Estimation of Surface Temperature and Heat Flux by Inverse Heat Transfer Methods using Internal Temperatures Measured while Radiantly Heating a Carbon/Carbon Specimen up to 1920°F

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OUTLINE

Problem Statement and Background

Computational Code Development

- Algorithm derivation
- Data filtering
- Mesh convergence
- Method validation

Analysis of LaRC Radiant Test Data

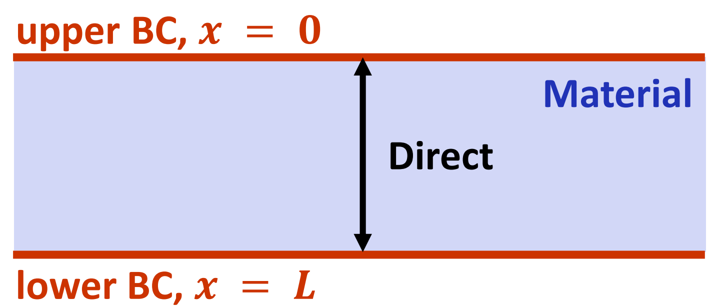
Concluding Remarks

Future Work

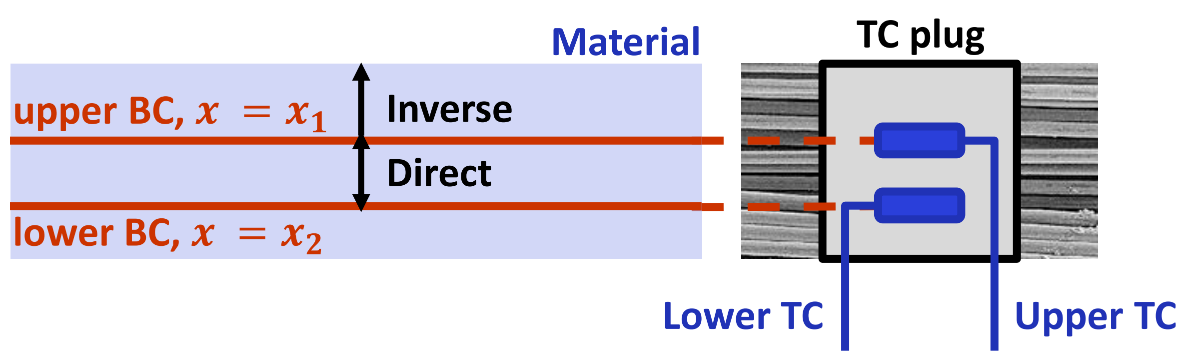
PROBLEM STATEMENT

- Purpose is to estimate surface temperature and heat flux values from internal temperatures measured while radiantly heating a carbon/carbon specimen up to 1920°F
- Initial and boundary conditions (BC) for T and/or q'' must be known

Ideally, BC's given on upper and lower surfaces of the material; **direct** problem needs only be solved



In some cases, BC's given internally to material surface; both **direct** and **inverse** problems must be solved



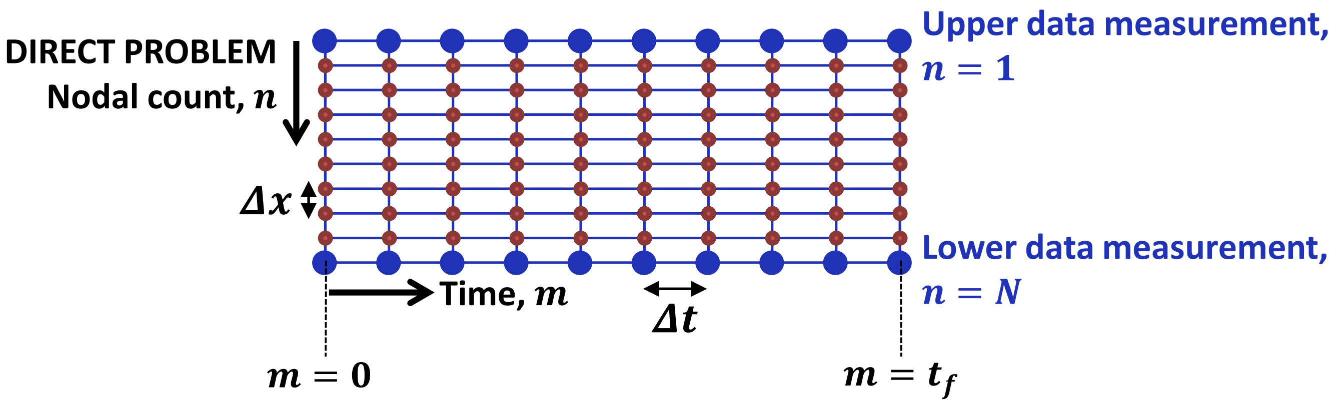
TC: thermocouple

COMPUTATIONAL METHOD

DIRECT SOLUTION

$$C_p(x,T)\rho(x)\frac{\partial T(x,t)}{\partial t} = \frac{\partial}{\partial x}\left(k(x,T)\frac{\partial T(x,t)}{\partial x}\right) + Q(x,t)$$

- Developed a program to first solve the direct heat conduction problem
  - **Direct Problem**: *solving between internal measurements*
    - Created a mesh between the upper and lower data measurements
    - Calculated T and q'' values at nodal points in the mesh and every time step using a one-dimensional, implicit, centered, finite volume method



$$\frac{T_n^{(m+1)} - T_n^{(m)}}{\Delta t} = \frac{\alpha_{n-1,n}^{(m)}}{2\Delta x}\left(\frac{T_{n-1}^{(m)} - T_n^{(m)} + T_{n-1}^{(m+1)} - T_n^{(m+1)}}{\Delta x}\right) + \frac{\alpha_{n,n+1}^{(m)}}{2\Delta x}\left(\frac{T_{n+1}^{(m)} - T_n^{(m)} + T_{n+1}^{(m+1)} - T_n^{(m+1)}}{\Delta x}\right)$$

$$q_n''^{(m+1)} = k_n^{(m)}\left(\frac{T_n^{(m)} - T_{n+1}^{(m)} + T_n^{(m+1)} - T_{n+1}^{(m+1)}}{2\Delta x}\right) + \rho c_{p\,n}^{(m)}\Delta x\left(\frac{T_n^{(m+1)} - T_n^{(m)}}{\Delta t}\right)$$

