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Title: Probabilistic fatigue damage prognosis using a surrogate model trained via 3D finite element analysis

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ABSTRACT

Utilizing inverse uncertainty quantification techniques, structural health monitoring can be integrated with damage progression models to form probabilistic predictions of a structure's remaining useful life. However, damage evolution in realistic structures is physically complex. Accurately representing this behavior requires high-fidelity models which are typically computationally prohibitive. In the present work, a high-fidelity finite element model is represented by a surrogate model, reducing computation times. The new approach is used with damage diagnosis data to form a probabilistic prediction of remaining useful life for a test specimen under mixed-mode conditions.

INTRODUCTION

Structural Health Monitoring (SHM) is motivated by the idea that knowledge of an individual structure's current health state increases reliability through a systematic reduction of uncertainty [1]. However, SHM in this sense is inherently reactive, as damage must occur before diagnosis of the structural health provides any useful information. This can limit the utility of SHM, especially when applied to fatigue loading where remaining useful life (RUL) is of interest. A more comprehensive approach would be to utilize SHM systems to detect and quantify damage, but then extend this information through a prediction of how the damage will propagate and ultimately impact the structural RUL. This is the motivation for damage prognosis.

In general, two primary challenges exist for damage prognosis. First, the problem is probabilistic in nature. SHM sensors cannot provide a deterministic assessment of the exact damage state, and the models used to predict how that damage progresses are, at best, approximations of the true underlying physics. Therefore, uncertainty

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quantification has a central role in damage prognosis. Second, the evolution of damage occurs at multiple length scales [2,3]. However, the macroscale is of primary interest for SHM-aided prognosis, since detectable damage typically falls into this regime.

There are many fracture mechanics-based, one-dimensional growth models for macroscale fatigue cracks [4]. Most of these models stem from the work of Paris et al. [5] in which the crack growth rate is a function of empirical parameters and either the stress intensity factor (SIF) or the strain energy release rate. For idealized geometries and boundary conditions, these values can be computed analytically. A great deal of work has been devoted to fatigue damage prognosis using these types of models for both composite and metallic structures; see [3,6] for examples. In the context of SHM, these works typically use sensors to acquire in-situ damage accumulation data that are then used to inversely quantify the uncertainty in model parameters via Markov Chain Monte Carlo (MCMC) techniques [7]. These uncertainties can then be propagated back through the models to form a probabilistic prediction of RUL [3,6,7].

For SHM to be an effective tool, however, real-world structures must be monitored. Damage evolution in these structures progresses in two or three dimensions with mixed-mode driving forces [8]. Finite element (FE) analysis is capable of capturing this behavior, but often at too high a computational cost for use with MCMC methods, which can require thousands to millions of model simulations to reliably sample from the target posterior distribution [7]. Parallel MCMC algorithms [9] constitute an active area of research that could eventually alleviate this burden. However, these methods depend on a set of Markov chains (i.e., each realization is dependent on its predecessor) and, therefore, still require an intractable number of simulations if utilizing FE analyses.

Surrogate models have garnered a great deal of attention for their ability to quantitatively represent the primary features of high-fidelity models at a fraction of the computational cost [7,10]. Research into high-fidelity damage prognosis over the last five years has focused primarily on employing surrogate modeling techniques [10-14]. Sankararaman et al. [10,11] used the FE software ANSYS to develop surrogate models capable of returning an equivalent SIF for a given three-dimensional crack configuration and multi-axial loading condition based on a characteristic plane approach. Ling and Mahadevan then integrated this approach with SHM data to develop probabilistic fatigue damage prognostics for aluminum test specimens [12]. Hombal et al. expanded on this work by presenting a two-stage technique for planar approximation of a non-planar crack [13]. The semi-analytical approaches in [10-14] retain some advantages of high-fidelity modeling but simplify the fundamental growth mechanics and restrict potential crack shapes.

More recently, Hombal et al. [14] offered an approach based on non-parametric representations of arbitrary crack fronts that were dimensionally reduced via Principal Component Analysis (PCA) [15] for use as input to a surrogate model. In this way, complex crack evolution can be modeled in a low-dimensional space, reducing computation times while retaining the mechanics of the non-planar growth. However, since the growth is internal to the surrogate model, the method is inherently dependent on a user-defined growth rate function. Without access to SIF information, this prohibits the consideration of the uncertainty in growth rate parameters, which are known to exhibit a high degree of scatter.

