain Approaches to SR

- Iterative Back Projection (IBP)
- Interpolations
 - Nearest Neighbor (NN)
 - Inverse Distance Weighted (IDW) Interpolation
- Maximum Likelihood Estimation (MLE)
- Interpolation using Radial Basis Functions (RBF)

Imaging Model

- High Resolution (HR) Image
 - Let x be the HR image sought (in 1-D vector form scanned in lexicographical order)
- Low Resolution(LR) Images
 - Let $y_1, y_2, \dots y_k$ be LR images that differ in translation and rotation w.r.t image y_1 (reference image)
 - Let Y be concatenation of $y_1, y_2, \dots y_k$
- Each of the LR images (y_k) is given by
 - $y_k = D_k H_k F_k x + v_k$
 - F_k : motion matrix
 - H_k : Blurring matrix
 - D_k : down sampling
 - V_k : noise term

Imaging Model (2)

- The previous equation can be written as

$$\underline{y} = Mx + \underline{V}$$

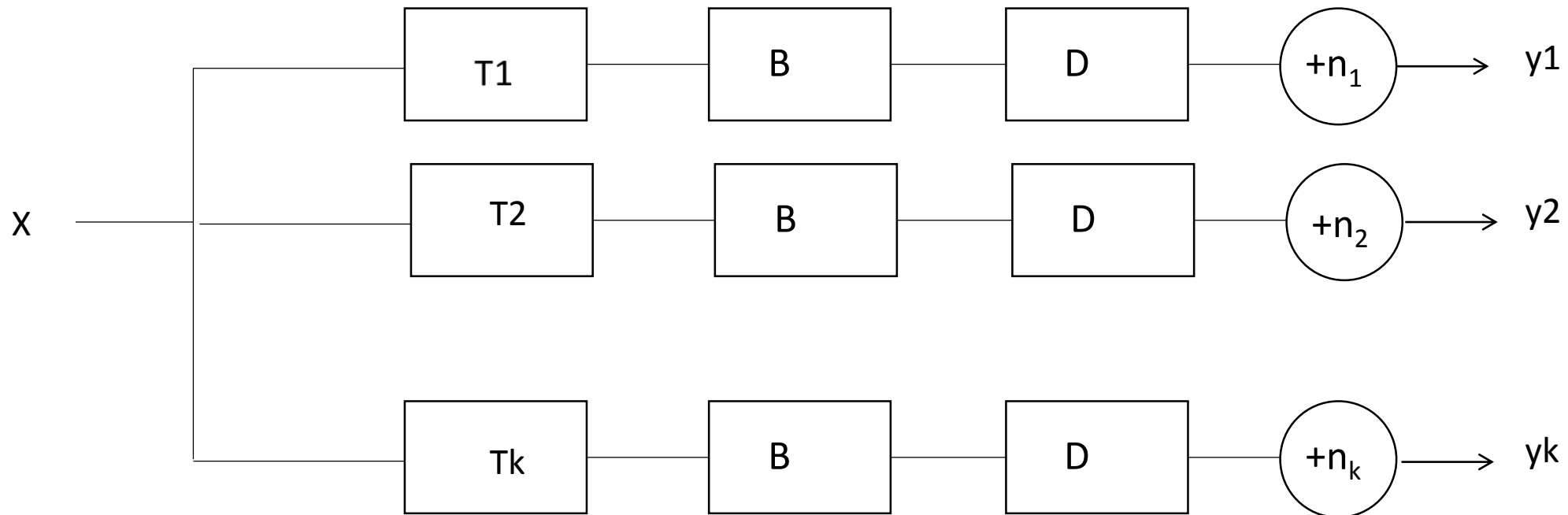
- $M = [D_1 H_1 F_1 \ D_2 H_2 F_2 \ \dots \ D_k H_k F_k]^t$
- \underline{y} is concatenation of y_1, \dots, y_k
- \underline{V} concatenation of noise components $V_1 \dots V_k$
- Given y , to determine x ; which is an ill-posed problem.
- Also in imaging system the matrices used in the above equations are not known.

