Signal to Noise Ratio Estimation of the ASCENDS CarbonHawk Experiment Simulator (ACES) for Atmospheric CO_2 Measurement

Songsheng Chen[1], Larry B. Petway[1], Yingxin Bai[2], Fenton W. Harrison[1], Joel F. Campbell[1], Byron L. Meadows[1], Bing Lin[1], Michael D. Obland[1], and Edward V. Browell[3]

[1]NASA Langley Research Center, 5 N Dryden Street, Hampton, VA 23681

[2]Science Systems & Applications Inc.,1 Enterprise Parkway, Hampton, VA 23666

[3]STARSS II Affiliate NASA Langley Research Center

SPIE REMOTE SENSING

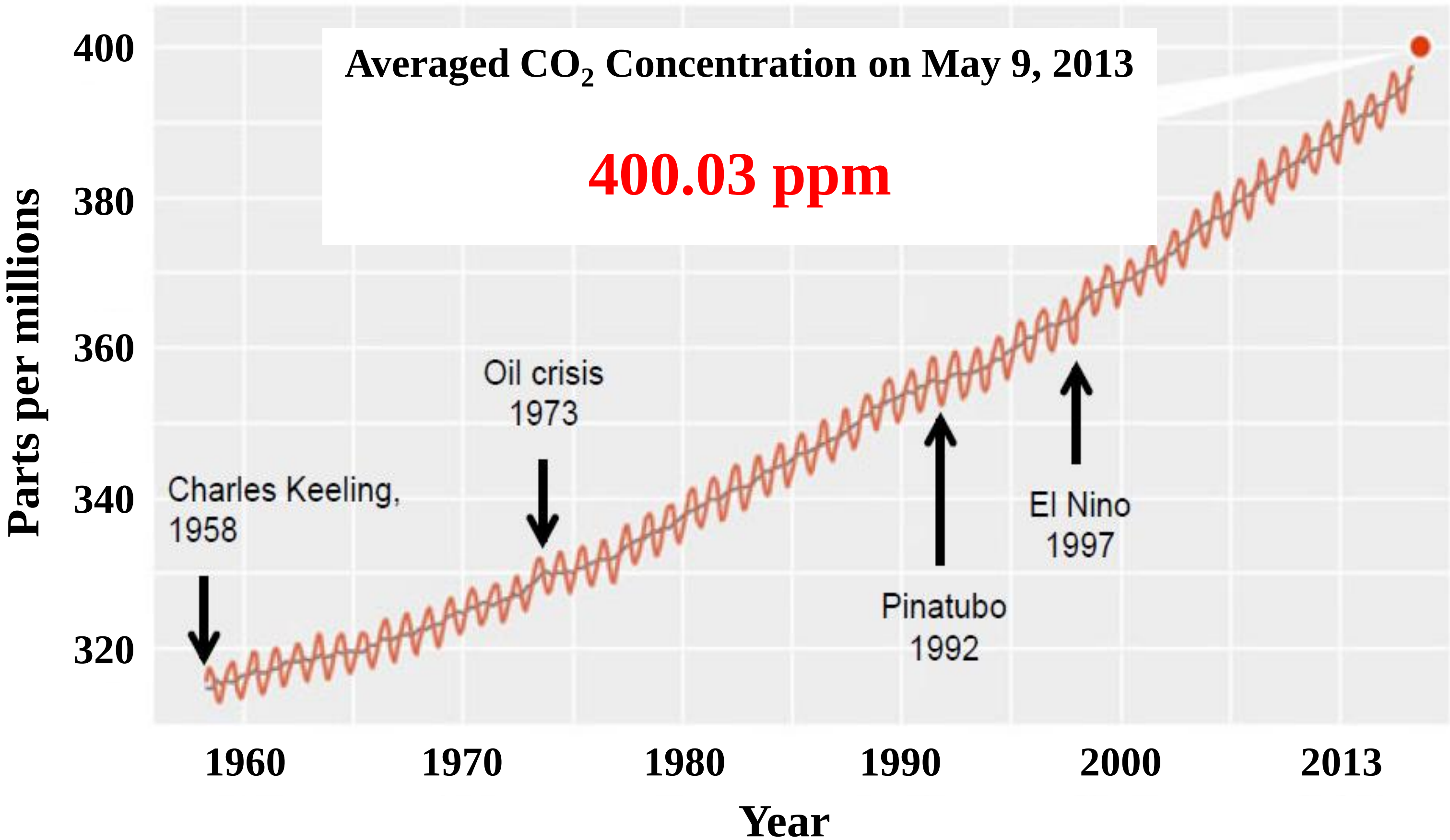
Toulouse France, Sept. 21-24, 2015

Outlines

- **Introduction**
- **Maximum Likelihood Estimation (MLE) for Multiple Linear Swept-Frequency Sine-Wave Detection**
- **An Airborne Multiple Swept-Frequency (SF) Intensity-Modulated Continuous-Wave (IM-CW) ASCENDS CarbonHawk Experiment Simulator (ACES)**
  - Multiple swept-frequency IM-CW laser transmitter
  - Multiple swept-frequency IM-CW digital detection
  - MLE of multiple swept-frequency digitized IM-CW signals
- **Conclusions and Discussions**

