Numerical Simulations of Thermographic Responses in Composites

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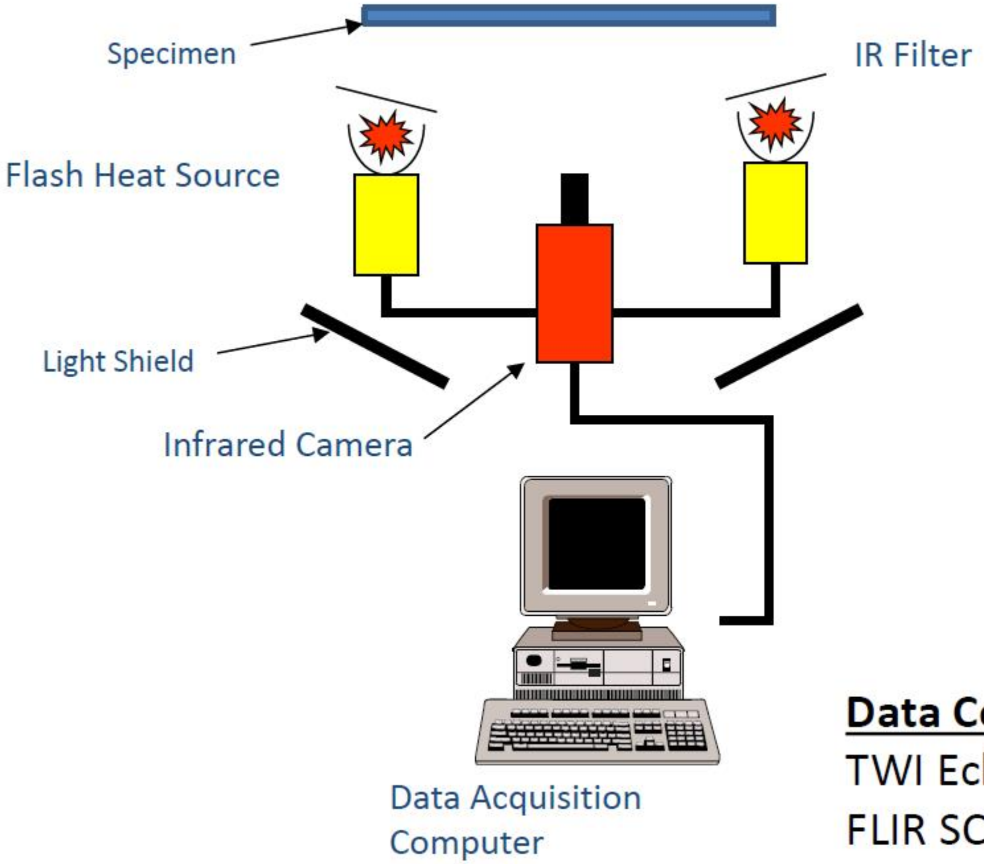
Outline

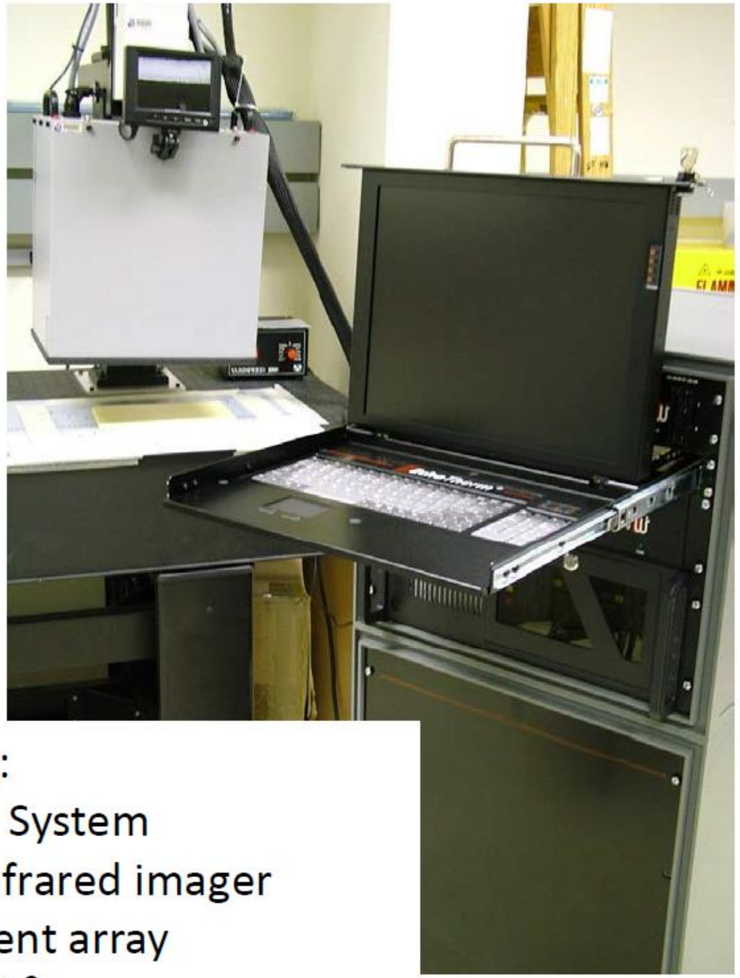
- Uses of Thermographic Simulations
- Description of Flash Thermographic Measurement
- Thermal Simulation of Heat Diffusion in Unidirectional and Quasi-isotropic composites
- Simulating delaminations in composites with quadrupole method

NASA Applications of Thermographic Simulations

- Optimization of thermographic techniques
- Determination of limitation of experimental and analysis techniques
- Generation of set of eigenvector of principle component analysis of thermal response
- Inversion of thermal responses to information on flaw sizes and locations
- Training neural networks for rapid reduction of thermal data

Flash Thermography Measurements





Data Collection:

TWI Echotherm System

FLIR SC6000® infrared imager

640 x 512 element array

12”x9” = ~0.75 ft^2

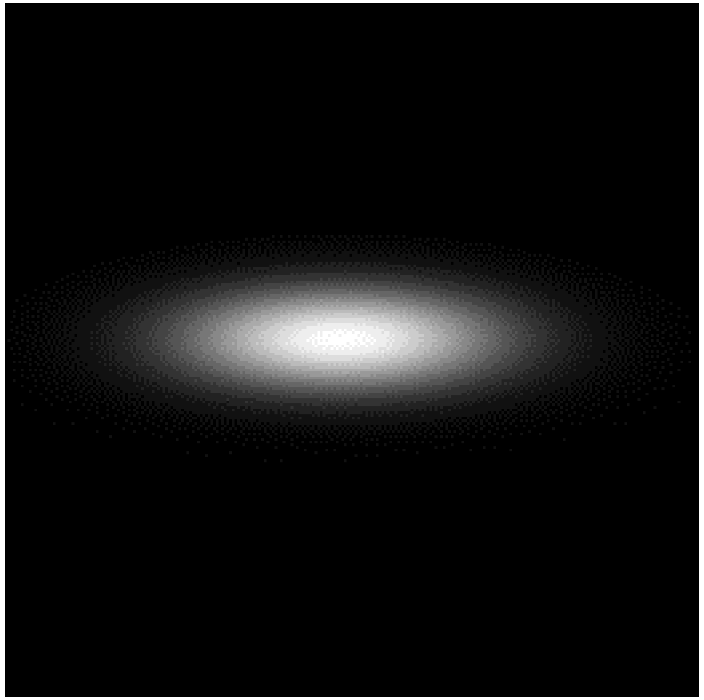
3-5 μm wavelength range

120 fps acquisition speed

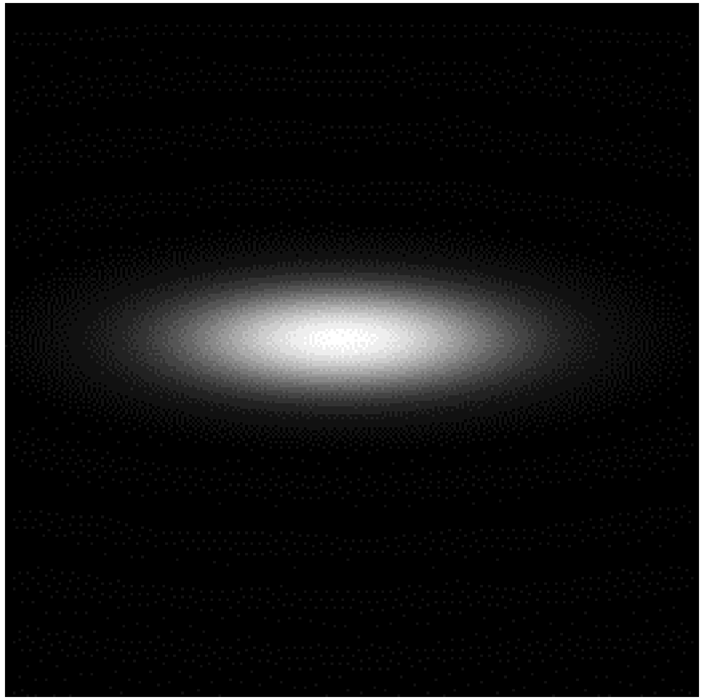
Simulation of Composites

- Finite Difference
  - Implicit and explicit
  - Initial flux at surface or initial temperature
  - "Finite Difference Methods in Heat Transfer", M. N. Ozisik
- Finite Element
  - Simpler for complex geometries
  - Initial flux at surface or initial temperature
  - Commercial packages are available
- Thermal Quadrupoles (examined in this paper)
  - Modification of method presented in "Thermal Quadrupoles, Solving the Heat Equation through Integral Transforms", Maillet, Andre, Bastsale, Degiovanni and Moyne

Thermal Response to an Impulse Flux Input at Single Point ($\delta(x)\ \delta(y)\ \delta(z)\ \delta(t)$) Unidirectional Composite ($\alpha_x = 10\ \alpha_y$) – 1/60 sec after pulse



Front surface temperature



Back surface temperature

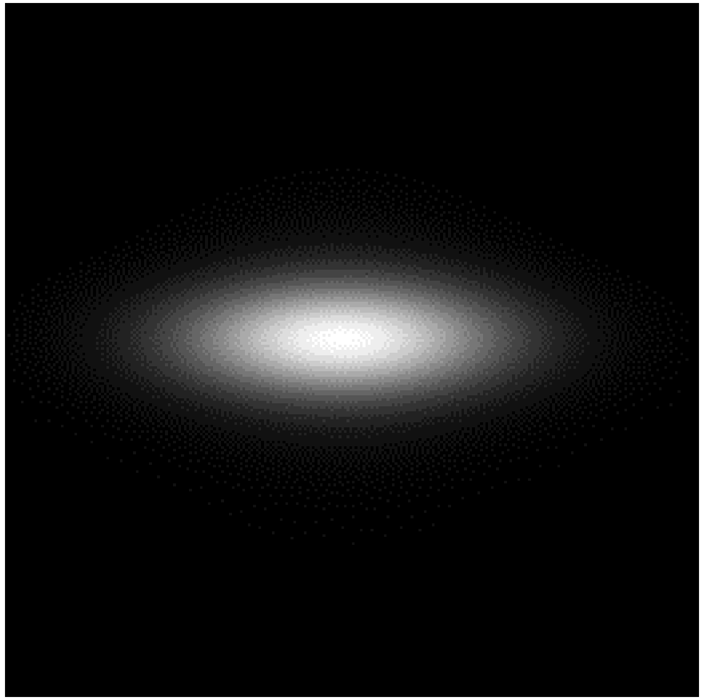
Front and back surface temperature are approximately the same

Fiber direction evident in thermal response

Single Ply 0.01 cm thick

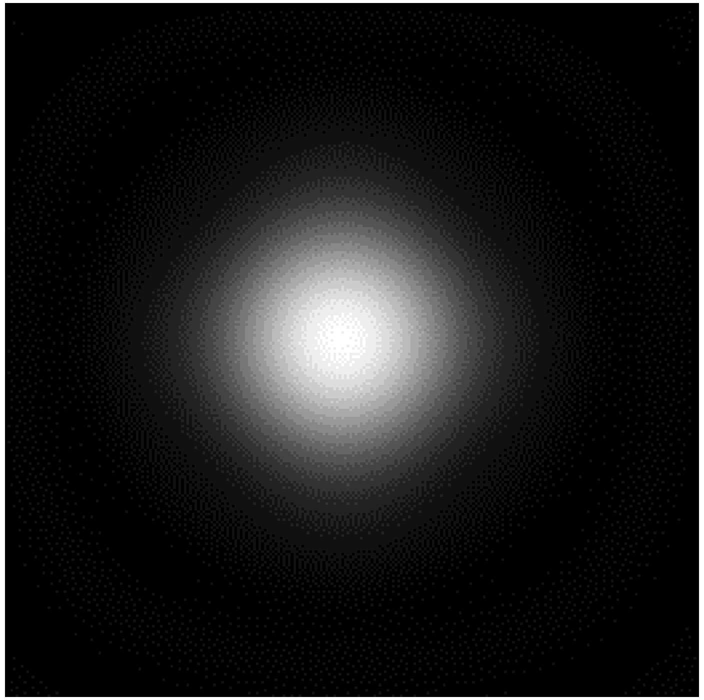
Thermal Response to $\delta(x)\ \delta(y)\ \delta(z)\ \delta(t)$ Quasi Isotropic Composite with Two Plies, Top ply ($\alpha_x = 10\ \alpha_y$), Second ply($10\ \alpha_x = \alpha_y$) – 1/60 sec after pulse