In the present work, a flexible surrogate model for high-fidelity fatigue crack growth simulations is developed based on a separation of the dynamic and quasi-static aspects of damage evolution. The proposed method enables modeling arbitrary crack geometries and does not restrict access to growth parameters, making it well suited for probabilistic prognosis. The remaining sections of this paper are organized as follows. First, the prognosis method is presented over three subsections discussing the growth algorithm, the surrogate modeling approach, and the SHM-based inverse uncertainty quantification problem, respectively. Next, an experiment to validate the new method is developed, followed by the presentation and discussion of results.

DAMAGE PROGNOSIS METHOD

Fatigue Crack Growth Algorithm

FRANC3D is a fracture mechanics code wherein arbitrary, three-dimensional, geometrically-explicit cracks are inserted into existing finite element meshes and grown via re-meshing. Stress intensity factors (SIFs) are calculated by evaluating the M-integral at the crack tip [16]. Unfortunately, a single crack growth simulation can take hours to complete, with the majority of this time consumed by the FE solution at each growth step. The goal of the presented work was to apply surrogate modeling techniques to develop an analog to FRANC3D which only takes seconds to run.

To accomplish this goal, the FE solutions, and, thus, the need for a mesh, were replaced by a surrogate model. The FRANC3D approach to modeling crack growth could then be adapted to represent a crack parametrically as a collection of *m* points in space (Figure 1). In the present work, only the front was modeled, although, in a more general approach, the crack surface could be represented explicitly. Crack growth is calculated on a point-by-point basis. An initial crack front is supplied by the user, and then the surrogate model provides the mode I, mode II, and mode III SIFs at each front point. It is important to note that, by definition, these values correspond to a local set of orthogonal axes at each crack front point. These axes are defined by the crack front normal ($\hat{e}_1$) and tangent ($\hat{e}_3$) unit vectors and their cross product ($\hat{e}_2$).

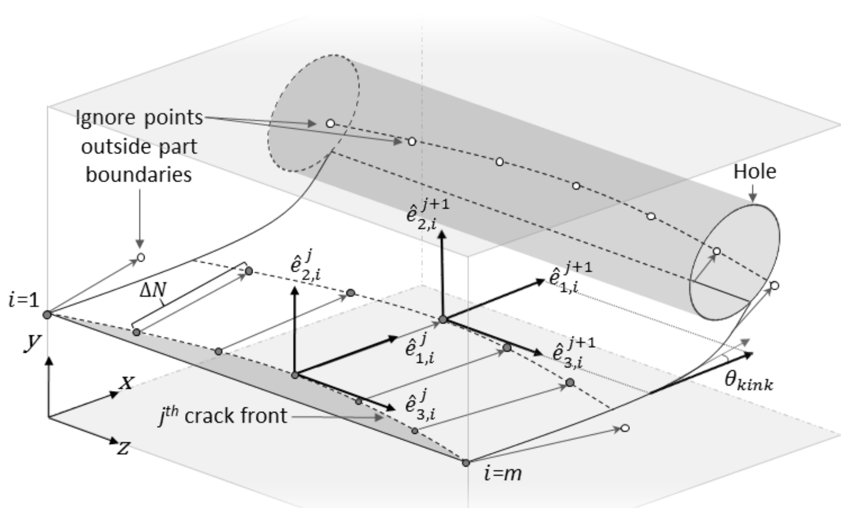

Figure 1: General illustration of the proposed fatigue crack growth process in an arbitrary volume.

If subjected to mixed-mode conditions, the crack can change directions or "kink." A kink angle, θ_{kink}, can be calculated via a user-defined algorithm. At each front point, θ_{kink} is used to rotate the local axes about $\hat{e}_3$, resulting in a set of m new growth directions. The SIFs are converted to an effective equivalent SIF range, ΔK_{ee}, which, for the present work, was defined as $K^r_{I,max} - K^r_{I,min}$, where these two terms correspond to the projection of the SIFs perpendicular to the new growth direction at maximum and minimum load, respectively. Finally, crack front points are advanced along the growth directions by assuming a constant growth rate over a small number of cycles, ΔN, such that the crack growth magnitude $\Delta a = da/dN \times \Delta N$, where da/dN is the one dimensional growth rate. Therefore, the accuracy of the model is dependent on the number of cycles per growth step, which, without a mesh, can be sufficiently small. The crack growth rate is defined by a user-specified crack growth law. Herein, Walker's modified Paris Law is used to incorporate *R*-ratio effects [17],

$$\frac{da}{dN} = C\left[\frac{\Delta K_{ee}}{(1-R)^{1-m}}\right]^n. \qquad (1)$$

