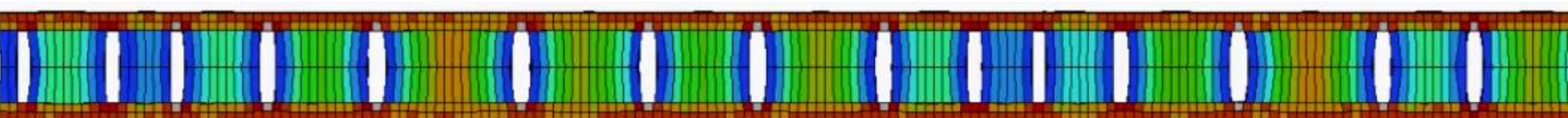


Mesh Convergence Requirements for Composite Damage Models

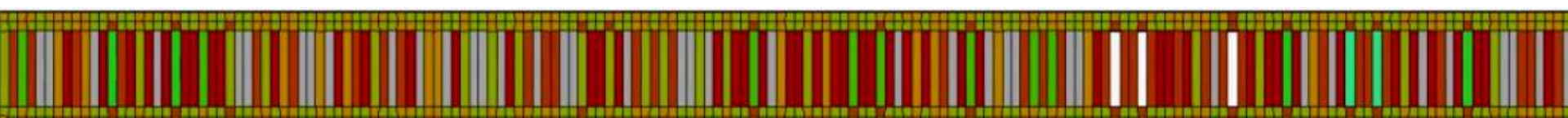


Carlos G. Dávila

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Hampton, VA



**Mechanical Engineering Dept.,
TU Eindhoven, NL
17 March 2016**

- **Strength or Toughness?**
- **Cohesive Laws**
- **Process Zones and Load Redistribution**
- **Continuum Damage Models**
- **Transverse Matrix Cracks**
- **Summary**

Scales of Damage in Composites

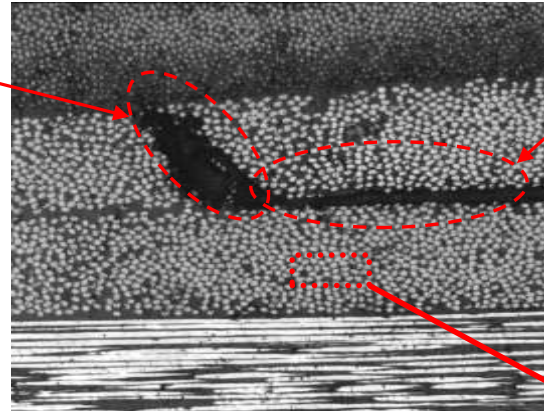


Structure

Microstructure



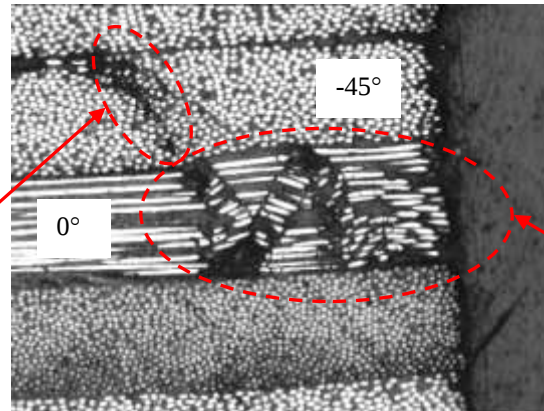
Matrix crack



Delamination



Matrix crack



Fiber kink

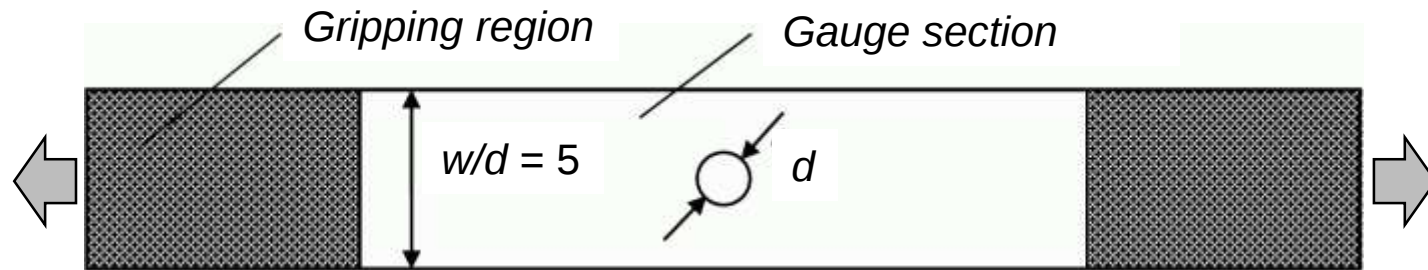
“Physics” of failure:

- At each scale, damage is described by different physical observations

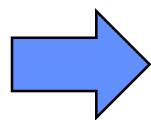
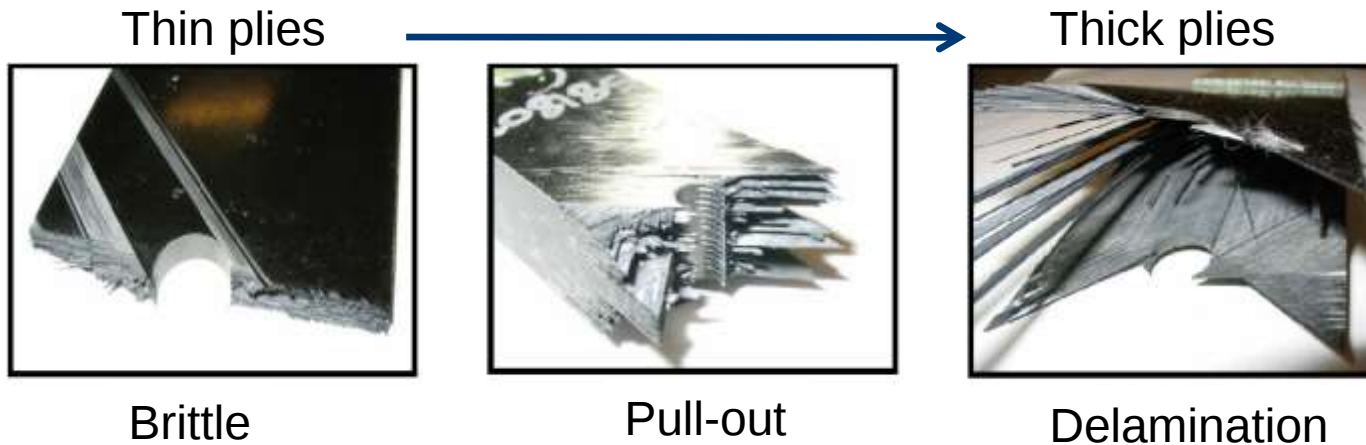
Effect of Material on Structural Response



- Quasi-isotropic laminates, three-dimensional (3-D) scaling
 - $[45_m/90_m/-45_m/0_m]_{ns}$, $m, n = 1, 2, 4, 8$
 - $d = 1/8$ in., $1/4$ in., $1/2$ in.



*Green and Wisnom, 2007



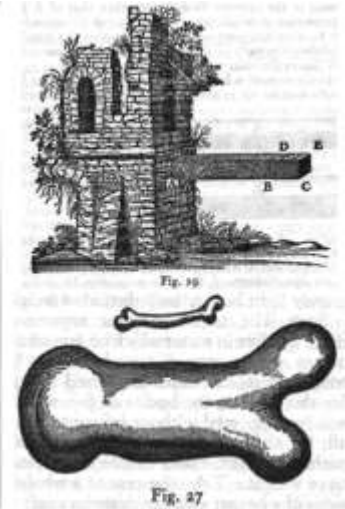
Identical material, different modes of failure!

Strength of Materials

(da Vinci, 1505)



(Galileo, 1638)

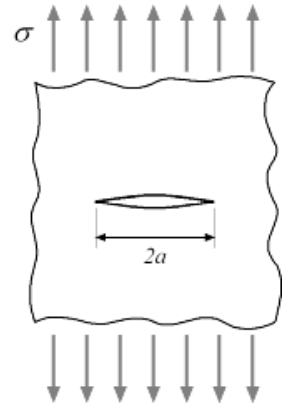


Fracture Mechanics

“weakness is due to the presence of flaws”

(Griffith, 1921)

$$\sigma_u = \sqrt{\frac{E G_c}{\pi a}}$$

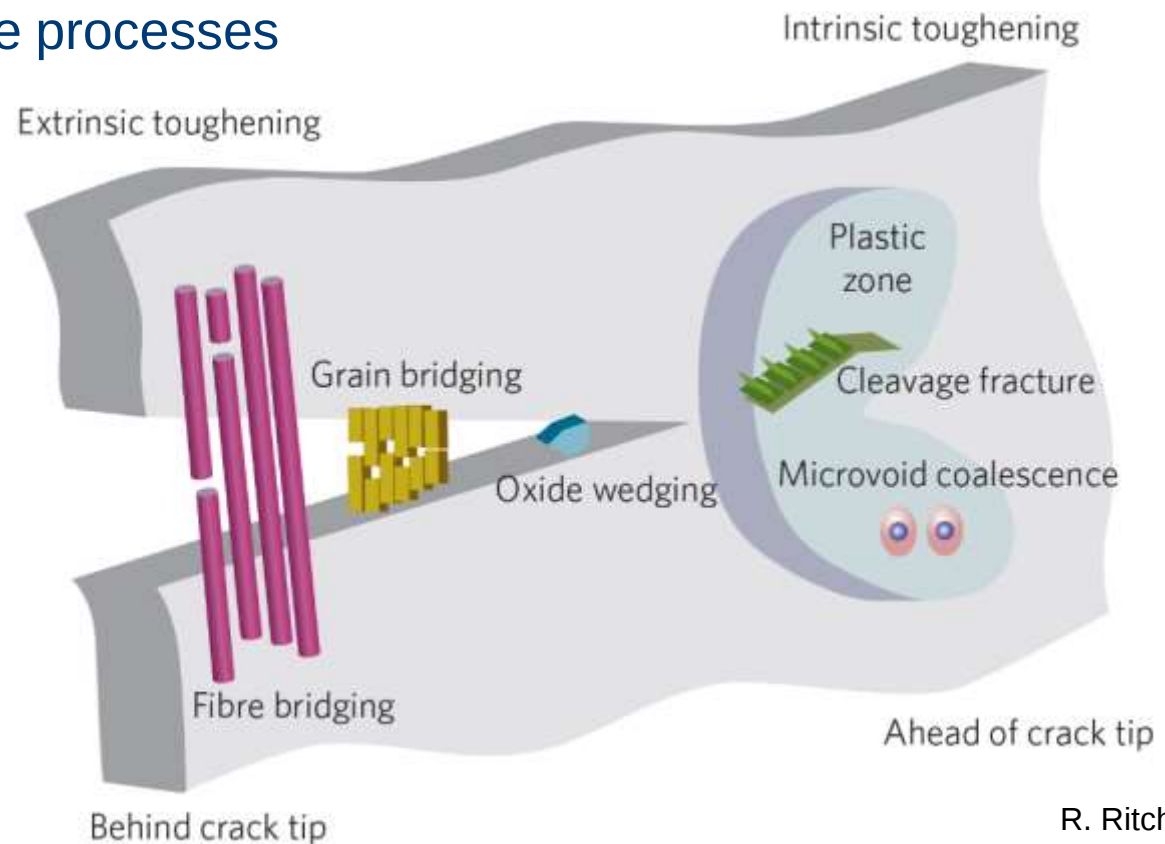


Statistical Theory of Size Effect

(Mariotte, 1686)

(Weibull, 1939)

Idealization of fracture processes



R. Ritchie, 2011

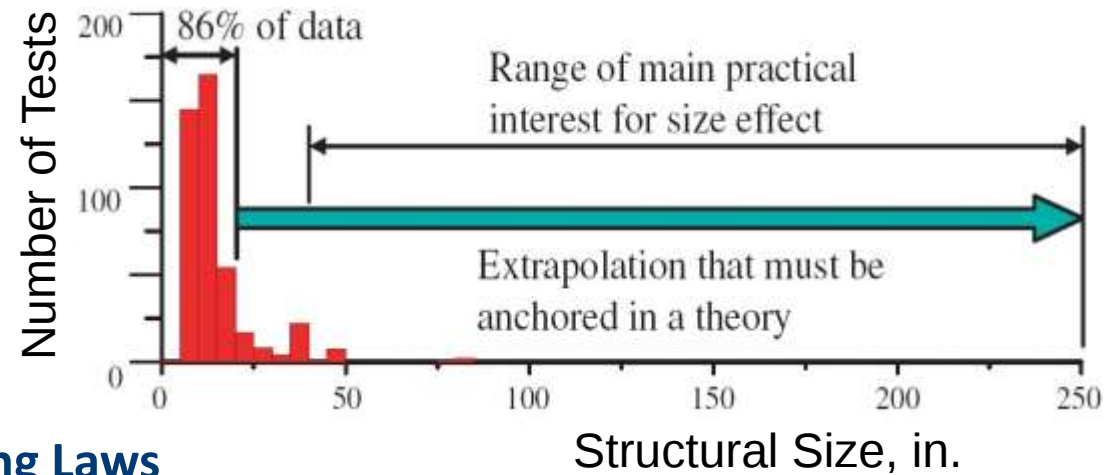
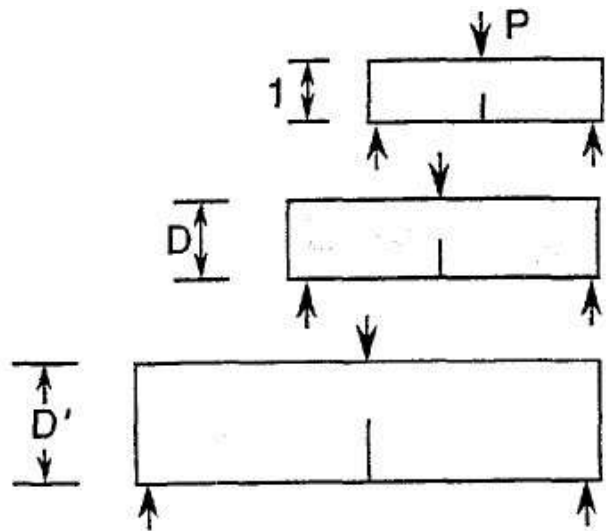
Strength versus toughness:

- Strength and toughness is the result of the interplay between a number of individual mechanisms originating at different length scales.
- Some damage mechanisms inhibit crack propagation.

