# Assessment of the Uniqueness of Wind Tunnel Strain–Gage Balance Load Predictions

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**A new test was developed to assess the uniqueness of wind tunnel strain–gage balance load predictions that are obtained from regression models of calibration data. The test helps balance users to gain confidence in load predictions of non–traditional balance designs. It also makes it possible to better evaluate load predictions of traditional balances that are not used as originally intended. The test works for both the Iterative and Non–Iterative Methods that are used in the aerospace testing community for the prediction of balance loads. It is based on the hypothesis that the total number of independently applied balance load components must always match the total number of independently measured bridge outputs or bridge output combinations. This hypothesis is supported by a control volume analysis of the inputs and outputs of a strain–gage balance. It is concluded from the control volume analysis that the loads and bridge outputs of a balance calibration data set must separately be tested for linear independence because it cannot always be guaranteed that a linearly independent load component set will result in linearly independent bridge output measurements. Simple linear math models for the loads and bridge outputs in combination with the variance inflation factor are used to test for linear independence. A highly unique and reversible mapping between the applied load component set and the measured bridge output set is guaranteed to exist if the maximum variance inflation factor of both sets is less than the literature recommended threshold of five. Data from the calibration of a six–component force balance is used to illustrate the application of the new test to real–world data.**

## Nomenclature

| | |
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| $AF$ | = axial force in the balance axis system |
| $\mathbf{C}$ | = matrix that has the linear regression coefficients of all load components of a balance |
| $\mathbf{C}_i$ | = vector that has coefficients of the regression model of a load component with index $i$ |
| $\mathbf{C1}$ | = square matrix that results from the application of the *Iterative Method* to balance data |
| $c_{\xi,i}$ | = regression coefficient of a fitted load component with index $i$ |
| $\mathbf{D}$ | = matrix that has the linear regression coefficients of all bridge outputs of a balance |
| $\mathbf{D}_i$ | = vector that has coefficients of the regression model of a bridge output with index $i$ |
| $d_{\xi,i}$ | = regression coefficient of a fitted bridge output with index $i$ |
| $i$ | = index of a load component or index of a bridge output |
| $j$ | = index of a bridge output |
| $k$ | = index of a data point |
| $M1$ | = pitching moment of a three–component canard balance in the balance axis system |
| $M2$ | = hinge moment of a three–component canard balance in the balance axis system |
| $n$ | = number of balance load components or bridge outputs |
| $n'$ | = number of linearly independent balance load components |
| $n''$ | = number of linearly independent bridge output measurements |
| $N1$ | = forward normal force of a force balance in the balance axis system |

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| $N2$ | = aft normal force of a force balance in the balance axis system |
| $NF$ | = normal force of a balance in the balance axis system |
| $p$ | = total number of data points that are used for the regression analysis |
| $PM$ | = pitching moment in the balance axis system |
| $q$ | = total number of state variables that describe physical changes of a balance |
| $\mathbf{R}$ | = rectangular matrix whose columns are defined by the regressors of a load component |
| $\mathbf{r}_i$ | = vector that has electrical output values of the bridge with index $i$ |
| $rAF$ | = output of the axial force bridge of a strain–gage balance |
| $RM$ | = rolling moment in the balance axis system |
| $rN1$ | = output of the forward normal force bridge of a force balance |
| $rN2$ | = output of the aft normal force bridge of a force balance |
| $rRM$ | = output of the rolling moment bridge of a strain–gage balance |
| $rS1$ | = output of the forward side force bridge of a force balance |
| $rS2$ | = output of the aft side force bridge of a force balance |
| $\mathbf{S}$ | = rectangular matrix whose columns are defined by the regressors of a bridge output |
| $\mathbf{s}_i$ | = vector that has load values of a load component with index $i$ |
| $S1$ | = forward side force of a force balance in the balance axis system |
| $S2$ | = aft side force of a force balance in the balance axis system |

