Modeling delamination migration: quasi-static and fatigue loading

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Motivation

Migration: The process by which a propagating delamination relocates to a new ply interface via matrix cracking

Impact


Skin-stringer pull off

1 Experiments: delamination migration test
2 Modeling approach
3 Validation
4 Summary
Experiments: delamination migration

Test Setup - Premise

**Delamination**
- ("positive" shear stress)

**Migration**
- ("negative" shear stress)

*adapted from Greenhalgh, 2009

*E.S. Greenhalgh, C. Rogers, P. Robinson. Fractographic observations on delamination growth and the subsequent migration through the laminate. Composites Science and Technology, 69:2345-2351, 2009.*
Experiments: delamination migration test

Test setup

- Cross-ply laminate
- “2D” migration process
- Pre-crack (Teflon insert) between 0° and 90° ply
- Variable load position (L)

Experiments: delamination migration test
Test setup - overview
Experiments: delamination migration test

Test setup - overview

Delamination

Migration
Experiments: delamination migration test

Test setup – validation data

Load - displacement

Migration location

Damage morphology
1 Experiments: delamination migration test

2 Modeling approach: Floating Node Method (FNM) and Virtual Crack Closure Technique (VCCT)

3 Validation

4 Summary
Floating Node Method

Same implementation strategy suitable for standard finite element architecture

X-FEM

Phantom Node Method (PNM)

Floating Node Method (FNM)

Remeshing

Same solution

Same solution

Floating Node Method (FNM)

- **Real node**
- **Floating node**
- **Coordinates of crack positions**

\[ K_q = Q \]
Floating Node Method (FNM)

Real node
Floating node
Coordinates of crack positions

\[ K_q = Q \]

\[ K_A q_A = Q_A \]

\[ K_B q_B = Q_B \]
Floating Node Method (FNM)

Key Characteristics:

- Floating Nodes are topologically related to each element with no initial position assigned.
- The position of the floating nodes is assigned only after the crack path is determined.
- The floating nodes are used to form sub-elements within the original element and accommodate crack networks.
- Ideally suited to represent multiple cracks and their intersection.
- Can be coupled with Virtual Crack Closure Technique (VCCT) and cohesive zone crack formulations to model crack propagation.
Virtual Crack Closure Technique (VCCT):

- **Mode I**
  \[
  G_I = \frac{1}{2\Delta a_1} F_n [q_n] \left( \frac{\Delta a_1}{\Delta a_2} \right)^{\frac{1}{2}}
  \]

- **Mode II**
  \[
  G_{II} = \frac{1}{2\Delta a_1} F_t [q_t] \left( \frac{\Delta a_1}{\Delta a_2} \right)^{\frac{1}{2}}
  \]
FNM & VCCT applied to cross-ply laminates:

Laminate

\[ [0^\circ/90^\circ_2/0^\circ] \]

1 FNM Element (multiple plies)

\[ \Omega \]

- Real node
- Floating node (DoF)
- Coordinates of crack positions

FNM & VCCT applied to cross-ply laminates:

**Laminate**

\([0^\circ/90^\circ_2/0^\circ]\)

---

**1 FNM Element**

\([0^\circ/90^\circ_2/0^\circ]\)

- Real node
- Floating node (DoF)
- Coordinates of crack positions
FNM & VCCT applied to cross-ply laminates:

Quasi-static

- Fracture Criterion:
  \[ f(G_I, G_{II}) = \frac{G_T}{G_{c,\text{Int}}} - 1 = 0 \]
- Mixed Mode exponential law:
  \[ G_{c,\text{Int}} = G_{Ic} + (G_{IIc} - G_{Ic}) \left( \frac{G_{II}}{G_T} \right)^\eta \]

Fatigue

\[
\frac{da}{dN} = A \left( G_{T,\text{max}} \right)^n
\]
\[ n = n_I + (n_{II} - n_I) \left( \frac{G_{II,\text{max}}}{G_T} \right) \]
\[ A = A_I + (A_{II} - A_I) \left( \frac{G_{II,\text{max}}}{G_T} \right) \]