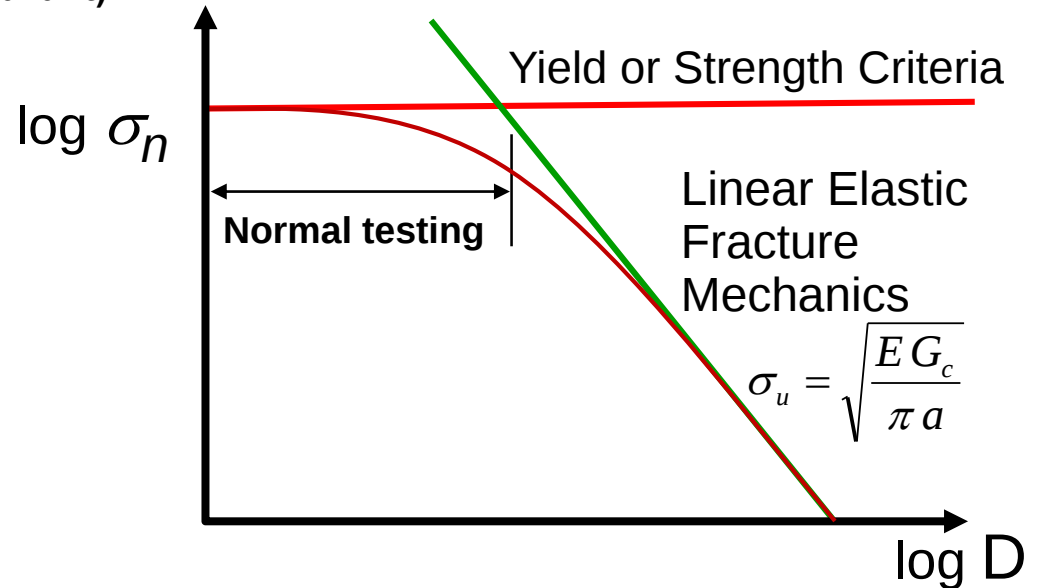
Size Effect - the Issue of Scale

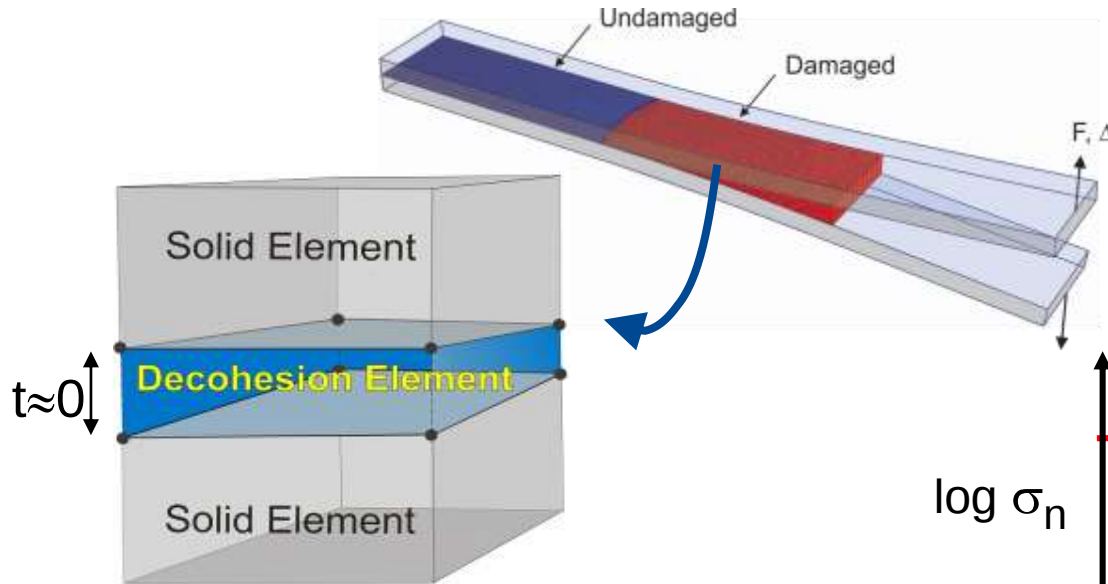


Scaling from test coupon to structure



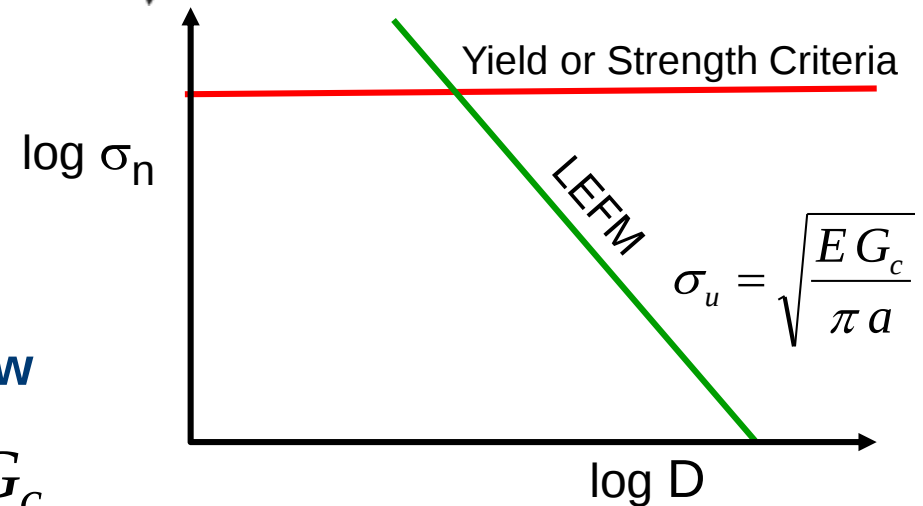
Scaling Laws (Z. Bažant)



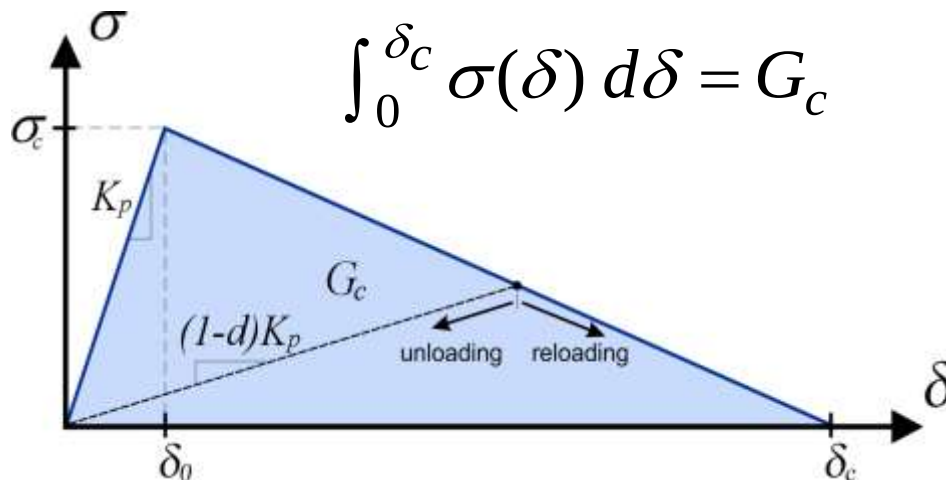


Two material properties:

- G_c Fracture toughness
- σ_c Strength



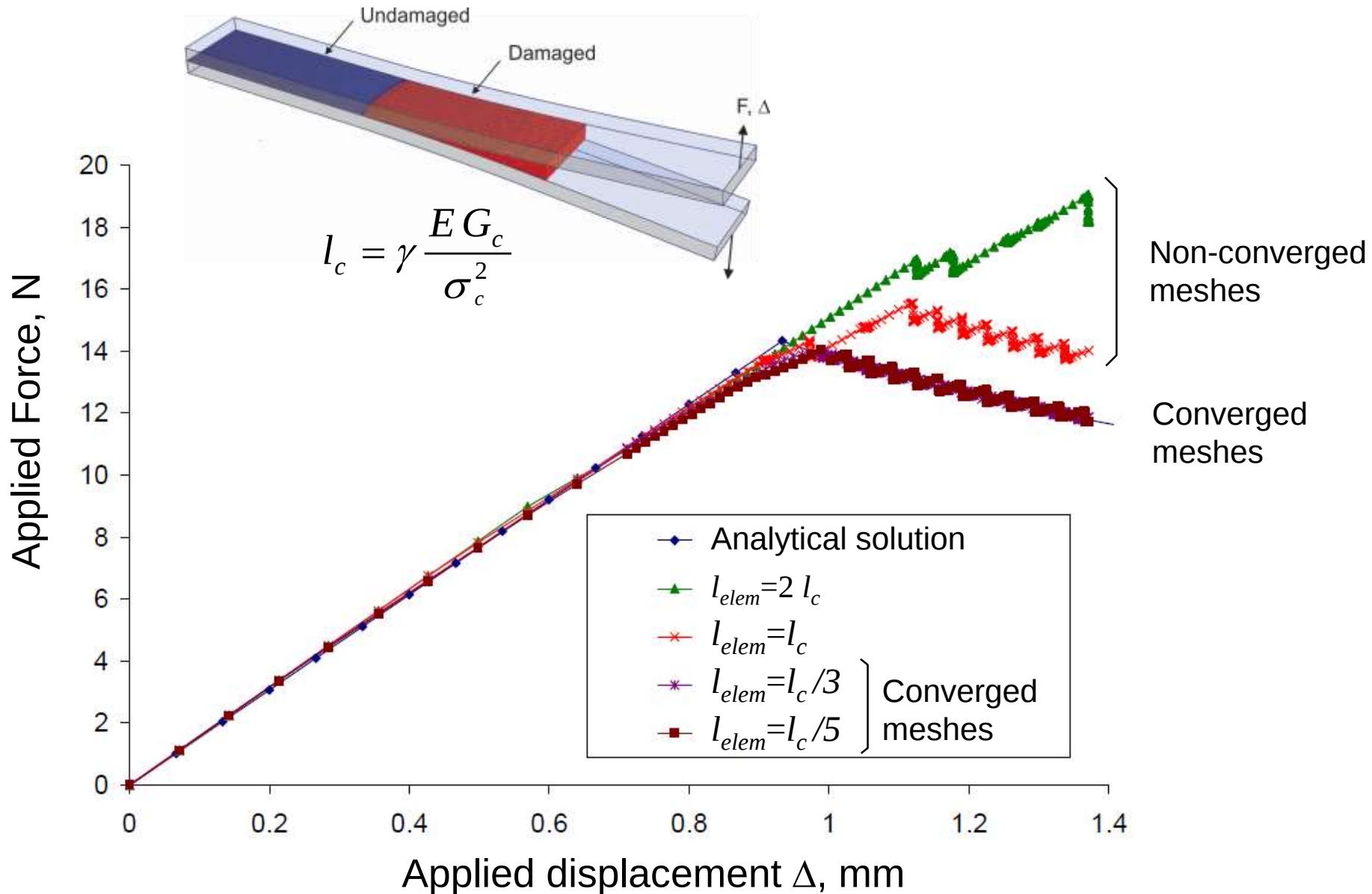
Bilinear Traction-Displacement Law



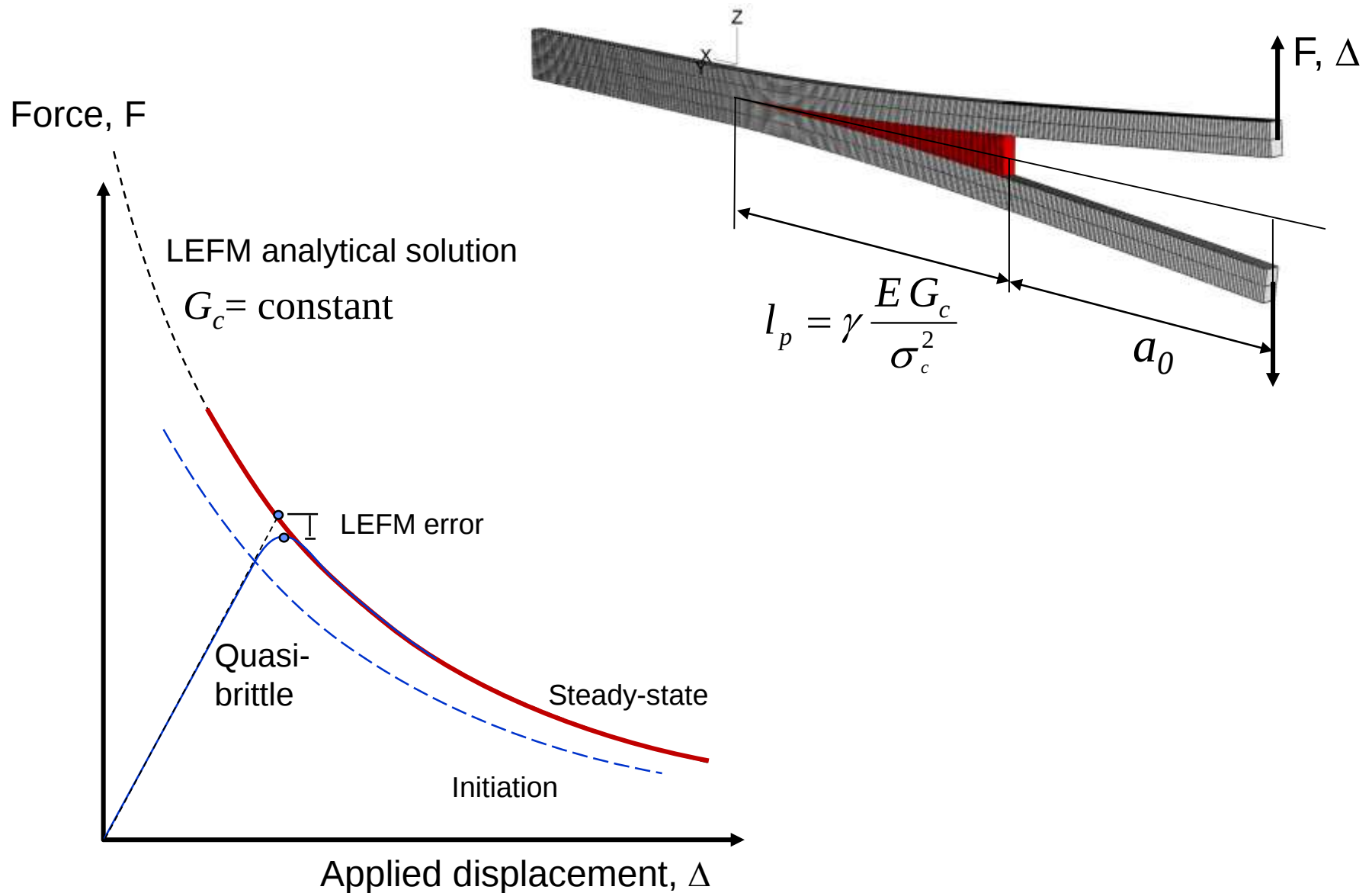
Characteristic Length:

$$l_c = \gamma \frac{E G_c}{\sigma_c^2}$$

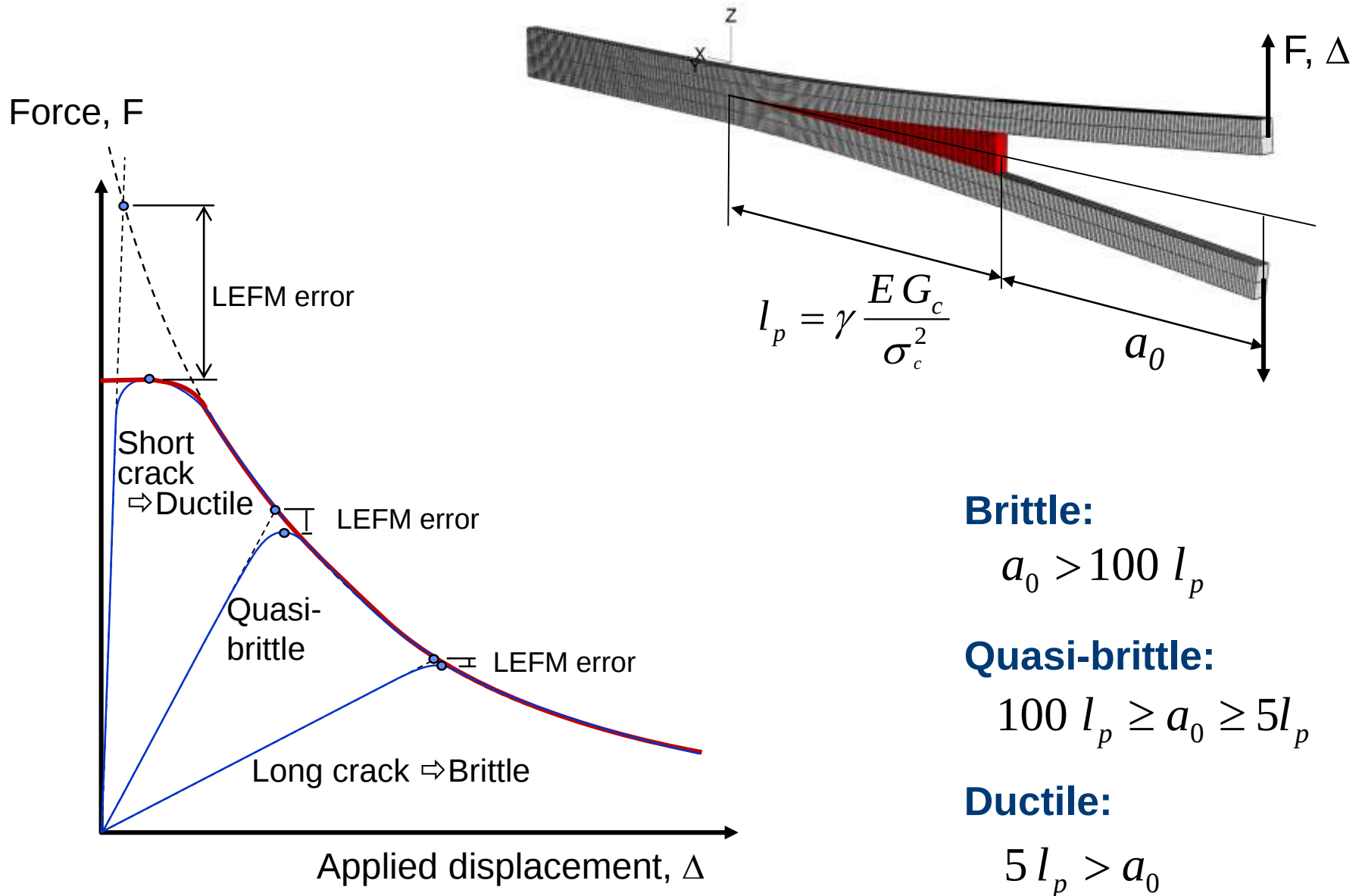
Mesh Convergence Studies



Crack Length and Process Zone



Crack Length and Process Zone



Brittle:

$$a_0 > 100 l_p$$

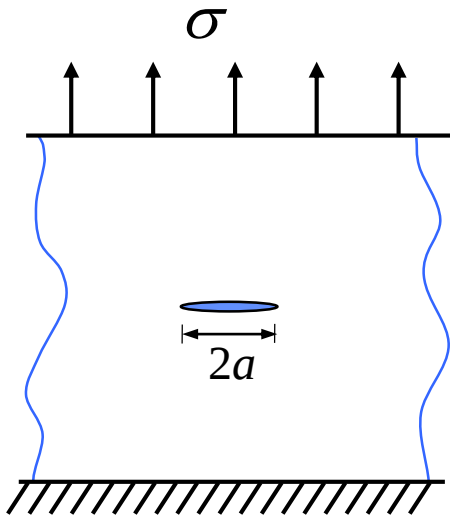
Quasi-brittle:

$$100 l_p \geq a_0 \geq 5 l_p$$

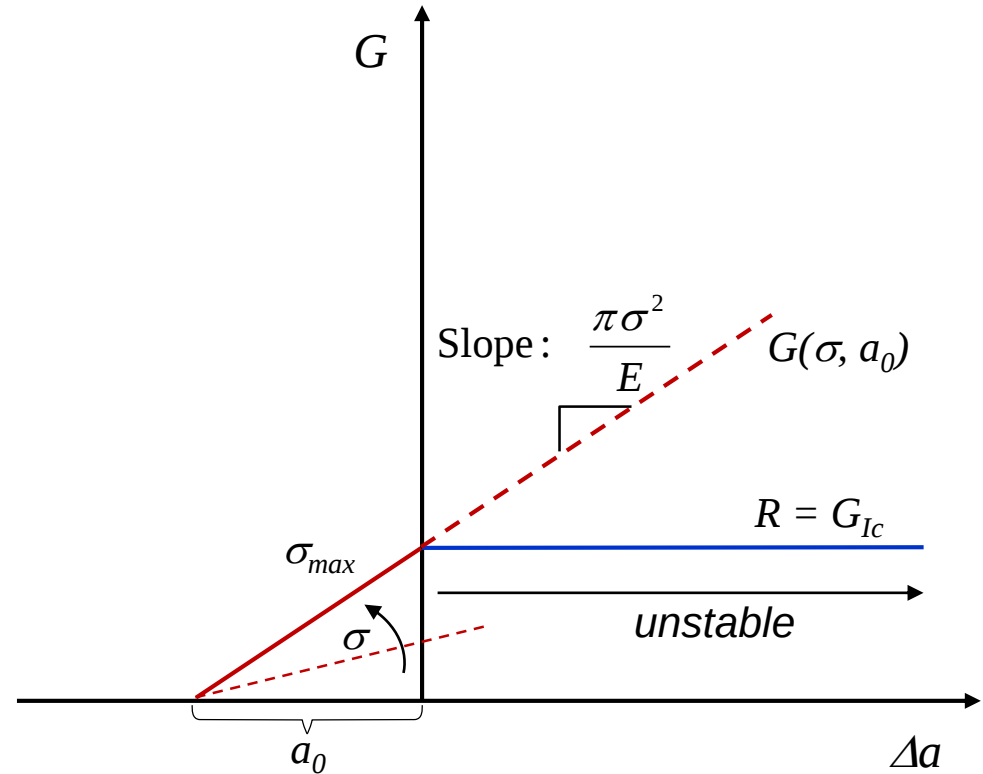
Ductile:

$$5 l_p > a_0$$

Fracture-Dominated Failure

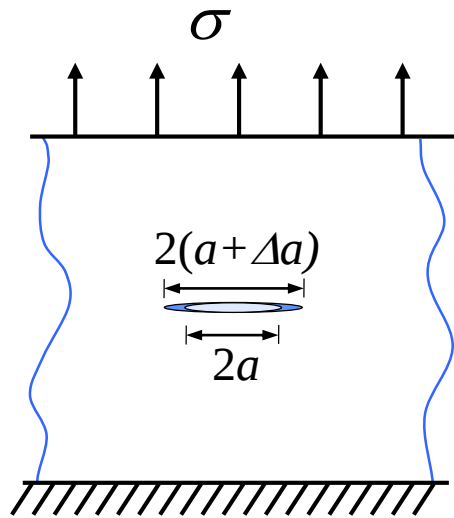


$$G = \frac{\pi \sigma^2 a}{E}$$

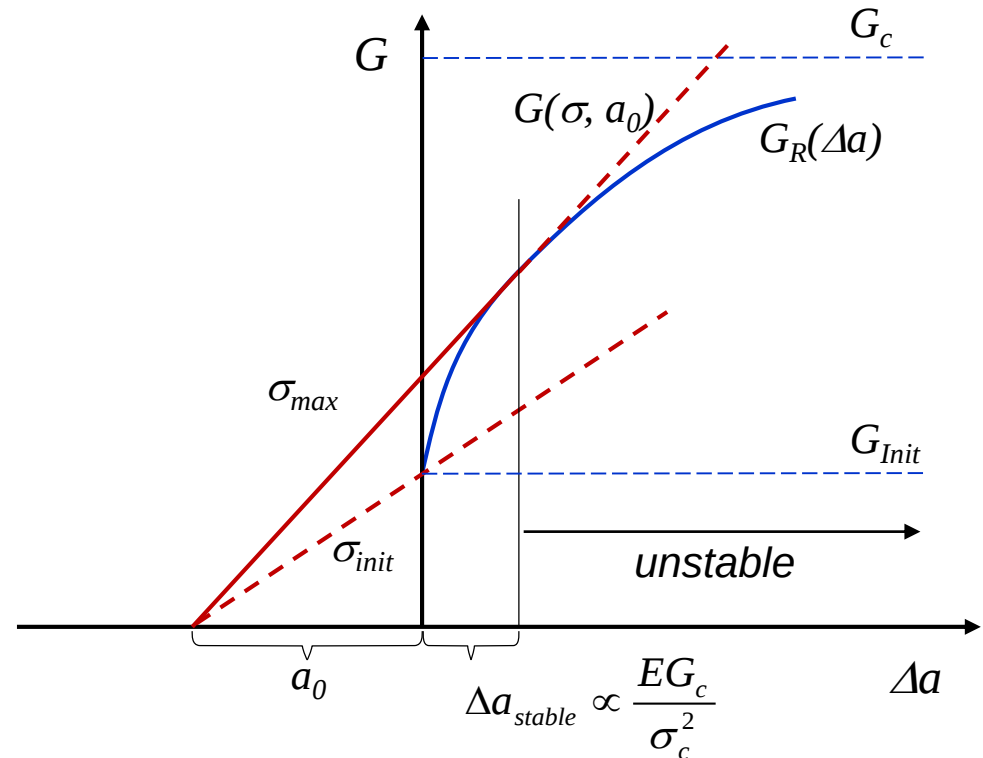


Crack propagates unstably once driving force $G(\sigma, a_0)$ reaches G_{Ic}

Fracture-Dominated Failure: R-Curve



$$G = \frac{\pi \sigma^2 a}{E}$$



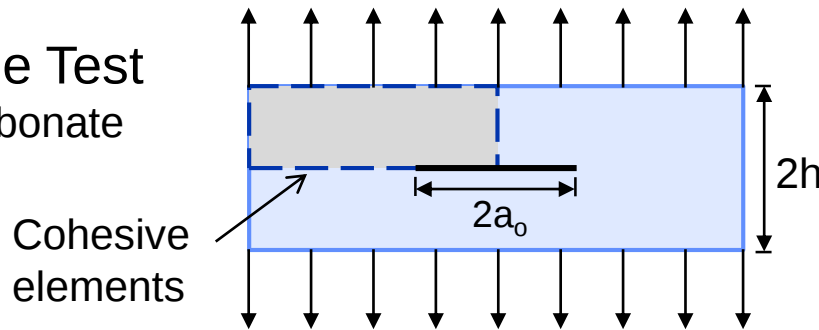
Crack propagates stably when driving force $G(\sigma, a_0) > G_{Init}$

