Eleventh USA/Europe Air Traffic Management Research and Development Seminar (ATM2015)

Characterizing the Effects of a Vertical Time Threshold for a Class of Well-Clear Definitions

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***Abstract*—A fundamental requirement for the integration of unmanned aircraft into civil airspace is the capability of aircraft to remain well clear of each other and avoid collisions. This requirement has led to a broad recognition of the need for an unambiguous, formal definition of *well clear*. It is further recognized that any such definition must be interoperable with existing airborne collision avoidance systems (ACAS). A particular class of well-clear definitions uses logic checks of independent distance thresholds as well as independent time thresholds in the vertical and horizontal dimensions to determine if a well-clear violation is predicted to occur within a given time interval. Existing ACAS systems also use independent distance thresholds, however a common time threshold is used for the vertical and horizontal logic checks. The main contribution of this paper is the characterization of the effects of the decoupled vertical time threshold on a well-clear definition in terms of (1) time to well-clear violation, and (2) interoperability with existing ACAS. The paper provides governing equations for both metrics and includes simulation results to illustrate the relationships. In this paper, interoperability implies that the time of well-clear violation is strictly less than the time a resolution advisory is issued by ACAS. The encounter geometries under consideration in this paper are initially well clear and consist of constant-velocity trajectories resulting in near-mid-air collisions.**

***Keywords*—sense and avoid; self separation; well clear; collision avoidance; unmanned aircraft systems; safety; interoperability**

I. Introduction

A widely accepted requirement for the safe integration of unmanned aircraft into civil airspace is the need for adequate detect-and-avoid (D&A) or sense-and-avoid (SAA)[1] systems [1,3–5]. The FAA-sponsored SAA Workshop for unmanned aircraft systems (UAS) defines *sense and avoid* as "the capability to remain *well clear* from and *avoid collisions* with other airborne traffic" [6]. In the context of remotely-piloted UAS, both EUROCONTROL [7] and ICAO [2] have also made reference to the *well-clear* and *collision-avoidance* functional aspects of SAA. This terminology is also paralleled in the existing regulatory framework governing manned aircraft.

Under visual flight rules (VFR) for manned aircraft operations, onboard pilots are required to remain *well clear* of other aircraft when complying with the particular rules addressing right-of-way [8,9]. Furthermore, the pilot always has the responsibility for avoiding collisions by not "operating an aircraft so close to another aircraft as to create a collision hazard" [10,11]. In this context, the ability to remain well clear and avoid collisions depends, in part, upon the perception and judgement of a human pilot.

In the absence of an onboard pilot to make a determination of being well clear, the need for an unambiguous, quantitative definition has been identified [12–18]. Additionally, the need for compatibility/interoperability between existing airborne collision avoidance systems (ACAS) and future UAS SAA functionality is a well-established problem concerning the integration of UAS into civil airspace [1,14–16,19–22]. In general, maneuvers to maintain well clear are made much earlier than collision avoidance maneuvers [14–16,19]. Hence, a well clear determination should be large enough to avoid collision advisories, yet not so large as to cause undue concern for traffic aircraft [20].

The particular ACAS considered in this paper is Traffic alert and Collision Avoidance System (TCAS) II [23]. In the US, TCAS II is mandated for use in aircraft having greater than 30 seats or maximum takeoff weight (MTOW) greater than 33,000 lbs. In the EU, ACAS II[2] is mandated for use by aircraft authorized to carry more than 19 passengers or MTOW greater than 5700 kg [24]. The high-level logic of TCAS II is based on checks of time and distance variables against predefined thresholds in the horizontal and vertical dimensions, which vary as a function of the sensitivity level (SL) at which the aircraft is operating, where the SL is determined by the altitude of the aircraft. The time threshold, Tau, and the respective relative horizontal and vertical distance thresholds, DMOD and ZTHR, jointly characterize a Collision Avoidance Threshold (CAT), whereby simultaneous violations in the vertical and horizontal dimensions triggers a TCAS II traffic advisory (TA) or resolution advisory (RA) [25]. Different sets of threshold values are used to determine which of these two types of advisories is issued. This paper restricts attention to the more serious case of TCAS II RAs, which are intended to provide guidance to pilots to maintain or increase vertical separation from other aircraft. A listing of the time and distance thresh-

[1] As in [1,2], this paper uses the terms *detect and avoid* and *sense and avoid* interchangeably.

[2] At the time of this paper, ACAS II is realized as TCAS II.

olds used by TCAS II for RAs at each SL is given in Table I [23]. As shown in Table I, the horizontal and vertical time variables are compared to a single time threshold, Tau. It should be noted that there are two types of TCAS II RA: *preventive* and *corrective*, where the particular type of RA to be issued is determined by further logic checks. Preventive RAs do not require a change in the aircraft's vertical speed, and corrective RAs do require such a change [25]. This paper considers interoperability with respect to both types of RA, although the corrective RA is of particular practical concern [20].

The necessity for interoperability with TCAS II has led to proposals that a formal definition for well clear be explicitly based on time and distance thresholds [15,17,18]. Such thresholds then characterize a well-clear boundary (WCB). That is, crossing the WCB signifies a loss of well clear, i.e., a well-clear violation (WCV) has occurred. The mathematical definitions for such a class of WCB models have been summarized in [18]. An important feature of these WCB models is the explicit decoupling of the time threshold into horizontal and vertical components, TTHR and TCOA, respectively. This decoupling represents a high-level distinction from the TCAS II logic. A primary motivation for this paper is to present the resulting effects of this decoupling in terms of time to WCV and TCAS II interoperability.

The models in [18] also allow for predictions of future WCVs, assuming constant velocities. Furthermore, a high-level specification of the TCAS II RA logic is given in [26], where it is possible to predict the time at which a TCAS II RA will be issued. A key property maintained by a subset of the WCB models in [18] is that there exists some choice of time and distance thresholds such that the resulting WCB is guaranteed to lie outside of the CAT defined by TCAS II for any sensitivity level. For the analysis in this paper, TCAS II interoperability implies that a WCV occurs prior to a TCAS II RA. However, it should be noted that the selection of thresholds which guarantee such TCAS II interoperability may not always be practical from an operational standpoint. Thus, there are perhaps situations where it is operationally acceptable for a TCAS II RA to be issued prior to a WCV. Furthermore, there may also exist practical mitigating factors (e.g., air traffic control) which could act to prevent such scenarios. This paper does not address these last two considerations, thus the scope of the paper is limited to *mathematical* interoperability of *unmitigated* aircraft-pair encounters. This choice is partially motivated by the absence of a history of operational experience with UAS in civil airspace to substantiate the exact extent to which such mitigations may be relied upon [19].

TABLE I
TCAS II SENSITIVITY LEVEL (SL) VERSUS RESOLUTION ADVISORY (RA) THRESHOLDS

Own Altitude (ft)	SL	Tau (s)	DMOD (nmi)	ZTHR (ft)
$<$ 1000 (AGL)	2	N/A	N/A	N/A
1000 - 2350 (AGL)	3	15	0.20	600
2350 - 5000	4	20	0.35	600
5000 - 10000	5	25	0.55	600
10000 - 20000	6	30	0.80	600
20000 - 42000	7	35	1.10	700
$>$ 42000	7	35	1.10	800

This paper characterizes the effects of decoupling the vertical time threshold from the horizontal time threshold in terms of time to WCV and mathematical interoperability with TCAS II, both for unmitigated, constant-velocity, near-mid-air collision (NMAC) encounters. Because existing aircraft encounter models do not sufficiently model UAS flight profiles [21], such encounters were intended to represent a base case of particular interest. The main contribution of this paper is the provision of both analytical and Monte Carlo simulation results, which together give a closed-form representation and an intuitive visualization of the effects of selecting different values for TCOA for a particular WCB definition with respect to the time to WCV and the mathematical interoperability with TCAS II, for the given encounter model.

