DIRECT NUMERICAL SIMULATION OF TURBULENT COUETTE-POISEUILLE FLOW WITH ZERO SKIN FRICTION

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Abstract The near-wall scaling of mean velocity $U(y_w)$ is addressed for the case of zero skin friction on one wall of a fully turbulent channel flow. The present DNS results can be added to the evidence in support of the conjecture that U is proportional to $\sqrt{y_w}$ in the region just above the wall at which the mean shear $\mathrm{d}U/\mathrm{d}y = 0$.

INTRODUCTION AND APPROACH

The subject of this study is wall-bounded turbulence adjacent to a no-slip surface with vanishing mean shear stress. It extends our earlier investigations [1, 2] of wall layers subjected to favorable and adverse streamwise pressure gradients (FPG and APG, respectively), and specifically of how these gradients affect near-wall similarity relationships such as the logarithmic law. As in those earlier studies, we use direct numerical simulation (DNS) of Couette-Poiseuille flow, which contains shear-stress gradients of opposite signs on its two sides, and thus efficiently emulates an FPG layer on one side and an APG layer on the other. For the present case, the balance between the streamwise pressure gradient $\mathrm{d}(P/\rho)/\mathrm{d}x$ and streamwise wall velocity U_w is adjusted such that the mean velocity gradient on the APG wall approaches zero and therefore the non-dimensional pressure gradient $p^+ \equiv \mathrm{d}\tau^+/\mathrm{d}y^+ = [\mathrm{d}(P/\rho)/\mathrm{d}x]\left[\nu/u_\tau^3\right] \to \infty$, where u_τ is the wall friction velocity and ν the kinematic molecular viscosity. For the Reynolds number used here, $h\Delta U/\nu = 20,000$ (where $2h$ is the full channel height and $\Delta U = 2U_w$ is the wall-velocity difference), this requires $\mathrm{d}(P/\rho)/\mathrm{d}x = -0.00365U_w^2/h$, which results in $p^+ \approx -0.0006$ on the FPG side (corresponding to a pure Poiseuille flow with $\mathrm{Re}_\tau = u_\tau h/\nu \equiv 1/p^+ \approx 1700$).

The simulation uses the Fourier/Chebyshev spectral channel code described in [1], with real-space resolution of $n_x \times n_y \times n_z = 576 \times 193 \times 576$ and domain of size $4\pi h \times 2h \times 2\pi h$ in the streamwise x, wall-normal y and spanwise z directions, respectively. The 2/3 rule is applied in x and z for dealiasing. In wall units from the FPG side (i.e.,the side with larger u_τ), the time step is $\Delta t^+ \approx 0.4$ and the streamwise and spanwise grid spacing are $\Delta x^+ \approx 19$ and $\Delta z^+ \approx 9.5$, while the wall-normal distance of the 10th grid point is $y_{10}^+ \approx 9$. Thus, although the FPG side is slightly under-resolved [3], these values are small enough to indicate the resolution for the $u_\tau \to 0$ side – upon which we focus our attention here – will be more than adequate.

Straford [4] and Townsend [5] proposed that the mean velocity U above a wall with vanishing skin friction will be proportional to $\sqrt{y_w}$ in a region between the wall, $y_w = 0$, and the outer flow, for the following reasons. If it is assumed (as it was in the derivation of the log law) that the wall-normal velocity gradient $\mathrm{d}U/\mathrm{d}y_w$ depends solely on y_w, $\mathrm{d}(P/\rho)/\mathrm{d}x$ and ν, that leads to

$$\frac{\mathrm{d}U^-}{\mathrm{d}y^-} = \mathcal{F}(y^-), \tag{1}$$

where $U^- = U/u_p$, $y^- = y_w u_p/\nu$ and $u_p^3 = \nu\mathrm{d}(P/\rho)/\mathrm{d}x$ (cf. [6]). In the large-y^- 'overlap' region, where presumably the viscosity is irrelevant, dimensional analysis implies (1) can be written as

$$\frac{\mathrm{d}U^-}{\mathrm{d}y^-} = \frac{A}{\sqrt{y^-}}. \tag{2}$$

The $y_w^{1/2}$ relationship then follows, with

$$U^- = B\sqrt{y^-} + C, \tag{3}$$

where A, $B = 2A$ and C are, within the present idealization, universal non-dimensional constants. The functional form of (3) will be tested using the Couette-Poiseuille results shown below.