The Generative Model of LR Images

$$y_i = T_i B D(x) + n_i,$$

T is sub-pixel translation, B is blurring, and D down sample operators

n_i is noise



Spatial Domain Approach to SR

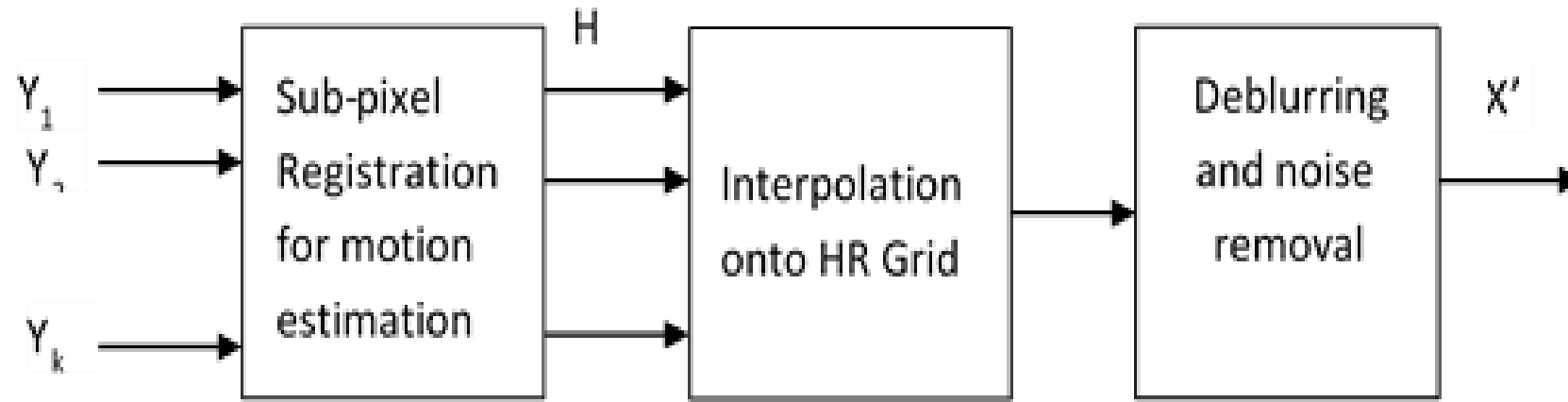


Figure 1. Block diagram reconstructing HR image from LR images

Sub-pixel Registration

- Uses Discrete Wavelet Transforms (DWT)
- Decomposes the Images into LR image and High Frequency Components in Horizontal and Vertical Directions (LL,LH,HL,HH) in multi-resolution fashion into a number of levels thus building a hierarchy low resolution images
- The decomposition is done for both (reference and input) images that are to be registered.
- At each level of decomposition features are extracted from low-frequency and high-frequency subbands for reference and input images and transformation function is computed using correlations.
- The transformation function is improved iteratively using images from the hierarchy.

Interpolation onto HR Grid

- Interpolation algorithms used to project the LR images onto HR
 - Nearest Neighbor (NN) Interpolation
 - Inverse Distance Weighted (IDW) Interpolation
 - Radial Basis Functions (RDF) Interpolation

Deblurring and Noise filter

- Simulation of LR images from Original Image includes a known Point Spread Function (PSF) and a noise component as shown in generative model LR images
- The reconstructed image deblurring is obtained by deconvolving using Wiener filter
- A low pass filter is used on the reconstructed SR image to filter out high frequency noise present in LR images

Statistical Methods in Computing SR image

- Maximum Likelihood Estimation (MLE)
 - In this method the SR image is estimated from a given set of LR images differing in sub-pixel shifts
 - $$p(x|y_1, y_2, \dots, y_k) = \frac{p(y_1, y_2, \dots, y_k|x) p(x)}{\int_x p(y_1, y_2, \dots, y_k|x) p(x)}$$
 - x is SR image, y_1, y_2, \dots, y_k are LR images
 - Log Likelihood function $L(x) = \log(p(y_1, y_2, \dots, y_k|x) p(x))$
 - MLE assumes flat a prior $p(x)$
 - $$x' = \underset{x}{\operatorname{argmax}} L(x)$$
- Every pixel on HR grid is the one that maximizes the $L(x)$