Front surface temperature



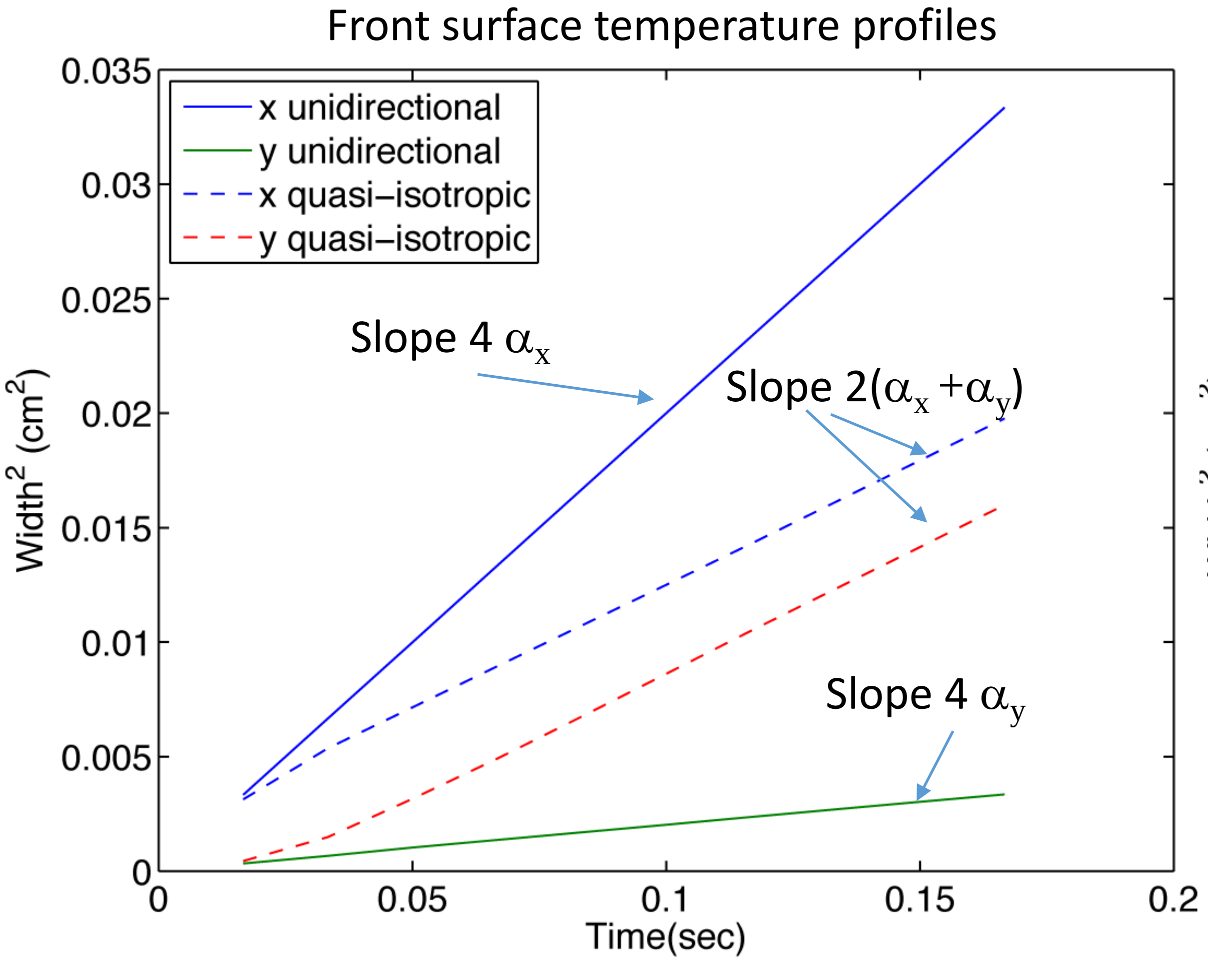
Two Plies, both 0.01 cm thick

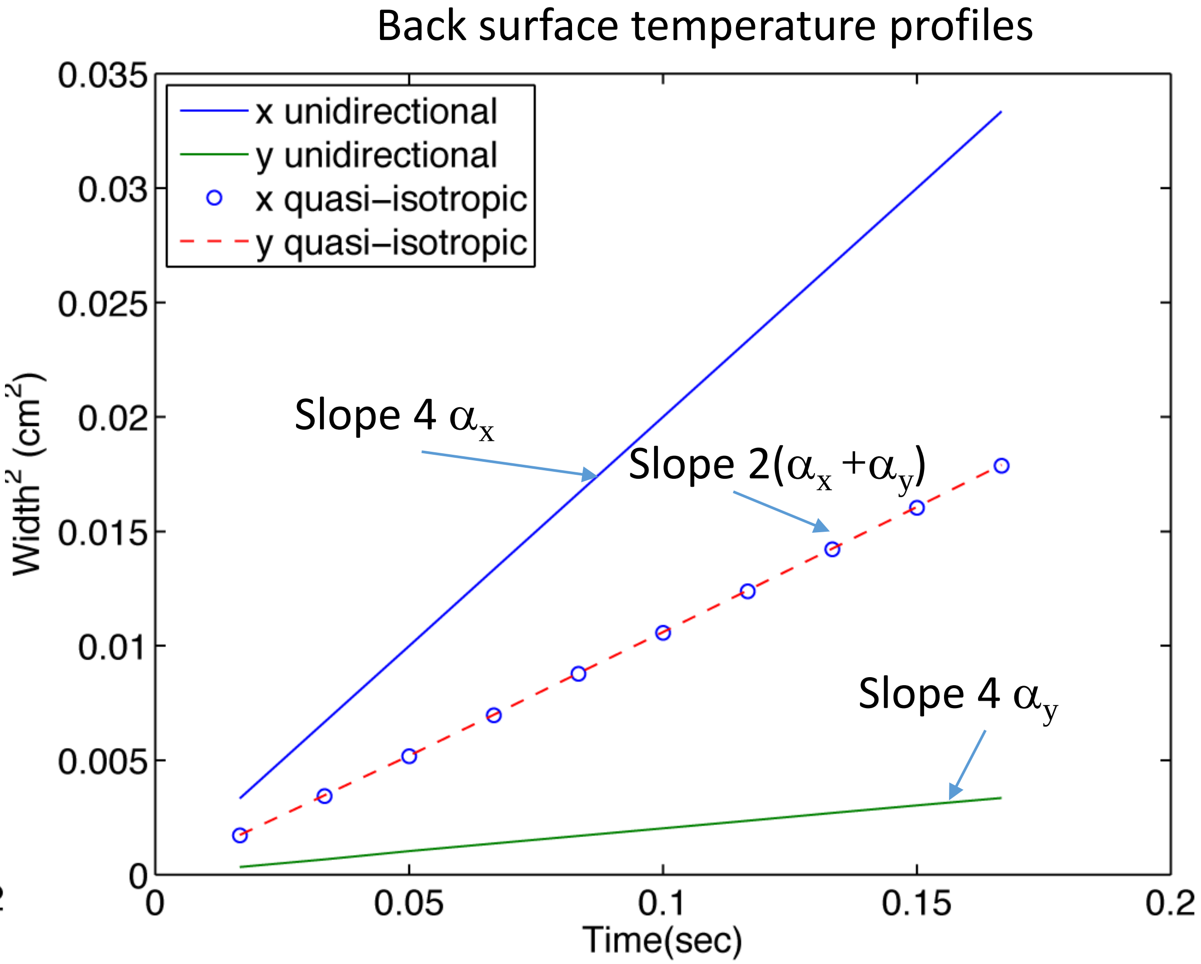
Back surface temperature



- Initial front surface temperature indicates upper ply fiber direction and back surface temperature has appearance of isotropic in-plane heat flow.
- Back surface response is more indicative of flaw response

