Damage Instability and Transition from Quasi-Static to Dynamic Fracture

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Loading Phases:

- 0) to A) – Quasi-static (QS) loading
- A) to B) – Dynamic response

Snapback behavior:
- More strain energy available than necessary for fracture
Failure Criteria and Material Degradation

Progressive Failure Analysis

Benefits
• Simplicity (no programming needed)
• Convergence of equilibrium iterations

Drawbacks
• Mesh dependence
• Dependence on load increment
• Ad-hoc property degradation
• Large strains can cause reloading
• Errors due to improper load redistributions
Failure Criteria and Material Degradation

Progressive Failure Analysis

Progressive Damage Analysis – Regularized Softening Laws
Strength-Dominated Failure

Before damage

\[ F = A \sigma = E A \frac{\Delta}{L + \frac{E}{K}} \]

After damage

\[ F = A \sigma = EA \left( \frac{\Delta - \frac{2Gc}{\sigma_c}}{\frac{2EGc}{\sigma_c^2} + \frac{E}{K}} \right) \]

For stable fracture under \( \Delta \) control:

\[ \frac{\partial F}{\partial \Delta} \leq 0 \]

\[ L \leq \frac{2EGc}{\sigma_c^2} \]

For “long” beams, the response is unstable, dynamic, and independent of \( Gc \)
Fracture-Dominated Failure

\[ G = \frac{\pi \sigma^2 a}{E} \]

Crack propagates unstably once driving force \( G(\sigma, a_0) \) reaches \( G_{IC} \)
Fracture-Dominated Failure

\[ G = \frac{\pi \sigma^2 a}{E} \]

Crack propagates stably when driving force \( G(\sigma, a_0) > G_{\text{Init}} \)

Unstable propagation initiates at \( G_{\text{Init}} < G \leq G_c \)
Mechanics of Crack Arrest

Crack arrest due to decreasing $G$
Large strain rates often result in lower fracture toughness and delayed arrest.
Griffith growth criterion

\[
\frac{\partial \Pi_{\text{total}}}{\partial a_i} = \frac{\partial (\Pi_{\text{int}} + \Pi_{\text{ext}})}{\partial a_i} + G_{c,i} = \begin{cases} 
> 0 & \text{no growth} \\
0 & \text{equilibrium growth} \\
< 0 & \text{dynamic growth}
\end{cases}
\]

Stability of equilibrium propagation

\[
\frac{\partial^2 \Pi_{\text{total}}}{\partial a_i^2} = \begin{cases} 
> 0 & \text{stable} \\
< 0 & \text{unstable}
\end{cases}
\]

Wimmer & Pettermann
J of Comp. Mater, 2009
Curved laminate with through-the-width delamination

\[
\frac{\partial^2 \Pi_{\text{total}}}{\partial a_i^2} = \begin{cases} 
> 0 & \text{stable} \\
< 0 & \text{unstable}
\end{cases}
\]

Wimmer & Pettermann
J of Comp. Mater, 2009
Scaling: The Effect of Structure Size on Strength

Scaling from test coupon to structure

Scaling Laws
(Z. Bažant)

Structural size, in.

\[ \log \sigma_n \]

Yield or Strength Criteria

Normal testing

Linear Elastic Fracture Mechanics (LEFM)

Range of main practical interest for size effect

Extrapolation that must be anchored in a theory

86% of data

Number of Tests
Cohesive Laws

Two material properties:
- $G_c$ Fracture toughness
- $\sigma_c$ Strength

Bilinear Traction-Displacement Law

$$\int_{0}^{\delta_c} \sigma(\delta) \, d\delta = G_c$$

Characteristic Length:

$$l_p = \gamma \frac{E G_c}{\sigma_c^2}$$
Crack Length and Process Zone

Brittle: $a_0 > 100 \, l_p$

Quasi-brittle: $100 \, l_p \geq a_0 \geq 5l_p$

Ductile: $5 \, l_p > a_0$

$G_c = \text{constant}$

$F, \Delta$

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Crack Length and Process Zone

Brittle:
\[ a_0 > 100 \, l_p \]

Quasi-brittle:
\[ 100 \, l_p \geq a_0 \geq 5l_p \]

Ductile:
\[ 5 \, l_p > a_0 \]
Strength and Process Zone

As the strength $\sigma_c$ decreases,

1. the length $l_p$ of the process zone increases
2. the error of the Linear LEFM solution increases

$G_c = \text{constant}$

$F, \Delta$

Force, $F$

Applied displacement, $\Delta$

$\sigma_c$

$\alpha_0$

$\alpha_0 + l_p$

LEFM error

Decreasing

$l_p = \gamma \frac{EG_c}{\sigma_c^2}$
Size Effect and Material Softening Laws

Damage Evolution Laws:

Each damage mode has its own softening response

Two material properties:

- $\sigma_c$ Strength
- $G_c$ Fracture toughness

Material length scale

$$l_c \approx \gamma \frac{EG_c}{\sigma_c^2}$$
**Progressive Damage Analysis**  (Maimí/Camanho 2007)

**Damage Modes:**

- Tension $F^+$
- Compression $F^-$

**Damage Evolution:**

Thermodynamically-consistent material degradation takes into account energy release rate and element size for each mode.