Unstable propagation initiates at $G_{Init} < G \leq G_c$

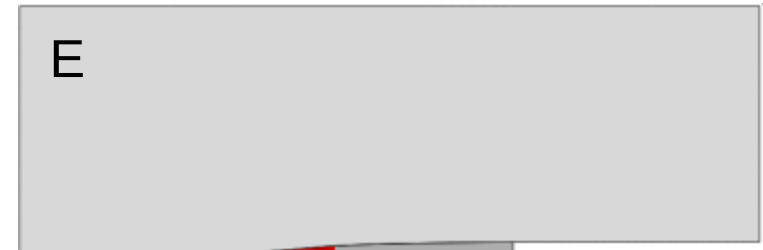
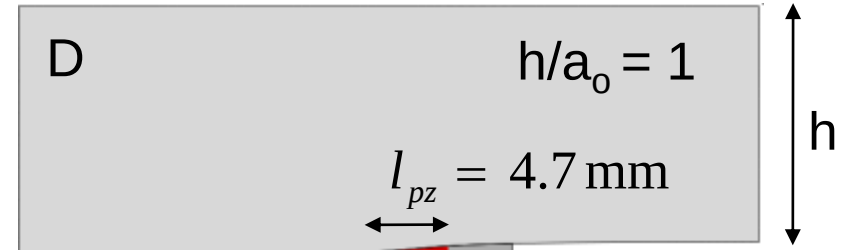
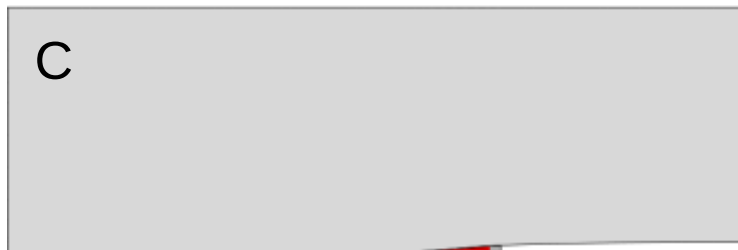
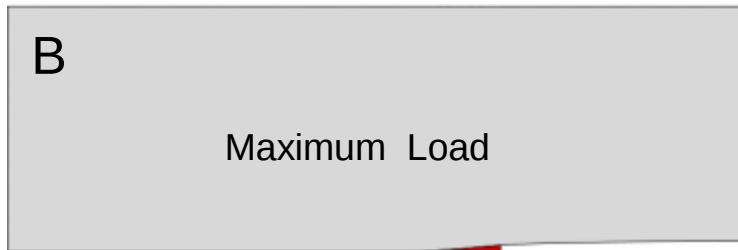
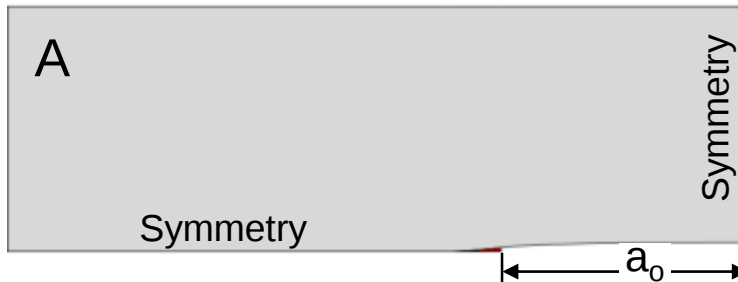
Length of the Process Zone



Short Tensile Test
Lexan Polycarbonate



$$l_c \approx 0.6 \frac{E G_c}{\sigma_c^2} = 3.4 \text{ mm}$$

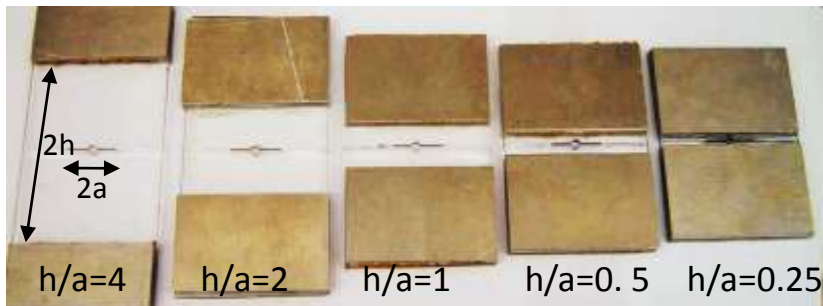


Cohesive Laws - Prediction of Scale Effects

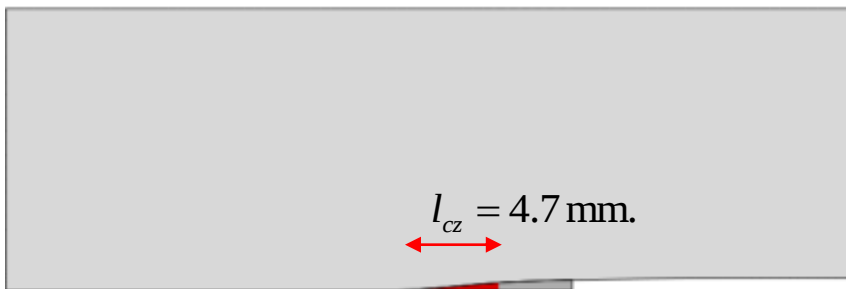


- The use of cohesive laws to predict the fracture in complex stress fields is explored
- The bulk material is modeled as either elastic or elastic-plastic

Lexan Plexiglass tensile specimens (CT Sun)

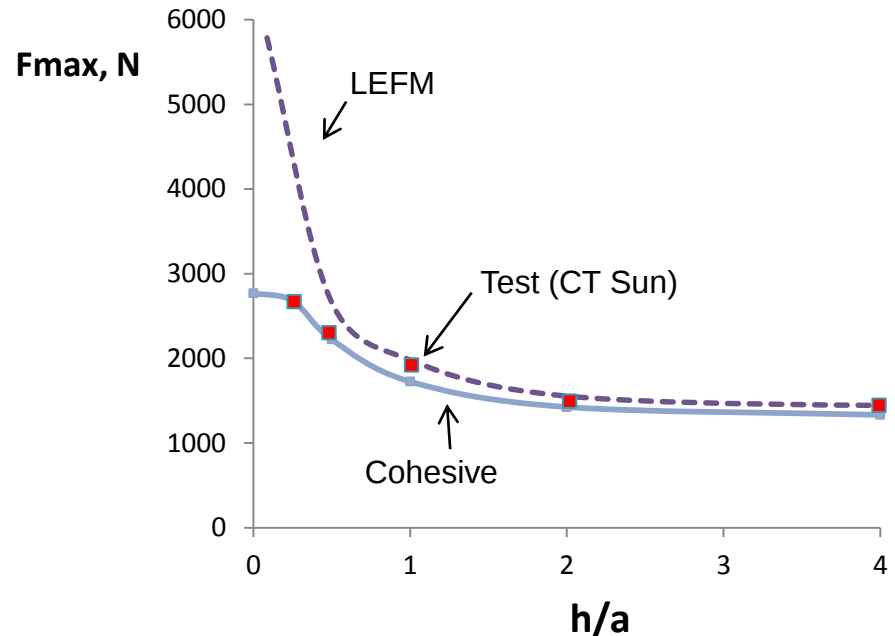


$h/a=1$ (short process zone)



Observations:

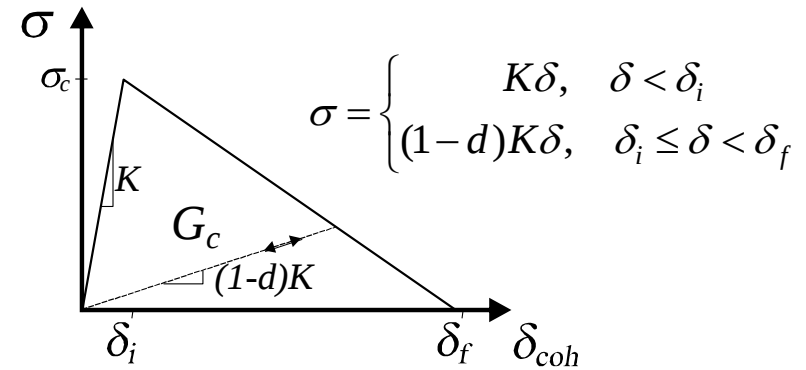
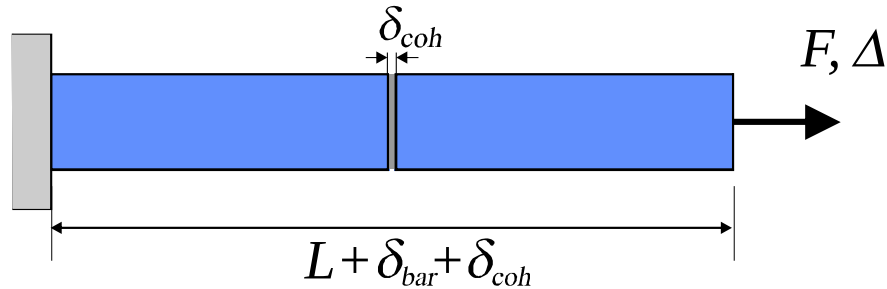
- LEFM overpredicts tests for $h/a < 1$



$h/a = 0.25$ (long process zone)



Quasi-Brittle Response and Fracture

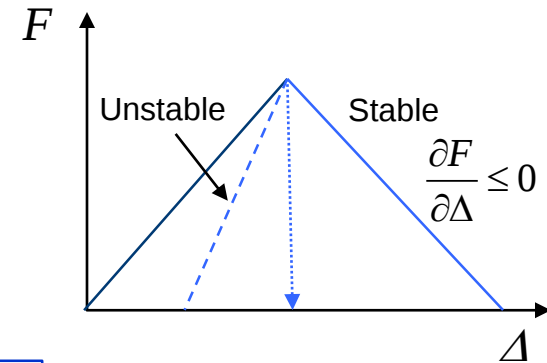


Before damage

$$F = A\sigma = EA \frac{\Delta}{L + \frac{E}{K}}$$

After damage

$$F = A\sigma = EA \frac{\Delta - \frac{2G_c}{\sigma_c}}{L - \frac{2EG_c}{\sigma_c^2} + \frac{E}{K}}$$

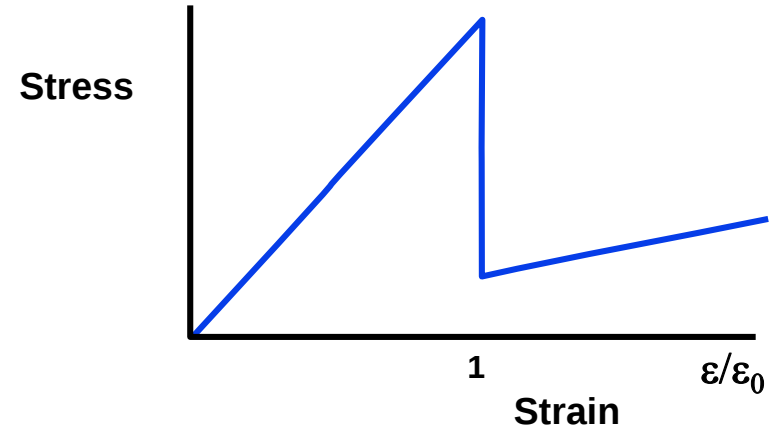
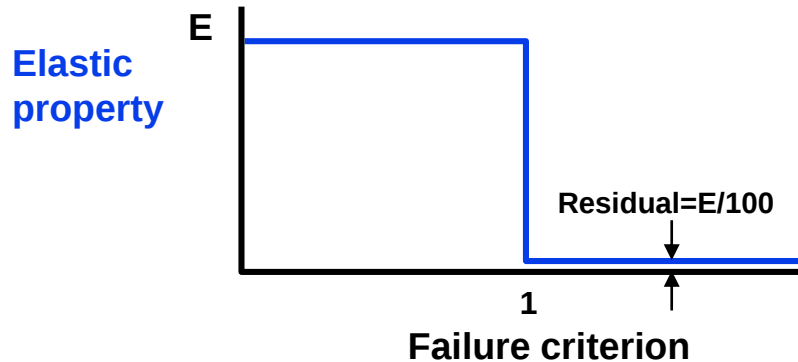


For stable fracture under Δ control: $\frac{\partial F}{\partial \Delta} \leq 0 \Rightarrow \boxed{L \leq \frac{2EG_c}{\sigma_c^2}}$

For "long" beams, the response is unstable, dynamic, and independent of G_c

- Strength or Toughness?
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Progressive Failure Analysis



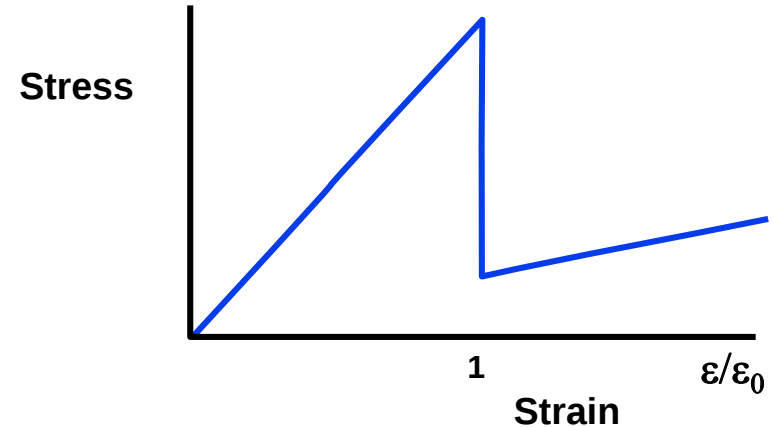
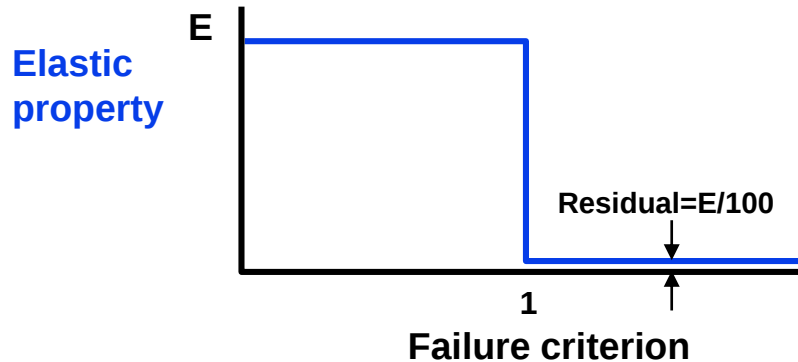
Benefits