The remainder of the paper is organized as follows. Section II gives a summary of the WCB models under consideration [18]. Section III derives a set of closed-form expressions and relationships for understanding the effects of a vertical time threshold on time to WCV. Section IV provides a set of closed-form expressions conjectured to define a worst case TCAS II RA interoperability metric for arbitrary values of a vertical time threshold. Section V discusses the design and results of the Monte Carlo simulation, which quantifies the analytical results presented in Sections III and IV. Finally, Section VI provides concluding remarks.

II. A CLASS OF WELL-CLEAR BOUNDARY DEFINITIONS

In the subsequent mathematical development, letters in **bold-face** denote two-dimensional (2-D) vectors. Vector operations such as addition, subtraction, scalar multiplication, dot product, i.e., $\mathbf{s} \cdot \mathbf{v} \equiv s_x v_x + s_y v_y$, the square of a vector, i.e., $\mathbf{s}^2 \equiv \mathbf{s} \cdot \mathbf{s}$, and the norm of a vector, i.e., $\|\mathbf{s}\| \equiv \sqrt{\mathbf{s}^2}$, are defined in a 2-D Euclidean geometry. Furthermore, all development presented in this paper and marked as a *Lemma*, *Corollary*, or *Theorem* has been *formally* specified and verified in the Prototype Verification System (PVS) [27]. In particular, a formal proof in PVS provides a mathematical guarantee that a stated relationship, equation, algorithm, etc, holds exactly as specified. Such formal methods are routinely used in mathematics, engineering, computer science, and particularly in areas demanding a high degree of certainty in the results, such as safety-critical systems. However, to make the following presentation accessible to non-PVS users, this paper uses mathematical notation instead of PVS concrete syntax.

The class of WCB models considered in this paper are related by their dependence on time and distance thresholds in the horizontal and vertical dimensions. The models in this class differ only in the particular horizontal time variable, denoted t_{var}, used to check against a horizontal time threshold TTHR. Henceforth, $\text{WCB}_{t_{\text{var}}}$ denotes the particular WCB model for the time variable t_{var}. As in [18], this paper considers two aircraft, called *ownship* and *intruder* aircraft, whose states are given

by position and velocity vectors in a Euclidean coordinate system. Thus, a pairwise $\text{WCB}_{t_{\text{var}}}$ violation is then completely characterized by the Boolean formula

$$\begin{aligned}\texttt{WCV}_{t_{\text{var}}}(\mathbf{s}, s_z, \mathbf{v}, v_z) \equiv \texttt{HWCV}_{t_{\text{var}}}(\mathbf{s}, \mathbf{v}) \text{ and} \\ \texttt{VWCV}(s_z, v_z),\end{aligned} \tag{1}$$

where $\mathbf{s}, \mathbf{v} \in \mathbb{R}^2$ are the respective ownship-intruder relative horizontal position and velocity vectors, and $s_z, v_z \in \mathbb{R}$ are the respective ownship-intruder relative vertical position and velocities. The horizontal and vertical checks given in (1) are respectively defined as

$$\begin{aligned}\texttt{HWCV}_{t_{\text{var}}}(\mathbf{s}, \mathbf{v}) \equiv \|\mathbf{s}\| \leq \text{DTHR or} \\ (d_{\text{cpa}}(\mathbf{s}, \mathbf{v}) \leq \text{DTHR and } 0 \leq t_{\text{var}}(\mathbf{s}, \mathbf{v}) \leq \text{TTHR}),\end{aligned} \tag{2}$$

$$\begin{aligned}\texttt{VWCV}(s_z, v_z) \equiv |s_z| \leq \text{ZTHR or} \\ 0 \leq t_{\text{coa}}(s_z, v_z) \leq \text{TCOA},\end{aligned} \tag{3}$$

where TTHR and DTHR are the respective time and distance thresholds in the horizontal dimension, and TCOA and ZTHR are the respective time and distance thresholds in the vertical dimension. The function d_{cpa} in (2) computes the projected distance between the ownship and intruder at their closest point of approach, assuming constant relative horizontal velocity, $\mathbf{v}$, and is formally defined as

$$d_{\text{cpa}}(\mathbf{s}, \mathbf{v}) \equiv \|\mathbf{s} + t_{\text{cpa}}(\mathbf{s}, \mathbf{v})\mathbf{v}\|, \tag{4}$$

where t_{cpa} is the time at closest point of approach, which is defined as

$$t_{\text{cpa}}(\mathbf{s}, \mathbf{v}) \equiv \begin{cases} -\frac{\mathbf{s}\cdot\mathbf{v}}{\mathbf{v}^2} & \text{if } \|\mathbf{v}\| \neq 0, \\ 0 & \text{otherwise.} \end{cases} \tag{5}$$

Note that $t_{\text{cpa}}(\mathbf{s}, \mathbf{v}) > 0$ when the aircraft are horizontally converging, $t_{\text{cpa}}(\mathbf{s}, \mathbf{v}) < 0$ when the aircraft are horizontally diverging, and $t_{\text{cpa}}(\mathbf{s}, \mathbf{v}) = 0$ when the aircraft are at the closest point of approach. The function t_{coa} in (3) computes the time to co-altitude assuming constant relative vertical speed v_z and is defined as

$$t_{\text{coa}}(s_z, v_z) \equiv \begin{cases} -\frac{s_z}{v_z} & \text{if } s_z v_z < 0, \\ -1 & \text{otherwise.} \end{cases} \tag{6}$$

Similar to the horizontal case, the product $s_z v_z$ characterizes whether the aircraft are vertically diverging, i.e., $s_z v_z > 0$, or vertically converging, i.e., $s_z v_z < 0$. For completeness, this paper defines time to co-altitude as -1 when the aircraft are not vertically converging.

The function t_{var} in (1) is a horizontal time variable determined by the particular WCB model being used. Possible choices for the time variable t_{var} are *tau* (τ), defined as range over closure rate, *modified tau* [28] (τ_{mod}), which is a variant of tau used in the TCAS II alerting logic, time to closest point of approach (t_{cpa}), and time to violation of DTHR (t_{ep}). The time variable t_{cpa} is as defined in (5), and formal definitions of τ and t_{ep} are found in [18]. Modified tau is a time variable particularly relevant to this paper, and in vector notation, is defined as

$$\tau_{\text{mod}}(\mathbf{s}, \mathbf{v}) \equiv \begin{cases} \frac{\text{DTHR}^2 - \mathbf{s}^2}{\mathbf{s}\cdot\mathbf{v}} & \text{if } \mathbf{s}\cdot\mathbf{v} < 0, \\ -1 & \text{otherwise.} \end{cases} \tag{7}$$

As is the case for the other time variables, τ_{mod} is defined as -1 when the aircraft are not converging. For a given sensitivity level, the TCAS II RA logic corresponds to the Boolean formula $\texttt{WCV}_{\tau_{\text{mod}}}$, as defined in (1), where t_{var} is instantiated with τ_{mod}. Furthermore, the thresholds DTHR, TTHR, ZTHR, and TCOA appearing in (2) and (3) are respectively equal to the TCAS II RA threshold values DMOD, Tau, ZTHR, and Tau in Table I.