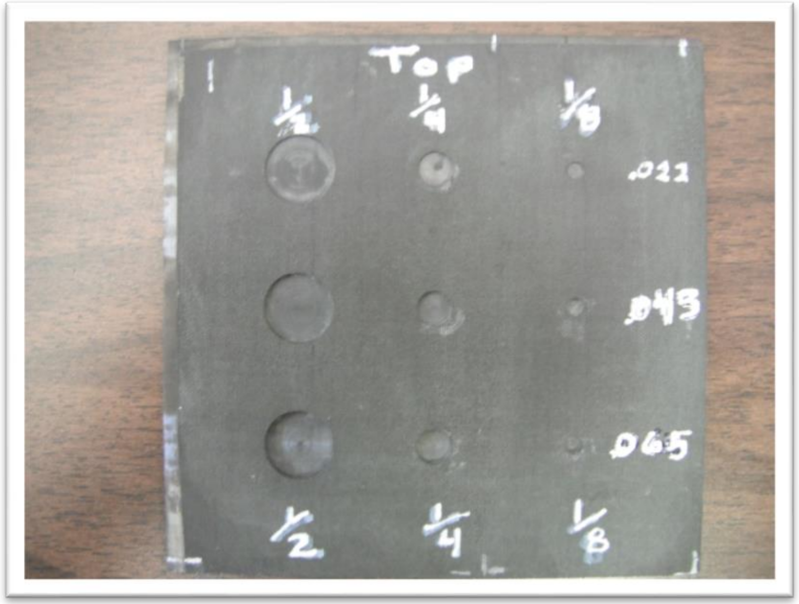
Gaussian Fit of Temperature Profiles in Simulated Responses as Function of Time

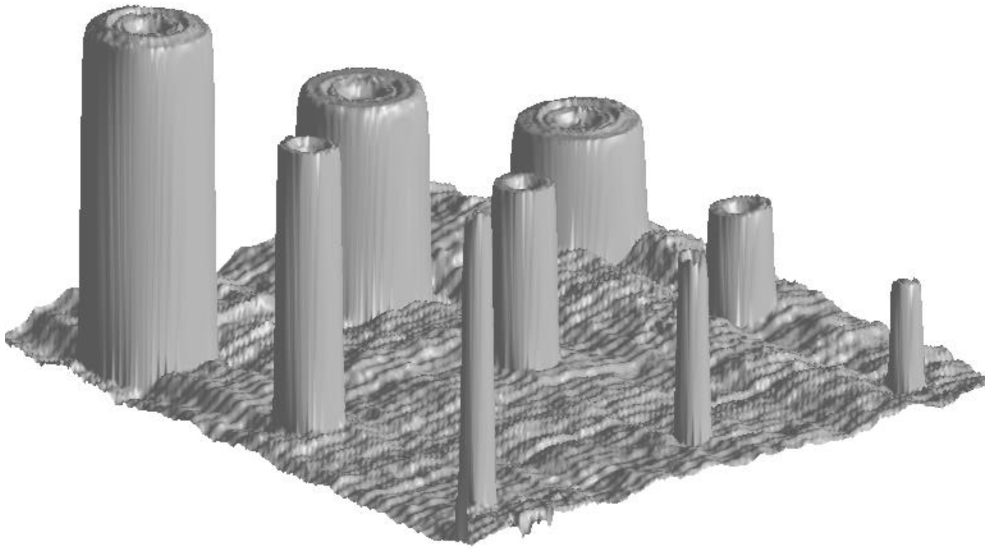




For quasi-isotropic layup the in-plane diffusivity can be considered to be approximately isotropic