COMPUTATIONAL METHOD

INVERSE SOLUTION

- Developed a program to then solve the inverse heat conduction problem
  - **Inverse Problem**: *using direct problem solution to march to the surface of the material*
    - Used work presented by A.S. Carasso, 1992 to select a space marching scheme
    - Space marching allows for the estimation of surface T and q"

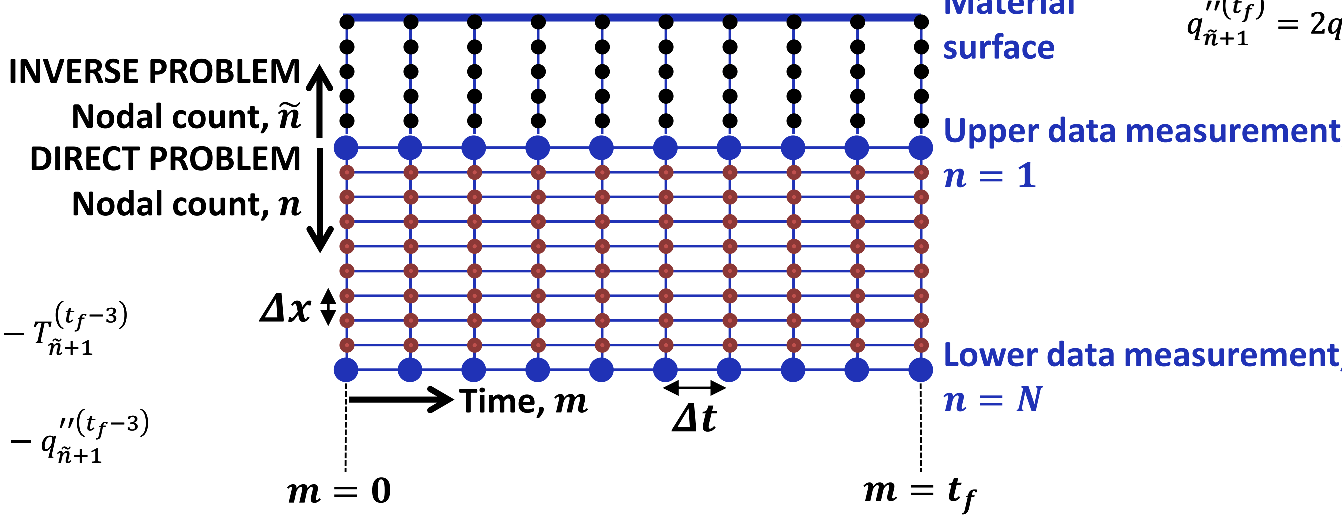
THE FUTURE 0 SPACE MARCHING TECHNIQUE WAS SELECTED FOR THE INVERSE ANALYSIS.

$$T_{\tilde{n}+1}^{(m)} = T_{\tilde{n}}^{(m)} + \Delta x \frac{q_{\tilde{n}}^{\prime\prime(m)}}{k_{\tilde{n}}^{(m)}}$$

$$q_{\tilde{n}+1}^{\prime\prime(m)} = q_{\tilde{n}}^{\prime\prime(m)} + \Delta x \rho {c_p}_{\tilde{n}}^{(m)} \left(\frac{4T_{\tilde{n}}^{(m+1)} - T_{\tilde{n}}^{(m+2)} - 3T_{\tilde{n}}^{(m)}}{2\Delta t} \right)$$

$$T_{\tilde{n}+1}^{(t_f)} = 2T_{\tilde{n}+1}^{(t_f-1)} - T_{\tilde{n}+1}^{(t_f-2)}$$

$$q_{\tilde{n}+1}^{\prime\prime(t_f)} = 2q_{\tilde{n}+1}^{\prime\prime(t_f-1)} - q_{\tilde{n}+1}^{\prime\prime(t_f-2)}$$



$$T_{\tilde{n}+1}^{(0)} = T_{\tilde{n}}^{(0)}$$

$$q_{\tilde{n}+1}^{\prime\prime(0)} = q_{\tilde{n}}^{\prime\prime(0)}$$

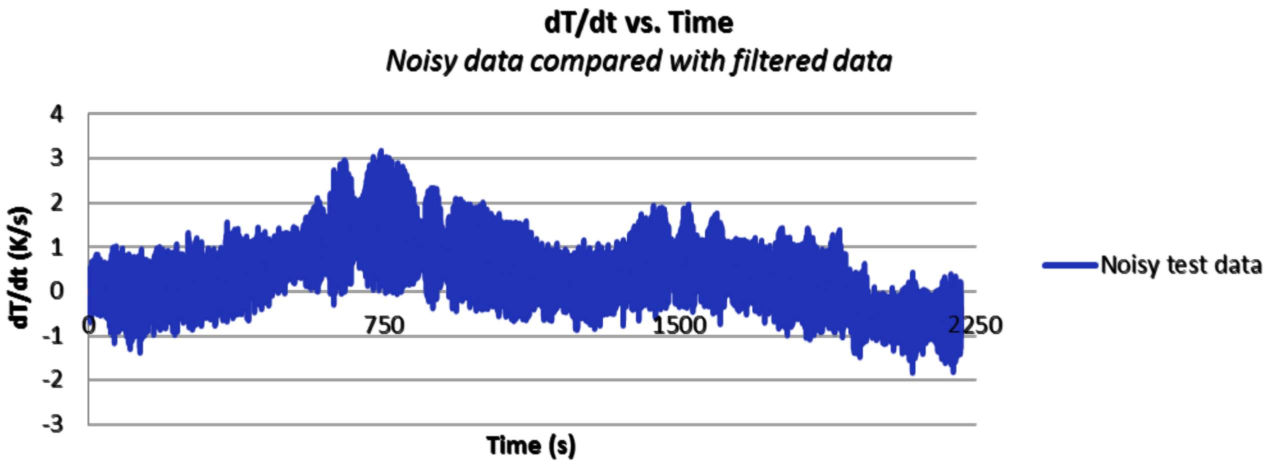
$$T_{\tilde{n}+1}^{(t_f-1)} = 2T_{\tilde{n}+1}^{(t_f-2)} - T_{\tilde{n}+1}^{(t_f-3)}$$

$$q_{\tilde{n}+1}^{\prime\prime(t_f-1)} = 2q_{\tilde{n}+1}^{\prime\prime(t_f-2)} - q_{\tilde{n}+1}^{\prime\prime(t_f-3)}$$

Carasso, A. S. "Space Marching Difference Schemes in the Nonlinear Inverse Heat Conduction Problem." Inverse Problems 8.1 (1992): 25-43. Print.