Statistical Methods in Computing SR image (2)

- Maximum A Posterior (MAP) method
- $p(x|y_1, y_2, \dots, y_k) = \frac{p(y_1, y_2, \dots, y_k|x) p(x)}{\int_x p(y_1, y_2, \dots, y_k|x) p(x)}$
- $x' = \underset{x}{\operatorname{argmax}} p(y_1, y_2, \dots, y_k|x)p(x)$
- This method allows inducing priors in estimating x'
- The results of MLE and MAP are similar

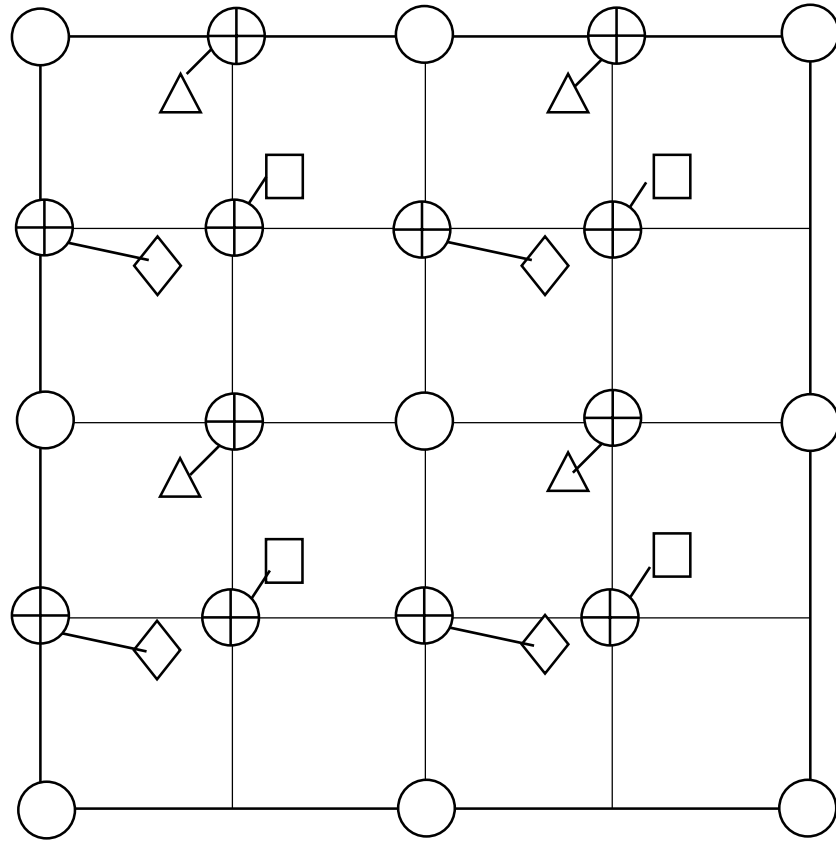
Iterative Back Projection (IBP)

- Initial HR (x') image is reconstructed from the reference LR image (y_1) by replicating pixels
- From x' the LR images are generated using the regenerative model (y'_1, y'_2, \dots, y'_k)
- $$x' = x' + \sum_{i=2}^k (y_i - y'_i)$$

Projection of LR images on HR Grid

- Assume the HR grid is $2n \times 2n$ where LR images are $n \times n$
- Number of LR images differing in sub-pixel shift are 2^2
- HR grid $k \times k$, requires at k^2 LR images for optimal reconstruction
- All LR images are assumed to differ in subpixel shifts that are different from each other.
- The subpixel shifts are global