Introduction-1

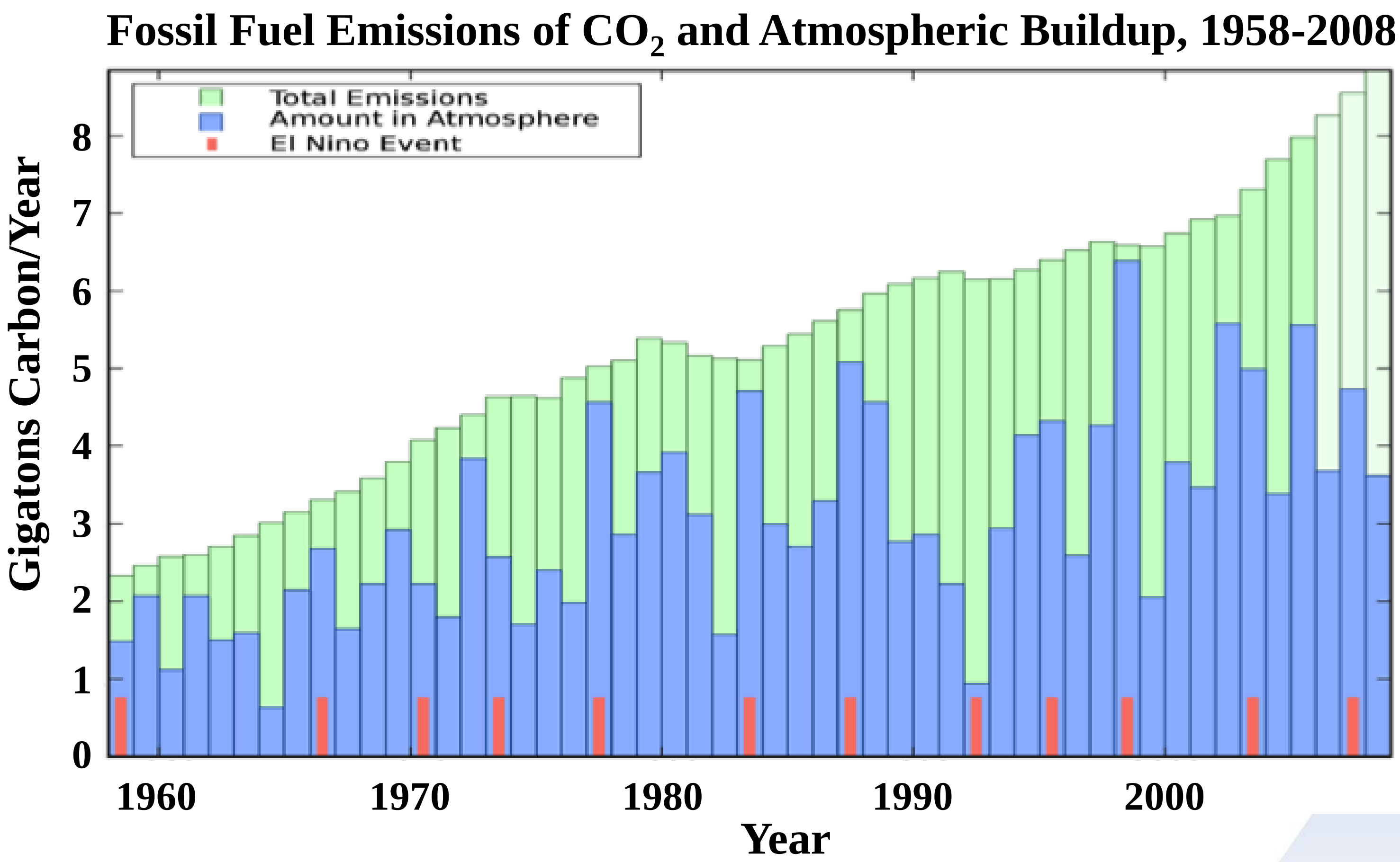
Carbon Dioxide (CO_2) Concentration in the Atmosphere (Increase)



Based on NOAA/Scripps Institution of Oceanography

Introduction-2

CO_2 Sources and Sinks (Missing?)



Based on LeQuere et al., 2009

σ^2

Maximum Likelihood Estimation (MLE) -1

- MLE for analysis of the parameters of multiple-frequency digitized sine-wave signals based on a statistical model, all the digitized signals, $S(n)$, is sum of the expectation value of the signals and Gaussian-distributed random noise with the probability,

$$\Pr(S(n)) = \frac{1}{\sqrt{2\pi\sigma_n^2}} \exp\left(-\frac{\left(S(n) - E\langle S(n)\rangle\right)^2}{2\sigma_n^2} \right) \qquad n = 0, 1, 2, \ldots N_s - 1$$

$E\langle S(n)\rangle$ is expected value of $S(n)$ and σ_n^2 is variance of the noise.

- MLE formula

$$\underset{over\ all\ variables}{\arg\max}(L) = \underset{over\ all\ variables}{\arg\max}\left(\prod_{n=0}^{N_s-1} \left(\frac{1}{\sqrt{2\pi\sigma_n^2}} \right) \exp\left(-\frac{\left(S(n) - E\langle S(n)\rangle\right)^2}{2\sigma_n^2} \right) \right)$$

Maximum Likelihood Estimation (MLE) -2

- Maximum Log-Likelihood Estimation Function l

$$\underset{over\ all\ variables}{\arg\max}(L) = \exp\left(\underset{over\ all\ variables}{\arg\max}(\ln(L))\right)$$

$$= \exp\left(\underset{over\ all\ variables}{\arg\max}\left(-\frac{1}{2}\sum_{n=0}^{N_s-1}\left(2\pi\sigma_n^2\right) - \sum_{n=0}^{N_s-1}\left(\frac{\left(S(n) - E\langle S(n)\rangle\right)^2}{2\sigma_n^2}\right)\right)\right)$$

$$\equiv \exp\left(\underset{over\ all\ variables}{\arg\max}(l')\right)$$

$$l' = \left(-\frac{1}{2}\sum_{n=0}^{N_s-1}\left(2\pi\sigma_n^2\right) - \sum_{n=0}^{N_s-1}\left(\frac{\left(S(n) - E\langle S(n)\rangle\right)^2}{2\sigma_n^2}\right)\right)$$

Maximum Likelihood Estimation (MLE) -3

- Multiple Digitized Sine Wave Signals (numbers of k)

$$S(n)=\frac{1}{f_s}\left(S_c^0+\sum_{i=1}^{k}S_i^0\cos\left(2\pi\left(f_i^m+\frac{f_\Delta^m}{2}\frac{n}{f_s}\right)\frac{n}{f_s}+\varphi_i\right)+N_G\left(\frac{n}{f_s}\right)\right)$$

$$=\frac{1}{f_s}\sum_{i=1}^{k}\left(S_c^0+S_i^c\cos\left(2\pi\left(f_i^m+\frac{f_\Delta^m}{2}\frac{n}{f_s}\right)\frac{n}{f_s}\right)+S_i^s\sin\left(2\pi\left(f_i^m+\frac{f_\Delta^m}{2}\frac{n}{f_s}\right)\frac{n}{f_s}\right)+N_G\left(\frac{n}{f_s}\right)\right)$$

$$S_i^c=S_i^0\cos(\varphi_i)\ \ and\ \ S_i^s=-S_i^0\sin(\varphi_i)$$

$$E\langle S(n)\rangle=S_c^0+\sum_{i=1}^{k}\left(S_i^c\cos\left(2\pi\left(f_i^m+\frac{f_\Delta^m}{2}\frac{n}{f_s}\right)\frac{n}{f_s}\right)+S_i^s\sin\left(2\pi\left(f_i^m+\frac{f_\Delta^m}{2}\frac{n}{f_s}\right)\frac{n}{f_s}\right)\right)$$

f_i^m, φ_i : frequency and phase of the i^{th} swept-frequency signals

f_s: sampling frequency; S_c^0: constant dc-term

$N_G\left(\frac{n}{f_s}\right)$: Gaussian-distributed noise term

Maximum Likelihood Estimation (MLE) -4

- Maximum Log-Likelihood Estimation Function l for Multiple Digitized Sine Wave Signals with constant Frequencies

$$\underset{over\ all\ variables}{\arg\max}\ (l)$$

$$= \underset{S_1^c,\ldots S_k^c,\, S_c^0,\, S_1^s,\ldots S_k^s}{\arg\max}\left(-\frac{1}{2}\sum_{n=0}^{N_s-1}\left(2\pi\sigma_n^2\right)-\sum_{n=0}^{N_s-1}\left(\frac{\left(S(n)-E\langle S(n)\rangle\right)^2}{2\sigma_n^2}\right)\right)$$

This is the formula for the computer simulation of the MLE of the airborne multiple swept-frequency IM-CW ACES system

An Airborne Multiple SF IM-CW ACES

➢ **Advances of airborne technologies in support of NASA ASCENDS mission**

Airborne Platforms:


Hu-25


C-130


DC-8


GlobalHawk

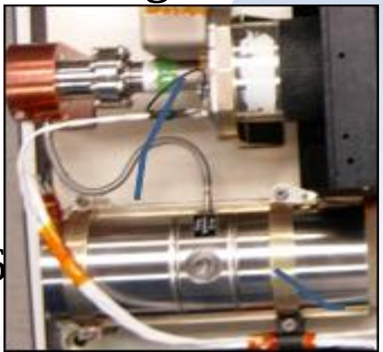
- High and large altitude range: ~1.0 km - ~20 km

EDFA-based high-power Laser transmitter:

up to 3x10W (Avg.)

