STATISTICAL ISSUES FOR CALCULATING REENTRY HAZARDS

John B. Bacon(1) and Mark Matney (2)

(1)*Orbital Debris Program Office, NASA Johnson Space Center, Houston, TX, USA, john.bacon-1@nasa.gov*
(2)*Orbital Debris Program Office, NASA Johnson Space Center, Houston, TX, USA, mark.matney-1@nasa.gov*

ABSTRACT

A number of statistical tools have been developed over the years for assessing the risk of reentering object to human populations. These tools make use of the characteristics (e.g., mass, shape, size) of debris that are predicted by aerothermal models to survive reentry. This information, combined with information on the expected ground path of the reentry, is used to compute the probability that one or more of the surviving debris might hit a person on the ground and cause one or more casualties.

The statistical portion of this analysis relies on a number of assumptions about how the debris footprint and the human population are distributed in latitude and longitude, and how to use that information to arrive at realistic risk numbers. This inevitably involves assumptions that simplify the problem and make it tractable, but it is often difficult to test the accuracy and applicability of these assumptions.

This paper builds on previous IAASS work to re-examine one of these theoretical assumptions.. This study employs empirical and theoretical information to test the assumption of a fully random decay along the argument of latitude of the final orbit, and makes recommendations how to improve the accuracy of this calculation in the future.

1. INTRODUCTION

One of the hazards of the space age is that many objects in orbit eventually reenter the Earth's atmosphere. For many large objects, components can survive reentry and pose a hazard to persons on the ground. A whole science has developed around predicting what portions of a satellite survive the violent forces and heating of reentry.

For controlled reentries, the reentry target zone can be chosen to avoid populated areas on the Earth. For uncontrolled reentries, however, statistical tools must be used to map human distributions on the Earth under the satellite orbit. The exact time and location for uncontrolled reentries are notoriously difficult to predict with any accuracy. In the hours immediately before a reentry, it may be possible to narrow the possible ground tracks of the reentering object. But when the risk calculation is needed weeks or months before reentry, essentially any location on the Earth over which the satellite flies is a potential reentry landing site. Nevertheless, it is possible to use even this vague information to compute meaningful risk statistics.

2. REENTRY MODEL

Under the assumption that a satellite is reentering randomly, computations to date have assumed that there is no preferred position along the orbit that the reentry occurred. Using the simplification of a circular orbit, this assumption is that the argument of latitude (equal to the argument of perigee plus the true anomaly) is randomly distributed. This is a very good assumption if the Earth were a perfect sphere.

Using the satellite inclination, i, and the argument of latitude θ, the latitude λ can be computed

$$\lambda = ArcSin(Sin(i)\ Sin(\theta))$$

Assuming that any possible value of the argument of latitude is equally likely, the distribution in argument of latitude will be

$$P(\theta)d\theta = \frac{d\theta}{2\pi}$$

which corresponds to a normalized distribution in latitude of

$$P(\lambda)d\lambda = \frac{1}{\pi}\frac{Cos(\lambda)}{\sqrt{Sin(i)^2 - Sin(\lambda)^2}}d\lambda$$

where

$$-Sin(i) \le Sin(\lambda) \le Sin(i)$$

This distribution in randomized latitude is integrated over the latitude distribution of population on the Earth to get an average number of human beings underneath a particular orbit. This can be combined with assumptions on the size of the surviving debris pieces and the average size of human beings to compute a casualty expectation, E_c. This value is then used to determine the risk to humans on the ground [1],[2].

This set of assumptions assumes the Earth's gravitational field is that of a perfect sphere. In actuality, the Earth's gravitational field can be described by spherical harmonic terms that become more important to an orbit as it gets closer to the Earth's surface, which is what actually happens in the final stages of orbital decay.

In addition, the Earth itself is not a perfect sphere, but an oblate spheroid, with the equatorial radius slightly larger than the polar radius. Because the atmospheric density is primarily a function of altitude above the planet's surface, that means that a satellite crosses an enhancement of atmospheric density, and corresponding atmospheric drag, as it crosses the equator – an atmospheric "speed bump" that may affect the location of the final reentry.