**LaRC04 Criteria**

- In-situ matrix strength prediction
- Advanced fiber kinking criterion
- Prediction of angle of fracture (compression)
- Criteria used as activation functions within framework of continuum damage mechanics (CDM)

$$d_i = 1 - \frac{1}{f_i} \exp(A_i(1 - f_i))$$

$f_i$: LaRC04 failure criteria as activation functions

$$i = F^+; F^-; M^{y+}; M^{y-}; M^s$$

**Bazant Crack Band Theory:**

$$A_i = \frac{2l^*X_i^2}{2E_iG_i - l^*X_i^2}$$

Critical (maximum) finite element size:

$$l^* \leq \frac{2E_iG_i}{X_i^2}$$
Prediction of size effects in notched composites

- Stress-based criteria predict no size effect
- CDM damage model predicts scale effects w/out calibration
  (P. Camanho, 2007)
Process Zone and Scale Effect in Open Hole Tension

Scale effect is due to relative size of process zone

Cohesive law

Stress distribution

(P. Camanho, 2007)
Length of the Process Zone (Elastic Bulk Material)

Short Tensile Test
Lexan Polycarbonate

Cohesive elements

\[ l_c \approx 0.6 \frac{EG_c}{\sigma_c^2} = 3.4 \text{ mm} \]

A

Symmetry

2h

B

Maximum Load

C

D

h/a_0 = 1

E

l_{pz} = 4.7 \text{ mm}

F
The use of cohesive laws to predict the fracture in complex stress fields is explored.

The bulk material is modeled as either elastic or elastic-plastic.

**Lexan Plexiglass tensile specimens (CT Sun)**

<table>
<thead>
<tr>
<th>h/a</th>
<th>Width</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>7.4mm</td>
</tr>
<tr>
<td>2</td>
<td>7.4mm</td>
</tr>
<tr>
<td>1</td>
<td>7.4mm</td>
</tr>
<tr>
<td>0.5</td>
<td>7.4mm</td>
</tr>
<tr>
<td>0.25</td>
<td>7.4mm</td>
</tr>
</tbody>
</table>

**Observations:**
- LEFM overpredicts tests for h/a<1

**Cohesive**

\[ F_{\text{max}}, \text{N} \]

- h/a = 0.25 (long process zone)
- h/a = 1 (short process zone)

\[ l_{cz} = \text{Width} \]

\[ l_{cz} = 4.7 \text{ mm.} \]
Study of size effect: measuring the R-curve

Double-notched compression specimens

By FEM analysis

\[ G = \phi \left( \frac{a}{w} \right) \frac{\sigma_u^2 a}{E^{\text{eff}}} \]

(Similar to \( G = \frac{\pi \sigma^2 a}{E} \))

\( \sigma_u^{-2}, \text{MPa}^{-2} \times 10^{-5} \)

Catalanotti, et al., Comp A, 2014

Increasing \( \frac{a}{w} \)
Characterization of Through-Crack Cohesive Law

Compact Tension (CT) Specimen

Characterization Procedure:

1. Measure R-curve from CT test

\[ G_R = \frac{P^2}{2t} \frac{\partial C}{\partial a} \]

2. Assuming a trilinear cohesive law, fit analytical R-curve to the measured R-curve

\[ \eta = \sum_{i} |j_{fit}^i - G_R^i| \]

3. Obtain the cohesive law by differentiating the analytical R-curve

\[ \sigma(\delta) = \frac{\partial J_{fit}}{\partial \delta} \]

Experimental setup

Bergan, 2014

 Thin multidirectional laminate

\[ \begin{array}{c}
\begin{array}{c}
1.18W \\
y \\
x
\end{array} \\
\begin{array}{c}
a_0 \\
\Delta a \\
W
\end{array}
\end{array} \]

Trilinear cohesive law
Size-Dependence of R-Curve

Displacements measured through digital image correlation (DIC)

Plotting the R-curve as a function of the notch displacement removes the size-dependency
R-Curve Effect in Fiber Fracture

\[ J_R = \int_{0}^{\delta_c} \sigma(\delta) d\delta \]

Curve fit assuming bilinear \( \sigma(\delta) \)