RESULTS

The mean velocity and total shear-stress profiles are shown in figure 1. Both reveal the desired $\mathrm{d}U/\mathrm{d}y \approx 0$ behavior at the APG wall, $y = +h$. The linear stress profile, whose slope agrees well with the constant pressure gradient $\mathrm{d}(P/\rho)/\mathrm{d}x$, indicates that the mean statistics are reasonably close to a fully converged stationary state.

A fairly compelling affirmative answer to the $U \sim \sqrt{y_w}$ conjecture (3) is provided in figure 2. Also shown, via the straight broken line, is the curve fit defined by $B = 1.8$ and $C = -2.05$. The latter coefficient is within the $-3.2 \leq C \leq 2.2$ range given in Schlichting & Gersten [6] for this flow, but the former is well below the $2.5 \leq B \leq 5$ they provide. As of

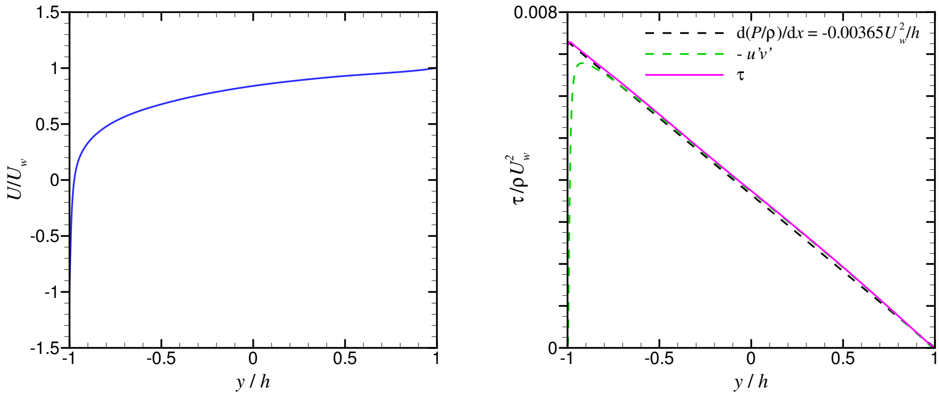

Figure 1. Left, mean velocity; right, Reynolds shear stress.

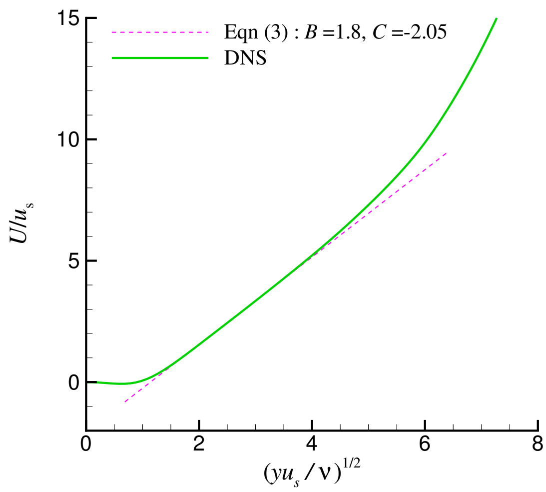

Figure 2. Mean velocity for vanishing-skin-friction wall.

now, the degree of universality of the square-root law (3) is thus an open question, requiring further experimental and/or simulation studies.

The final paper will include finer resolution, and an extensive study of the apparent conflict with experiment in our results so far. Other theoretical sources will be explored. The Reynolds stresses and other aspects of the turbulence will also be presented.

References

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