The family of WCB models resulting from the time variables listed above is formally studied in [18]. For example, it is proved that for the same choice of threshold values, the following containment relationships hold:

$$\text{WCB}_{\tau} \subseteq \text{WCB}_{t_{\text{cpa}}} \subseteq \text{WCB}_{\tau_{\text{mod}}} \subseteq \text{WCB}_{t_{\text{ep}}}. \tag{8}$$

These relationships guarantee that for the models $\text{WCB}_{\tau_{\text{mod}}}$ and $\text{WCB}_{t_{\text{ep}}}$, there exists a choice of parameters that makes the boundaries larger than the TCAS II RA boundary.

Another property studied in [18] is local convexity. A boundary model is *locally convex* if for nonmaneuvering, i.e., constant-velocity, encounters there exists at most one time interval of well-clear violation. It is formally proved in [18] that, with the notable exception of WCB_{τ}, all the other models in (8) are locally convex. The lack of the local-convexity property in WCB_{τ} is related to well-known problems with the definition of tau [28], which makes it impractical to use this particular model as a basis for a SAA concept. Henceforth, this paper only considers the WCB models that are locally convex.

For locally convex models, it is possible to define a detection algorithm which computes a time interval of well-clear violation for nonmaneuvering pairwise encounters; the generic algorithm $\texttt{WCD}_{t_{\text{var}}}$ in (9) has as inputs a relative state and a lookahead time interval $[B, T]$. This algorithm returns a time interval when a violation of $\text{WCB}_{t_{\text{var}}}$ will occur and is defined as

$$\begin{aligned}&\texttt{WCD}_{t_{\text{var}}}(\mathbf{s}, s_z, \mathbf{v}, v_z, B, T) \equiv \\ &\quad \text{let } [t_1, t_2] = \texttt{VWCD}(s_z, v_z, B, T) \text{ in} \\ &\qquad \text{if } t_1 > t_2 \text{ then } \emptyset \\ &\qquad \text{elsif } t_1 = t_2 \text{ and } \texttt{HWCV}_{t_{\text{var}}}(\mathbf{s} + t_1\mathbf{v}, \mathbf{v}) \\ &\qquad\ \text{then } [t_1, t_1] \\ &\qquad \text{elsif } t_1 = t_2 \text{ then } \emptyset \\ &\qquad \text{else} \\ &\qquad\ \text{let } [t_{\text{in}}, t_{\text{out}}] = \texttt{HWCD}_{t_{\text{var}}}(\mathbf{s} + t_1\mathbf{v}, \mathbf{v}, t_2 - t_1) \text{ in} \\ &\qquad\quad [t_{\text{in}} + t_1, t_{\text{out}} + t_1] \\ &\qquad \text{endif},\end{aligned} \tag{9}$$

where $\texttt{HWCD}_{t_{\text{var}}}$ and VWCD are functions that compute time intervals of violation for the horizontal and vertical dimensions, respectively. Precise definitions of VWCD and $\texttt{HWCD}_{t_{\text{var}}}$, for

$t_{\text{var}} \in \{\tau_{\text{mod}}, t_{\text{cpa}}, t_{\text{ep}}\}$, are provided in [18], and these definitions are assumed in this paper. Furthermore, $\emptyset$ denotes that no well-clear violation is predicted to occur.

Since the vertical check VWCV is independent of t_{var}, the function VWCD that computes the time interval for a vertical well-clear violation is the same for any $\text{WCB}_{t_{\text{var}}}$ model. On the other hand, the horizontal check $\text{HWCV}_{t_{\text{var}}}$ depends on the definition of the time variable t_{var}. Hence, a horizontal detection function $\text{HWCD}_{t_{\text{var}}}$ for each particular definition of t_{var} is necessary. The following theorem, which has been formally proved in [18], states that $\text{WCD}_{t_{\text{var}}}$ completely characterizes the time interval of $\text{WCB}_{t_{\text{var}}}$ violation within the lookahead time interval $[B, T]$, for nonmaneuvering pairwise encounters.

Theorem 1. Let t_{var} be one of τ_{mod}, t_{cpa}, or t_{ep}. For all relative states denoted by $\mathbf{s}, s_z, \mathbf{v}, v_z$, time interval $[B, T]$, with $0 \leq B < T$, and $t \in [B, T]$, $\text{WCV}_{t_{\text{var}}}(\mathbf{s}+t\mathbf{v}, s_z+tv_z, \mathbf{v}, v_z)$ is equal to true if and only if $t \in [t_{\text{in}}, t_{\text{out}}]$, where

$$[t_{\text{in}}, t_{\text{out}}] = \text{WCD}_{t_{\text{var}}}(\mathbf{s}, s_z, \mathbf{v}, v_z, B, T). \tag{10}$$

If $t_{\text{in}} \leq t_{\text{out}}$ in (10), t_{in} and t_{out} denote the first and last times within $[B, T]$ when a violation of $\text{WCB}_{t_{\text{var}}}$ occurs in a nonmaneuvering encounter. Henceforth, these times to WCV will be denoted $t_{\text{in}}^{\mathcal{M}}$ and $t_{\text{out}}^{\mathcal{M}}$, where $\mathcal{M}$ is a particular locally convex, well-clear boundary model.

III. Effects of TCOA on Time to Well-Clear Violation

Consider a model $\text{WCB}_{t_{\text{var}}}$ as defined in Section II, and let this model have two instantiations that coincide in their threshold values DTHR, TTHR, and ZTHR but differ in the particular choice of the vertical time threshold, TCOA. Let these instantiations be denoted by $\mathcal{M}_0$ and $\mathcal{M}_1$, having TCOA_0 and TCOA_1, respectively, such that $\text{TCOA}_0 < \text{TCOA}_1$. The metric $\Delta t_{\text{in}} \equiv t_{\text{in}}^{\mathcal{M}_0} - t_{\text{in}}^{\mathcal{M}_1}$ then characterizes the difference in time to well-clear violation between two models, based exclusively on the particular choices of TCOA. In the following development, a closed-form expression for an upper bound on Δt_{in} is derived for the case of unmitigated, nonmaneuvering encounters resulting in a near mid-air collision.

As discussed in Section II, a well-clear violation results from simultaneous violations in the vertical and horizontal dimensions. In particular, if a violation is predicted to occur in the vertical dimension within the given lookahead time, the horizontal dimension is checked. If a violation is also predicted to occur within the given lookahead in the horizontal dimension, a time interval for the well-clear violation is computed by the function $\text{WCD}_{t_{\text{var}}}$. The following lemma gives a closed-form expression for the difference in time to a vertical well-clear violation between $\mathcal{M}_0$ and $\mathcal{M}_1$, denoted Δt_{in}^z.