Projection of LR images on HR Grid: NN Reconstruction



$$x' = U^2(y_1)$$

U^2 is upsample operator by 2

$$x' = U^2(y_1) + \sum_{k=2}^4 U^2(T_k(y_k))$$

T_k Translation operator

Projection LR images on HR Grid using IDW

- Unlike in NN, all the LR images contribute to the estimation of every missing pixels of HR grid
- Pixel value depends on distance of the LR image pixels from the missing HR grid point by inverse distance
- $x' = U^2(y_1) + \sum_{k=2}^4 U^2(w_k y_k)$
- w_k is inverse distance of y_k from missing positions of x'
- $\sum_{k=2}^4 w_k = 1$

Interpolation using Radial Basis Functions (RBF)

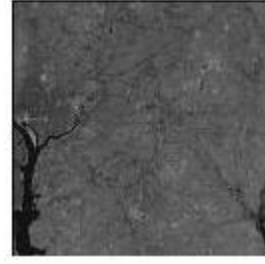
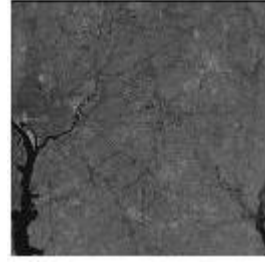
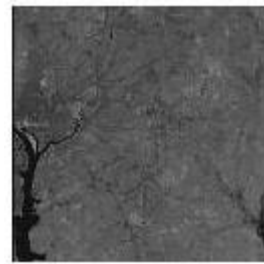
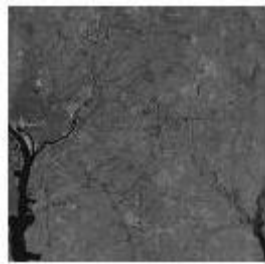
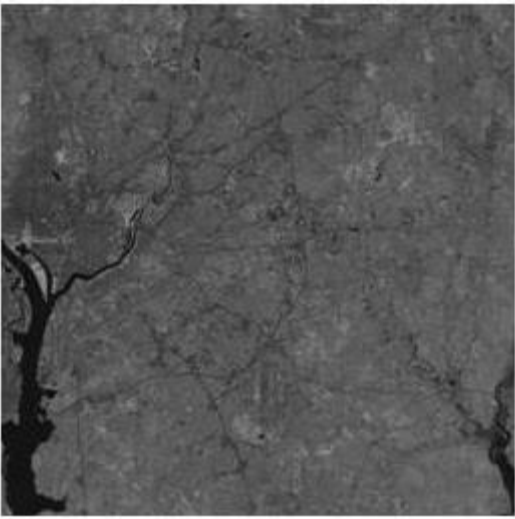
- A radial basis function (RBF) is a real valued function whose value depends on the distance from a given point x_i (x a variable not to be confused with image variable used for HR image).
- $\phi(x, x_i) = \phi(||x - x_i||)$
- When the distance function is Gaussian, the RBF is called Gaussian RBF
- $\phi(x, x_i) = e^{-(x-x_i)^2}$

Interpolation using Radial Basis Functions (RBF)

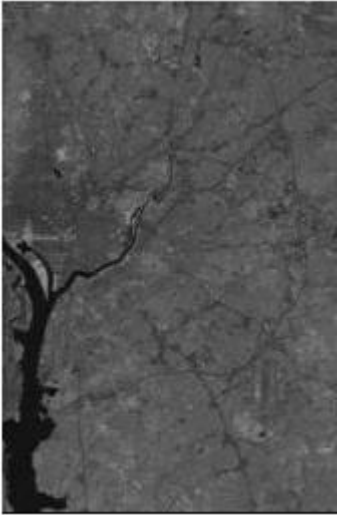
- The reference LR image($n \times n$) is used to fill up every other HR grid ($2n \times 2n$) pixels along the row and column.
- The remaining LR images are used to estimate the 3 missing cells of 2×2 HR grid as follows
- $x(i) = \sum_{k=2}^4 \phi(i - T(k)) * y_k$
- $x(i) = \sum_{j=2}^k e^{-(||i-T(j)||)} y_j$
- In the above equation vector i is the position of the missing pixel from HR grid (0,1 for horizontal, 1,0 for vertical and 1,1 for diagonal)
- $T(k)$ is the translation of LR image, y_k with respect to i .