COMPUTATIONAL METHOD

DATA FILTERING

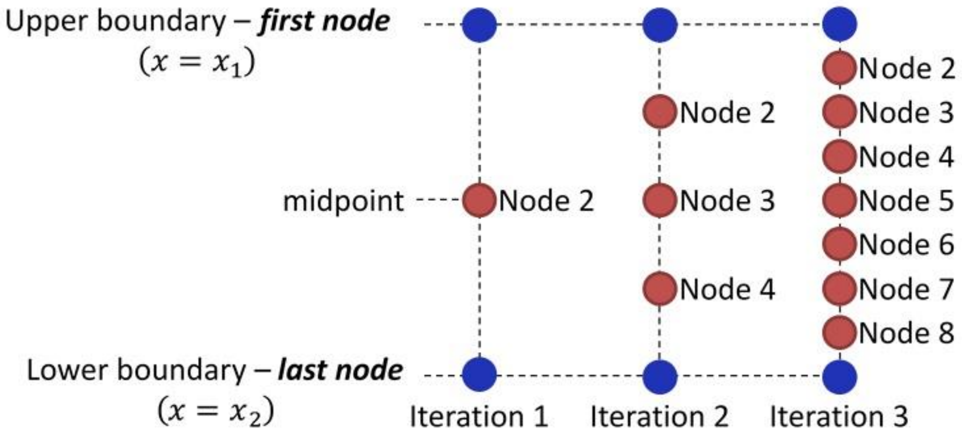


$$C_p(x,T)\rho(x)\frac{\partial T(x,t)}{\partial t} = \frac{\partial}{\partial x}\left(k(x,T)\frac{\partial T(x,t)}{\partial x}\right) + Q(x,t)$$

- In this study, local Blackman and Hamming windowed sinc filters were used to reduce noise in the data.
  - Noise is reduced at *each data point* using information from a **certain number** of data points depending on the selected bandwidth
    - Bandwidth of 0.1 uses 41 data points, 20 on either side of each data point
    - Bandwidth of 0.2 uses 21 data points, 10 on either side of each data point
  - Saves on computational time when there is an excessive amount of data points compared to using a global filter

COMPUTATIONAL METHOD
MESH CONVERGENCE

- Solutions were calculated using multiple iterations of increasing nodal counts (3, 5, 9, 17, 33, etc.)



- Variance between T values at the current and previous nodal count was calculated over the entire range of time at the midpoint between the upper and lower boundary
- Solution was considered to be converged once the variance was less than 0.01 K

METHOD VALIDATION

COMPARISON TO EXACT ANALYTICAL SOLUTION

- To solve for an exact, analytical solution the following assumptions were made:
  - Constant thermal properties
  - No internal sources of heat

$$\frac{\partial T(x,t)}{\partial t} = \alpha \frac{\partial^2 T(x,t)}{\partial x^2}, \alpha = \frac{k}{C_p \rho}$$

- Spatial range: $0 \leq x \leq L$
- Constant initial condition: $T(x,0) = T_0$
- Constant boundary conditions: $T(0,t) = T_1$ and $T(L,t) = T_2$
- Solved for the unique analytical solution over a range of $t = [0,60]$ seconds at varying spatial locations of $x = {}^{L}/_{4}, {}^{L}/_{2}$, and ${}^{3L}/_{4}$

TEMPERATURE $T(x,t) = T_1 + (T_2 - T_1)\frac{x}{L} + \frac{2}{\pi}\sum_{j=1}^{\infty}\frac{1}{j}[(-1)^j(T_2 - T_0) - (T_1 - T_0)]\sin\left(\frac{j\pi x}{L}\right)e^{-k\left(\frac{j\pi}{L}\right)^2 t}$

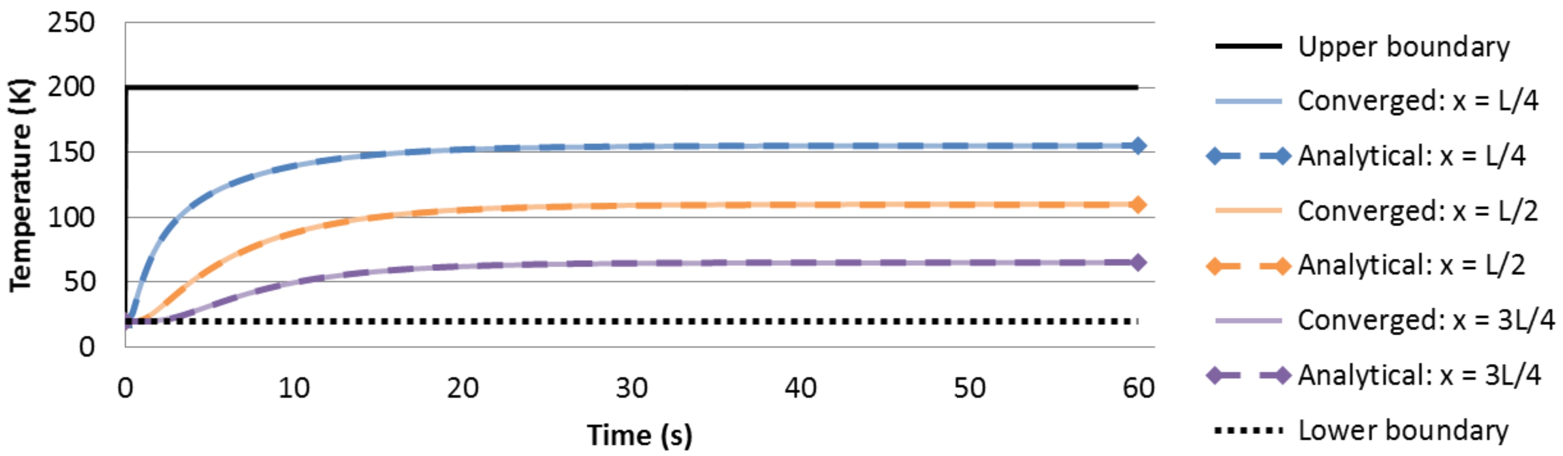
HEAT FLUX $q''(x,t) = -k\left[(T_2 - T_1)\frac{1}{L} + \frac{2}{\pi}\sum_{j=1}^{\infty}\frac{1}{j}[(-1)^j(T_2 - T_0) - (T_1 - T_0)]\left(\frac{j\pi}{L}\right)\cos\left(\frac{j\pi x}{L}\right)e^{-k\left(\frac{j\pi}{L}\right)^2 t}\right]$