In the case of a near-polar circular orbit, the difference is 21.4 km in local altitude from the pole to the equator, in a final decay region where the density scale height is only 5.4 km. This would result in a theoretical 50-fold difference in the local deceleration as the satellite crosses the equator. In the simplifying assumption of a delta function of density at the equator, one sees that the resulting decay would behave like a series of Hohmann transfers that would tend to force the final perigee of the orbit to the equator.

3. REENTRY MODEL CALCULATIONS

In an effort to quantify the effects of these effects, we have simulated the decay of satellites in NASA's Generalized Mission Analysis Tool (GMAT): a high-fidelity mission-certified software environment using the 4^{th} order Earth spherical harmonics, the MSIS-E 90 atmosphere, and a 9^{th} order Runge-Kutta orbit propagator. Decaying orbits were simulated from 200 km altitude with initial conditions set to minimize J_2 spherical harmonic effects on the radius vector: i.e., as nearly circular as the gravitational perturbations would allow. For moderate ballistic number, this resulted in approximately 40 orbits before final decay. A typical relative density along the decaying orbit is shown in Figure 1, with the density plotted as a function of latitude in the gradually decaying orbit, and Figure 2, with relative density plotted as a function of time, showing a significant pulsing near the equatorial regions of the drag rate that a satellite will experience as it decays.

The initial mass of the satellite was slowly varied over a total range of less than ±1.1% from the median value in uniform small steps of 0.025% to create a full 360 degree spread along the orbit of final decay locations ("decay" defined as 85 km). With such small steps over a limited range, a uniform step in decay location around the orbit would be expected, and any deviation from that spread should be the contribution of non-uniformities in the spherical-Earth model, such as the atmospheric bulge and gravitational perturbations.

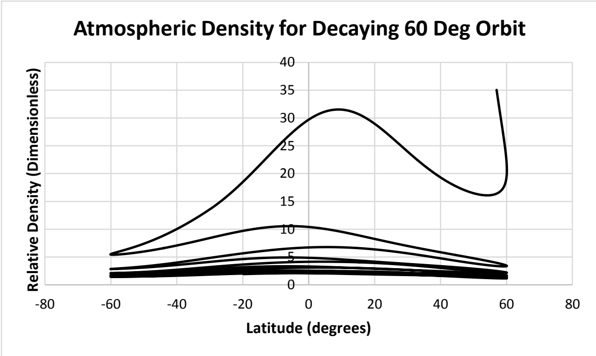

Figure 1 – As a satellite decays, it encounters varying atmospheric density. Here the density is plotted as a function of satellite latitude. Note how the atmospheric density is higher near 0º degrees latitude, near the equator.

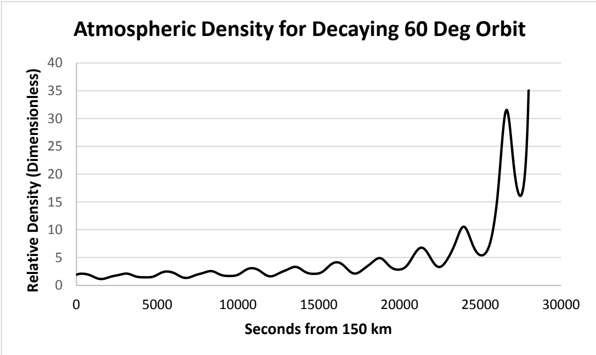

Figure 2 – This plot shows the changing atmospheric density as in Figure 1, but here it is plotted as a function of time, showing the repeating "pulses" in atmospheric density.

The integrated reentry location results show a bias of decay latitude towards the equator that increases with orbit inclination. The bias is symmetric such that there is no apparent tendency to reenter on the approach to the equator compared to departure from it. The difference between the simple spherical Earth model and the numerical integration is shown in Figure 3. The difference between the two models is approximated very well by a pure sine wave ±90º of arc around a nodal crossing. The perturbation is sinusoidal at twice the orbit rate, such that the perturbation is zero at the latitude extremes and at the equator.