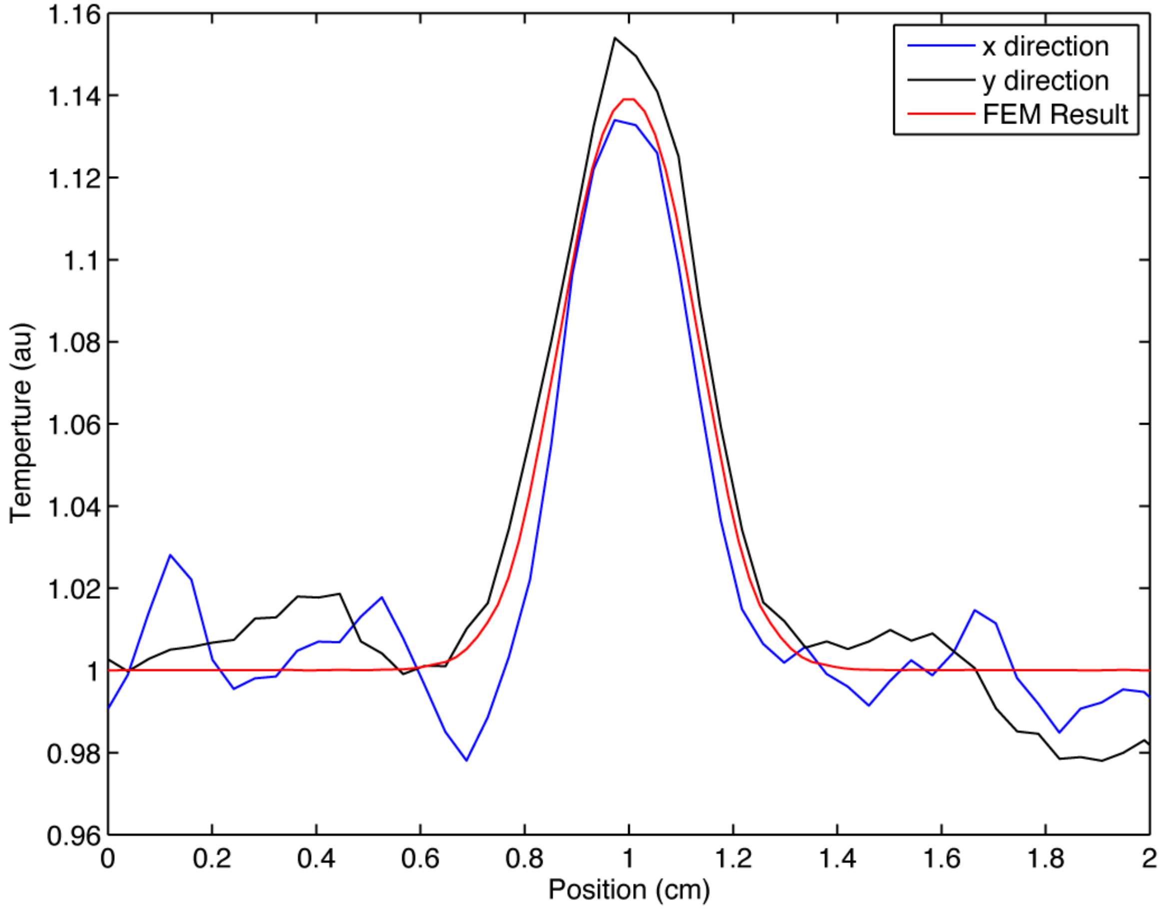
Composite "Flat" Bottom Hole Specimen



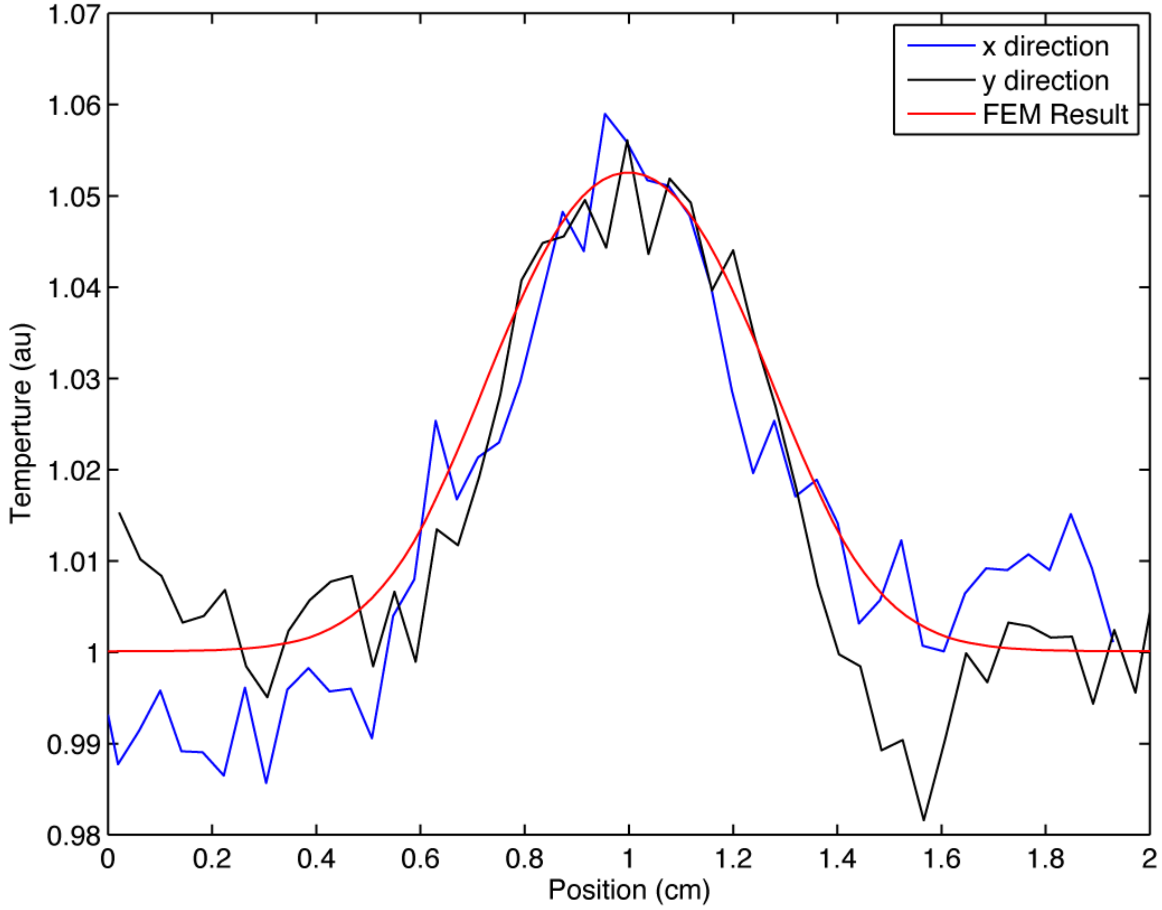


Approximate Depths of Flat Bottom Holes 0.05, 0.1, 0.15 cm
Approximate Diameters 1.27, 0.63, 0.32 cm
Quasi-isotropic ply layup

Temperature Profiles in X and Y directions over Holes in Composite Specimen With FEM Simulation Assuming Isotropic In-plane Heat Conduction

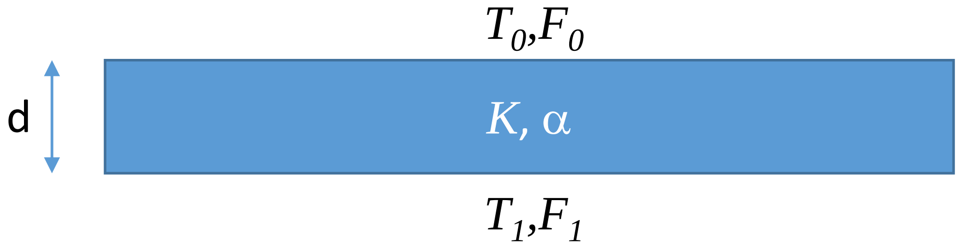


0.32 cm diameter hole 0.05 cm below surface



0.64 cm diameter hole 0.1 cm below surface

Thermal Quadrupoles for Single Layer



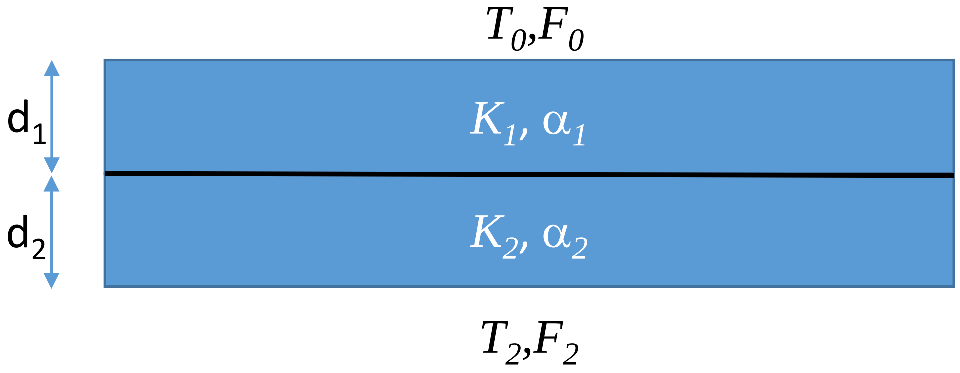
$$\begin{pmatrix} \cosh(q\,d) & -\dfrac{\sinh(q\,d)}{Kq} \\ -Kq\sinh(q\,d) & \cosh(q\,d) \end{pmatrix} \begin{pmatrix} T_0 \\ F_0 \end{pmatrix} = \begin{pmatrix} T_1 \\ F_1 \end{pmatrix}$$

$$q = \sqrt{s/\alpha}$$

- T_0- Laplace transform of front surface temperature
- F_0-Laplace transform of front surface flux
- T_1- Laplace transform of back surface temperature
- F_1-Laplace transform of back surface flux
- α – Thermal Diffusivity
- K - Thermal Conductivity
- d – layer thickness