Lemma 1. For any relative vertical state denoted by s_z and v_z such that the vertical time interval of violations for $\mathcal{M}_0$ and $\mathcal{M}_1$ in the lookahead time interval $[B, T]$ are nonempty, the vertical metric Δt_{in}^z satisfies the following equalities.

$$\Delta t_{\text{in}}^z = \begin{cases} 0 & \text{if } v_z = 0 \text{ or } \Theta_0 \leq B, \\ \Theta_0 - \Theta_1 & \text{if } \Theta_1 > B, \\ \Theta_0 - B & \text{otherwise,} \end{cases} \tag{11}$$

where, for $i = 0, 1$,

$$\Theta_i \equiv \frac{-\text{sign}(v_z) H_i - s_z}{v_z}, \tag{12}$$

$$H_i \equiv \max(\text{ZTHR}, \text{TCOA}_i |v_z|). \tag{13}$$

The proof of Lemma 1 proceeds by case analysis according to the definition of VWCD provided in [18]. The following lemma, which is a consequence of Lemma 1, states that the initial time to well-clear violation for the vertical dimension computed for $\mathcal{M}_1$ is less than or equal to that computed for $\mathcal{M}_0$, i.e., $\Delta t_{\text{in}}^z \geq 0$. It also gives the maximum value that Δt_{in}^z can take, given two arbitrary values of TCOA.

Lemma 2. For any relative vertical state denoted by s_z and v_z such that the vertical time interval of violations for $\mathcal{M}_0$ and $\mathcal{M}_1$ in the lookahead time interval $[B, T]$ are nonempty, the vertical metric Δt_{in}^z satisfies the following relations:

$$0 \;\leq\; \Delta t_{\text{in}}^z \;\leq\; \begin{cases} 0 & \text{if } v_z = 0, \\ \frac{H_1 - H_0}{|v_z|} & \text{otherwise,} \end{cases} \tag{14}$$

where H_i, for $i = 0, 1$, is defined as in (13).

The specification of $\text{WCD}_{t_{\text{var}}}$ in Section II uses the vertical time interval of violation in the computation of the horizontal time interval of violation. In particular, the time interval $[t_1, t_2]$ in (9), which denotes the time interval of violation in the vertical dimension, appears in the expression $\text{HWCD}_{t_{\text{var}}}(\mathbf{s}+t_1\mathbf{v}, \mathbf{v}, t_2-t_1)$, which computes a time interval of violation in the horizontal dimension. In that expression, the first and second parameters represent relative 2-dimensional position and velocity vectors, respectively. The last parameter represents a horizontal lookahead time. The following lemma gives a relationship between $\text{HWCD}_{t_{\text{var}}}(\mathbf{s}, \mathbf{v}, T)$ and $\text{HWCD}_{t_{\text{var}}}(\mathbf{s} + t_1\mathbf{v}, \mathbf{v}, t_2 - t_1)$ for an arbitrary definition of a locally convex $\text{WCB}_{t_{\text{var}}}$.

Lemma 3. Consider the definition of $\text{WCD}_{t_{\text{var}}}$ in Section II for a locally convex $\text{WCB}_{t_{\text{var}}}$. For any relative horizontal state denoted by $\mathbf{s}, \mathbf{v}$, lookahead time T, and times $0 \leq t_1 < t_2 \leq T$, let $[\alpha_0, \beta_0] = \text{HWCD}_{t_{\text{var}}}(\mathbf{s}, \mathbf{v}, T)$ and $[\alpha_1, \beta_1] = \text{HWCD}_{t_{\text{var}}}(\mathbf{s}+t_1\mathbf{v}, \mathbf{v}, t_2 - t_1)$ be such that $\alpha_0 \leq \beta_0$, $\alpha_1 \leq \beta_1$, and $t_1 \leq \alpha_0$. Then, the following equation holds:

$$\alpha_0 = t_1 + \alpha_1. \tag{15}$$

The proof of Lemma 3 uses the fact that the function $\text{WCD}_{t_{\text{var}}}$ in (9) completely characterizes the time interval of violation for any locally convex $\text{WCB}_{t_{\text{var}}}$. This lemma provides sufficient conditions under which the time to horizontal well-clear violation linearly shifts at the same rate as time progresses in a nonmaneuvering pairwise encounter.

Now, the following theorem combines the results of Lemma 2 and Lemma 3 to give an overall upper bound for Δt_{in} for the three-dimensional case.

Theorem 2. Consider the definition of $\texttt{WCD}_{t_{\text{var}}}$ in Section II for a $\text{WCB}_{t_{\text{var}}}$, where $t_{\text{var}} \in \{t_{\text{cpa}}, \tau_{\text{mod}}, t_{\text{ep}}\}$, i.e., locally convex. For any relative state denoted by $\mathbf{s}$, $\mathbf{v}$, s_z, and v_z such that $B \le t_{\text{in}}^{\mathcal{M}_0} \le t_{\text{out}}^{\mathcal{M}_0} \le T$, the metric Δt_{in} satisfies the following relations.

$$0 \;\le\; \Delta t_{\text{in}} \;\le\; \begin{cases} 0 & \text{if } v_z = 0, \\ \frac{H_1 - H_0}{|v_z|} & \text{otherwise,} \end{cases} \tag{16}$$

where H_i, for $i = 0, 1$, is defined as in (13).

The proof of Theorem 2 proceeds by case analysis on the definition of $\texttt{WCD}_{t_{\text{var}}}$ in (9). In each case, Lemma 2 and Lemma 3 are used as appropriate.

The following corollary addresses the special case when $\text{TCOA}_0 = 0$.

Corollary 1. Let $\mathcal{M}_0, \mathcal{M}_1$ be such that $\text{TCOA}_0 = 0$ and $\text{TCOA}_1 \ge 0$. In this case, the upper bound in (16) reduces to

$$\Delta t_{\text{in}} \;\le\; \begin{cases} \frac{\text{TCOA}_1 |v_z| - \text{ZTHR}}{|v_z|}, & \text{if } \text{TCOA}_1 |v_z| > \text{ZTHR}, \\ 0, & \text{otherwise.} \end{cases} \tag{17}$$

A consequence of Corollary 1 is that the smallest vertical closure rate, μ, such that a particular choice of TCOA_1 for $\mathcal{M}_1$ is able to provide any earlier time to well-clear violation over $\mathcal{M}_0$ is

$$\mu \;>\; \frac{\text{ZTHR}}{\text{TCOA}_1}. \tag{18}$$

Fig. 1 illustrates the theoretical upper bound on Δt_{in} given by Corollary 1 for $\text{WCB}_{\tau_{\text{mod}}}$ when $B = 0$ and $T = 1201$ s. The particular threshold values for TTHR, DTHR, and ZTHR are 35 s, 1.5 nmi, and 450 ft, respectively. Furthermore, $\text{TCOA}_0 = 0$, and TCOA_1 is allowed to vary as indicated by the legend. By inspection of (17), for given values TCOA_1 and ZTHR, the maximum value for Δt_{in} is

$$\lim_{|v_z| \to \infty} \Delta t_{\text{in}} = \text{TCOA}_1. \tag{19}$$

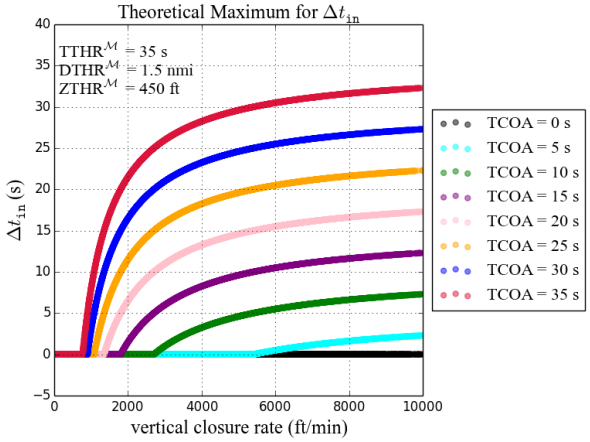

Fig. 1. Theoretical upper bound on Δt_{in} for $\text{WCB}_{\tau_{\text{mod}}}$.