As shown in Figure 4, the amplitude of the sinusoidal perturbation geometrically increases with the square of the orbit inclination, and reaches a peak amplitude of 11º bias in the case of a polar orbit, but only 3.5º for a 51.6º inclination orbit typical of Russian spacecraft. These calculations are all for the same 4050 kg, 14.6 m^2 hypothetical object decaying from low altitude with free molecular Cd of 2.07 (Ballistic Number = 134 kg/m^2). Initial studies show that the amplitude of the perturbation is less when the object is exceptionally low ballistic number, executing its final plunge from an altitude where the density scale height is larger, and the relative equatorial density perturbation consequently lower. The case of decay latitude for a 134 kg/m^2 object is shown for a polar orbit in Figure 3.

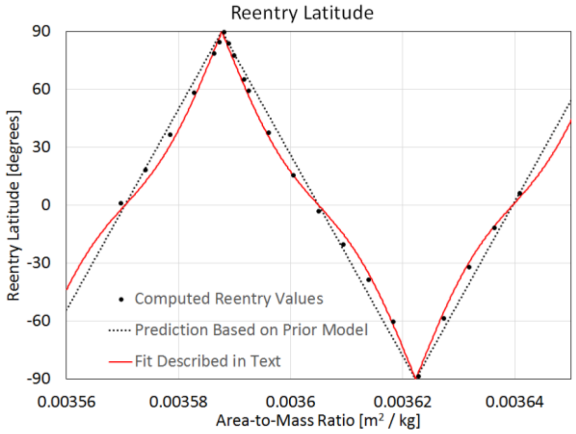

Figure 3 – Comparison of the calculated reentry locations for a polar orbit compared to the "prior model" based on a spherical Earth.

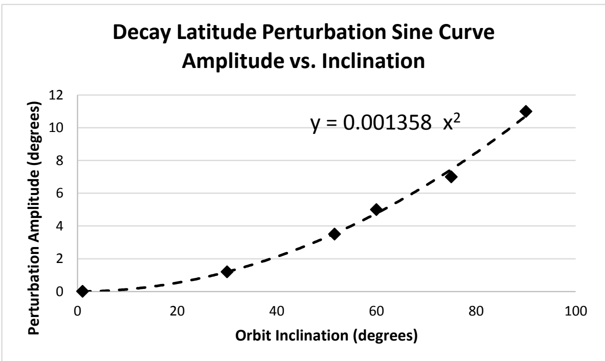

Figure 4 – The amplitude of the difference between the calculated reentry locations and the "prior model" based on a spherical Earth.

An extensive statistical study over different values of argument of latitude and inclinations gives the following empirical equation for the realistic decay latitude λ for a 134 kg/m^2 object.

$$\lambda = ArcSin\big(Sin(i)\,Sin(\theta)\big) - 0.001358°\,(i°)^2\,Sin(2\theta)$$

Our future work is to more fully evaluate the bias as a function of object ballistic number, and to evaluate a broader range of initial eccentricities and arguments of perigee.

4. EMPIRICAL DATA

In order to test the calculations against actual data, information on actual reentries were gathered from the final TIP decay messages for satellites in groups of common inclination orbits available from the Space-Track public database of the US Joint Space Operations Command's (JSpOC) [3]. Only TIP messages that posted after the declared decay time were gathered. There is reason to believe that not all the final reentry locations are reported accurately, and therefore may contain noise. However, the overall distribution of the reentry locations should provide useful data.