Two of T_0,F_0 ,T_1,F_1 are defined, inverse transform solved analytically

One Dimensional Thermal Quadrupoles for Two Layers



$$\begin{pmatrix} \cosh(q_2 d_2) & -\dfrac{\sinh(q_2\, d_2)}{Kq} \\ -K_2 q_2 \sinh(q_2 d_2) & \cosh(q_2 d_2) \end{pmatrix}$$

$$\begin{pmatrix} \cosh(q_1 d_1) & -\dfrac{\sinh(q_1 d_1)}{K_1 q_1} \\ -K_1 q_1 \sinh(q_1 d_1) & \cosh(q_1 d_1) \end{pmatrix} \begin{pmatrix} T_0 \\ F_0 \end{pmatrix} = \begin{pmatrix} T_2 \\ F_2 \end{pmatrix}$$

$$q_n = \sqrt{s/\alpha_n}$$

- T_0- Laplace transform of front surface temperature
- F_0-Laplace transform of front surface flux
- T_2- Laplace transform of back surface temperature
- F_2-Laplace transform of back surface flux
- α_1,α_2– Thermal Diffusivities
- K_1, K_2 - Thermal Conductivities
- d_1,d_2 – layer thicknesses

Two of T_0,F_0 ,T_2,F_2 are defined, inverse transform solved numerically

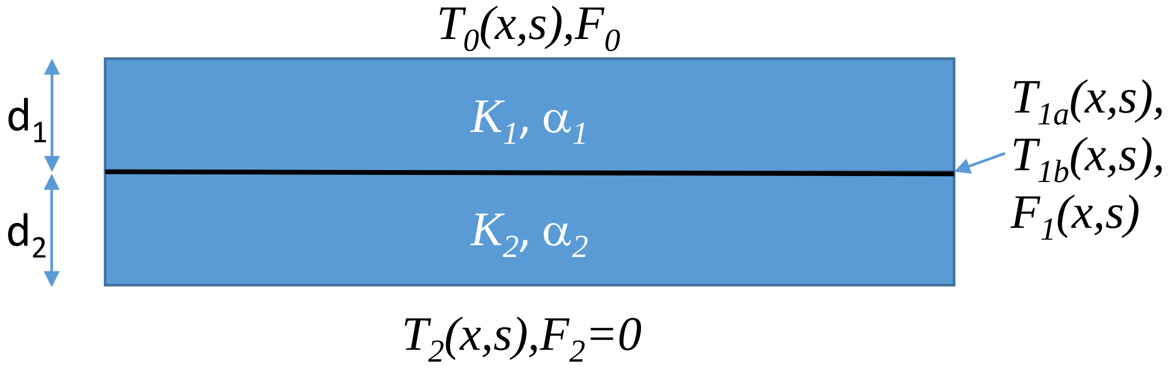
Laplace Transform Thermal Response in Plate

$$T(x,y,z,s) = \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} \cos\left(\frac{n\pi y}{L_y}\right) \cos\left(\frac{m\pi x}{L_x}\right) \left(T_{m,n}\cosh(qz) - F_{m,n}\frac{sinh(qz)}{qK_z}\right)$$

$$q = \sqrt{\frac{s}{\alpha_z} + \frac{\alpha_x}{\alpha_z}\left(\frac{\pi m}{L_x}\right)^2 + \frac{\alpha_y}{\alpha_z}\left(\frac{\pi n}{L_y}\right)^2}$$

- α_x, α_y, α_z, K_z, diffusivities in x, y, z directions and thermal conductivity in z direction
- $T_{m,n}$ and $F_{m,n}$ Cosine Fourier coefficients for surface temperature and flux

Two Dimensional Thermal Quadrupoles for Two Layers with Zero Flux at Back Surface and No Spatial Variation in Front Surface Flux(F_0)



$$\begin{pmatrix} \cosh(q_2 d_2) & -\dfrac{\sinh(q_2\, d_2)}{Kq} \\ -K_2 q_2 \sinh(q_2 d_2) & \cosh(q_2 d_2) \end{pmatrix} \begin{pmatrix} T_{1bm} \\ F_{1m} \end{pmatrix} = \begin{pmatrix} T_{2m} \\ F_{2m} \end{pmatrix}$$

$$\begin{pmatrix} \cosh(q_1 d_1) & -\dfrac{\sinh(q_1 d_1)}{K_1 q_1} \\ -K_1 q_1 \sinh(q_1 d_1) & \cosh(q_1 d_1) \end{pmatrix} \begin{pmatrix} T_{0m} \\ F_{0m} \end{pmatrix} = \begin{pmatrix} T_{1am} \\ F_{1m} \end{pmatrix}$$

$$q = \sqrt{\frac{s}{\alpha_z} + \frac{\alpha_x}{\alpha_z}\left(\frac{\pi m}{L_x}\right)^2}$$

T_0, T_{1a} , T_{1b} , T_2 can be defined in terms of F_0 and F_1

- T_0, T_{1a}, T_{1b}, T_2- Laplace transform of temperature at front surface, above and below interface and back surface
- F_0, F_1, F_2-Laplace transform of front surface interface and back surface flux
- α_1,α_2– Thermal Diffusivities
- K_1, K_2 - Thermal Conductivities
- d_1,d_2 – layer thicknesses
- *m*- refers to Fourier cosine series coefficient

Solving for Temperature at Front Surface

- Instead of solving for Fourier cosine series coefficients, discretize the temperature and fluxes at surfaces and interface