Here, $C, m,$ and n are empirical parameters, and $R = K^r_{I,min}/K^r_{I,max}$. Any crack growth model can be used in place of (1). Once the crack growth increment and direction are defined, the current crack front can be propagated forward in time for ΔN cycles. The new crack front points are then compared to the geometric boundaries of the host component. A spline is fit through the front and trimmed anywhere the crack has met a free surface, as shown in Figure 1. A new set of m crack front points is then interpolated along the spline, comprising the new crack front. The crack growth process is iterated, and the crack front at each subsequent step is stored until a stopping condition is met (e.g., $K_I > K_{I,critical}$).

Surrogate Model

Surrogate modeling involves representing complex physical models as an input-output (IO) relationship. In general, as the number of inputs to the surrogate model increases, the training set required to characterize the parameter space grows exponentially. The total number of parameters to describe a single crack front in three dimensions is 3m, resulting in an intractable IO relationship. However, as in [14], Principal Component Analysis (PCA) can be used to solve this issue. PCA is a statistical technique for dimensional reduction of a dataset consisting of correlated variables where data are transformed to a new set of variables called the principal components. These components are uncorrelated and ordered such that the first few typically account for the majority of the variance in the original dataset [15]. PCA was used to reduce the dimensions of both the crack fronts and their associated SIF profiles. As an example, crack fronts with $3m$=60 were dimensionally reduced to only two parameters while still accounting for 99% of the variance.

Furthermore, reducing complexity in the IO relationship itself is necessary for tractability. By isolating the crack growth mechanics and only using the surrogate model to capture the quasi-static evaluation of the SIFs, the modeled relationship becomes simple, direct, and well-defined. Crack geometry and loading serve as the inputs and the output is a PCA-reduced representation of the corresponding SIFs. The PCA reduction can then be inverted to recover the original SIF solution for a given geometry with a mean absolute percentage error on the order of 0.5% at each front point. Gaussian Process Regression (GPR) was chosen to build the surrogate model because of its ability to both capture this complex IO behavior and to quantify the uncertainty in the surrogate model predictions [7]. For use with MCMC, the surrogate model requires training data that are dispersed over the infinite number of potential crack geometries, which is feasible due to three training-specific advantages of the proposed method. First, a crack growth simulation consisting of S steps produces S training data points since the IO relationship is independent of cycle count. Second, training data can be generated in parallel, reducing the upfront computational burden. Finally, training data can be added if the initial training set is deemed insufficient.

SHM and Inverse Uncertainty Quantification

In SHM, damage diagnosis data collected over the life of an individual component or structure can be used to inform the prognosis through what is known as Bayes' Theorem of Inverse Problems [7,18]. Assuming unbiased, independent and identically distributed measurement errors (i.e., $\varepsilon_i \sim N(0, \sigma^2)$), the relationship between the model and the experimental observations is expressed as

$$\Upsilon_i = f_i(Q) + \varepsilon_i, \qquad i = 1, \ldots, k, \tag{2}$$

where Υ_i, $f_i(Q)$, and Q are random variables representing the experimental measurements, model response, and model parameters, respectively. The number of observations is denoted by k. The goal is to determine the posterior density of Q given the observed realizations, υ_{obs}, of Υ_i. Considering q to be the realizations of Q, Bayes' Theorem can be used to formulate the inverse problem as follows:

$$\pi(q, \upsilon_{obs}) = \frac{\pi(\upsilon_{obs}|q)\pi_0(q)}{\pi(\upsilon_{obs})} = \frac{\pi(\upsilon_{obs}|q)\pi_0(q)}{\int_{\mathbb{R}^p} \pi(\upsilon_{obs}|q)\pi_0(q)dq} \tag{3}$$

where $\pi(q, \upsilon_{obs})$ is the posterior density of interest, $\pi_0(q)$ is the prior density in which any *a priori* knowledge of the parameters can be incorporated, and p is the number of parameters. The SHM data influence the problem through the likelihood, $\pi(\upsilon_{obs}|q)$,

$$\pi(\upsilon|q) = \frac{1}{(2\pi\sigma^2)^{n/2}} e^{\frac{-\sum_{i=1}^{n}[\upsilon_i - f_i(q)]^2}{2\sigma^2}} . \tag{4}$$

The denominator of (3) is intractable for most problems, especially when p is large. However, Markov Chain Monte Carlo (MCMC) techniques, whose stationary distribution is the posterior density in (3), allow for a solution to the inverse problem.