Mock Temperature and Thermal Values

	$T_1\ (K)$	$T_2\ (K)$	$T_0\ (K)$	$L\ (m)$	$k\left(\frac{W}{mK}\right)$	$C_p\left(\frac{J}{kgK}\right)$	$\rho\left(\frac{kg}{m^3}\right)$	$\alpha\left(\frac{m^2}{s}\right)$
Test I	200	100	20	0.01	4	1500	1580	$1.69E-6$
Test II	200	20	20	0.01	4	1500	1580	$1.69E-6$

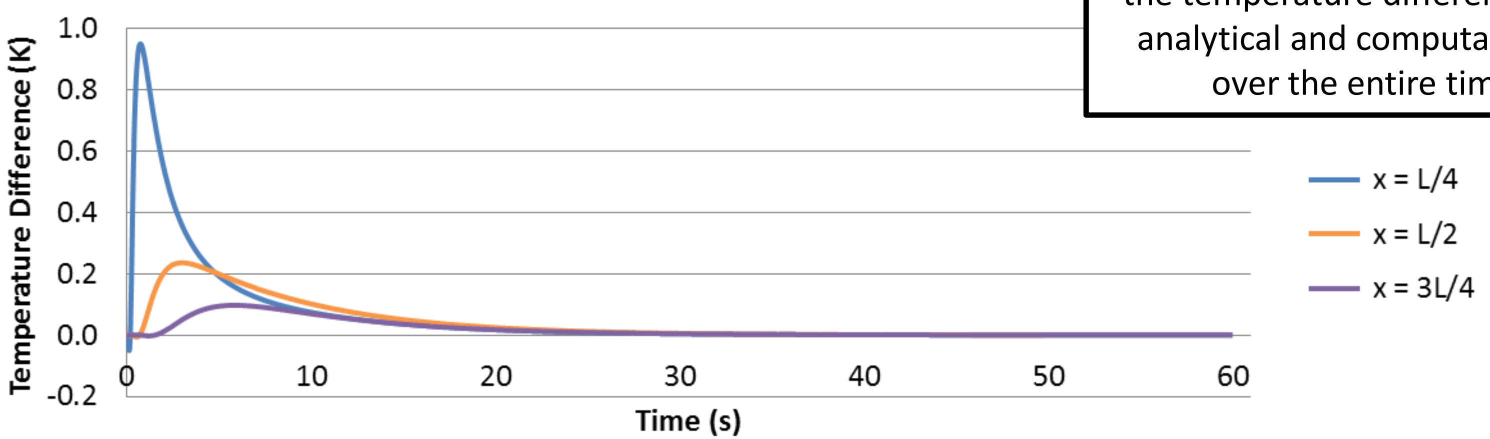
METHOD VALIDATION
COMPARISON RESULTS

Temperature vs. Time
Test II, Δt = 0.05 seconds

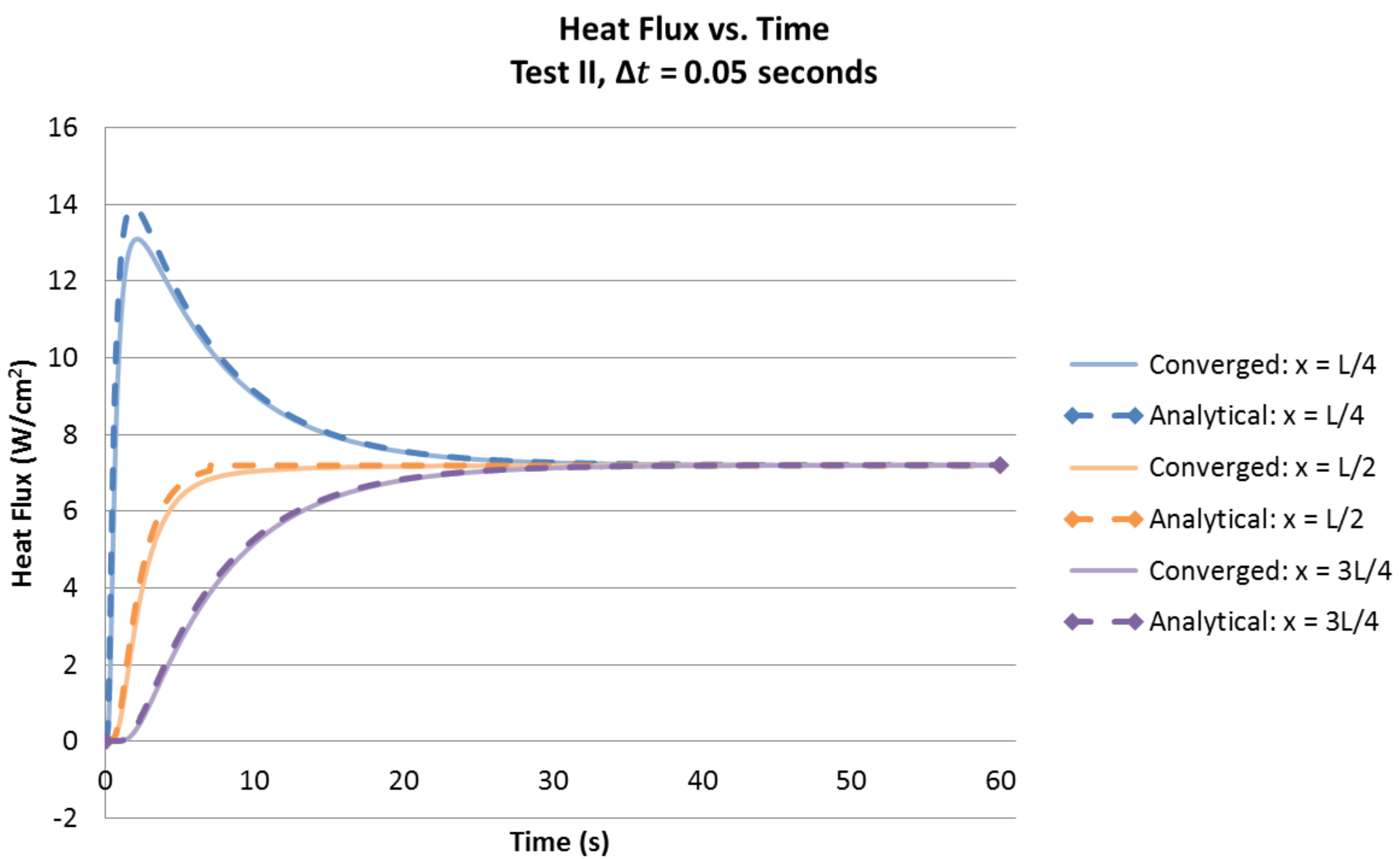


Temperature Difference vs. Time
Test II, Δt = 0.05 seconds

This temperature difference plot depicts the temperature difference between the analytical and computational solutions over the entire time domain.



METHOD VALIDATION
COMPARISON RESULTS

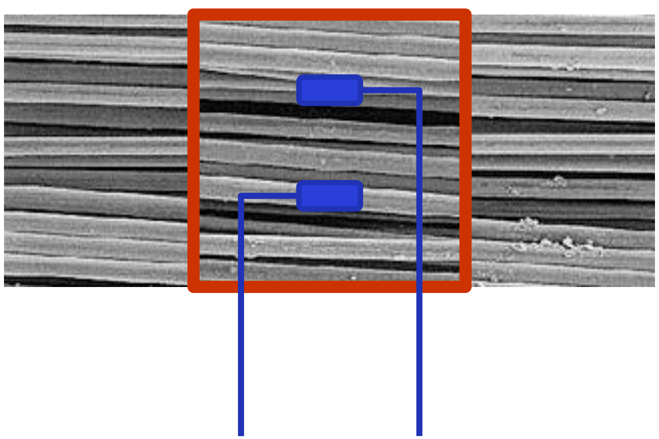


TC PLUG DESCRIPTIONS

- Carbon/carbon (C/C) material, slightly different for each plug
- For accuracy of solution
  - Material properties must be accurate
  - TC depths must be accurate
  - **Analysis must represent the physics** (1D analysis, 1D physics)