  $T_{0b}(xn), T_{1a}(x_n), T_{1b}(x_n)\ , T_2(x_n)$, and $F_1(x_n)$

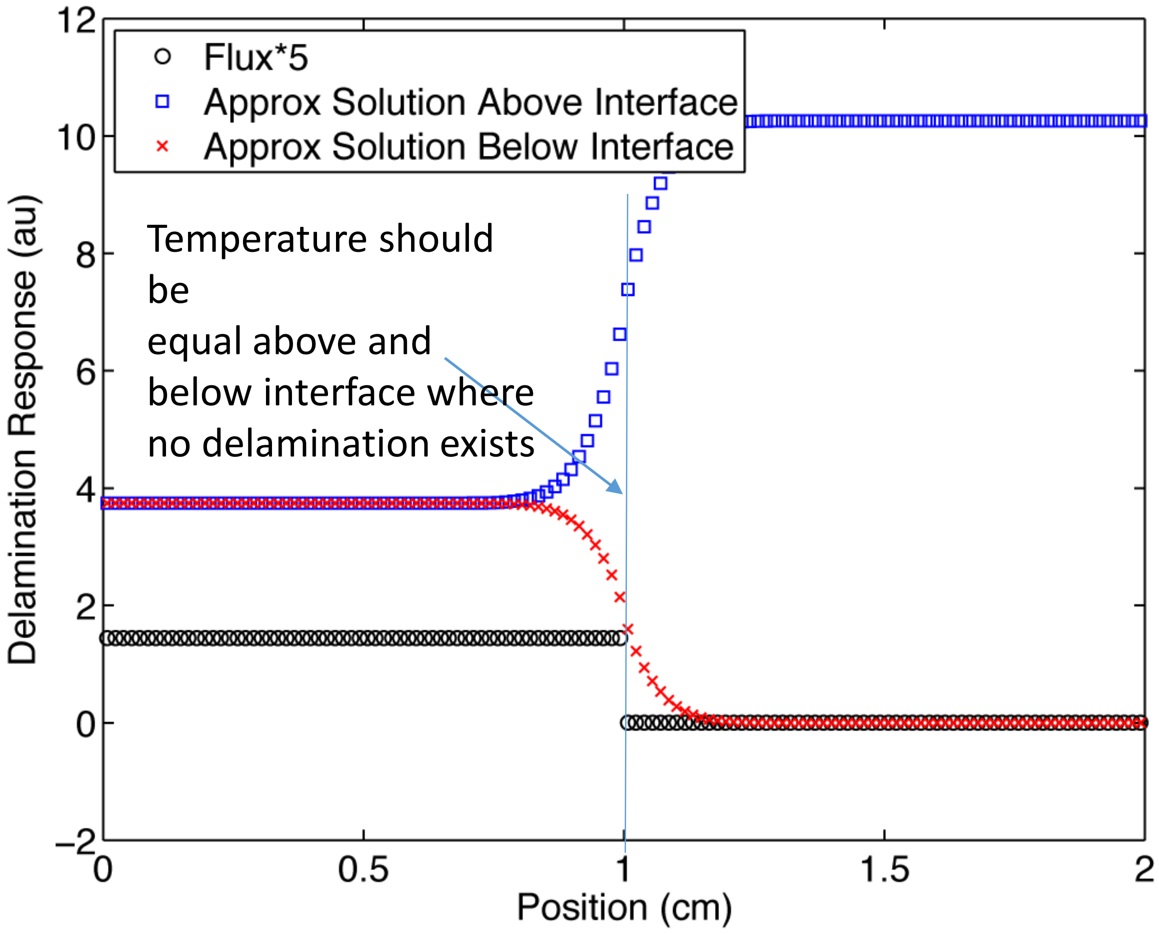
  - Represent the all temperatures and fluxes as vectors, then relate temperatures to fluxes with matrix equations

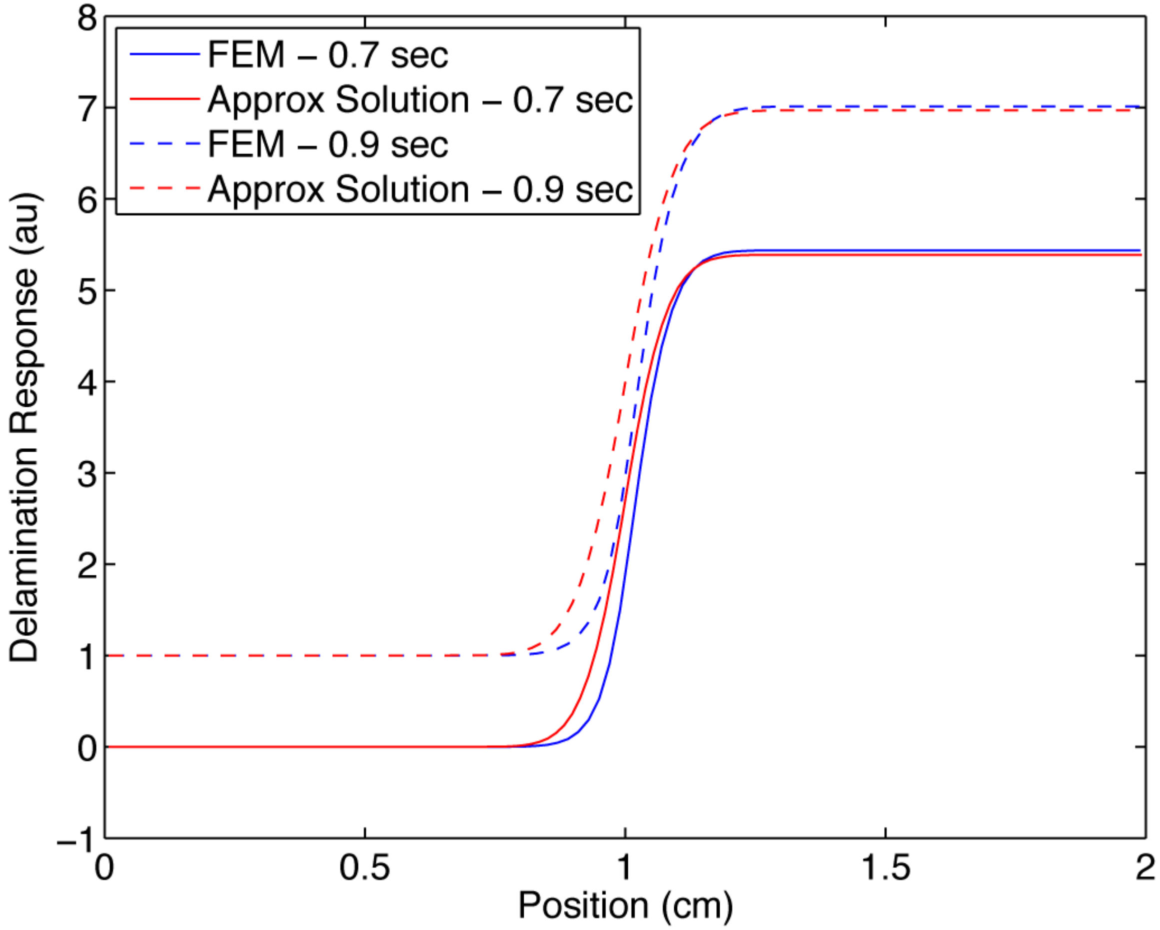
$$T_0 = M_0 * F_1 + F_0 \frac{coth(\sqrt{\frac{s}{\alpha_z}}d_1)}{\sqrt{s/\alpha_z}}$$

- $T_{1b} = M_{1a} * F_1 + F_0 \frac{csch(\sqrt{\frac{s}{\alpha_z}}d_1)}{\sqrt{s/\alpha_z}}$ $\qquad T_{1b} = M_{1b} * F_1$

$$F_1 = Tranpose([F_1(x_0), F_1(x_2), \dots, F_1(x_n)])$$

Estimating the Flux at the Interface as Zero Flux over Delamination and 1 Dimensional Flux Value in Region with No Delamination