HgCdTe APD Array-based high-sensitive detector:

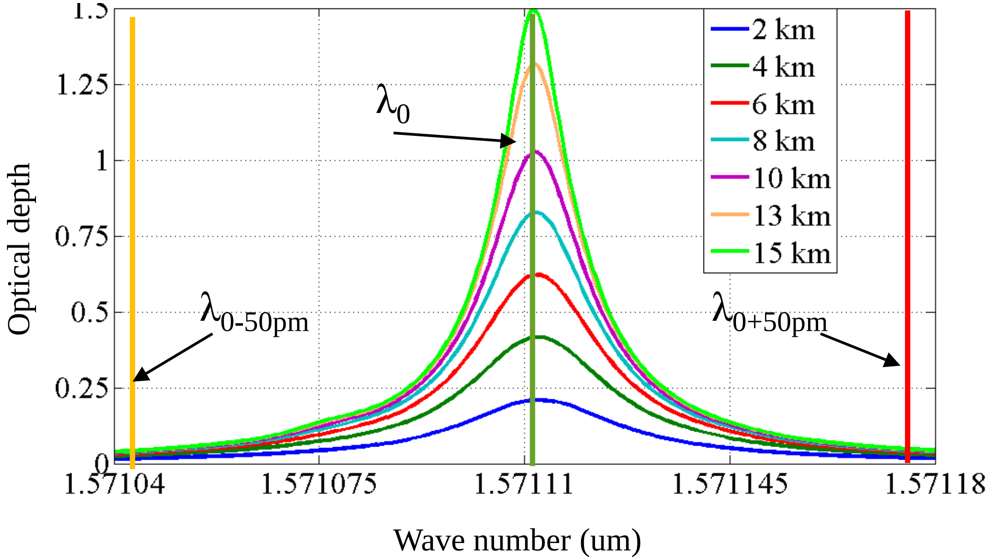
NEP: 2.4fW/Hz$^{1/2}$
Temperature ≤ 77k
Bandwidth:
~4.9 MHz @ gain 10E6



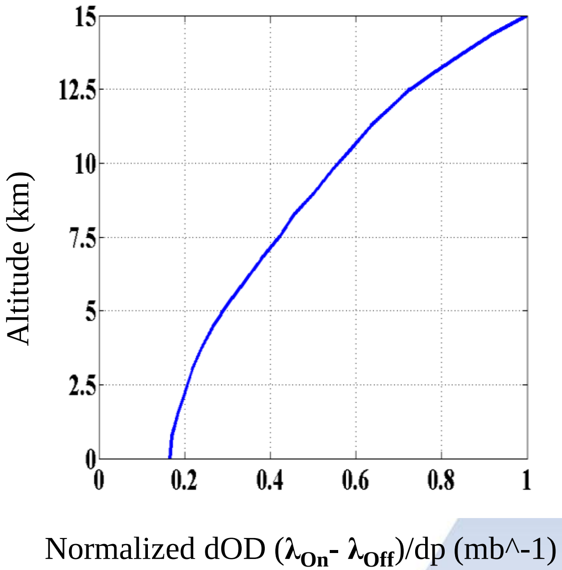
An Airborne Multiple SF IM-CW ACES

- **Airborne multiple SF IM-CW ACES operated at Three Different Wavelengths around CO_2 Absorption Line of 1571nm**

Two-Way Optical Depth at CO_2 Absorption Line



nline Weighting Functions at of 1571nm



Calculated based on HITRAN 2008 Database, US standard atmosphere

$\lambda_{On}=\lambda_0$; $\lambda_{Off1}=\lambda_0$-50pm; $\lambda_{Off2}=\lambda_0$+50pm