Delamination

- Real node
- Floating node (DoF)
- Coordinates of crack positions
FNM & VCCT applied to cross-ply laminates:
Migration onset

**Quasi-static**

\[
\frac{G_T}{G_c^\mathrm{i}(F_t)} > \frac{G_T}{G_c^{\mathrm{Inter}}} \geq 1
\]

\[
G_c^\mathrm{i} = \begin{cases} 
G_c^A, & F_t < 0 \\
G_c^B, & F_t > 0 
\end{cases}
\]

**Fatigue**

\[
\left( \frac{da}{dN} (F_t) \right)_i > \left( \frac{da}{dN} \right)_{\mathrm{Inter}}
\]

\[
\left( \frac{da}{dN} \right)_i = \begin{cases} 
\left( \frac{da}{dN} \right)_A, & F_t < 0 \\
\left( \frac{da}{dN} \right)_B, & F_t > 0 
\end{cases}
\]
FNM & VCCT applied to cross-ply laminates: Migration onset – quasi-static

\[ \frac{G_T}{G_c^i(F_t)} > \frac{G_T}{G_{\text{Inter}}} \geq 1 \]

\[ G_c^i = \begin{cases} 
G_c^A, & F_t < 0 \\
G_c^B, & F_t > 0 
\end{cases} \]

Material A

Material B

No growth
FNM & VCCT applied to cross-ply laminates: Migration onset – quasi-static

\[ \frac{G_T}{G_c^i (F_t)} > \frac{G_T}{G_{\text{Inter}}} \geq 1 \]

\[ G_c^i = \begin{cases} G_c^A, & F_t < 0 \\ G_c^B, & F_t > 0 \end{cases} \]

Diagram showing the relationship between \( G_T/G_c \) and \( G_T/G_{\text{Inter}} \) for different material configurations.
FNM & VCCT - application to composites: Migration onset - fatigue

\[
\left( \frac{da}{dN} \right)_{F_t} > \left( \frac{da}{dN} \right)_{Inter}
\]

\[
\left( \frac{da}{dN} \right)_i = \begin{cases} 
\left( \frac{da}{dN} \right)_A, & F_t < 0 \\
\left( \frac{da}{dN} \right)_B, & F_t > 0
\end{cases}
\]

Material A

Material B

\[
\left( \frac{da}{dN} \right)_{Inter}
\]
FNM & VCCT applied to cross-ply laminates:

**Quasi-static**

\[ f \left( G_I, G_{II} \right) = \frac{G_T}{G_{Ic}} - 1 = 0 \]

**Fatigue**

\[ \frac{da}{dN} = A_I (G_{Tmax})^{n_I} \]

**Maximum tangential stress criterion:**

\[ \theta = 2 \tan^{-1} \left( \frac{1}{4} \left[ \left( \frac{G_I}{G_{II}} \right) \pm \sqrt{\left( \frac{G_I}{G_{II}} \right)^2 + 8} \right] \right) \]
• Topological criterion
  - local delamination is onset when matrix crack reaches interface
Fatigue algorithm

1. Determine the growth rate for each crack

\[ \left( \frac{da}{dN} \right)_i \]

2. Determine the number of cycles needed to propagate each crack by one element, and the crack which propagates in fewest cycles

\[ \delta N_{\text{inc}}^i = \delta a_{1\text{el}}^i \left( \frac{da}{dN} \right)_i - \delta N_{\text{acc}}^i \]

\[ \delta N_{\text{inc}}^n = \min \left\{ \delta N_{\text{inc}}^i \right\} \]

3. Propagate the crack

\[ a^n = a^n + \delta a_{1\text{el}}^n \]