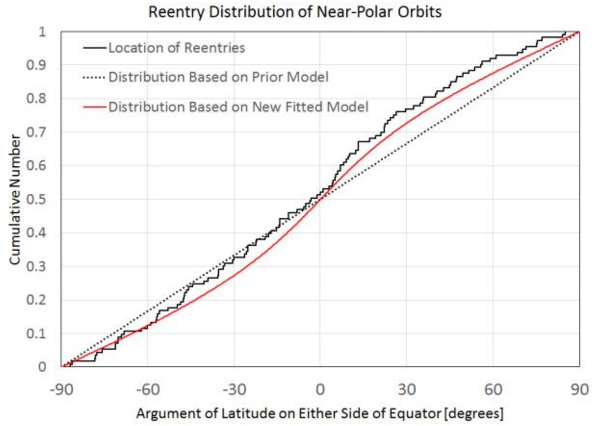

Figure 5 – The ordered cumulative distribution of 113 historical reentry locations for a set of near-polar orbiting satellites are shown here. To remove the inclination differences between the objects, the reentry latitudes have been converted to argument of latitude, and the range converted to between -90° (the southernmost point in the orbit) and +90° (the northernmost point in the orbit). Also shown are the straight-line curve expected under the spherical-Earth approximation and the equator-biased distribution described in the text. Statistical tests on the distribution indicate the data is outside the 2-sigma standard deviation for the prior model, but well within the 1-sigma range for the new fitted model, indicating that the distribution of actual reentries are consistent with the model described in the text.

Reentry data on 113 historical near-polar reentries are plotted in Figure 5, with the latitude converted into the argument of latitude in order to remove the effects of the different inclinations, and limited to ±90° to be analogous to latitude. Also shown are the prior model predictions assuming a spherical Earth, and the distribution computed using the formula described above, with reentries biased toward the equator. Under the prior model, the data should lie along a straight line with minor statistical scatter.

Using the Kuiper statistical test (described in [2]), the data curve lies outside the 2-sigma standard deviation for the prior model expected from sampling error, indicating that there is a statistically significant bias away from the prior model. When the same statistical test is applied to the comparison of the data to the new model described in the text, the data is now within 1-sigma. It appears that a bias similar to what we predict is being observed in actual reentry latitudes.

5. REENTRY GEOMETRY

Although the latitude bias varies geometrically, the actual perturbation along track varies more linearly as orbit inclination increases. This is because from simple geometry the flight path length to achieve a given latitude bias varies like the size of the bias divided by the sine of the inclination. The peak "along track" perturbation ranges from about 800 km to 1220 km as inclination increases. To see how significant this effect really is, one can ask what sort of ΔV would be required to achieve the same result. Under Clohessy-Wiltshire relations in zero atmosphere, one can see that downrange deflection of 500 km (500,000 meters) in a 51.6º inclination orbit, initiated one-half orbit before (approximately 2600 seconds downrange at low decay altitude) would result from an impulsive ΔV of about 64 meters/second. This is 42% of the ΔV accumulated during the final 2600 seconds of decay before the orbit reaches 85 km altitude.

The authors propose that this integral effect is significant enough that spacecraft with finite but inadequate ΔV for a direct plunge to a targeted entry may be able to strongly influence the final target location through minor orbit shaping that would take advantage of the driving density "bump" during the final orbit. In general, this would make it easier to target regions closer to the equator than farther from it, given natural biasing of the decay location towards there.

6. POPULATION RISK

The next obvious question is how this biasing in the reentry location affects the average population density beneath the orbit. Figure 6 shows the population density as a function of latitude once the longitude dependence has been averaged out. Note the preponderance of population in the northern hemisphere and the lack of population near the poles [4],[5].

To find the average density of humans beneath an orbit, this population density is integrated over the probability that the satellite will reenter at each latitude. Figures 7 and 8 show the result of this calculation using both the spherical-Earth random-angle approximation and the new biased equation derived from the GMAT simulations. The two curves are plotted for 2020 population data.

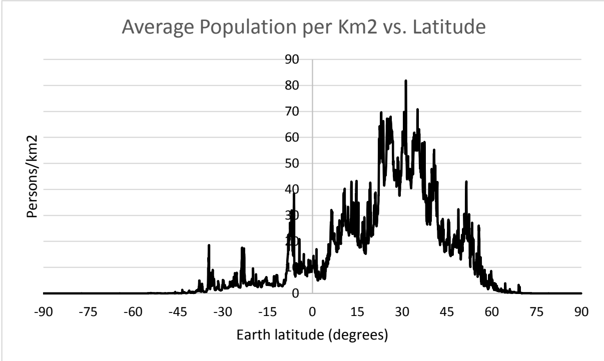

Figure 6 – the average population density as a function of latitude on the Earth (longitude dependence removed) based on the SEDAC World Population database. (2020 estimates are shown.)