λ_0=1571.111nm ---Center Wavelength

An Airborne Multiple SF IM-CW ACES

➢ **EDFA-Based high-power laser transmitter**

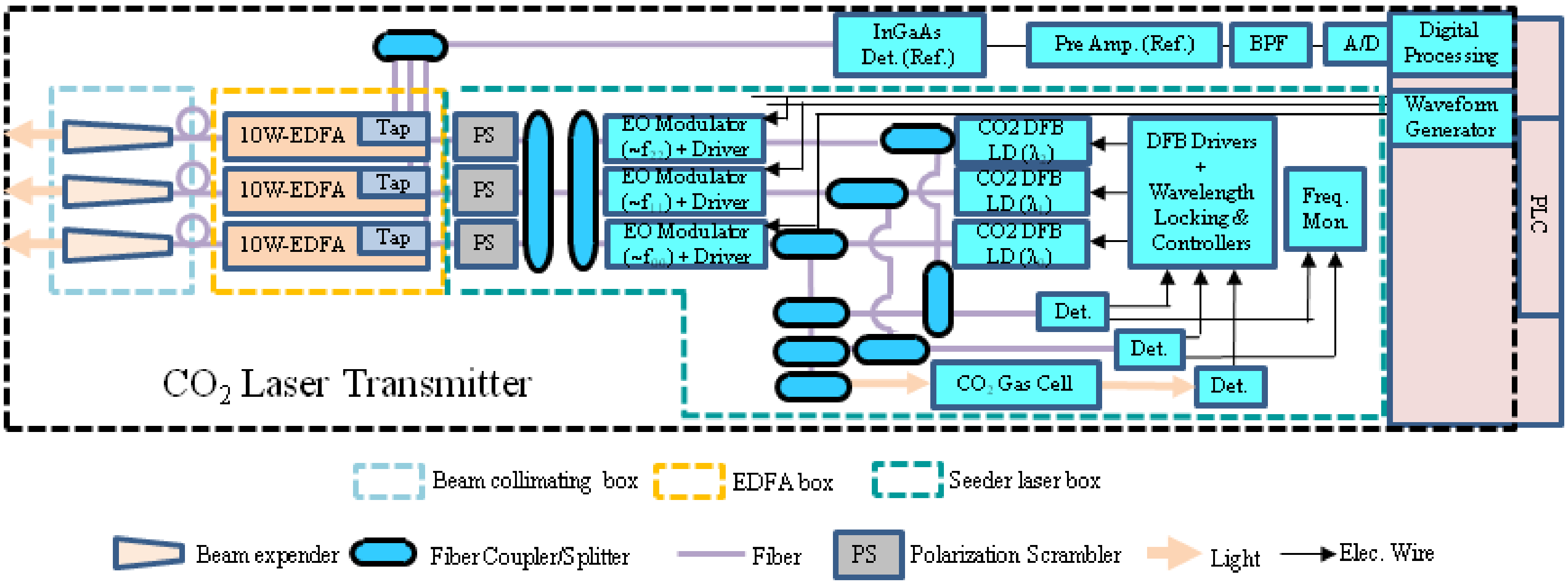


Diagram of EDFA-based Laser Transmitter of ACES

An Airborne Multiple SF IM-CW ACES

- **Multiple SF IM-CW ACES optical and electrical digitized receiver**

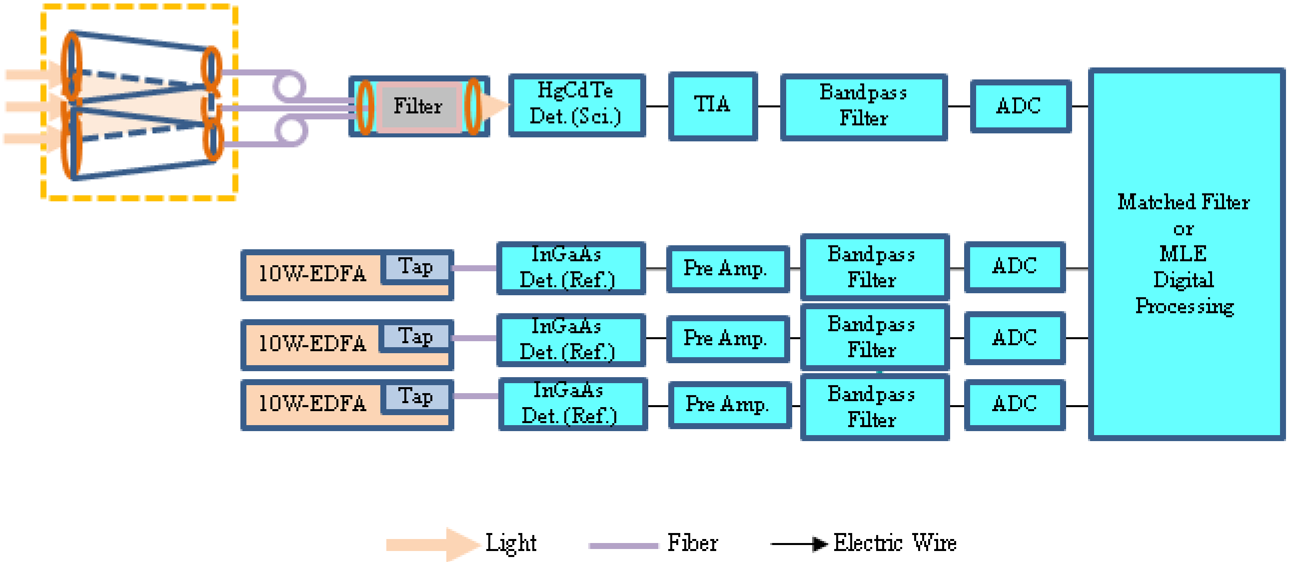


Diagram of ACES Optical and Electrical Digitized Receiver

An Airborne Multiple SF IM-CW ACES

- **LIDAR Equation for the airborne multiple SF IM-CW ACES at three different aavelengths**
  - Total Received Signals under the Intrinsic Limit of Shot Noise

$$S_T^R(t) = \frac{D_T^2}{\pi R^2}\eta_q \eta_{opt} r_s \sum_{i=1}^{3} S_{\lambda_i}^L(t)\frac{hc}{\lambda_i}\tau_{\lambda_i}^2 + S_{Solar}^B + S_{Dark}^B$$

$$= \frac{D_T^2}{\pi R^2}\eta_{opt} r_s \sum_{i=1}^{3} S_{\lambda_i}^L(t)\eta_q \frac{hc}{\lambda_i}\tau_{\lambda_i}^2 + J\frac{\pi D_T^2}{4}\frac{\pi}{4}(FOV)^2 \eta_q \frac{hc}{\lambda_0}\eta_{opt}\tau_{eff}^a(\Delta\lambda) + S_{Dark}^B$$

$$S_{\lambda_i}^L(t) = S_i^0(1+m_i)os\left(2\pi\left(f_i^m + \frac{f_\Delta^m}{2}t\right)t + \varphi_i\right) \quad and \quad OD_{\lambda_i} = -\ln\left(\tau_{\lambda_i}^2\right)$$

$S_{\lambda_i}^L(t)$ S_i^0 m_i : Laser signals, Signal amplitude, Modulation index at λi ;

$\tau_{\lambda_i}^2$ *and* OD_{λ_i}: Two-way atmospheric transmittance and optical depth at λi ;

η_q *and* η_{opt} : Detector quantum efficiency and system optical efficiency;

D_T, r_s, τ_{eff}^a, *and* $(\Delta\lambda)$: Dia. Of Telescope, Surface reflectance, Effective atmospheric transmittance and bandwidth near CO_2 absorption line at 1571 nm

R, J, θ, (Δλ): Transmitter-Target range, Solar background induced photon radiance and scattering, and FOV of telescope

An Airborne Multiple SF IM-CW ACES

- **Maximum Likelihood Estimation of the Digitized Multiple-Frequency IM-CW Signals at Three Different Wavelengths**
  - Gaussian noise assumption: Detected laser signal photons, solar background photons, and detector dark current – contributed numbers of electric charges (electron) during one sampling period are more than 15-20

  Total detected signal in a simple format:

$$S(t) = S_c^0 + \sum_{i=1}^{3} S_i^0 \cos\left(2\pi f_i^{\,m} t + \varphi_i\right) + N_G(t)$$

$$= S_c^0 + \sum_{i=1}^{3}\left(S_i^c \cos\left(2\pi f_i^{\,m} t\right) + S_i^c \sin\left(2\pi f_i^{\,m} t\right)\right) + N_G(t)$$

An Airborne Multiple SF IM-CW ACES

- **Maximum Likelihood Estimation of the digitized multiple SF IM-CW Signals at three different wavelengths**
  - Gaussian noise assumption: Detected laser signal photons, solar background photons, and detector dark current – contributed numbers of electric charges (electron) during one sampling period are more than 15-20
  - Gaussian noise with constant variance: Simpler case for solar background photon and detector dark current contributed shot noise is much larger than signal shot noise $\sigma_n^2 = \sigma^2$ proportional to total detected solar background photons at low detector dark current. Simple MLE formula

$$\underset{S_1^c,\ldots S_k^c,\, S_c^0,\, S_1^s,\ldots S_k^s}{\arg\max}(l) \equiv \underset{S_1^c,\ldots S_k^c,\, S_c^0,\, S_1^s,\ldots S_k^s}{\arg\max}\left(-\sum_{n=0}^{N_s-1}\left(\frac{\left(S(n)-E\langle S(n)\rangle\right)^2}{2\sigma^2}\right)\right)$$