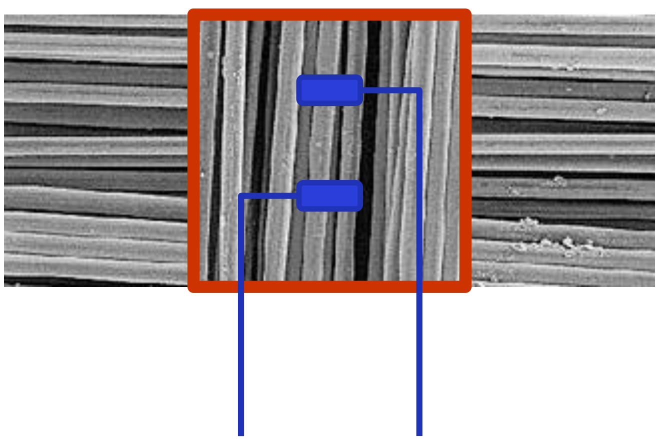
Horizontal-ply plug

SAME MATERIAL *and*
SAME FIBER ORIENTATION



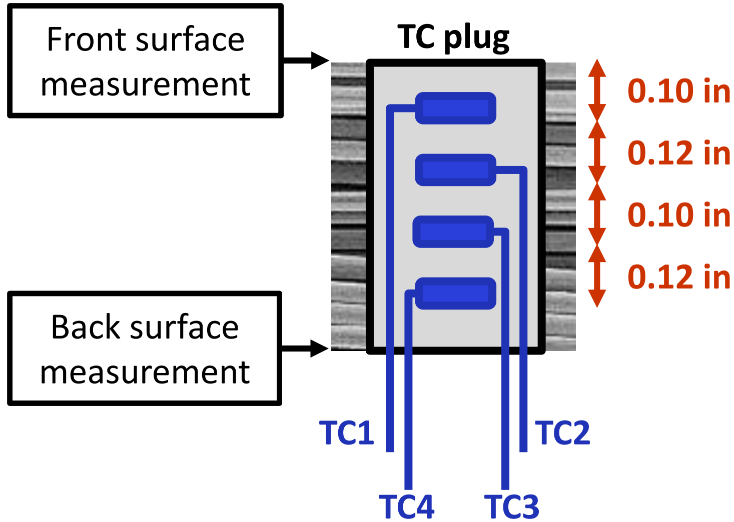
Vertical-ply plug

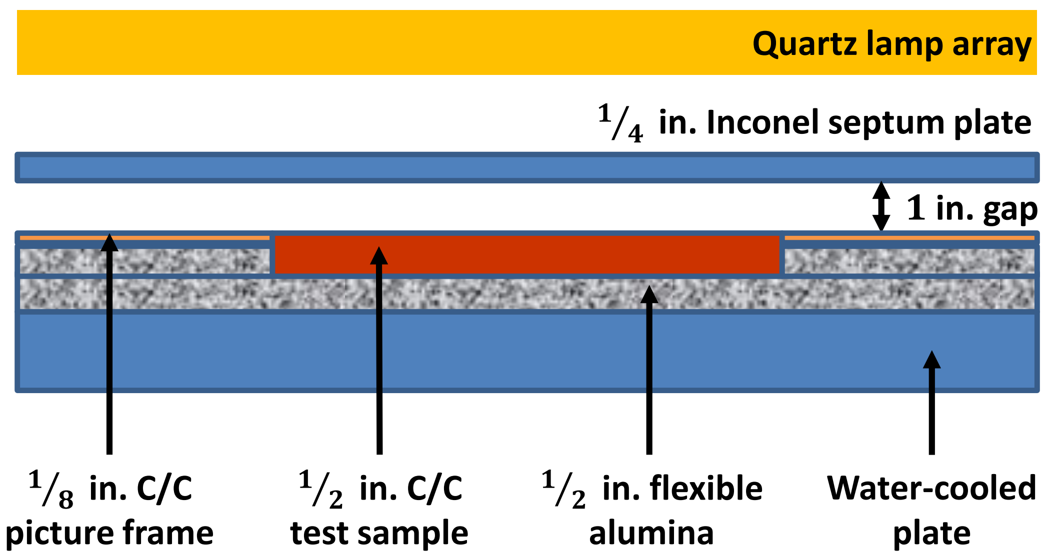
DIFFERENT MATERIAL *and/or*
DIFFERENT FIBER ORIENTATION



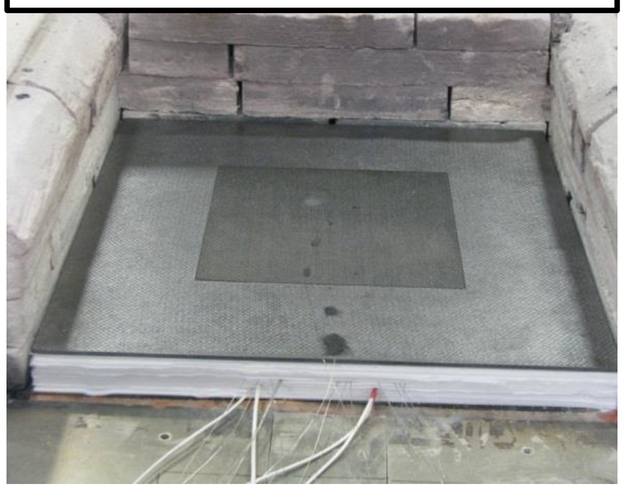
LaRC RADIANT TEST SETUP

- 6 in. × 6 in. × ½ in. carbon/carbon test sample placed directly on ½ in. alumina insulation on top of water-cooled plate; heated with quartz lamps
- Measured internal T using horizontal-ply and vertical-ply TC plugs, each with 4 embedded TC's; data sampled at 10 Hz
- 12 different tests conducted with target T of 500°F to 1920°F, Test 9 results discussed