Although values beyond $|v_z| = 10{,}000$ ft/min are not presented in Fig. 1, this limiting behavior can still be seen in the figure. Furthermore, Fig. 1 can be used to compare $\mathcal{M}_0$ and $\mathcal{M}_1$ when both models use nonzero TCOA. In particular, when $|v_z| > \frac{\text{ZTHR}}{\text{TCOA}_0}$, it can be verified that

$$\Delta t_{\text{in}} = \text{TCOA}_1 - \text{TCOA}_0. \tag{20}$$

For $|v_z|$ satisfying $\frac{\text{ZTHR}}{\text{TCOA}_1} < |v_z| < \frac{\text{ZTHR}}{\text{TCOA}_0}$, it can be shown that

$$\Delta t_{\text{in}} = \frac{\text{TCOA}_1 |v_z| - \text{ZTHR}}{|v_z|}, \tag{21}$$

and for $|v_z| < \frac{\text{ZTHR}}{\text{TCOA}_1}$, Δt_{in} is identically zero.

IV. Interoperability with TCAS II

The subsequent development presents the mathematical interoperability of a WCB model with the TCAS II RA logic. In particular, interoperability in the context of this paper implies that if a well-clear violation is predicted to occur within a given lookahead interval then the resulting time to WCV will precede the time to TCAS II RA, regardless of whether the RA is *preventive* or *corrective*.

As discussed in Section II, the high-level TCAS II RA logic for a given sensitivity level can be represented by a model $\mathcal{R} = \text{WCB}_{\tau_{\text{mod}}}$, where the threshold values of $\mathcal{R}$, denoted $\text{DTHR}^{\mathcal{R}}$, $\text{TTHR}^{\mathcal{R}}$, $\text{ZTHR}^{\mathcal{R}}$, and $\text{TCOA}^{\mathcal{R}}$, are set to the TCAS II threshold values from Table I, i.e., $\text{DTHR}^{\mathcal{R}} = \text{DMOD}$, $\text{ZTHR}^{\mathcal{R}} = \text{ZTHR}$, and $\text{TTHR}^{\mathcal{R}} = \text{TCOA}^{\mathcal{R}} = \text{Tau}$. Now, for any model $\mathcal{M} = \text{WCB}_{t_{\text{var}}}$, where $t_{\text{var}} \in \{\tau_{\text{mod}}, t_{\text{ep}}\}$, there exists a choice of threshold values of $\mathcal{M}$, denoted $\text{DTHR}^{\mathcal{M}}$, $\text{TTHR}^{\mathcal{M}}$, $\text{ZTHR}^{\mathcal{M}}$, and $\text{TCOA}^{\mathcal{M}}$, that makes $\mathcal{M}$ compatible with $\mathcal{R}$. To provide a measure of interoperability of $\mathcal{M}$ with the TCAS II RA logic, the time to WCV in the model $\mathcal{M}$, i.e., $t_{\text{in}}^{\mathcal{M}}$, is computed and subtracted from the time to TCAS II RA, i.e., $t_{\text{in}}^{\mathcal{R}}$, for a given encounter geometry. The resulting metric of interoperability is given as $\Delta t_{\text{in}}^{\mathcal{R}} \equiv t_{\text{in}}^{\mathcal{R}} - t_{\text{in}}^{\mathcal{M}}$. Interoperability requires that $\Delta t_{\text{in}}^{\mathcal{R}} > 0$. This definition motivates investigation of interoperability in terms of worst-case, i.e., minimum, bounds on $\Delta t_{\text{in}}^{\mathcal{R}}$.

Because TCAS II uses aircraft altitude to determine the sensitivity level, which in turn specifies the thresholds used to trigger an RA, there are a large number of interoperability cases to consider. However, the following conjecture is a preliminary result, which gives a general minimum bound on $\Delta t_{\text{in}}^{\mathcal{R}}$ for any sensitivity level, assuming unmitigated, constant-velocity NMAC trajectories, absent any TCAS II RA suppression logic. In particular, it is conjectured that the following four inequalities govern the minimum value $\Delta t_{\text{in}}^{\mathcal{R}}$:

- If $|v_z| \le \min(\frac{\text{ZTHR}^{\mathcal{M}}}{\text{TCOA}^{\mathcal{M}}}, \frac{\text{ZTHR}}{\text{Tau}})$,

$$\Delta t_{\text{in}}^{\mathcal{R}} \ge \frac{\text{ZTHR}^{\mathcal{M}} - \text{ZTHR}}{|v_z|}. \tag{22}$$

- If $\frac{\text{ZTHR}^{\mathcal{M}}}{\text{TCOA}^{\mathcal{M}}} \leq |v_z| \leq \frac{\text{ZTHR}}{\text{Tau}}$,

$$\Delta t_{\text{in}}^{\mathcal{R}} \geq \frac{\text{TCOA}^{\mathcal{M}}|v_z| - \text{ZTHR}}{|v_z|}. \tag{23}$$

- If $\frac{\text{ZTHR}}{\text{Tau}} \leq |v_z| \leq \frac{\text{ZTHR}^{\mathcal{M}}}{\text{TCOA}^{\mathcal{M}}}$,

$$\Delta t_{\text{in}}^{\mathcal{R}} \geq \frac{\text{ZTHR}^{\mathcal{M}} - \text{Tau}|v_z|}{|v_z|}. \tag{24}$$

- If $|v_z| \geq \max(\frac{\text{ZTHR}^{\mathcal{M}}}{\text{TCOA}^{\mathcal{M}}}, \frac{\text{ZTHR}}{\text{Tau}})$,

$$\Delta t_{\text{in}}^{\mathcal{R}} \geq \text{TCOA}^{\mathcal{M}} - \text{Tau}. \tag{25}$$

A key assumption in this conjecture is that the horizontal thresholds for $\mathcal{M}$ are larger than those for $\mathcal{R}$, and thus, lack of interoperability is due to artifacts in the vertical dimension, i.e., $\text{DTHR}^{\mathcal{M}} \geq \text{DMOD}$ and $\text{TTHR}^{\mathcal{M}} \geq \text{Tau}$.