- Simplicity (no programming needed)
- Convergence of equilibrium iterations

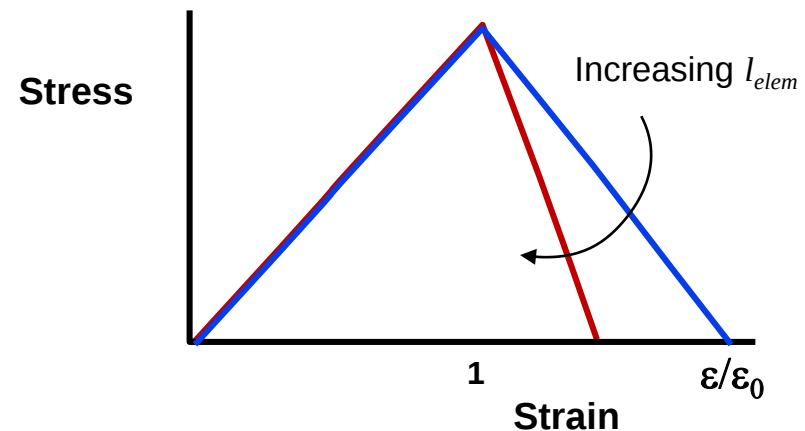
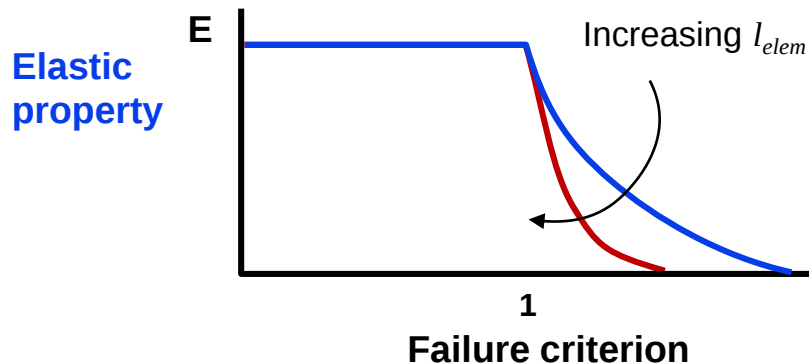
Drawbacks

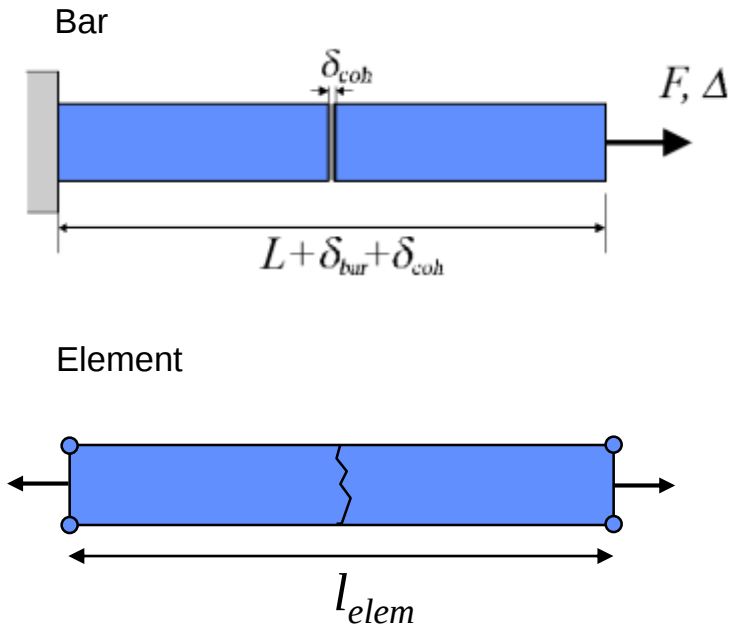
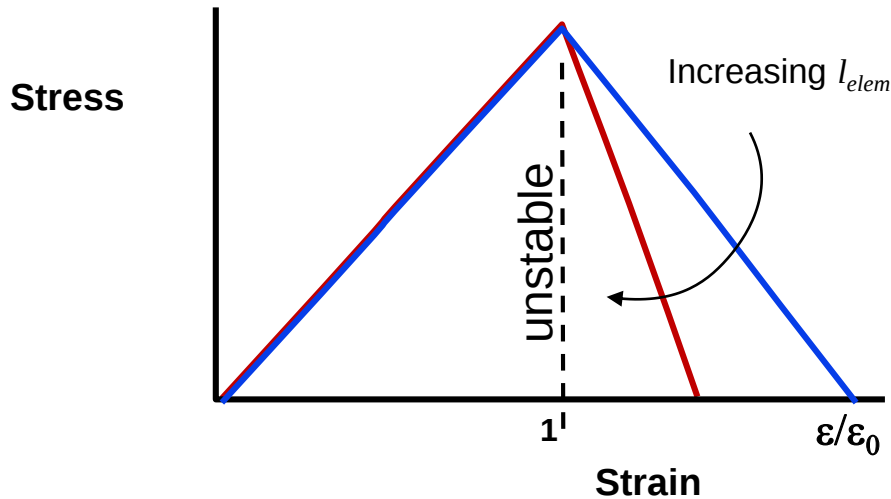
- Mesh dependence
- Dependence on load increment
- Ad-hoc property degradation
- Large strains can cause reloading
- Errors due to improper load redistributions

Progressive Failure Analysis



Progressive Damage Analysis – Regularized Softening Laws





Critical Element size (snap-back):

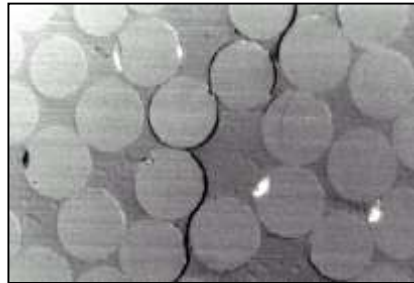
$$l_{elem} \leq \frac{2EG_c}{\sigma_c^2}$$

Failure Criteria: equations based on stresses and strengths that represent the initiation of damage mechanisms

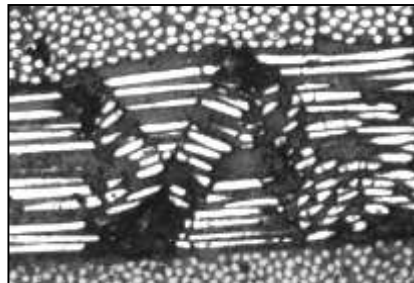
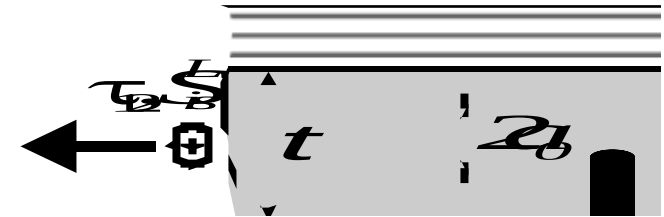
LaRC04 Criteria

- In-situ matrix strength prediction
- Advanced fiber kinking criterion
- Prediction of angle of fracture

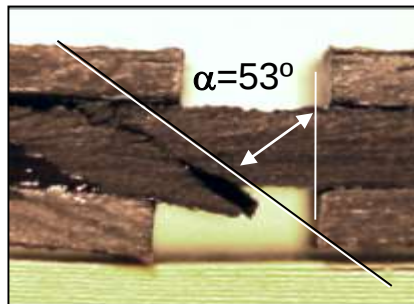
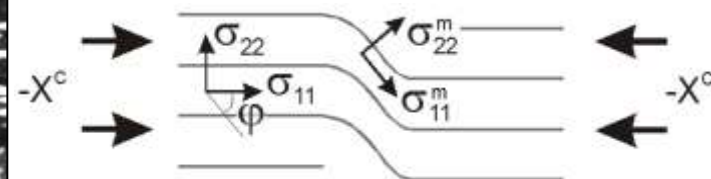
LaRC02 - Dávila, Rose et al., 2003
LaRC04 - Pinho, et al., 2005



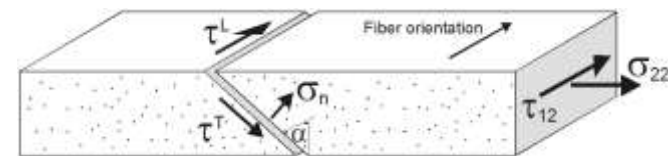
Matrix Tension & Shear



Fiber Compression



Matrix Compression and Shear

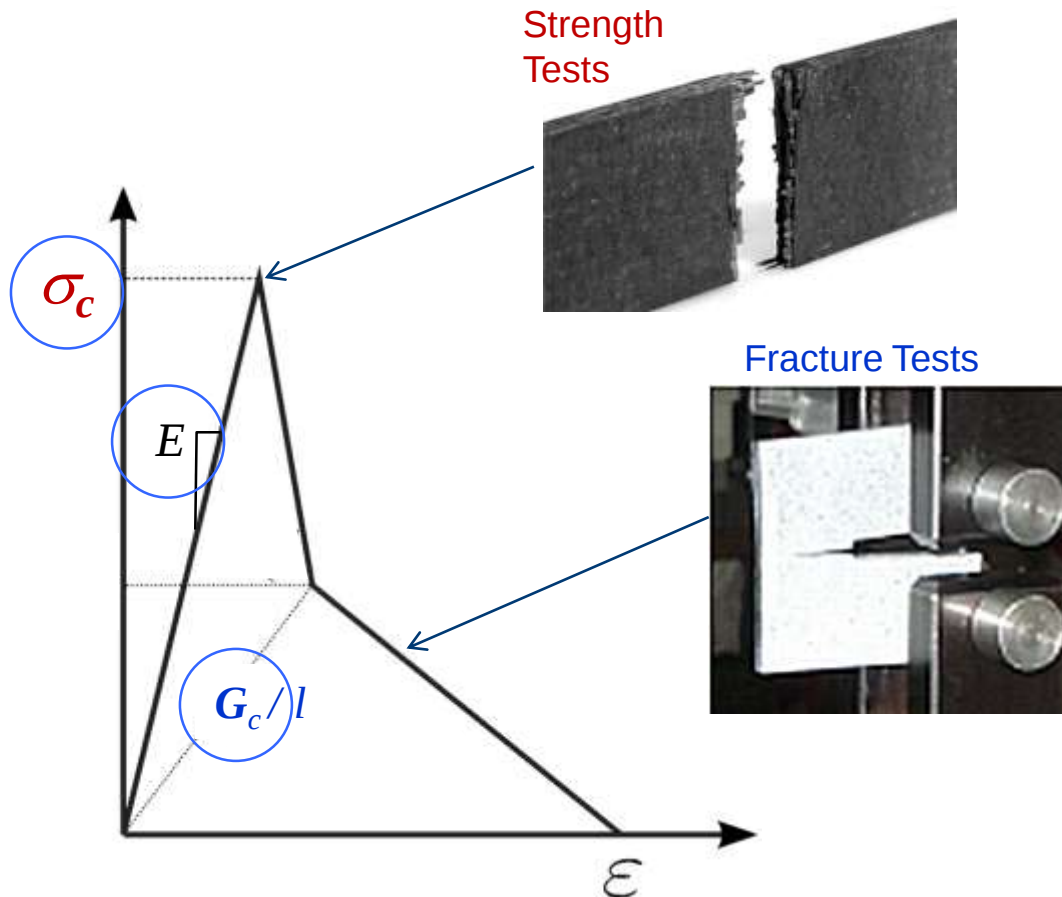


Size Effect and Material Softening Laws



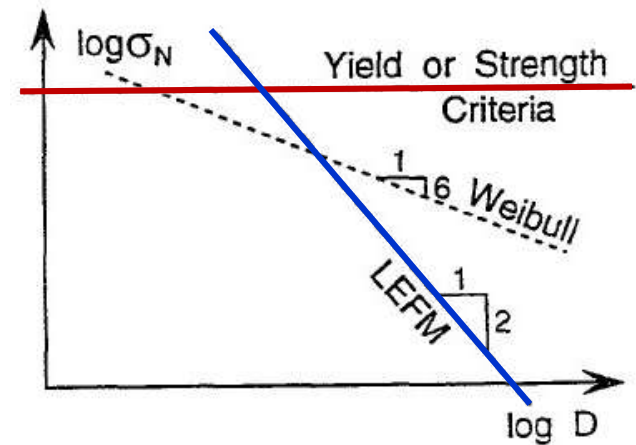
Damage Evolution Laws:

Each damage mode has its own softening response



Two material properties:

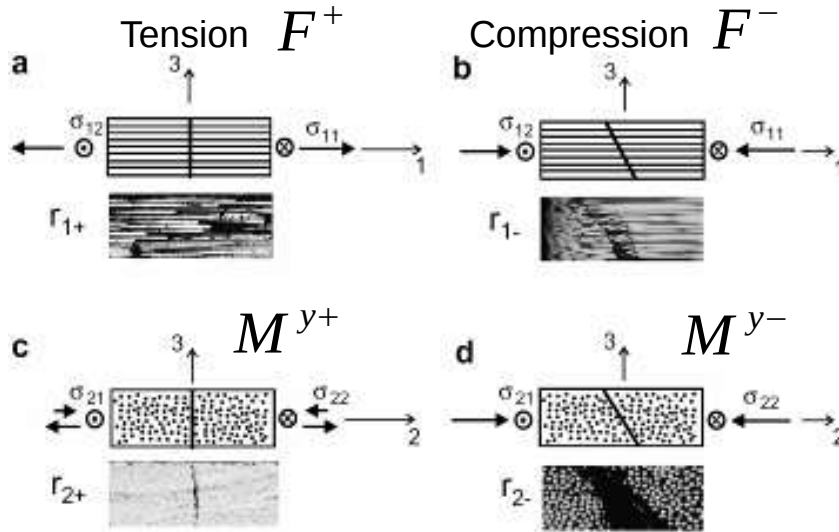
- σ_c Strength
- G_c Fracture toughness



Material length scale

$$l_c \approx 0.6 \frac{E G_c}{\sigma_c^2}$$

Damage Modes:



LaRC04 Criteria

- In-situ matrix strength prediction
- Advanced fiber kinking criterion
- Prediction of angle of fracture (compression)
- Criteria used as activation functions within framework of damage mechanics

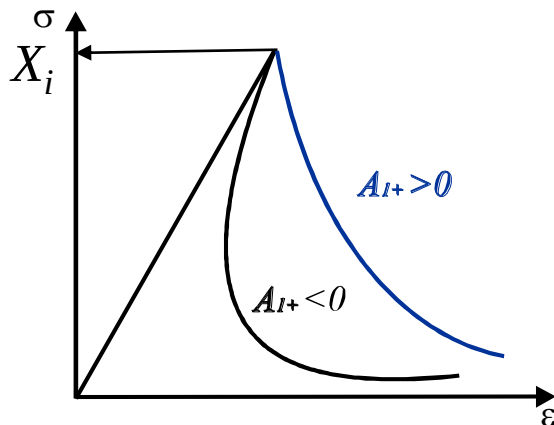
$$d_i = 1 - \frac{1}{f_i} \exp(-A_i(1 - f_i))$$

f_i : LaRC04 failure criteria as activation functions

$$i = F^+; F^-; M^{y+}; M^{y-}; M^s$$

Damage Evolution:

Thermodynamically-consistent material degradation takes into account energy release rate and element size for each mode.