Experimental Results

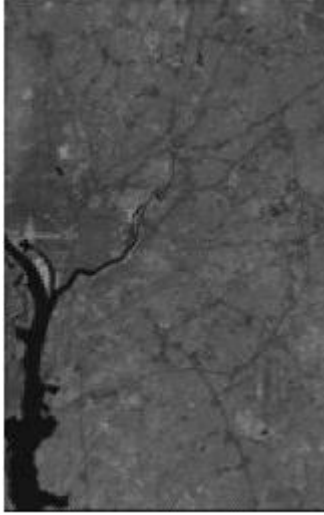
- Simulation of LR images from Chesapeake Bay Landsat Image



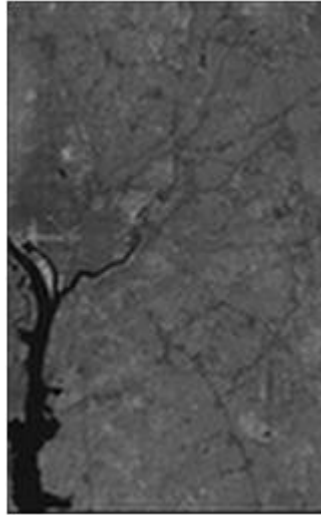
Results



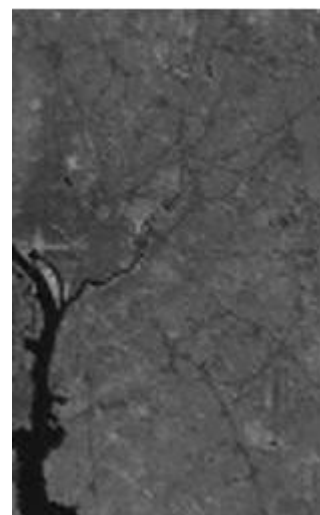
NN



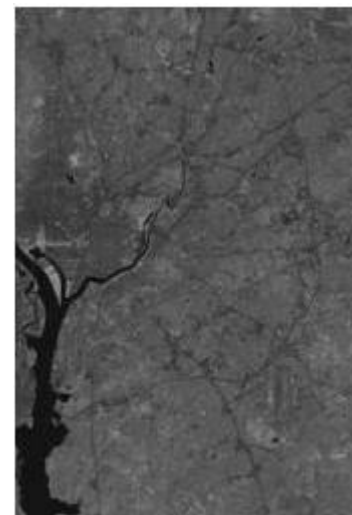
IDW



MLE



IBP



RBF

Results (SSE)

y2 : tr2 = 0.9, tc2 = 0.24

y3: tr3 = 0.12, tc3 = 0.82

y4: tr4 = 0.9, tc4 = 0.55

Algorithm	SSE
NN(Nearest Neighbor) interpolation	87799
IDW (inverse Distance Weighted)	128624
MLE (Maximum Likelihood)	167781
IBP (Iterative Back Projection)	220468
RBF (Radial Basis Function) Interpolation	77732

Results (SSE)

y2: tr2 = 0.82, tc2 = 0.08

y3: tr3 = 0.17, tc3 = 0.84

y4: tr4 = 0.64, tc4 = 0.56

Algorithm	SSE
NN(Nearest Neighbor) interpolation	886717
IDW (inverse Distance Weighted)	120953
MLE (Maximum Likelihood)	167938
IBP (Iterative Back Projection)	226565
RBF (Radial Basis Function) Interpolation	71946

Concluding Remarks

- Various algorithms are implemented to compare their performance in reconstructing HR image from a set of LR images differing subpixel shifts
- Interpolation using RBFs performed the best in most cases.
- HR reconstruction accuracy depends on the subpixel shifts and number of LR images
- We have experimented with four LR images to increase the spatial resolution by a factor of 2
- Further research is required to improve the spatial resolution by an arbitrary number with a given number of LR images