Fig. 2 shows the theoretical minimum values for $\Delta t_{\text{in}}^{\mathcal{R}}$ and $\text{WCB}_{\tau_{\text{mod}}}$ as a function of $|v_z|$ for the particular case of TCAS II sensitivity level 4 and threshold values for $\mathcal{M}$ as indicated in the legend. Note that the plots for $\text{TCOA} = 0$ s and $\text{TCOA} = 5$ s coincide for the displayed range. Inspection of Fig. 2 illustrates that there are vertical closure rates where interoperability is not mathematically guaranteed for certain choices of TCOA. In particular, choosing $\text{TCOA}^{\mathcal{M}} \leq \text{Tau} = 20$ s results in the situation that mathematical interoperability is never guaranteed. Conversely, for $\text{TCOA}^{\mathcal{M}} > \text{Tau}$, mathematical interoperability is guaranteed beyond certain vertical closure rates. Furthermore, inspection of (22)-(25) shows that there are threshold values for $\mathcal{M}$ such that mathematical interoperability is always guaranteed. However, the fact that a choice of thresholds guarantees mathematical interoperability with the TCAS II RA logic does not necessarily lead to an operationally desirable definition for well clear.

V. Simulation

A. Design Approach

The Monte Carlo simulation design approach was primarily motivated by the lack of aircraft encounter models in civil airspace that sufficiently incorporate UAS flight characteristics [21] and the lack of broad operational experience with UAS in an integrated civil airspace [19]. While the encounter space was limited to unmitigated, constant-velocity NMAC trajectories as the base case of interest, the overall goal was to otherwise not overly constrain the simulation with assumptions beyond maintaining feasibility in the encounters. In particular, parameters were chosen from uniform distributions, where the upper and lower bounds were selected such that the full trade space of possible encounters could be explored. Furthermore, the simulation results afford a means to visually convey the formally verified effects of TCOA on Δt_{in} given in Section III and the conjectured effects of TCOA on $\Delta t_{\text{in}}^{\mathcal{R}}$ given in Section IV. In the absence of established operational experience with such encounters in the context of UAS in the civil airspace, these results may provide some basis for selecting a particular value for TCOA in a formal definition of well clear, or perhaps for omitting the vertical time threshold, entirely.

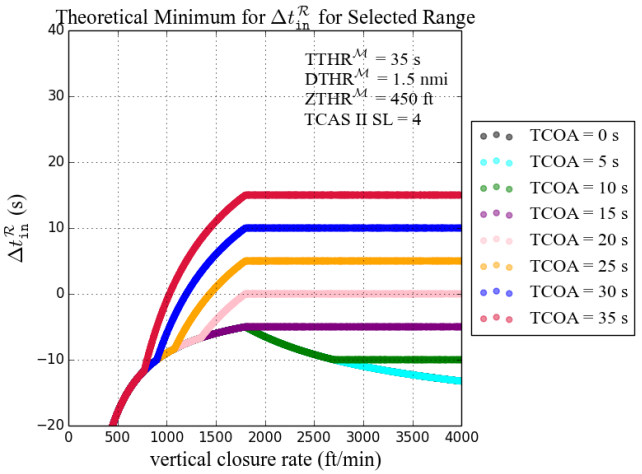

Fig. 2. Minimal vertical TCAS II RA interoperability.

The simulation was composed of eight independent Monte Carlo runs, where each of the eight runs considered a different value for TCOA. The first run set TCOA to zero, subsequent runs incremented TCOA by 5 seconds, and the final run set $\text{TCOA} = 35$ s. Each of the eight runs consisted of 5000 randomly generated aircraft-pair encounters determined by sampling from several characterizing uniform random variables as follows:

1) encounter duration, $t_{\text{e}} \in (0, 1200]$ s,
2) the two-dimensional relative horizontal speed, $\mathbf{v} \in (0, 1185]$ kn,
3) a point inside a collision volume, centered on the origin, and having parameters (r, ϕ, z), where $r \in [0, 500]$ ft, $\phi \in [0, 360]^\circ$, $z \in [-100, 100]$ ft,
4) an angle, θ, with respect to the horizontal plane such that $\theta \in [-90, 90]^\circ$,
5) the relative vertical speed, $v_z \in (0, 10000]$ ft/min,

where the choice to limit $|v_z|$ to 10,000 ft/min was based on the design of TCAS II, which is capable of detecting vertical closure rates up to 10,000 ft/min [23].

The parameters r,ϕ,z and θ were together used to determine the relative velocity vector and terminating point inside the collision volume, and the parameter t_{e} was used to set the initial relative position such that at time t_{e}, the final relative position was $-(r, \phi, z)$, i.e., the intruder position was the terminating point inside the collision volume. The lookahead interval for $\text{WCB}_{\tau_{\text{mod}}}$ was set to $[0, t_{\text{e}} + 1]$. Furthermore, only cases of *future* WCVs were considered, so that if the randomly generated scenario placed the aircraft into an immediate WCV, the scenario was discarded. The process was repeated until a given trial accumulated 5000 WCVs.

For each randomly generated encounter scenario, three models were considered: (1) the $\text{WCB}_{\tau_{\text{mod}}}$ model having $\text{TCOA} \geq 0$, i.e., $\mathcal{M}_1$, (2) the $\text{WCB}_{\tau_{\text{mod}}}$ model having $\text{TCOA} = 0$, i.e., $\mathcal{M}_0$, and (3) the $\text{WCB}_{\tau_{\text{mod}}}$ model having the TCAS II RA threshold values, i.e., $\mathcal{R}$. The metric Δt_{in} was determined by subtracting $t_{\text{in}}^{\mathcal{M}_1}$ from $t_{\text{in}}^{\mathcal{M}_0}$, and metric $\Delta t_{\text{in}}^{\mathcal{R}}$ was determined

by subtracting $t_{\text{in}}^{\mathcal{M}_1}$ from $t_{\text{in}}^{\mathcal{R}}$. The parameters for $\text{WCB}_{\tau_{\text{mod}}}$ were set as: $\text{DTHR}^{\mathcal{M}} = 1.5$ nmi; $\text{ZTHR}^{\mathcal{M}} = 450$ ft; and $\text{TTHR}^{\mathcal{M}} = 35$ s. The TCAS II parameters used in the simulation corresponded to sensitivity level 4, i.e., DMOD $= 0.35$ nmi; ZTHR $= 600$ ft; and Tau $= 20$ s. The greater TCAS II ZTHR value allows for TCAS II RAs to be issued prior to a WCV for certain vertical closure rates, however, a 500 ft vertical miss distance is generally acceptable for safe air traffic procedures flying under VFR [29]. This tradeoff in selecting ZTHR was made in order to provide a practical visualization of the implications associated with selecting particular values for TCOA.

B. Simulation Results and Discussion

Fig. 3 illustrates the simulation results for Δt_{in}, as derived in Section V-A, for several values of TCOA. In particular, $\mathcal{M}_0$ has TCOA $= 0$ and $\mathcal{M}_1$ has TCOA as indicated in the legend. The results provide a means to compare how different models $\mathcal{M}_1$ compare to a nominal model, $\mathcal{M}_0$, which does not use a vertical time threshold. The simulated encounters obey the theoretical upper bounds illustrated in Fig. 1.

It can be verified that for any TCOA_1 satisfying $0 < \text{TCOA}_1 < \text{TTHR}^{\mathcal{M}}$ there is some vertical closure rate, μ, such that an earlier time to WCV is possible. In Section III, it was shown that for any TCOA, it is the case that $\mu > \text{ZTHR}^{\mathcal{M}}/\text{TCOA}_1$. When $\text{TCOA}_1 = 0$, Δt_{in} is identically zero, and no earlier time to WCV is possible. For the case when $\text{TCOA}_1 = \text{TTHR}^{\mathcal{M}}$, the simulation results in occasions when the upper bound earlier time to WCV is not achieved. This case illustrates the subtractive effect of the horizontal component on Δt_{in}, a consequence of Lemma 3. However, Fig. 3 also shows that Δt_{in} closely follows the theoretical upper bound. Some key results illustrated in Fig. 3 follow.