EXPERIMENTAL VALIDATION, RESULTS & DISCUSSION

Two tension-tension fatigue crack growth experiments were conducted using edge-notched, Al 2024-T3 specimens. Adapted from work by Ingraffea et al. [19], holes were drilled (Figure 2) to induce localized mix-mode conditions, causing nearby cracks to kink. A constant amplitude stress of 5.95 ksi was applied with R = 0.1 at a frequency of 10 Hz. The first test was stopped once the crack growth rate exceeded 1×10^{-5} in/cycle. This was assumed to correspond to a conservative end of life (EOL) condition, $K_I > K_{I,EOL}$, where $K_{I,EOL}$ = 26.8 ksi·in$^{1/2}$ was determined by recreating the final, as-measured crack in FRANC3D, applying the maximum load, and computing K_I. From the second experiment, a crack breaching the hole was determined to arrest for at least 3 million cycles before reinitiating, which was accounted for in RUL calculations. The observed fatigue crack path for the first specimen is presented in Figure 2. While no sensor-based SHM data were available for these experiments, visual crack tip measurements were taken over the specimen lifespan. To simulate the capabilities of guided wave interrogation, Gaussian white noise was added to the *x* and y measurements with variances, σ^2, of 0.0004 and 0.002 in^2, respectively. The five measurements used for prognosis are represented by circled 'x' marks in Figure 2.

To train the surrogate adequately over the wide range of potential crack starting locations, thirty initial y locations were simulated, uniformly spaced from 0.0 to 0.3782 inches, each with a length of 0.08 inches. The origin is as shown in Figure 2. Training simulations were carried out using FRANC3D with median extension steps of 0.008 inches and a crack front template radius of 0.004 inches. In both the training and prognosis simulations, growth increments and kink angles were calculated using the Walker model (1) and Maximum Tangential Stress (MTS) theory [16], respectively. The final set of training data consisted of 3,000 first order relationships between a given crack front geometry and the corresponding SIFs. While the proposed methodology allows for the inclusion of loading as an input, this was not necessary in the present example. The training data were fit with a set of GPR models using the scikit-learn module for machine learning in Python [21]. Using the surrogate model, run times for crack growth simulations were reduced from approximately 3 hours to under 12 seconds – nearly three orders of magnitude faster. A verification of the surrogate model was conducted but will not be discussed further.

The Bayesian inverse problem was solved utilizing the acquired SHM data and an adaptive MCMC algorithm based on the Python module PyMC [22]. A burn-in of 5,000 samples was used to encourage sampling from the true posterior density, and then 10,000 samples were drawn. The resulting chains were thinned by retaining every tenth sample, and errors were assumed to be unbiased, independent and identically distributed. For illustrative purposes, only two model parameters were considered to be random variables: (1) the starting location of the crack, y_{loc}, and (2) the Walker exponential parameter, n. A global sensitivity analysis [23] would typically be conducted to inform these choices but was excluded for simplicity. Uninformative prior densities were assumed such that $y_{loc} \sim U(0.01, 0.365)$ and $n \sim U(1.0, 6.0)$. Updated parameter probability density functions (PDF) were inversely determined, from which 1,000 model realizations were constructed using a Monte Carlo sampling method.

These realizations were used to generate a normalized histogram for the RUL, as well as prediction and confidence bounds for the anticipated path, as seen in Figure 3a and Figure 3b, respectively. Since it is possible that the crack will grow into the hole and arrest, the histogram of predicted RUL in Figure 3a is bimodal (i.e., two distinct regions of probability exist). While the predicted mean RUL is less than the observed value by 77,537 cycles, the observed value still falls within the predicted PDF. This discrepancy is likely due to the noise level and sparse nature of the data. Furthermore, in Figure 3b, it appears that the predicted mean path agrees more with the data than the observed path, which was viewed as a validation of the proposed methodology.