Bazant CBT:

$$A_i = \frac{2l^* X_i^2}{2E_i G_i - l^* X_i^2}$$

Critical (maximum) finite element size:

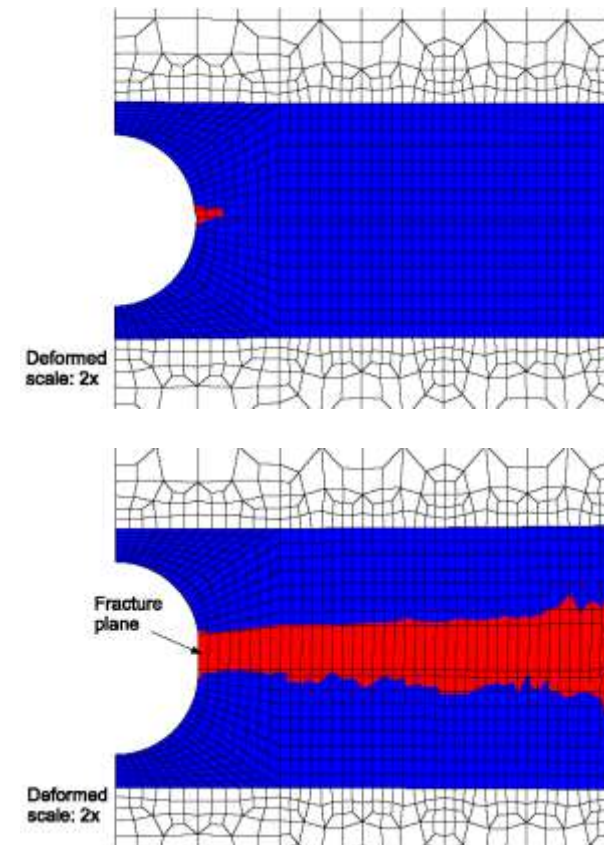
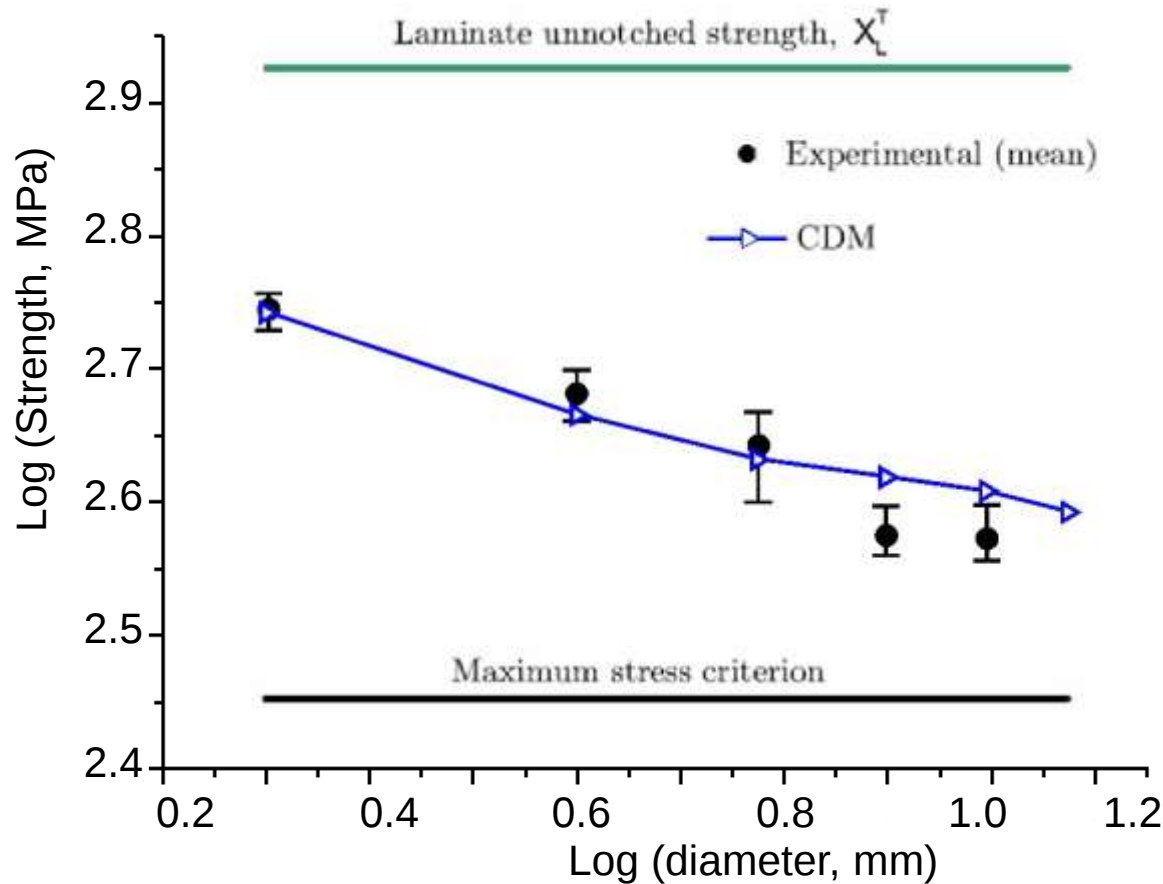
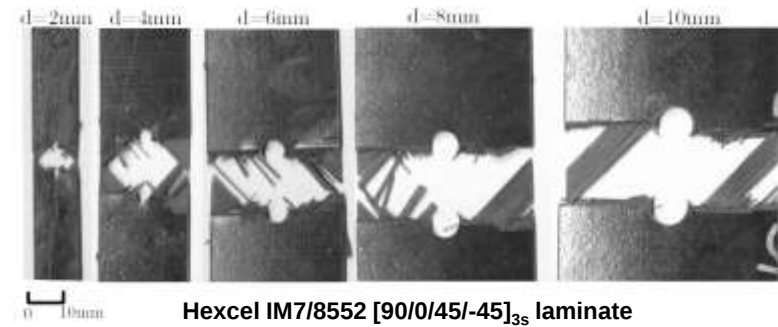
$$l^* \leq \frac{2E_i G_i}{X_i^2}$$

Scale Effect in Open Hole Tension (OHT)

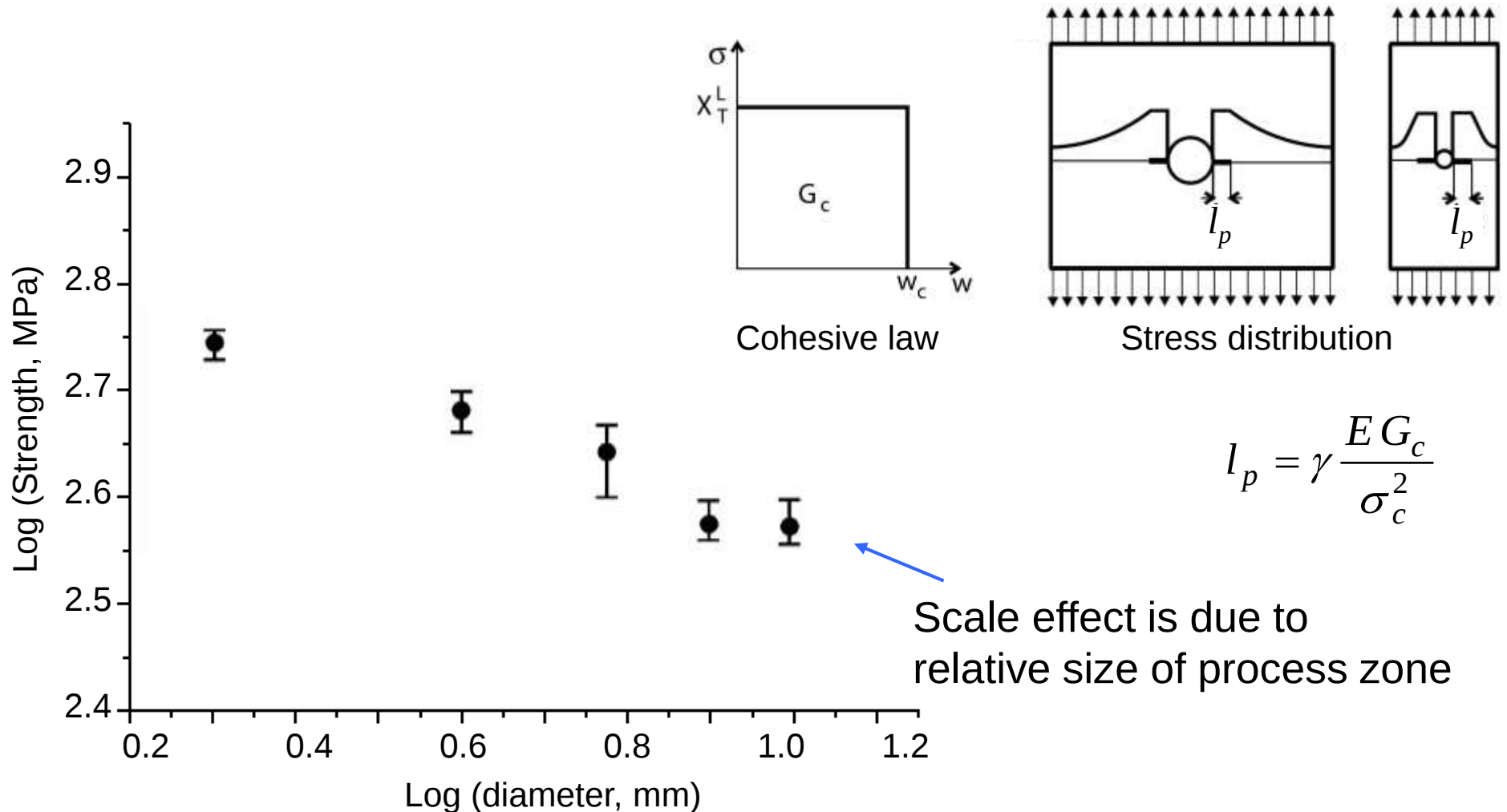


Prediction of size effects in notched composites

- Stress-based criteria predict no size effect.
- CDM damage model predicts scale effects w/out calibration.
(P. Camanho, 2007)



Process Zone and Scale Effect in OHT



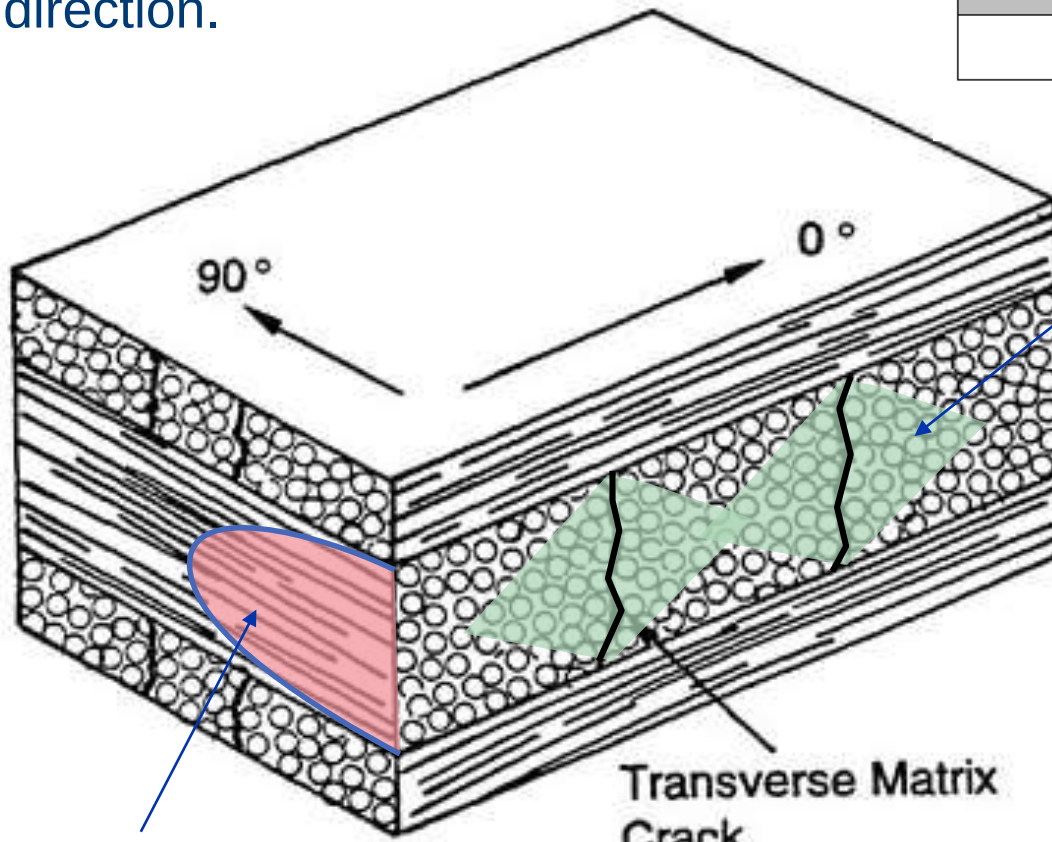
“Both energy and stress criteria are necessary conditions for fracture but neither one nor the other are sufficient.”