For the case when $\text{TCOA}_1 = 25$ s, the smallest vertical closure rate such that $\mathcal{M}_1$ can provide an earlier time to WCV is greater than 1080 ft/min. Furthermore, for $\mathcal{M}_1$ a 16 s earlier time to WCV over $\mathcal{M}_0$ is not only possible, but is the case for vertical closure rates of 3000 ft/min. Moreover, for a 3000 ft/min vertical closure rate, $\mathcal{M}_0$ gives a time to WCV 7 s prior to NMAC, while $\mathcal{M}_1$ gives a time to WCV 23 s prior to NMAC.

Now, for the same threshold values, if the vertical closure rate increases to 6000 ft/min then $\mathcal{M}_0$ gives a time to WCV 3.5 s before NMAC, and $\mathcal{M}_1$ gives a time to WCV 24 s before NMAC. Although pilot or automation reaction time is beyond the scope of this paper, it is assumed that there is some need to account for such latencies in threshold selection, and that an earlier time to WCV can provide some means of mitigation. Conversely, too large a vertical time threshold may have undesirable operational implications.

Next, Fig. 4 shows the results of the Monte Carlo simulation with respect to $\Delta t_{\text{in}}^{\mathcal{R}}$. In particular, the results depicted in Fig. 4 correspond to the same encouter set used to generate Fig. 3. At higher vertical closure rates, the value $\Delta t_{\text{in}}^{\mathcal{R}}$ is dominated by TCOA $-$ Tau and thus exhibits somewhat constant behavior. This limiting behavior first occurs when $|v_z| > \max(\text{ZTHR}/\text{Tau}, \text{ZTHR}^{\mathcal{M}}/\text{TCOA})$, or 1800 ft/min.

To better illustrate the critical points determined by TCOA, a restricted set of data is presented in Fig. 5. Now, comparing Fig. 5 with the conjectured theoretical minima shown in Fig. 2, it can be verified that the conjectured minima are satisfied. Furthermore, at lower vertical closure rates the overall interoperability picture is similar for all values of TCOA. Further note that for $|v_z| \geq 1800$ ft/min, the conjectured minima in the limit not only hold but are also useful approximations of the simulation results for $\Delta t_{\text{in}}^{\mathcal{R}}$. Finally, the particular threshold values used for TCAS II also illustrate that there exist some mathematical interoperability implications for any choice of TCOA.

Fig. 6 further restricts the plotted data set from the same Monte Carlo simulation to two particular TCOA values of interest, i.e., $\text{TCOA}_0 = 0$ s and $\text{TCOA}_1 = 25$ s. Fig. 6 also reinforces the general picture of similar TCAS II interoperability for both models at lower vertical closure rates. However, as

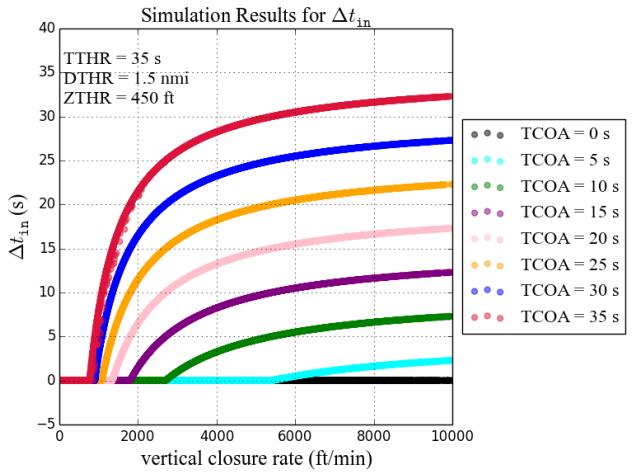

Fig. 3. Sensitivity of Δt_{in} to TCOA for the $\text{WCB}_{\tau_{\text{mod}}}$ Model.

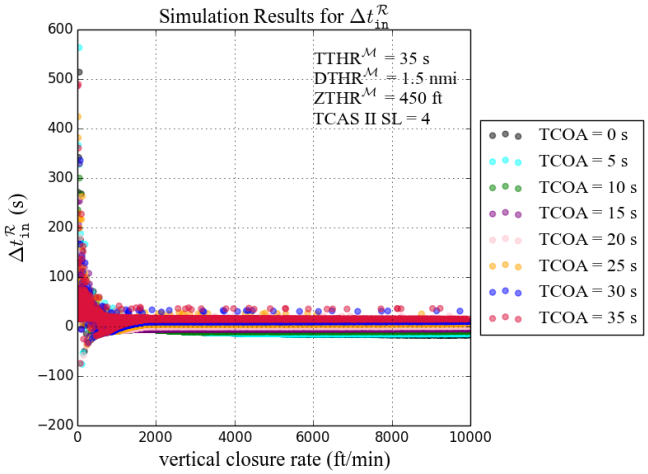

Fig. 4. Interoperability of the $\text{WCB}_{\tau_{\text{mod}}}$ Model with TCAS II.

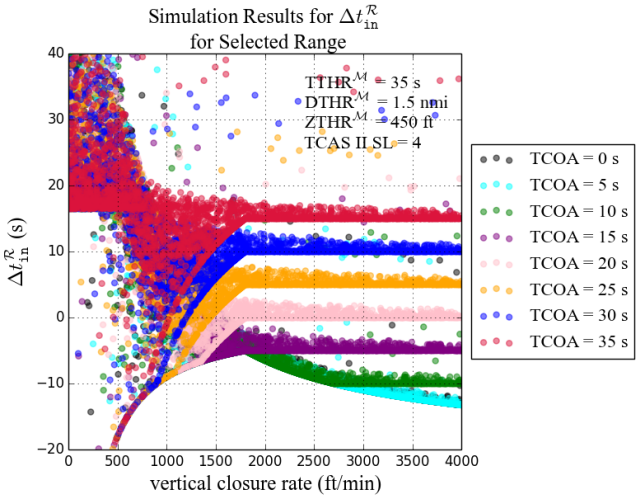

Fig. 5. Selected Range for the Interoperability of the WCB$_{\tau_{\text{mod}}}$ Model with TCAS II.

the vertical closure rate increases, the dissimilarity of the two models becomes pronounced. For TCOA$_1$ = 25 s, guaranteed TCAS II interoperability first occurs at $|v_z| > 1440$ ft/min and is maintained for all higher vertical closure rates, while for TCOA$_0$ = 0 s, TCAS II interoperability is never guaranteed. Indeed, the simulation results illustrate that TCOA$_0$ = 0 s is closely approximated by the theoretical minimum where TCAS II RAs will be issued prior to a WCV.

At higher vertical closure rates, the interoperability differences associated with TCOA$_0$ and TCOA$_1$ become clear. For example, at 3000 ft/min vertical closure rate, TCOA$_1$ provides, at a minimum, a time to WCV 5 s prior to TCAS II RA, whereas TCOA$_0$ provides, at a minimum, a time to WCV 11 s *after* a TCAS II RA has been issued. Furthermore, consider the case of a 6000 ft/min vertical closure rate. In this case, TCOA$_1$ still generally provides a 5 s earlier time to WCV than TCAS II RA, however, the situation for TCOA$_0$ worsens to $\Delta t_{\text{in}}^{\mathcal{R}} = -17.75$ s.