An Airborne Multiple SF IM-CW ACES

- **Maximum Likelihood Estimation of the digitized multiple SF IM-CW Signals at three different wavelengths**

$$
\begin{bmatrix}
\Re_{11} & \Re_{12} & \Re_{13} & \Re_{14} & \Re_{15} & \Re_{16} & \Re_{17} \\
\Re_{21} & \Re_{22} & \Re_{23} & \Re_{24} & \Re_{25} & \Re_{26} & \Re_{27} \\
\Re_{31} & \Re_{32} & \Re_{33} & \Re_{34} & \Re_{35} & \Re_{36} & \Re_{37} \\
\Re_{41} & \Re_{42} & \Re_{43} & \Re_{44} & \Re_{45} & \Re_{46} & \Re_{47} \\
\Re_{51} & \Re_{52} & \Re_{53} & \Re_{54} & \Re_{55} & \Re_{56} & \Re_{57} \\
\Re_{61} & \Re_{62} & \Re_{63} & \Re_{64} & \Re_{65} & \Re_{66} & \Re_{67} \\
\Re_{71} & \Re_{72} & \Re_{73} & \Re_{74} & \Re_{75} & \Re_{76} & \Re_{77}
\end{bmatrix}
\begin{bmatrix}
S_1^c \\ S_2^c \\ S_3^c \\ S_c^0 \\ S_1^s \\ S_2^s \\ S_3^s
\end{bmatrix}
=
\begin{bmatrix}
\sum_{n=0}^{N_s-1} S_n \cos\left(2\pi \frac{f_1^m}{f_s} n\right) \\
\sum_{n=0}^{N_s-1} S_n \cos\left(2\pi \frac{f_2^m}{f_s} n\right) \\
\sum_{n=0}^{N_s-1} S_n \cos\left(2\pi \frac{f_3^m}{f_s} n\right) \\
\sum_{n=0}^{N_s-1} S_n \\
\sum_{n=0}^{N_s-1} S_n \sin\left(2\pi \frac{f_1^m}{f_s} n\right) \\
\sum_{n=0}^{N_s-1} S_n \sin\left(2\pi \frac{f_2^m}{f_s} n\right) \\
\sum_{n=0}^{N_s-1} S_n \sin\left(2\pi \frac{f_3^m}{f_s} n\right)
\end{bmatrix}
$$

An Airborne Multiple SF IM-CW ACES

$$\Re_{11}=\frac{1}{2}\sum_{n=0}^{N_s-1}\left(1+\cos\left(4\pi\left(f_1^m\frac{n}{f_s}+\frac{1}{2}f_\Delta^m\frac{n^2}{f_s^2}\right)\right)\right)$$

$$\Re_{15}=\frac{1}{2}\sum_{n=0}^{N_s-1}\left(\sin\left(4\pi\left(f_1^m\frac{n}{f_s}+\frac{1}{2}f_\Delta^m\frac{n^2}{f_s^2}\right)\right)\right)$$

$$\Re_{12}=\frac{1}{2}\sum_{n=0}^{N_s-1}\left(\cos\left(2\pi\left(\left(f_1^m+f_2^m\right)\frac{n}{f_s}+f_\Delta^m\frac{n^2}{f_s^2}\right)\right)+\cos\left(2\pi\left(\left(f_1^m-f_2^m\right)\frac{n}{f_s}\right)\right)\right)$$

$$\Re_{16}=\frac{1}{2}\sum_{n=0}^{N_s-1}\left(\sin\left(2\pi\left(\left(f_1^m+f_2^m\right)\frac{n}{f_s}+f_\Delta^m\frac{n^2}{f_s^2}\right)\right)-\sin\left(2\pi\left(\left(f_1^m-f_2^m\right)\frac{n}{f_s}\right)\right)\right)$$

$$\Re_{13}=\frac{1}{2}\sum_{n=0}^{N_s-1}\left(\cos\left(2\pi\left(\left(f_1^m+f_3^m\right)\frac{n}{f_s}+f_\Delta^m\frac{n^2}{f_s^2}\right)\right)+\cos\left(2\pi\left(\left(f_1^m-f_3^m\right)\frac{n}{f_s}\right)\right)\right)$$

$$\Re_{17}=\frac{1}{2}\sum_{n=0}^{N_s-1}\left(\sin\left(2\pi\left(\left(f_1^m+f_3^m\right)\frac{n}{f_s}+f_\Delta^m\frac{n^2}{f_s^2}\right)\right)-\sin\left(2\pi\left(\left(f_1^m-f_3^m\right)\frac{n}{f_s}\right)\right)\right)$$

$$\Re_{14}=\sum_{n=0}^{N_s-1}\cos\left(2\pi\left(f_1^m\frac{n}{f_s}+\frac{1}{2}f_\Delta^m\frac{n^2}{f_s^2}\right)\right)$$

$$\Re_{21}=\frac{1}{2}\sum_{n=0}^{N_s-1}\left(\cos\left(2\pi\left(\left(f_1^m+f_2^m\right)\frac{n}{f_s}+f_\Delta^m\frac{n^2}{f_s^2}\right)\right)+\cos\left(2\pi\left(\left(f_1^m-f_2^m\right)\frac{n}{f_s}\right)\right)\right)$$

$$\Re_{25}=\frac{1}{2}\sum_{n=0}^{N_s-1}\left(\sin\left(2\pi\left(\left(f_1^m+f_2^m\right)\frac{n}{f_s}+f_\Delta^m\frac{n^2}{f_s^2}\right)\right)+\sin\left(2\pi\left(\left(f_1^m-f_2^m\right)\frac{n}{f_s}\right)\right)\right)$$

$$\Re_{22}=\frac{1}{2}\sum_{n=0}^{N_s-1}\left(1+\cos\left(4\pi\left(f_2^m\frac{n}{f_s}+\frac{1}{2}f_\Delta^m\frac{n^2}{f_s^2}\right)\right)\right)$$

$$\Re_{26}=\frac{1}{2}\sum_{n=0}^{N_s-1}\left(\sin\left(4\pi\left(f_2^m\frac{n}{f_s}+\frac{1}{2}f_\Delta^m\frac{n^2}{f_s^2}\right)\right)\right)$$

$$\Re_{23}=\frac{1}{2}\sum_{n=0}^{N_s-1}\left(\cos\left(2\pi\left(\left(f_2^m+f_3^m\right)\frac{n}{f_s}+f_\Delta^m\frac{n^2}{f_s^2}\right)\right)+\cos\left(2\pi\left(\left(f_2^m-f_3^m\right)\frac{n}{f_s}\right)\right)\right)$$