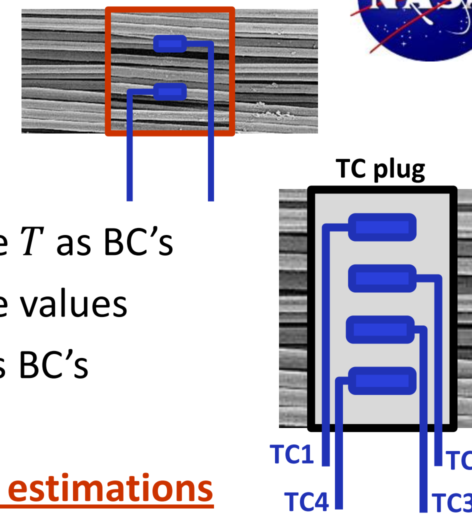


Photograph of experimental setup



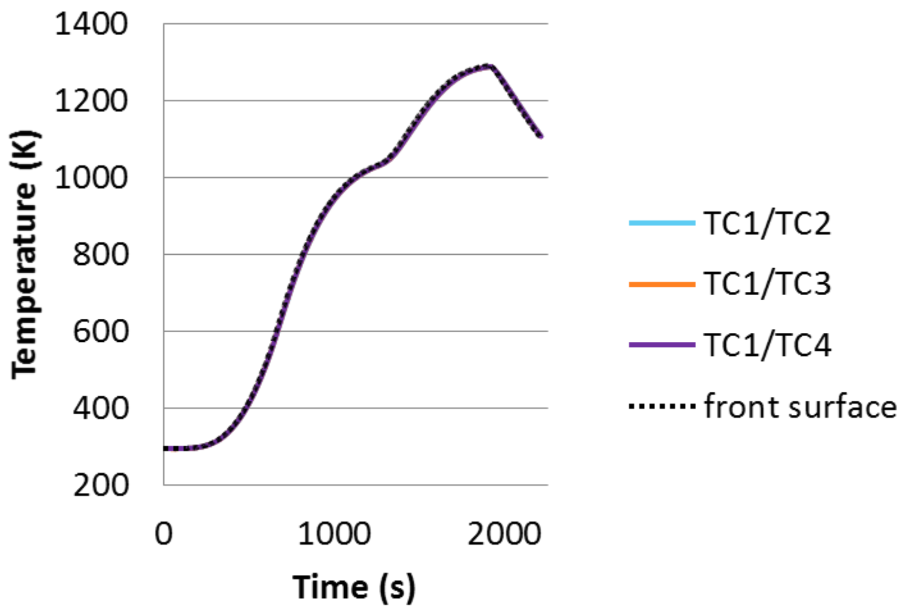
LaRC RADIANT TEST 9

HORIZONTAL-PLY RESULTS

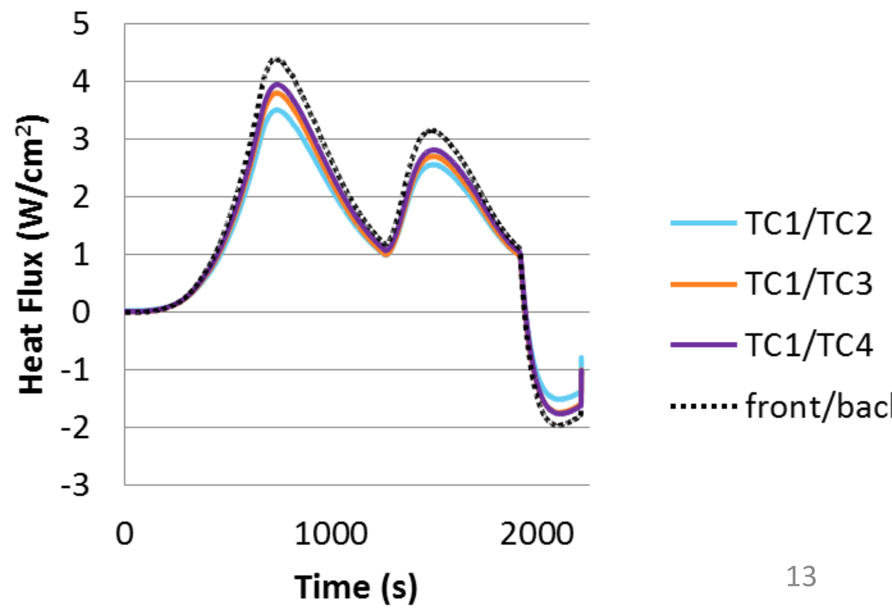


- Front surface q'' estimated by direct problem with front/back surface T as BC's
- Front surface T data, estimated q'' assumed to be the correct surface values
- TC plug data analyzed with combinations of TC1, TC2, TC3, and TC4 as BC's
  - Surface estimations compared to the assumed correct values