Cohesive elements w/ characterized cohesive law

Cohesive response for fiber failure

Bergan, 2014
Mode II-Dominated Adhesive Fracture

Tip of adhesive

Adhesive thickness: 0.13 mm

Teflon
ENF J-Integral from DIC

Mode II J-integral vs. Displacement Jump

\[ J_{ENF} \approx \frac{9}{16} \frac{F^2 a_0^2}{E b^2 t^3} + \frac{3}{8} \frac{F \delta_t}{b t} \]

Mode II cohesive law

\[ \frac{dJ}{d\delta_t} = \tau(\delta_t) \]

Shear Stress, \( \tau \) [MPa]

Displacement Jump, \( \delta_t \) [mm]
Nominally identical bonded MMB specimens sometimes fail in quasi-static mode and others dynamically. Why?
Double Delamination in MMB Tests

• Unexpected failure mechanism
• Two delamination fronts run in parallel: one in the adhesive, the other in the composite

• When the fiber bridge breaks, the crack grows unstably in the composite causing the drop in the load-displacement curve
A model was developed to evaluate the observed double delamination phenomenon.

The model contains two additional cohesive layers within the composite arms.

This failure mechanism is often observed in bonded joints.
Why Micromechanics?

Assumption:
"Micromechanics has more built-in physics because it is closer to the scale at which fracture occurs"

Why NOT Micromechanics? (Representative Volume Element [RVE])

• Problem of localization
• Randomness of unit cell configurations
• Lengthscales missing
• Characterization of material properties, especially the interface
• Computational expense
RVE: 1) Problem of Localization

Scale of RVE cannot be eliminated.

- Linear
- Hardening
- Softening

RVE, Schapery Theory, homogenization

Localization; regularized CDM, nonlocal methods

$\sigma$

$\varepsilon$

Hardening

Softening

Linear

Smeared

Localized

Scale of RVE cannot be eliminated
RVE: 2) Randomness of Unit Cell Configurations

Fracture is a combination of interacting discrete and diffuse damage mechanisms.

RVE: 3) Issue of Length Scales

RVE may not account for:
- Ply thickness
- Longitudinal crack length
- Crack spacing
Matrix Cracking – In Situ Effect

Transverse Strength of 90° Ply, GPa

- T300/944
- [±25/90]s
- [25/2/-25/2/90]s
- [90]s
- Onset of delamination
- [0/90/0]

Potential crack plane, with crack nucleus

- Thin ply model
- Thick ply model
- Unidirectional

Inner 90° Ply Thickness 2a, mm

Thickness propagation

Longitudinal propagation
Transverse Matrix Cracks w/ One Element Per Ply

Multi-element model: correct crack evolution

Conventional single-element: no opening w/out delam.

Modified single-element: correct Energy Release Rate

\[ K \approx \frac{4E_2}{\pi^2 t} \]

Van der Meer, F.P. & Dávila, C., JCM, 2013
Crack Initiation, Densification, and Saturation

\[ \sigma = 182 \text{ MPa} \]

\[ \sigma = 273 \text{ MPa} \]

\[ \sigma = 372 \text{ MPa} \]

\[ \sigma = 679 \text{ MPa} \]

Van der Meer, F.P. & Dávila, C., JCM, 2013
Initial crack density in a uniformly stressed laminate is strictly a function of material inhomogeneity

- Strength scaled by \( f \), Fracture toughness scaled by \( f^2 \)
- Constant \( f \) along each crack path

\( f(x) \)

Inhomogeneity applied to 3 levels of mesh refinement

- 10 elts.
- 2 elts.
- 1 elt.
Effect of Transverse Mesh Density on Crack Spacing

F Leone, 2015
Commercial finite element vendors and developers are providing more and more tools for progressive damage analysis.

But, if the load incrementation procedures do not converge…

… more analysis tools = more rope!
Techniques for Achieving Solution Convergence

- Viscoelastic Stabilization
  - Delayed damage evolution
- Implicit dynamics or Explicit solution
- Arc-length techniques
  - Dissipation-based arc-length

Constant energy dissipation in each load increment

QS Solution of Unstable OHT Fracture

Van der Meer, *Eng Fract Mech*, 2010
Open Questions

- Is the QS solution physical?
- Are the dynamic effects necessary?
- Which solution provides more insight into failure modes?

![Implicit](image1)

![Explicit](image2)
Concluding Remarks

• A typical structural tests usually consist of three stages:
  1. QS elastic response without damage
  2. QS response with damage accumulation
  3. Dynamic collapse/rupture

• Most structural failures exhibit size effects that depend on load redistribution that occurs during the QS phases
  • Correct softening laws based on strength and toughness considerations are required

• Dynamic collapse/rupture is a result of the interaction between damage propagation and structural response
  • A stable equilibrium state often does not exist after failure under either load or displacement control
  • Onset of instability (failure) occurs when more elastic strain energy can be released by the structure than is necessary for damage propagation
  • Simulation of unstable rupture is often needed to ascertain mode of failure and to compare to test results