AN APPROXIMATE EQUATION FOR THE

MULTIPLY SCATTERED CONTRIBUTION TO A LIDAR RETURN

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ABSTRACT

An approximate equation is developed which describes the contribution of N^{th} order scattering to a lidar return. This development assumes a homogeneous scattering medium characterized by a scattering function sharply peaked in the forward direction and relatively insensitive to angle near the backscatter direction. The derivation includes the effects of finite divergence of the transmitted laser beam, finite receiver field of view, finite separation between the laser and the receiver and nonparallel system alignment.

The derivation presented uses small angle approximations to reduce the time dependent multiple scattering problem to a time independent form which is then solved with techniques previously developed for multiple small angle nuclear scattering. This derivation results in the following equation for power received due to N^{th} order small angle scattering:

$$P_N = E_o \frac{C}{2} \beta \frac{A_r}{r} \frac{P(\pi)}{4\pi} e^{-2\beta r} \left(\frac{\beta r}{2}\right)^{N-1} \sum_{m=0}^{N-1} \frac{1}{m!(N-m-1)!} e^{-a\psi} \int_0^{a\delta_n^2} e^{-u} I_o(2\psi\sqrt{au})\,du \quad (1)$$

where:

$$a = \frac{3}{3\langle\delta^2\rangle + \langle\theta_s^2\rangle(3N-2m-3)}$$

$\psi = \frac{s}{r} + \gamma$

P_N = Power received due to N^{th} order scattering

E_o = Transmitter energy

c = Speed of light

β = Scattering cross section per unit volume

r = Range

A_r = Area of receiver

$P(\pi)$ = Backscatter phase function (see Deirmendjian, 1969)

N = Order of scattering

$\langle \delta^2 \rangle$ = Mean square divergence angle of transmitting laser

$\langle \theta_s^2 \rangle$ = Mean square wisth of the forward scattering peak in the phase function

I_o = Modified Bessel function of zero order

δ_r = Half-angle receiver field of view

s = Lateral separation between laser and receiving telescope

γ = Angular misalignment of the laser and telescope axis (this misalignment is assumed to be in the plane defined by the telescope mirror and the axis of the laser)

Equation 1 provides a simple means of estimating multiple scattering during operational lidar work. Notice that in the special case of a nearly coaxial system geometry where: $\psi<<1$, $I_o(2\psi\sqrt{au})\simeq 1$ for all $u \leq a\delta_r^2$. In this case the integral in equation 1 reduces to a simple exponential and 'slide rule' solutions can be obtained for P_N.

Examples of the multiply scattered lidar return predicted by equation 1 will be presented and compared to preliminary measurements.

References

Deirmendjian, Electromagnetic Scattering on Spherical Polydispersions; Elsevier (1969), N.Y.