Attempting to alleviate such interoperability concerns for $\mathcal{M}_0$ by adjusting other threshold parameters, e.g., ZTHR$^{\mathcal{M}}$, is problematic. For example, consider the case of raising ZTHR$^{\mathcal{M}}$ for $\mathcal{M}_0$ and $\mathcal{M}_1$ to 700 ft. to be above the TCAS II ZTHR at sensitivity level 4. Fig. 7 illustrates the results. These results indicate that while positive interoperability is offered for both models at modest vertical closure rates, $\mathcal{M}_0$ again faces interoperability problems at vertical closure rates of approximately 2000 ft/min. Mitigating feasible vertical closure rate encounters in such a way may also be operationally prohibitive. The picture for $\mathcal{M}_1$ improves to guaranteeing interoperability over all vertical closure rates.

The simulation results demonstrate that for unmitigated, constant-velocity NMAC trajectories, a well-clear definition using horizontal and vertical time and distance thresholds, such that TCOA $<$ Tau, generally does not offer a protective barrier outside of the TCAS II collsion avoidance logic. That is, in such encounter geometries, the well-clear boundary collapses to the TCAS II RA threshold. Thus, under these circumstances, the formal definition of well clear does not serve as a mitigation to the collision avoidance function. These results may provide some basis in exploring tradeoffs between desireable, verifiable safety properties and practical operational considerations in the selection of an appropriate vertical time threshold in a formal definition of well clear.

VI. Conclusion

The safe integration of UAS into civil airspace will require that unmanned aircraft be capable of detecting and avoiding other aircraft in a way that is compatible with existing manned aircraft operations. That is, unmanned aircraft should behave in a way that is relatively predictable to other aircraft, particularly manned aircraft, in terms of well clear and collision avoidance functionality. The class of WCB models considered

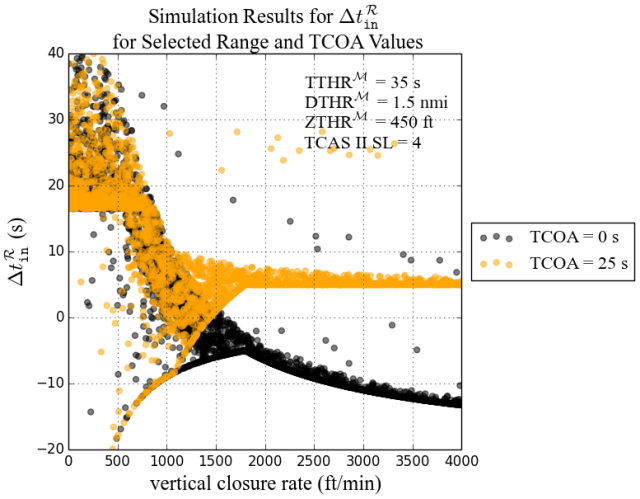

Fig. 6. Selected Range and Thresholds for the Interoperability of the WCB$_{\tau_{\text{mod}}}$ Model with TCAS II.

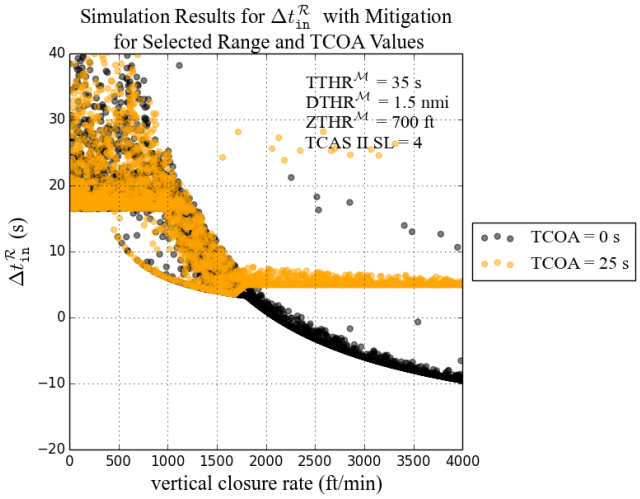

Fig. 7. Selected Range and Thresholds for the Interoperability of the WCB$_{\tau_{\text{mod}}}$ Model with TCAS II, new ZTHR$^{\mathcal{M}}$.

in this paper were investigated for their suitability for meeting such interoperability requirements with respect to a vertical time threshold. The primary motivation for the investigation resulted from the observation that the well-clear definitions explicitly allowed for decoupling of the horizontal and vertical time thresholds. This decoupling makes it possible to eliminate either or both of the time thresholds. Since elimination of a horizontal time threshold was not considered to be a likely characteristic in any practical definition of well clear, the investigation took the approach of directly analyzing the role of the vertical time threshold in the formal definition. A primary goal of the paper was to provide suitable intuition to the separation community and designers of detect and avoid systems on the interoperability consequences associated with particular choices of vertical time thresholds.

A closed-form, upper-bound relationship for earlier time to WCV was given as a function of vertical-closure rate for different choices of vertical time thresholds. This relationship was formally verified to hold for unmitigated, constant-velocity NMAC trajectories, and simulation results were shown to illustrate the consistency of the theory with the randomly generated encounters. The simulation reveals some important operational tradeoff considerations in terms of interoperability with manned aircraft expectations of what is well clear, particularly for higher vertical time thresholds. For example, at a vertical closure rate of 3000 ft/min, a UAS relying on the formal definition can still be well clear 7 s before NMAC, well after a human pilot has likely initiated collision avoidance maneuvers, absent intent information, i.e., unmitigated. The closed-form expressions provided in this paper may serve as tools for further assessment of the implications of particular choices of vertical time thresholds on a formal well-clear definition.

Furthermore, a preliminary result was given, in which a set of closed-form relationships was conjectured to govern the mathematical interoperability with existing TCAS II issuance of RAs for unmitigated, constant-velocity NMAC trajectories. While corrective RAs are of particular concern, this paper investigated interoperability for both cases of corrective and preventative RAs at TCAS II sensitivity level 4. However, further analysis is possible at sensitivity levels of interest through the conjectured theoretical minima for the TCAS II interoperability metric. It was demonstrated that these minima are satisfied by all of the simulated encounters and that they were reasonably approximated by $\Delta t_{\text{in}}^{\mathcal{R}}$ at higher vertical closure rates. At TCAS II sensitivity level 4, all choices of TCOA result in roughly the same mathematical interoperability characteristics at lower vertical closure rates. For these higher vertical closure rates, mathematical interoperability for the particular thresholds considered is guaranteed when TCOA $>$ Tau. It was also shown that increasing ZTHR may offer some improvement for interoperability, however these gains are lost at higher vertical closure rates.

The mathematical development presented in this paper, including definitions and theorems, has been specified and verified in the interactive theorem prover, PVS. A theorem prover is a computer program that provides a specification language and a logic engine that checks every deduction step of a mathematical proof. This verification process is particularly time consuming, but justified by the safety-critical nature of sense and avoid in the future integration of UAS into civil airspace.

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