$$\Re_{27}=\frac{1}{2}\sum_{n=0}^{N_s-1}\left(\sin\left(2\pi\left(\left(f_2^m+f_3^m\right)\frac{n}{f_s}+f_\Delta^m\frac{n^2}{f_s^2}\right)\right)-\sin\left(2\pi\left(\left(f_2^m-f_3^m\right)\frac{n}{f_s}\right)\right)\right)$$

$$\Re_{24}=\sum_{n=0}^{N_s-1}\cos\left(2\pi\left(f_2^m\frac{n}{f_s}+\frac{1}{2}f_\Delta^m\frac{n^2}{f_s^2}\right)\right)$$

$$\Re_{31}=\frac{1}{2}\sum_{n=0}^{N_s-1}\left(\cos\left(2\pi\left(\left(f_1^m+f_3^m\right)\frac{n}{f_s}+f_\Delta^m\frac{n^2}{f_s^2}\right)\right)+\cos\left(2\pi\left(\left(f_1^m-f_3^m\right)\frac{n}{f_s}\right)\right)\right)$$

$$\Re_{35}=\frac{1}{2}\sum_{n=0}^{N_s-1}\left(\sin\left(2\pi\left(\left(f_1^m+f_3^m\right)\frac{n}{f_s}+f_\Delta^m\frac{n^2}{f_s^2}\right)\right)+\sin\left(2\pi\left(\left(f_1^m-f_3^m\right)\frac{n}{f_s}\right)\right)\right)$$

$$\Re_{32}=\frac{1}{2}\sum_{n=0}^{N_s-1}\left(\cos\left(2\pi\left(\left(f_2^m+f_3^m\right)\frac{n}{f_s}+f_\Delta^m\frac{n^2}{f_s^2}\right)\right)+\cos\left(2\pi\left(\left(f_2^m-f_3^m\right)\frac{n}{f_s}\right)\right)\right)$$

$$\Re_{36}=\frac{1}{2}\sum_{n=0}^{N_s-1}\left(\sin\left(2\pi\left(\left(f_2^m+f_3^m\right)\frac{n}{f_s}+f_\Delta^m\frac{n^2}{f_s^2}\right)\right)+\sin\left(2\pi\left(\left(f_2^m-f_3^m\right)\frac{n}{f_s}\right)\right)\right)$$

$$\Re_{33}=\frac{1}{2}\sum_{n=0}^{N_s-1}\left(1+\cos\left(4\pi\left(f_3^m\frac{n}{f_s}+\frac{1}{2}f_\Delta^m\frac{n^2}{f_s^2}\right)\right)\right)$$

$$\Re_{37}=\frac{1}{2}\sum_{n=0}^{N_s-1}\sin\left(4\pi\left(f_3^m\frac{n}{f_s}+\frac{1}{2}f_\Delta^m\frac{n^2}{f_s^2}\right)\right)$$

$$\Re_{34}=\sum_{n=0}^{N_s-1}\cos\left(2\pi\left(f_3^m\frac{n}{f_s}+\frac{1}{2}f_\Delta^m\frac{n^2}{f_s^2}\right)\right)$$

$$\Re_{41}=\sum_{n=0}^{N_s-1}\cos\left(2\pi\left(f_1^m\frac{n}{f_s}+\frac{1}{2}f_\Delta^m\frac{n^2}{f_s^2}\right)\right)\qquad \Re_{42}=\sum_{n=0}^{N_s-1}\cos\left(2\pi\left(f_2^m\frac{n}{f_s}+\frac{1}{2}f_\Delta^m\frac{n^2}{f_s^2}\right)\right)\qquad \Re_{43}=\sum_{n=0}^{N_s-1}\cos\left(2\pi\left(f_3^m\frac{n}{f_s}+\frac{1}{2}f_\Delta^m\frac{n^2}{f_s^2}\right)\right)\qquad \Re_{44}=N_s$$

$$\Re_{45}=\sum_{n=0}^{N_s-1}\sin\left(2\pi\left(f_1^m\frac{n}{f_s}+\frac{1}{2}f_\Delta^m\frac{n^2}{f_s^2}\right)\right)\qquad \Re_{46}=\sum_{n=0}^{N_s-1}\sin\left(2\pi\left(f_2^m\frac{n}{f_s}+\frac{1}{2}f_\Delta^m\frac{n^2}{f_s^2}\right)\right)\qquad \Re_{47}=\sum_{n=0}^{N_s-1}\sin\left(2\pi\left(f_3^m\frac{n}{f_s}+\frac{1}{2}f_\Delta^m\frac{n^2}{f_s^2}\right)\right)$$

An Airborne Multiple SF IM-CW ACES

$$\Re_{51}=\frac{1}{2}\sum_{n=0}^{N_s-1}\left(\sin\left(2\pi\left(\left(f_1^m+f_3^m\right)\frac{n}{f_s}+f_\Delta^m\frac{n^2}{f_s^2}\right)\right)-\sin\left(2\pi\left(\left(f_1^m-f_3^m\right)\frac{n}{f_s}\right)\right)\right)$$

$$\Re_{55}=\frac{1}{2}\sum_{n=0}^{N_s-1}\left(1-\cos\left(4\pi\left(f_3^m\frac{n}{f_s}+\frac{1}{2}f_\Delta^m\frac{n^2}{f_s^2}\right)\right)\right)$$

$$\Re_{52}=\frac{1}{2}\sum_{n=0}^{N_s-1}\left(\sin\left(2\pi\left(\left(f_2^m+f_3^m\right)\frac{n}{f_s}+f_\Delta^m\frac{n^2}{f_s^2}\right)\right)-\sin\left(2\pi\left(\left(f_2^m-f_3^m\right)\frac{n}{f_s}\right)\right)\right)$$

$$\Re_{56}=\frac{1}{2}\sum_{n=0}^{N_s-1}\left(\cos\left(2\pi\left(\left(f_2^m+f_3^m\right)\frac{n}{f_s}+f_\Delta^m\frac{n^2}{f_s^2}\right)\right)-\cos\left(2\pi\left(\left(f_2^m-f_3^m\right)\frac{n}{f_s}\right)\right)\right)$$

$$\Re_{53}=\frac{1}{2}\sum_{n=0}^{N_s-1}\left(\sin\left(4\pi\left(f_3^m\frac{n}{f_s}+\frac{1}{2}f_\Delta^m\frac{n^2}{f_s^2}\right)\right)\right)$$

$$\Re_{57}=\frac{1}{2}\sum_{n=0}^{N_s-1}\left(\cos\left(2\pi\left(\left(f_1^m+f_3^m\right)\frac{n}{f_s}+f_\Delta^m\frac{n^2}{f_s^2}\right)\right)-\cos\left(2\pi\left(\left(f_1^m-f_3^m\right)\frac{n}{f_s}\right)\right)\right)$$