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Transverse Matrix Cracks

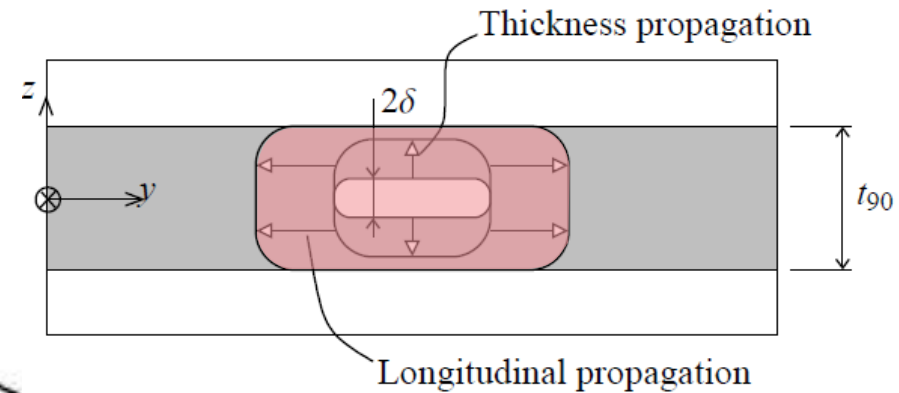


Transverse matrix cracks propagate in thickness direction and in the fiber direction.



Propagation

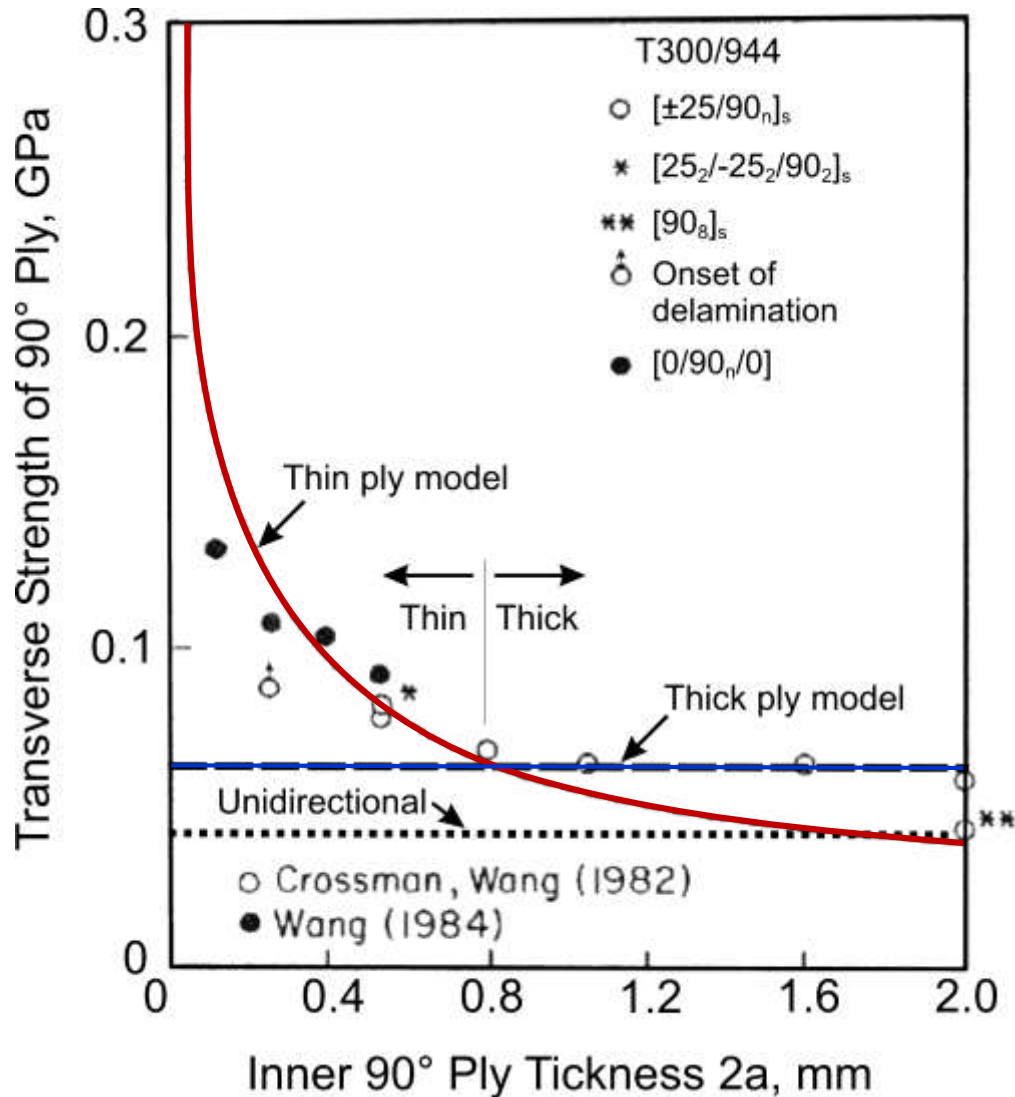
Transverse Matrix Crack



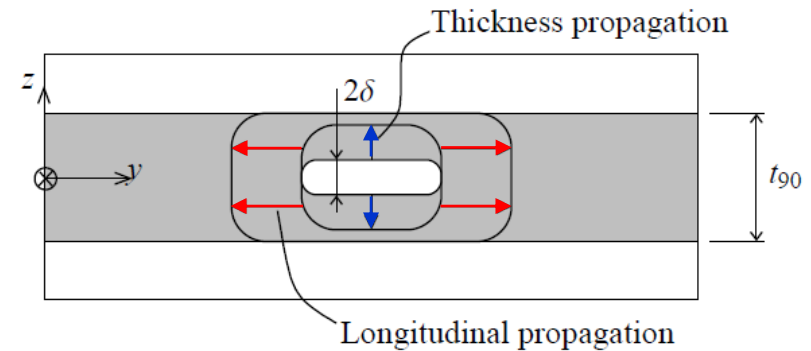
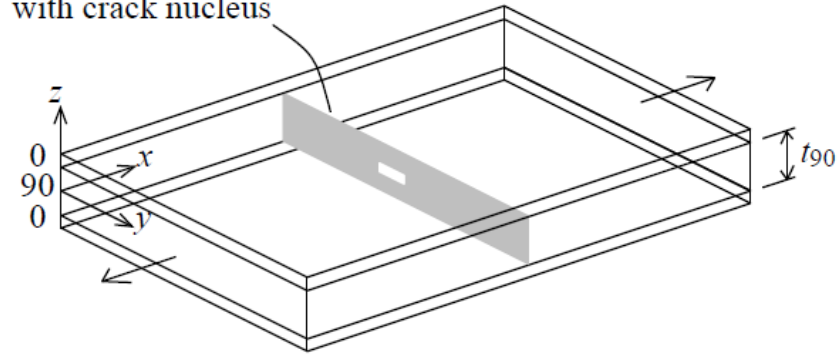
Shielded area

Cracks shield the adjacent material and prevent new cracks.

Matrix Cracking – In Situ Effect



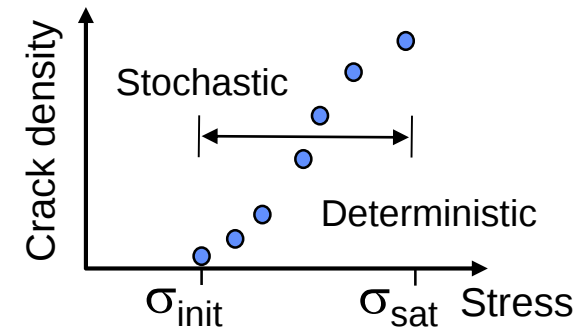
Potential crack plane, with crack nucleus



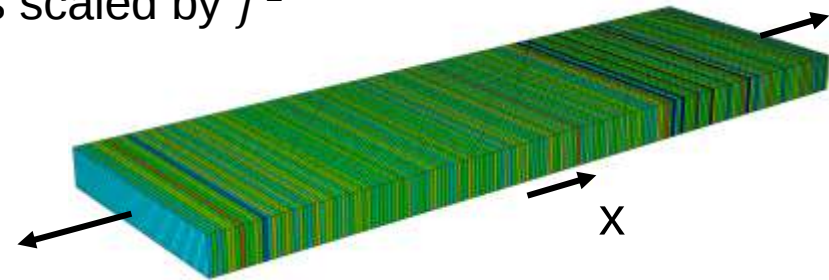
Material Inhomogeneity



Initial crack density in a uniformly stressed laminate is strictly a function of material inhomogeneity

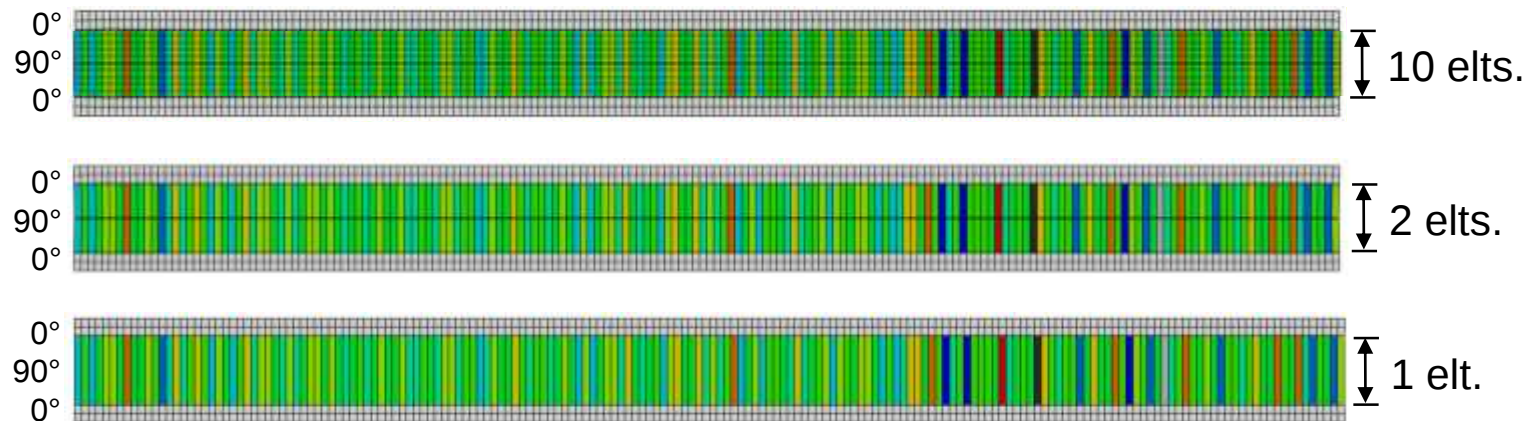
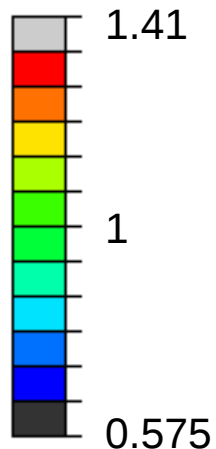


- Strength scaled by f , Fracture toughness scaled by f^2
- Constant f along each crack path



$f(x)$

Inhomogeneity applied to 3 levels of mesh refinement

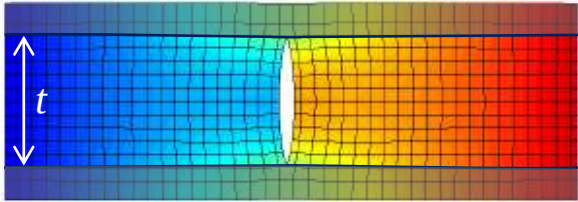


Analyses with 3 levels of mesh refinement

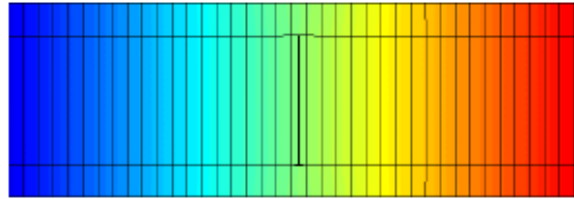


Transverse Matrix Cracks w/ One Element Per Ply

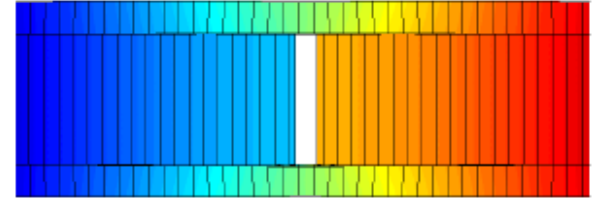
Multi-element model:
correct crack evolution



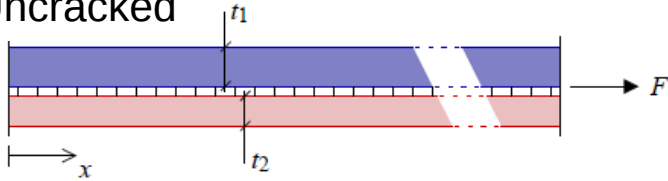
Conventional single-element:
no opening w/out delam.