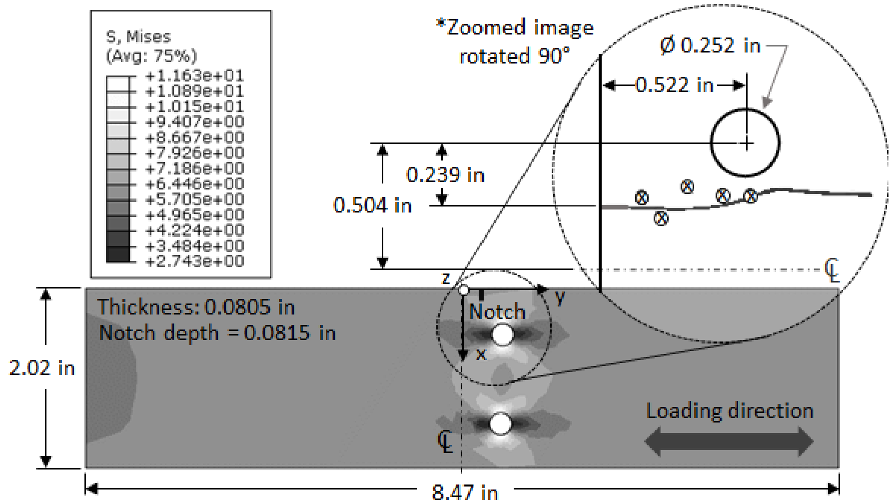

Figure 2: Abaqus [20] FE model, experimental crack path, and SHM data, designated by the 'x' marks.

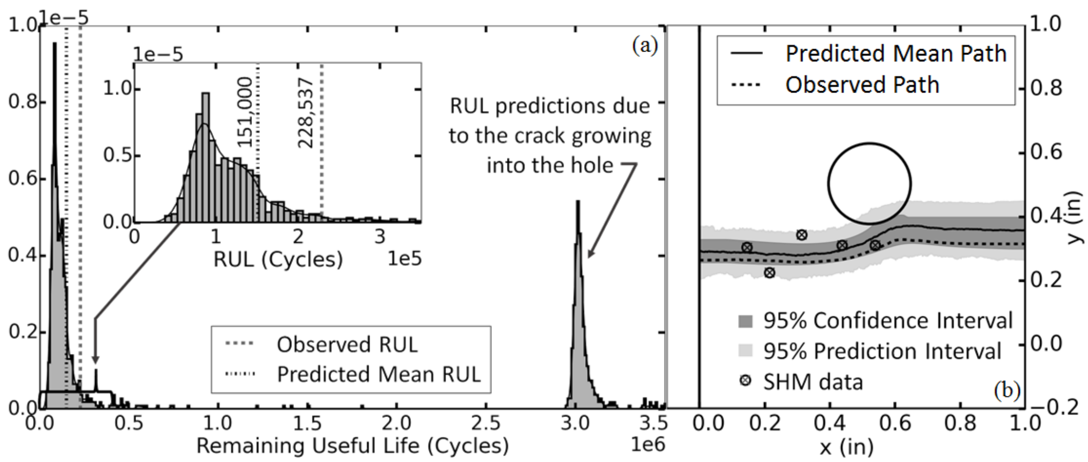

Figure 3: (a) Histogram of RUL samples compared with the observed RUL and (b) the predicted crack path with 95% confidence and prediction bounds compared with the observed crack path.

SUMMARY

A new method for probabilistic prognosis of fatigue crack growth was demonstrated. Prohibitively expensive stress intensity factor computations were replaced by an efficient surrogate model trained via high-fidelity finite element simulations. The proposed approach was validated through fatigue crack growth experiments with induced, localized, mixed-mode conditions. Simulated SHM data were used to inversely quantify the uncertainty in model parameters, including those associated with the crack growth rate. These uncertainties were then propagated through the modeling framework to successfully predict the observed RUL and crack growth path. Model run times were reduced from around three hours to under 12 seconds. Although the experiment was relatively simple, the modeling framework was developed generally, and the same techniques can be applied to a variety of more complex, three-dimensional SHM problems requiring high-fidelity prognosis.