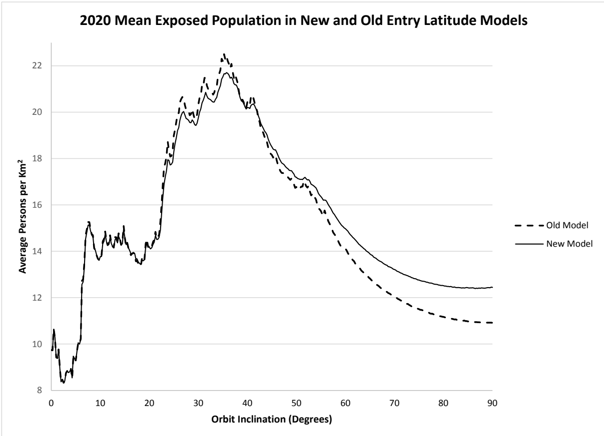

Figure 7 – Two curves that show the result of integrating orbits of different inclinations over the year 2020 population distribution in Figure 6, weighting the calculation by the probability that a satellite will reenter at a particular latitude. "Old model" refers to the spherical-Earth approximation, and "New model" refers to the new equations described in this paper. Note that for orbits with inclinations above 90º, the result will be the same as those with inclination equal to 180º minus the inclination.

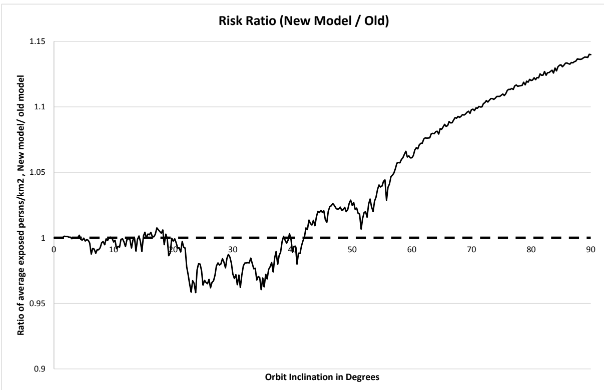

Figure 8 – This curve shows the ratio between the two curves in Figure 7.

There are two primary changes in the computed population densities. Because of the biases away from the poles and toward the equator, the new model gives a higher population density for high-inclination orbits > 42º. On the other hand, the regions between about 20º and 40º inclination, where the population density is a maximum, the new equations give a slight reduction in population density below an orbit. Lower inclination orbits do not show major differences between the two models.

Note that the updated calculations show changes in population densities at mid-latitudes of no larger than 4.5%, but the high-latitude changes reach nearly 14%.

7. CONCLUSIONS

The calculations presented in this paper have shown that calculations using a realistic non-spherical Earth and atmosphere result in a bias toward reentries near the equator. This results in modifications in the computed average density of humans that would be under a reentering satellite. At high latitudes, this results in a noticeable increase in risk, but a small reduction at mid-latitudes. While further statistical studies are needed to confirm these results and to evaluate for a range of ballistic numbers and initial orbits, this study points to a need to include these effects in computing future ground risks.

This equatorial “bulge” in the atmosphere could also be used by satellites undergoing controlled reentry to enhance their reliability of reentering over uninhabited regions, especially in cases where limited ΔV is available for the disposal operation.

8. REFERENCES

1. Matney, M. J., “Statistical Issues for Uncontrolled Reentry Hazards,” 3rd IAASS Conference, Rome, Italy, 2008.

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3. Historical reentry data (TIPS) can be obtained from the space-track web site at *www.space-track.org*

4. The SEDAC World Population Databases and their documentation can be found on the internet at *http://sedac.ciesin.columbia.edu/*

5. Opiela, J. N., & Matney, M. J. (2003). Improvements to NASA’s Estimation of Ground Casualties from Reentering Space Objects. Space Debris and Space Traffic Management Symposium 2003, Bremen, Germany, In *Proceedings of the International Academy of Astronautics Space Debris and Space Traffic Management Symposium held in conjunction with the 54th International Astronautical Congress* (Ed. J. Bendisch), IAA 03-5.4.03, Volume 109, Science and Technology Series, Univelt Publishers, San Diego, California, pp385-392.