Comparison of FEM solution to approximate quadrupole solution

Setting Temperature Equal Below and Above Interface

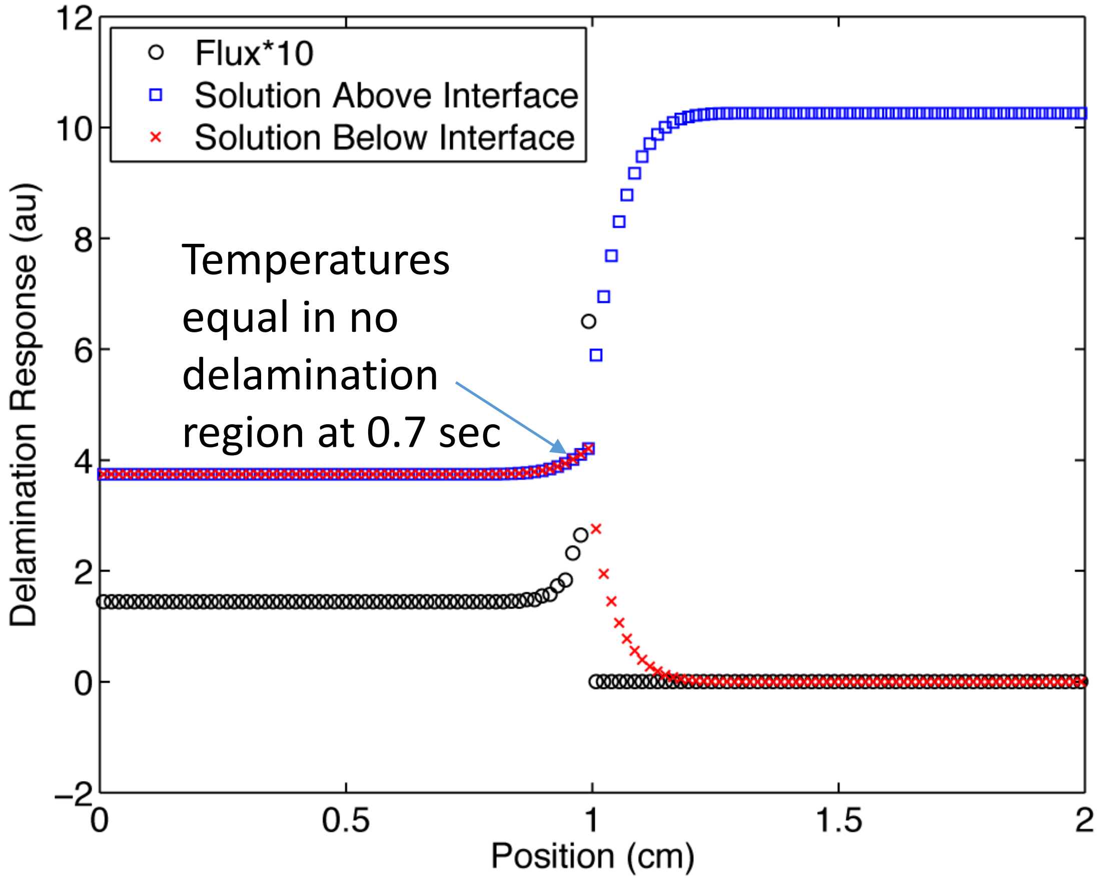
$$M_{1a} * F_1 + F_0 \frac{csch(\sqrt{\frac{s}{\alpha_z}} d_1)}{\sqrt{s/\alpha_z}} = M_{1b} * F_1$$

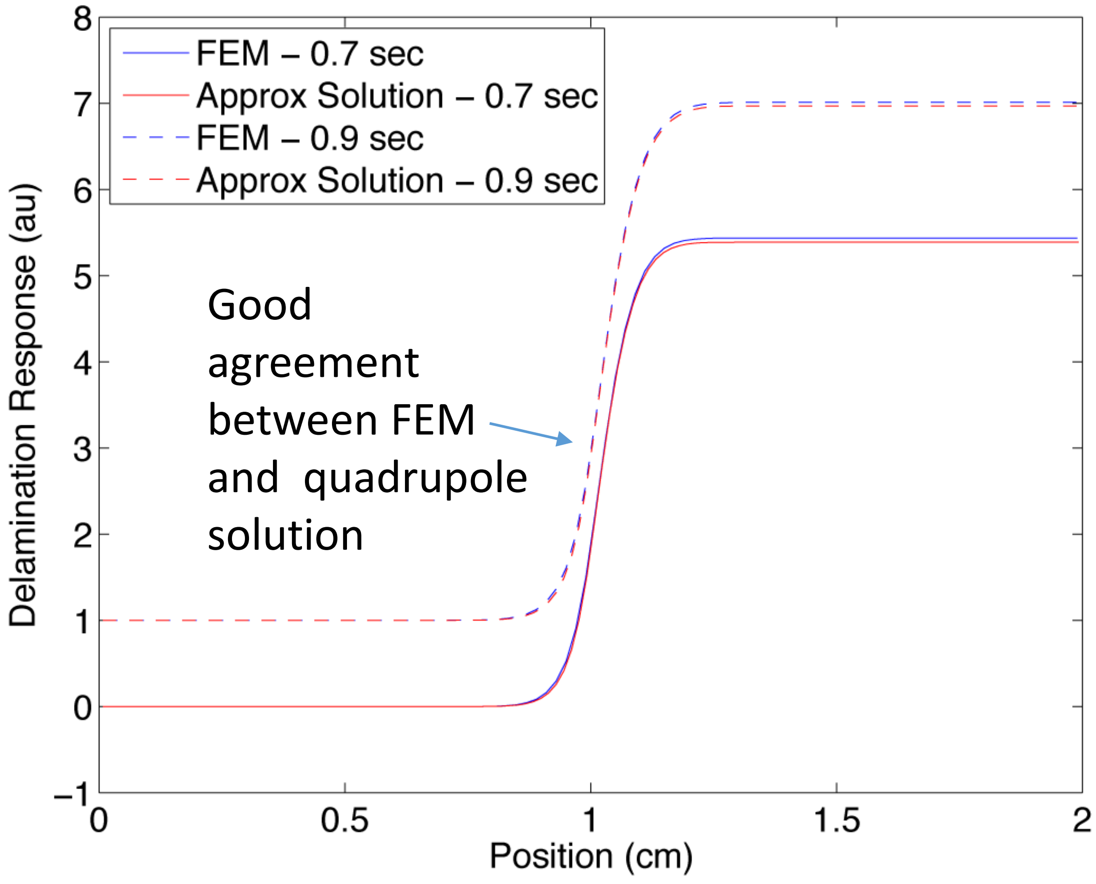
Resulting in the matrix equation

$$(M_{1a} - M_{1b}) * F_1 = F_0 \frac{csch(\sqrt{\frac{s}{\alpha_z}} d_1)}{\sqrt{s/\alpha_z}}$$

Matrix equation solved numerical

Temperatures and Fluxes Calculated for Interfaces and Front Surface





Quadrupole solution calculated in 0.3 sec, FEM solution in 30 sec

Summary

- Simulations assuming in-plane thermal conductivity is isotropic is a good approximation of quasi-isotropic composite layups

- Quadrupole method is a computationally efficient technique for simulating the thermal response of delaminations in composites