  - Results indicated **ACCEPTABLE** inverse analysis T and q'' <u>surface estimations</u>

Temperature vs. Time
Front Surface Comparison, LaRC Test 9

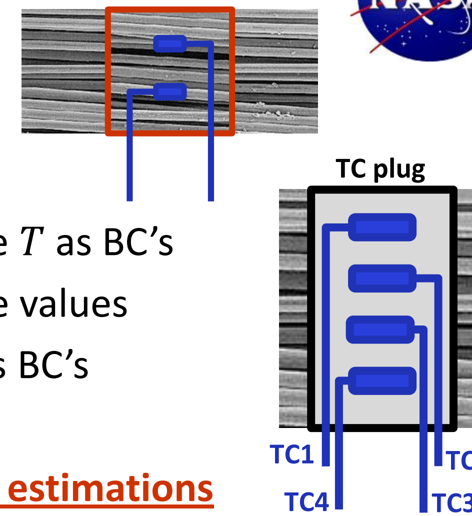


Heat Flux vs. Time
Front Surface Comparison, LaRC Test 9



LaRC RADIANT TEST 9

HORIZONTAL-PLY RESULTS



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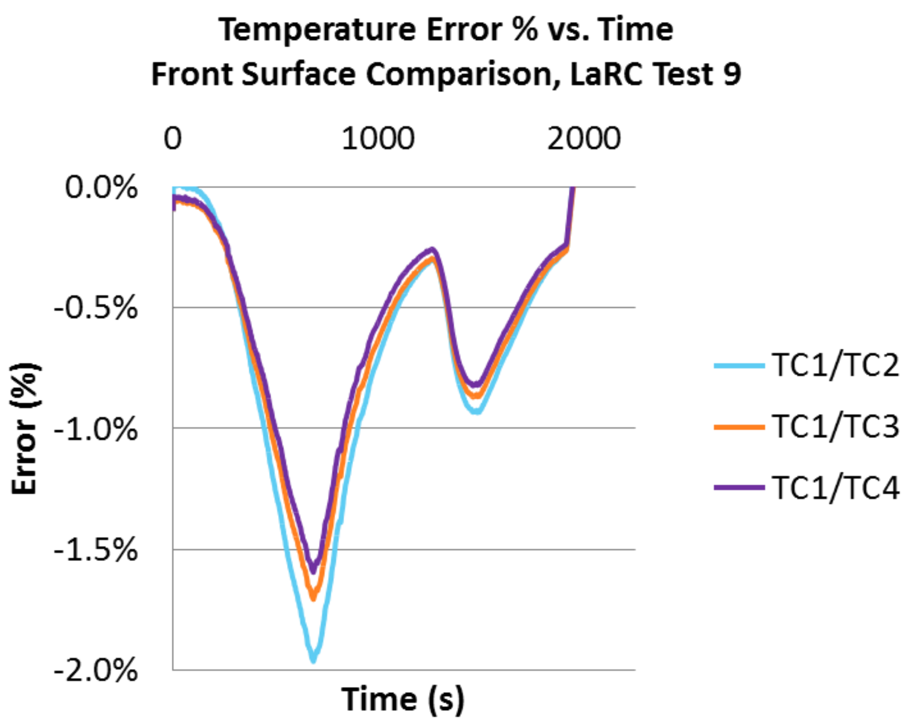
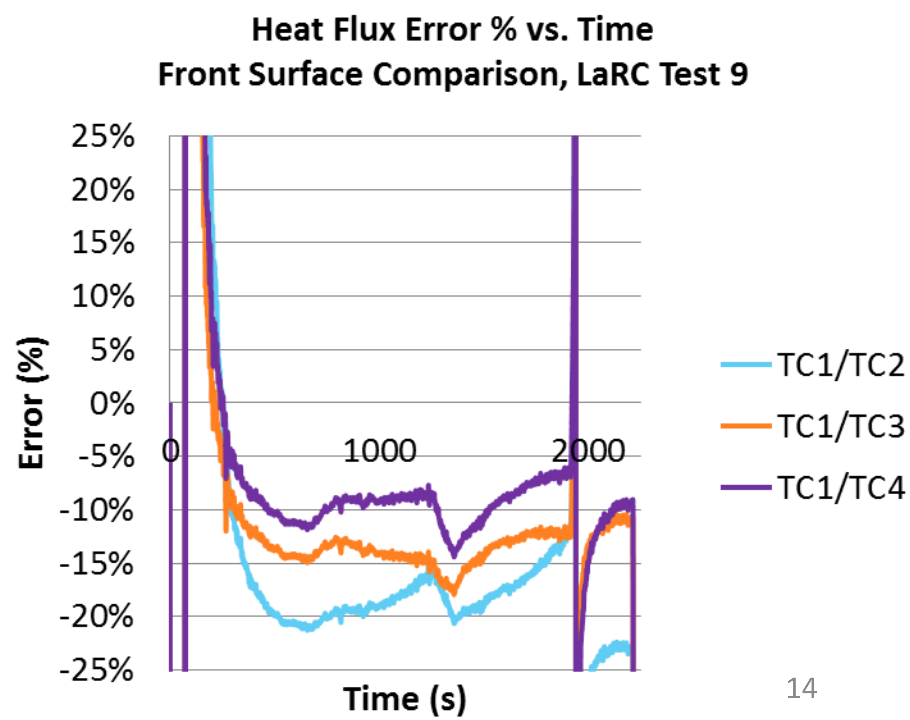
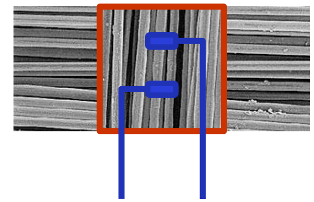
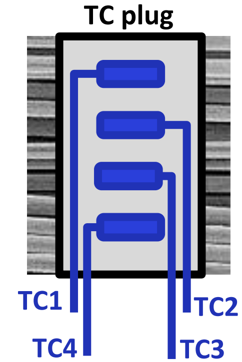