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REFERENCES

1. Farrar, C. R., and K. Worden. 2007. "An introduction to structural health monitoring," *Philos. Trans. R. Soc. A: Math. Phys. Eng. Sci.*, 365(1851):303-315.
2. Emery, J. M., J. D. Hochhalter, P. A. Wawrzynek, G. Heber, and A. R. Ingraffea. 2009. "DDSim: A hierarchical, probabilistic, multiscale damage and durability simulation system–Part I: Methodology and Level I," *Engineering Fracture Mechanics*, 76(10):1500-1530.
3. Chiachıo, J., M. Chiachio, A. Saxena, G. Rus, and K. Goebel. 2013. "An energy-based prognostics framework to predict fatigue damage evolution in composites," *Proc. Ann. Conf. of the Prognostics and Health Management Society*, 1:363-371.
4. Anderson, T. L. 2005. *Fracture mechanics: fundamentals and applications.* CRC Press, Taylor & Francis Group, pp. 451-507.
5. Paris, P. C., and F. Erdogan. 1963. "A critical analysis of crack propagation laws," *Journal of Fluids Engineering*, 85(4):528-533.
6. Peng, T., J. He, Y. Xiang, Y. Liu, A. Saxena, J. Celaya, and K. Goebel. 2015. "Probabilistic fatigue damage prognosis of lap joint using Bayesian updating," *J. Intell. Mat. Syst. Struct*., 26(8):965-979.
7. Smith, R. C. 2013. *Uncertainty Quantification: Theory, Implementation, and Applications*. SIAM Computational Science & Engineering Series, pp. 155-344.
8. Pascoe, J. A., R. C. Alderliesten, and R. Benedictus. 2013. "Methods for the prediction of fatigue delamination growth in composites and adhesive bonds – a critical review," *Eng. Fract. Mech.*, 112:72-96.
9. Neiswanger, W., C. Wang, and E. Xing. 2013. "Asymptotically exact, embarrassingly parallel MCMC," *arXiv preprint arXiv:1311.4780*.
10. Sankararaman, S., Y. Ling, C. Shantz, and S. Mahadevan. 2011. "Uncertainty quantification in fatigue crack growth prognosis," *Int. J. of Prognostics and Health Management*, 2(1):1-15.
11. Sankararaman, S., Y. Ling, and S. Mahadevan. 2011. "Uncertainty quantification and model validation of fatigue crack growth prediction," *Eng. Fract. Mech.*, 78(7):1487-1504.
12. Ling, Y., and S. Mahadevan. 2012. "Integration of structural health monitoring and fatigue damage prognosis," *Mechanical Systems and Signal Processing*, 28:89-104.
13. Hombal, V. K., Y. Ling, K. A. Wolfe, and S. Mahadevan. 2012. "Two-stage planar approximation of non-planar crack growth," *Eng. Fract. Mech.*, 96:147-164.
14. Hombal, V. K., and S. Mahadevan. 2013. "Surrogate modeling of 3D crack growth," *International Journal of Fatigue*, 47:90-99.
15. Jolliffe, I. 2002. *Principal component analysis*. John Wiley & Sons, Ltd.
16. *FRANC3D Verification Manual*, Version 6, Fracture Analysis Consultants, Inc., Ithaca, NY, 2011.
17. Walker, K. 1970. "The effect of stress ratio during crack propagation and fatigue for 2024-T3 and 7075-T6 aluminum," *Eff. of environ. and complex load history on fatigue life ASTM STP*, 462:1-14.
18. Kaipio, J. and E. Somersalo. 2005. *Statistical and Computational Inverse Problems*. Springer, NY
19. Ingraffea, A. R., M. D. Grigoriu, and D. V. Swenson. 1991. "Representation and probability issues in the simulation of multi-site damage," in *Structural Integrity of Aging Airplanes*, S. N. Atulri, S. G. Sampath, and P. Tong, eds. Springer Berlin Heidelberg, pp. 183-197.
20. *Abaqus/CAE user's manual*, Version 6.12, Dassault Systemes Simulia Corp., Providence, RI, 2012.
21. Pedregosa, F., G. Varoquaux, and A. Gramfort. 2011. "Scikit-learn: Machine learning in Python," *The Journal of Machine Learning Research*, 12:2825-2830.
22. Patil, A., D. Huard, and C. J. Fonnesbeck. 2010. "PyMC: Bayesian stochastic modelling in Python," *Journal of statistical software*, 35(4):1-81.
23. Saltelli, A., et al. 2008. *Global sensitivity analysis: the primer*. John Wiley & Sons, pp. 155-174.