4. Accumulate the cycles

\[ N = N + \delta N_{\text{inc}}^n \]

\[ \delta N_{\text{acc}}^i = \delta N_{\text{acc}}^i + \delta N_{\text{inc}}^n \]

\[ \delta N_{\text{acc}}^n = 0 \]
Verification – Static: DCB

Verification – Fatigue: DCB benchmark

1 Experiments: delamination migration test

2 Modeling approach: Floating Node Method (FNM) and Virtual Crack Closure Technique (VCCT)

3 Validation: modeling delamination migration

4 Summary
Validation: Delamination migration test

**Numerical model**

<table>
<thead>
<tr>
<th>Dimensions (mm)</th>
<th>$B$</th>
<th>$2h$</th>
<th>$C$</th>
<th>$S$</th>
<th>$a_0$</th>
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<tbody>
<tr>
<td>12.7</td>
<td>5.25</td>
<td>12.7</td>
<td>115</td>
<td>49</td>
<td></td>
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</tbody>
</table>

*B is the width of the specimen (out-of-the-page);
*90° - specimen width direction; 0° - specimen span direction

- Contact modeled between specimen and clamps/baseplate
- Clamping force applied in a first static step
- Abaqus/Standard (Implicit) + UEL
- All material properties obtained using standard/recommended test methods

Model details
Validation: delamination migration test

Results - migration process

Observations

- Correct sequence of events: delamination followed by migration
- Failure morphology well captured – including crack path through-thickness
Validation: delamination migration test

Results – load vs displacement

\[ L = 1.0a_0 \]:

**Observations**
- Max load: good agreement
- Delamination: unstable growth followed by arrest and subsequent unstable and stable growth
- Migration: predicted before delamination arrest
Validation: delamination migration test

Results – load vs displacement

\[ L = 1.1a_0 : \]

**Observations**
- Max load: good agreement
- Delamination: small region of stable growth prior to main load drop
- Migration: predicted within the main load drop
Validation: delamination migration test

Results – load vs displacement

\[ L = 1.2a_0: \]

**Observations**

- Max load: good agreement
- Delamination: stable delamination growth prior to main load-drop
- Migration: predicted within the main load drop
Validation: delamination migration test

Results – load vs displacement

Observations

- Max load: good agreement
- Delamination: stable growth prior to main load-drop
- Migration: predicted within the main load drop
Validation: delamination migration test

Results – Migration location

\[ L = a_0 \quad \text{and} \quad u_2 = V \]

\[ a_0 \quad \Delta M \]

\[ \Delta_M, \quad \text{mm} \]

Observations
- Trend well captured
- Conservative predictions
Fatigue - Preliminary results
Delamination growth and cycles to migration

Constant amplitude, $R = 0.1$ and $f = 5$ Hz:

Observations
- Load-offset affects fatigue life
1. Experiments: delamination migration test

2. Modeling approach: Floating Node Method (FNM) and Virtual Crack Closure Technique (VCCT)

3. Validation: modeling delamination migration

4. Summary
Summary

• Developed a finite element model based on the Floating Node Method combined with the Virtual Crack Closure Technique to capture the interaction between delamination and matrix-cracking.

• Identified and applied migration criteria for both quasi-static and fatigue loading.

• Compared simulations and experiments.
  – Good agreement observed for load-displacement, migration location, and path.

• Validation of the fatigue simulations are in progress.
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Backup Slides: cohesive zone elements

FNM: Formation of Non-Matching Mesh

- Global mesh with partitioning
- Cohesive zone elements before and after matrix cracking

“Non-matching mesh”

- Saturation crack density (experimental)
- Floating node method
- Experimental data points
Backup Slides: element integration
Backup Slides: Topological migration criterion, experimental evidence
Backup Slides: FNM vs PNM, convergence: $K_I$

![Diagram showing comparison between FNM and PNM for convergence of $K_I$.](image)

**Error in $K_I$**

- **Phantom Node Method (PNM) (Abaqus)**
- **FNM**

- Axes:
  - X-axis: Number of DoF
  - Y-axis: Error in $K_I$ (%)
Backup Slides: FNM vs PNM, accuracy: $K_l$, $K_{ll}$

$K_l, K_{ll}$ (MPa mm$^{1/2}$)

<table>
<thead>
<tr>
<th></th>
<th>FNM</th>
<th>PNM</th>
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<td>Int. 1</td>
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<td>Mode I</td>
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<tr>
<td>Mode II</td>
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Analytical
BENCHMARK  
SIMULATION