$$\Re_{54}=\sum_{n=0}^{N_s-1}\sin\left(2\pi\left(f_3^m\frac{n}{f_s}+\frac{1}{2}f_\Delta^m\frac{n^2}{f_s^2}\right)\right)$$

$$\Re_{61}=\frac{1}{2}\sum_{n=0}^{N_s-1}\left(\sin\left(2\pi\left(\left(f_1^m+f_2^m\right)\frac{n}{f_s}+f_\Delta^m\frac{n^2}{f_s^2}\right)\right)-\sin\left(2\pi\left(\left(f_1^m-f_2^m\right)\frac{n}{f_s}\right)\right)\right)$$

$$\Re_{65}=\frac{1}{2}\sum_{n=0}^{N_s-1}\left(\cos\left(2\pi\left(\left(f_1^m+f_2^m\right)\frac{n}{f_s}+f_\Delta^m\frac{n^2}{f_s^2}\right)\right)-\cos\left(2\pi\left(\left(f_1^m-f_2^m\right)\frac{n}{f_s}\right)\right)\right)$$

$$\Re_{62}=\frac{1}{2}\sum_{n=0}^{N_s-1}\sin\left(4\pi\left(f_2^m\frac{n}{f_s}+\frac{1}{2}f_\Delta^m\frac{n^2}{f_s^2}\right)\right)$$

$$\Re_{66}=\frac{1}{2}\sum_{n=0}^{N_s-1}\left(1-\cos\left(4\pi\left(f_2^m\frac{n}{f_s}+\frac{1}{2}f_\Delta^m\frac{n^2}{f_s^2}\right)\right)\right)$$

$$\Re_{63}=\frac{1}{2}\sum_{n=0}^{N_s-1}\left(\sin\left(2\pi\left(\left(f_2^m+f_3^m\right)\frac{n}{f_s}+f_\Delta^m\frac{n^2}{f_s^2}\right)\right)+\sin\left(2\pi\left(\left(f_2^m-f_3^m\right)\frac{n}{f_s}\right)\right)\right)$$

$$\Re_{67}=\frac{1}{2}\sum_{n=0}^{N_s-1}\left(\cos\left(2\pi\left(\left(f_2^m+f_3^m\right)\frac{n}{f_s}+f_\Delta^m\frac{n^2}{f_s^2}\right)\right)-\cos\left(2\pi\left(\left(f_2^m-f_3^m\right)\frac{n}{f_s}\right)\right)\right)$$

$$\Re_{64}=\sum_{n=0}^{N_s-1}\sin\left(2\pi\left(f_2^m\frac{n}{f_s}+\frac{1}{2}f_\Delta^m\frac{n^2}{f_s^2}\right)\right)$$

$$\Re_{71}=\frac{1}{2}\sum_{n=0}^{N_s-1}\left(\sin\left(4\pi\left(f_1^m\frac{n}{f_s}+\frac{1}{2}f_\Delta^m\frac{n^2}{f_s^2}\right)\right)\right)$$

$$\Re_{75}=\frac{1}{2}\sum_{n=0}^{N_s-1}\left(\cos\left(2\pi\left(\left(f_1^m+f_3^m\right)\frac{n}{f_s}+f_\Delta^m\frac{n^2}{f_s^2}\right)\right)-\cos\left(2\pi\left(\left(f_1^m-f_3^m\right)\frac{n}{f_s}\right)\right)\right)$$

$$\Re_{72}=\frac{1}{2}\sum_{n=0}^{N_s-1}\left(\sin\left(2\pi\left(\left(f_1^m+f_2^m\right)\frac{n}{f_s}+f_\Delta^m\frac{n^2}{f_s^2}\right)\right)+\sin\left(2\pi\left(\left(f_1^m-f_2^m\right)\frac{n}{f_s}\right)\right)\right)$$

$$\Re_{76}=\frac{1}{2}\sum_{n=0}^{N_s-1}\left(\cos\left(2\pi\left(\left(f_1^m+f_2^m\right)\frac{n}{f_s}+f_\Delta^m\frac{n^2}{f_s^2}\right)\right)-\cos\left(2\pi\left(\left(f_1^m-f_2^m\right)\frac{n}{f_s}\right)\right)\right)$$

$$\Re_{73}=\frac{1}{2}\sum_{n=0}^{N_s-1}\sin\left(\sin\left(2\pi\left(\left(f_1^m+f_3^m\right)\frac{n}{f_s}+f_\Delta^m\frac{n^2}{f_s^2}\right)\right)+\sin\left(2\pi\left(\left(f_1^m-f_3^m\right)\frac{n}{f_s}\right)\right)\right)$$

$$\Re_{77}=\frac{1}{2}\sum_{n=0}^{N_s-1}\left(1-\cos\left(4\pi\left(f_1^m\frac{n}{f_s}+\frac{1}{2}f_\Delta^m\frac{n^2}{f_s^2}\right)\right)\right)$$

$$\Re_{74}=\sum_{n=0}^{N_s-1}\sin\left(2\pi\left(f_1^m\frac{n}{f_s}+\frac{1}{2}f_\Delta^m\frac{n^2}{f_s^2}\right)\right)$$

An Airborne Multiple SF IM-CW ACES

- **Simplified linear equations**

Coefficient matrix R: off-diagonal components can be zero under following cases,

*** Sampling frequency is much larger than modulation frequencies**

**** Specified sampling frequency, all modulation frequencies, and integration time periods.**

Our simulation condition:

SF signals at three different wavelengths:

Sampling frequency: 2 MHz

Swept frequencies: 100.4 kHz-600.4 kHz; 105.2 kHz-605.2 kHz; and 110.8 kHz-610.8 kHz

Frequency increase rate (chirp rate): 2500 MHz

Orthogonal sweeping time: 5 ms (25 swept)

Integration time periods: 5ms, 25ms, 50ms, 100ms

An Airborne Multiple SF IM-CW ACES

➢ **The Fractional/Relative Random Error of the CO_2 measurement**

$$\left(\frac{\delta\left(\overline{N_{CO2}}\right)}{\overline{N_{CO2}}}\right)_{On-Offi} = \sqrt{\left(\frac{\sigma\left(\delta\left(dOD_{On-Offi}\right)\right)}{\overline{dOD_{On-Offi}}}\right)^2 + \left(\frac{\sigma(\delta R)}{\overline{R}}\right)^2}; \qquad \frac{\sigma\left(\delta\left(dOD_{On-Offi}\right)\right)}{\overline{dOD_{On-Offi}}} = \frac{1}{dOD_{On-Offi}}\left(\frac{1}{SNR^R_{On}} + \frac{1}{SNR^R_{Offi}}\right)$$

$$\sigma(\delta R) \approx \frac{c}{2\sqrt{2}\pi}\sqrt{\left(\frac{1}{\overline{f_1^m} m_1 SNR^R_{On}}\right)^2 + \left(\frac{1}{\overline{f_{i+1}^m} m_{i+1} SNR^R_{Offi}}\right)^2} \approx \frac{c}{4\pi}\left(\frac{1}{\overline{f_1^m} m_1 SNR^R_{On}} + \frac{1}{\overline{f_{i+1}^m} m_{i+1} SNR^R_{Offi}}\right)$$

$$\overline{f_1^m}, \overline{f_{i+1}^m} \approx 350kHz; \quad i = 1, 2; \qquad c = 3*10^8 m/s$$