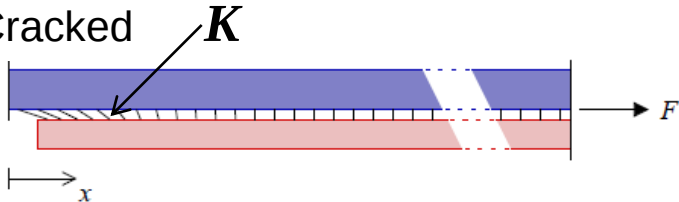
Modified single-element:
correct Energy Release Rate



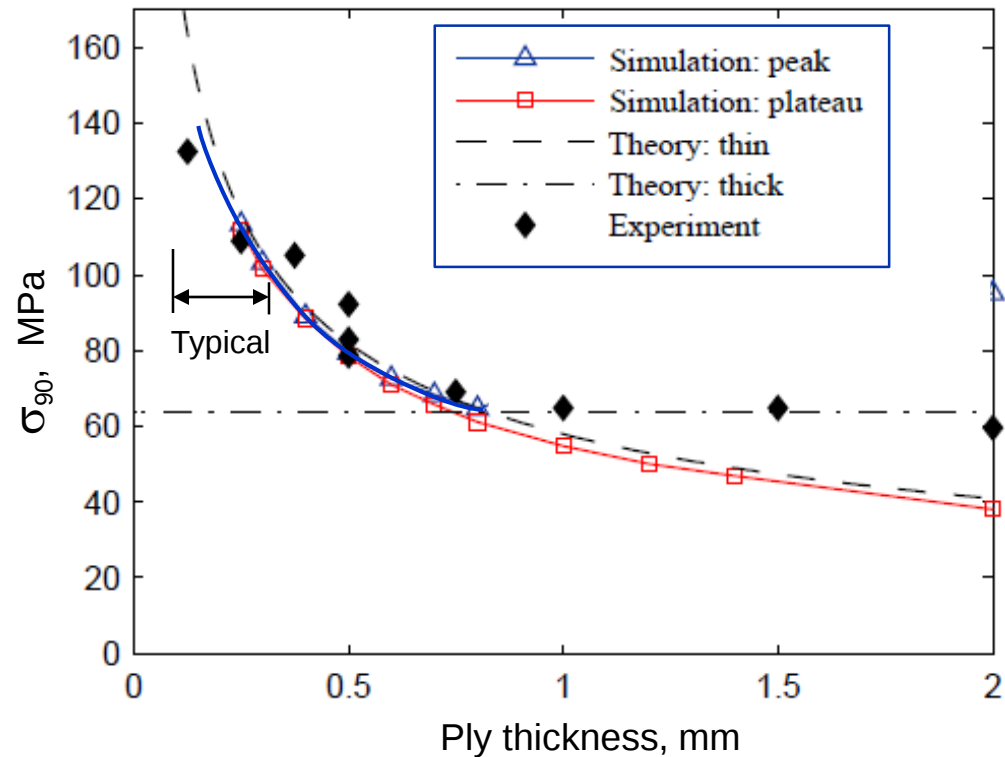
Uncracked



Cracked



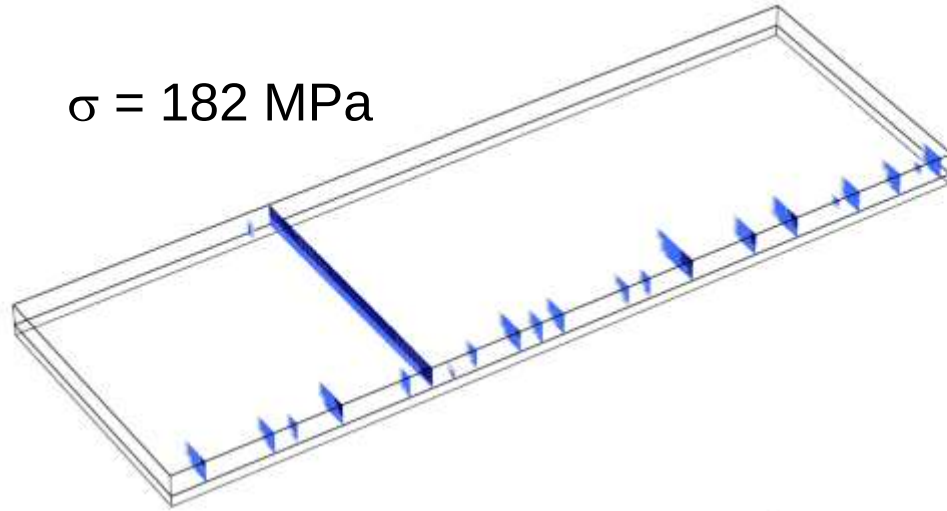
$$K \approx \frac{4E_2}{\pi^2 t}$$



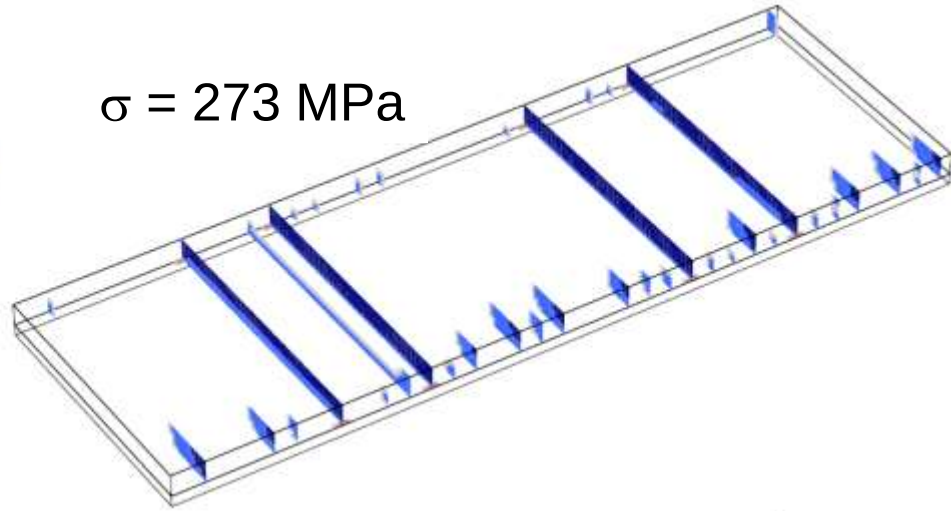
Crack Initiation, Densification, and Saturation



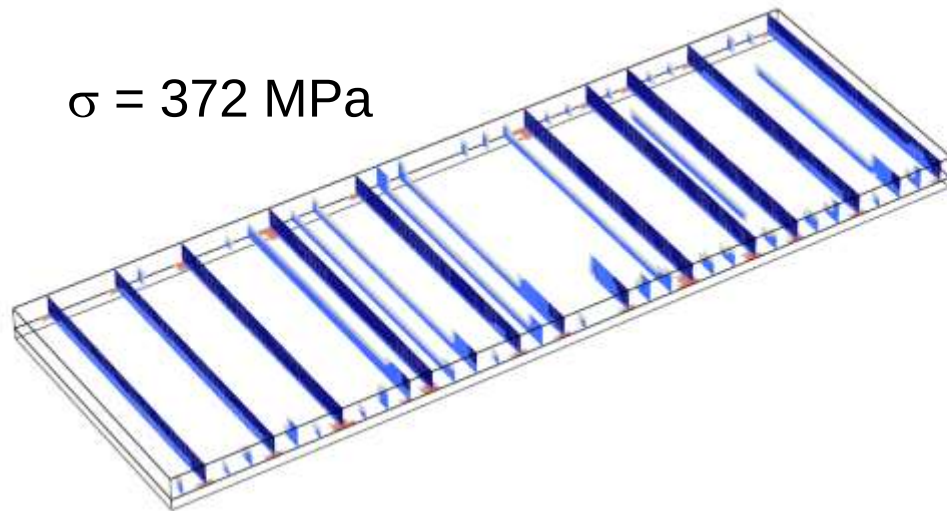
$\sigma = 182 \text{ MPa}$



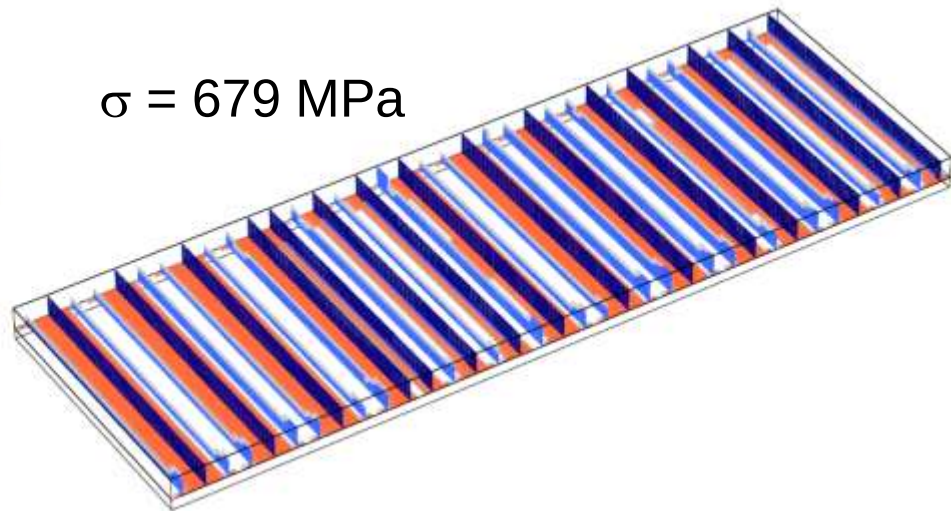
$\sigma = 273 \text{ MPa}$



$\sigma = 372 \text{ MPa}$



$\sigma = 679 \text{ MPa}$



Cohesive zone



Traction-free cohesive zone

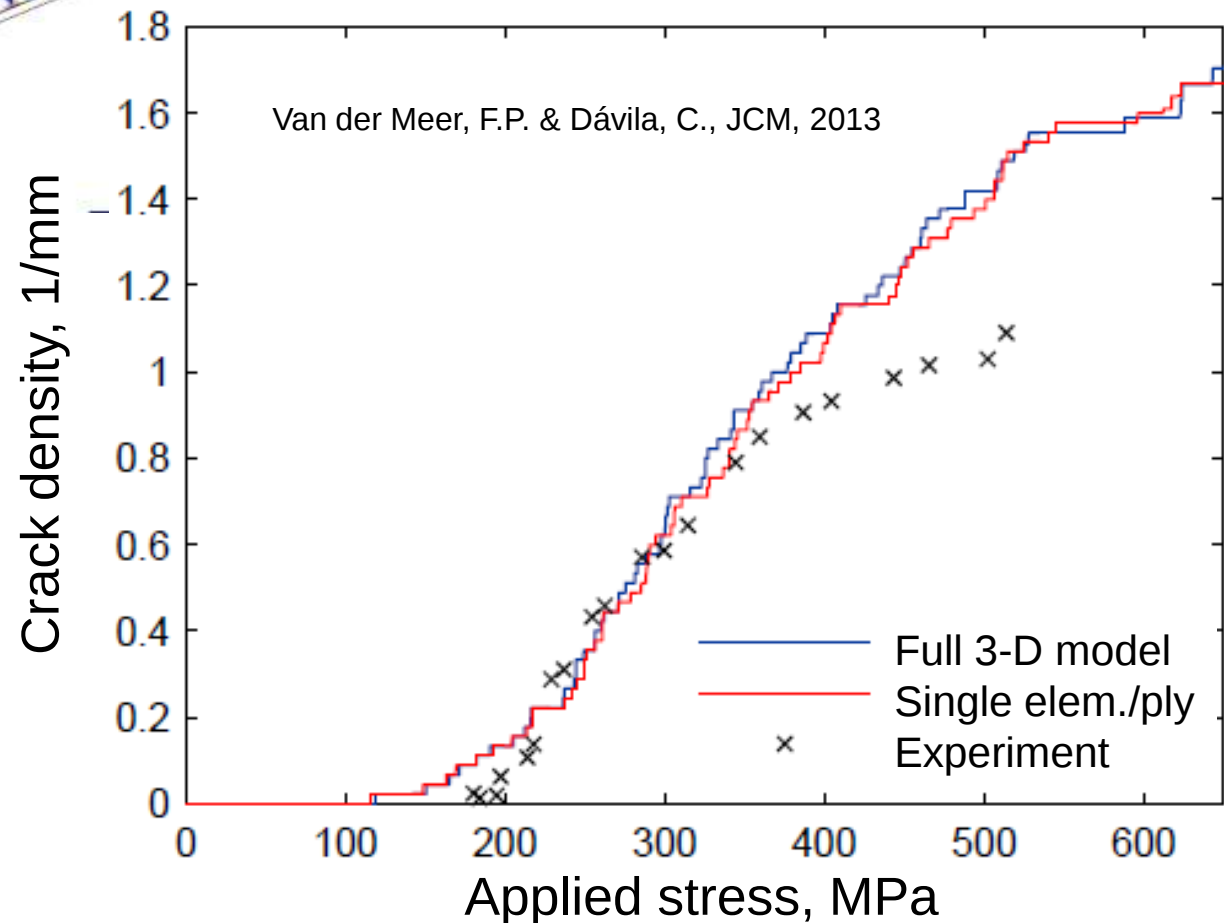
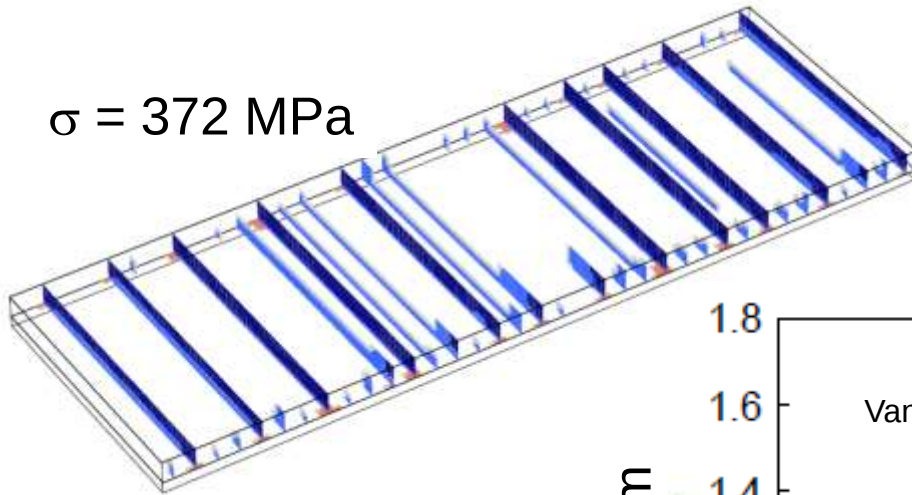


Delamination

Comparison of X-FEM Single-Ply Model and Full 3-D



$\sigma = 372$ MPa



- **Strength and fracture issues are:**
 - Interrelated
 - Subject to size effects
- **Cohesive laws:**
 - Account for strength and fracture toughness
 - More than 3 elements are required in the process zone
- **Continuum damage models**
 - Snap-back imposes maximum element size
- **Transverse matrix cracks:**
 - More than one element per ply is required
- **Other mesh issues?**

**Mesh Requirements
for Damage Models**