LaRC RADIANT TEST 9
VERTICAL-PLY RESULTS

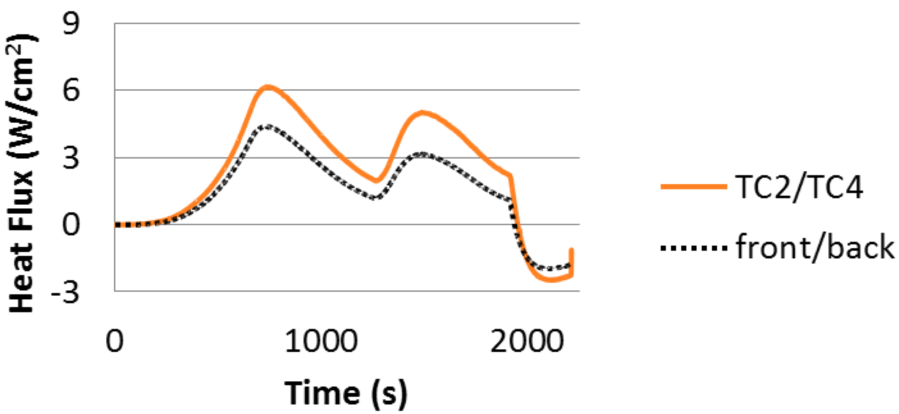




- Data analyzed using thermal properties of the plug and of the surrounding material with TC2/TC4 as BC's

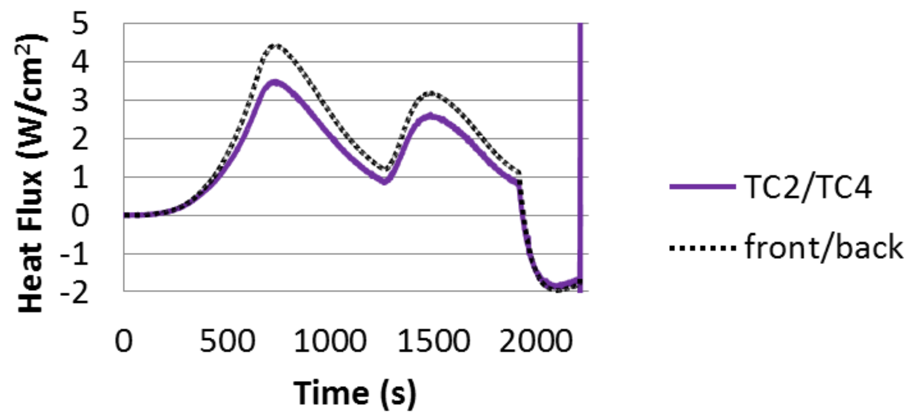
Analysis using ***thermal properties of plug***

Heat Flux vs. Time
Front Surface Comparison, LaRC Test 9



Analysis using ***thermal properties of surrounding material***

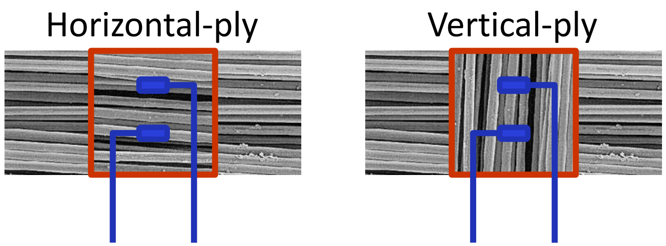
Heat Flux vs. Time
Front Surface Comparison, LaRC Test 9



Analysis demonstrated a 1D modeling technique can only <u>consistently</u> predict <u>accurate</u> surface T and q'' when the *TC plug properties and fiber orientation are the SAME as the surrounding material*, hence approximating 1D physics.

LaRC RADIANT TESTS
CONCLUDING REMARKS

- *Developed computational code to solve 1D direct and inverse heat conduction problems*
  - Used one-dimensional, implicit, centered, finite volume method for direct problem
  - Used space marching techniques for inverse problem
  - Validated direct problem accuracy by comparing to an exact analytical solution
  - Accounted for data filtering and mesh convergence
- *Applied 1D analysis to real experimental data*
- *Demonstrated solution has better accuracy when using TC plugs manufactured with same material and fiber orientation (approximately 1D physics) vs. when manufactured with different material and/or fiber orientation (non-1D physics)*
  - Analysis yielded small errors for T, q'' when using horizontal-ply plug
  - Analysis yielded large errors for T, q'' when using vertical-ply plug



FUTURE WORK

- Assume a BC on the upper surface of the material, $x = x_0$
- Using the direct problem computational formula (one-dimensional, implicit, centered, finite volume method), solve for thermal solution between the material surface and lower BC using x_0 and x_2 as BC's
- Adjust x_0 estimation until thermal solution at $x = x_1$ matches TC measurement