$\left(\frac{\delta\left(\overline{N_{CO2}}\right)}{\overline{N_{CO2}}}\right)_{On-Offi}$ Fractional/relative errors of the CO_2 column density measurement

$dOD_{On-Offi}$ Differential optical depth of the transmitted signals in the atmosphere between on-line and offline (1 or 2) wavelengths

SNR^L_{On} and SNR^L_{Offi} Signal to Noise Ratio (SNR) at on-line and off -line (1 or 2) wavelengths on the transmitter detector for laser power monitor

SNR^R_{On} and SNR^R_{Offi} Signal to Noise Ratio (SNR) at on-line and off-line (1 or 2) wavelengths on the detector of the receiver for signal measurement

$dOD_{On-Offi} = -\ln\left(\frac{\tau^2_{On}}{\tau^2_{Offi}}\right)$ τ^2_{On} and τ^2_{Offi} are two-way transmittance at on-line and off-line (1 or 2) wavelengths

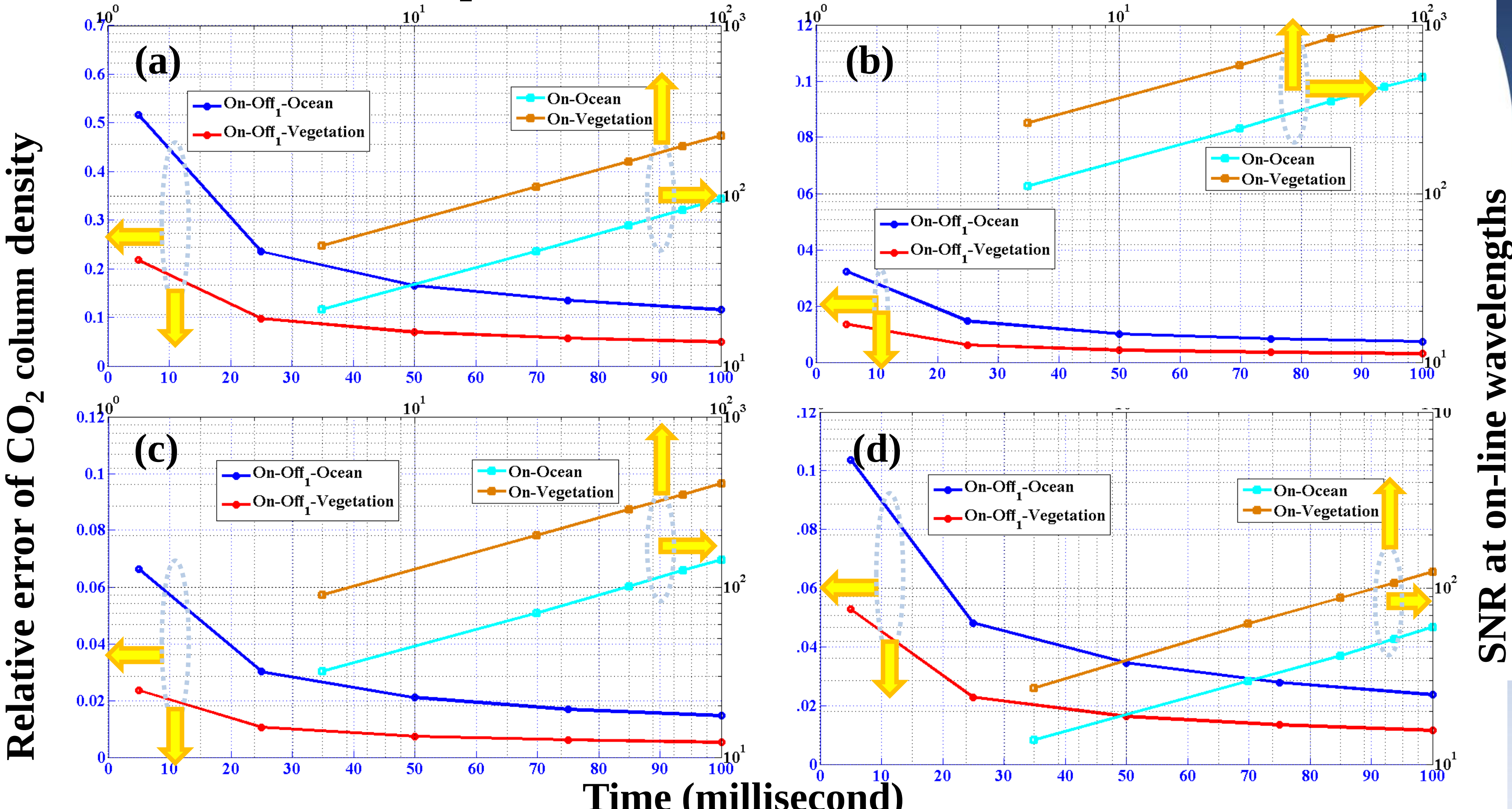
An Airborne Multiple SF IM-CW ACES

System Parameters for Computer Simulation of MLE

Altitude:	2-15 km
Receiving telescope:	
Effective Diameter:	3*(0.17/√3) m
(3 7” Ritchey-Chretien telescopes with center blocks in ~2” dia.)	
FOV (full angle):	495 μrad
Over all optical transmittance	8.5 %
Optical filter bandwidth:	2.7 nm
Laser transmitter:	
Beam divergence:	300 μrad
Modulation index:	0.9
Photodetector: (HgTeCd APD 8*8 array)	
Noise Equivalent Power (NEP):	2.4fW/Hz$^{1/2}$@~77K
Quantum efficiency:	0.8
Ground radiance near 1571 nm: (W/m2/sr/μm)	
ocean/vegetable	1.7/5.0
Surface reflectance:	
ocean/vegetable	0.08/0.31
Atmospheric transmittance for radiance	95%

An Airborne Multiple SF IM-CW ACES

Computer Simulated MLE Results



(a): Total Power: 5W@2km **(b): Total Power: 10W@6km**

(c): Total Power: 10W@10km **(d): Total Power: 30W@15km**

Total Power distributed to different wavelength based on the atmospheric transmissions

Conclusions and Discussions

- **MLE of multiple digitized swept-frequency (SF) Intensity wave signals with Gaussian-distributed noise**
- **SNRs and relative errors of the CO_2 column density measurement from airborne at different altitudes have been simulated for the airborne SF IM-CW ACES**
- **SNRs increases as integration time increases up to 0.1s (linearly in log scales)**