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| $\Delta p$ | = bellows pressure change of an air balance relative to a fixed reference pressure |
| $\Delta T$ | = balance temperature change relative to a fixed reference temperature |
| $\eta$ | = total number of terms of the regression model of a load component |
| $\lambda_i$ | = load component of a strain–gage balance with index $i$ |
| $\lambda_{i,A}$ | = load caused by aerodynamic effects on the wind tunnel model |
| $\lambda_{i,E}$ | = load caused by the total weight of the calibration equipment and the metric part |
| $\lambda_{i,G}$ | = load caused by an applied calibration load (e.g., by a set of known gravity weights) |
| $\lambda_{i,P}$ | = load caused by a propulsion simulator during a wind tunnel test |
| $\lambda_{i,W}$ | = load caused by the total weight of the wind tunnel model |
| $\mu$ | = total number of terms of the regression model of a bridge output |
| $\nu$ | = total number of load components (or bridge outputs) and state variables of a balance |
| $\rho_i$ | = measured electrical output of a bridge with index $i$ |
| $\sigma$ | = state variable; describes a physical change of the balance that influences bridge outputs |

## I. Introduction

A wind tunnel strain–gage balance measures loads, i.e., forces and moments, that act on a test article during a wind tunnel test. The specific design of a strain–gage balance is always a compromise as it is dictated by (i) the chosen number of load components, (ii) the expected load magnitudes on the model, (iii) load prediction accuracy requirements, and (iv) balance manufacturing and model installation constraints. In principle, the total number of independently measured load components can vary from one to six. Six load components is the theoretical maximum as the resultant force and moment vectors at the balance moment center can only have up to three independent components each in three–dimensional space.

One frequently used balance type is a six–component balance. Three design variations of this type exist: direct–read balance, force balance, and moment balance (see, e.g., Ref. [1] for a description of different balance types). In theory, each type should be designed such that the internal strain caused by six independently applied load components (three forces and three moments, or, five forces and one moment, or, five moments and one force) can be related to six independently measured bridge outputs. In other words, it should be possible to map balance loads that are described as a point in a six–dimensional "load space" to a set of measured bridge outputs that are described as a point in a six–dimensional "output space."

Figure 1 shows, for example, what a simplified description of an "unloaded" and "loaded" six–component single–piece balance could look like. Three basic parts of the balance can be identified: (i) the metric part, (ii) the non–metric part, and (iii) the transitional zone. The metric part attaches the balance to the wind tunnel model or the calibration body. In addition, it defines the balance axis system and, consequently, the exact location of the balance moment center in space. The non–metric part, on the other hand, attaches the balance to the model support system. It is also the location where all strain–gage wire sets are assembled in a wire harness that allows for the transmission of the electrical outputs of the bridges to a test facility's data acquisition system. The transitional zone is the physical interface between metric and non–metric part where flexures, gages, and Wheatstone bridges are located. Model loads act on the metric part. Reaction loads, on the other hand, act on the non–metric part. The simplified description shown in Fig. 1 is very general in nature. Therefore, it is applicable to all balance types that use strain–gages. Figure 2, for example, shows how the descriptions of Fig. 1 can be applied to an "unloaded" and "loaded" six–component multi–piece balance that has a "metric outer sleeve" instead of a "metric end."

The correct selection of the location of flexures and bridges relative to the expected test article loads is critical as far as balance design and the definition of a "unique mapping" between loads and bridge outputs is concerned ("unique mapping" $\equiv$ a point in the "load space" only maps to one point in the "output space" and vice versa). This selection attempts to obtain the highest possible outputs within the elastic range of the flexures assuming that loads act at or near the balance moment center. Figure 3a shows an "ideal" situation when a balance is calibrated while the balance moment center is near the model moment center. In this case, a linearly independent set of loads will result in a linearly independent set of bridge outputs because the balance is used as designed. Figure 3b shows an alternate calibration situation for the balance that is shown in Fig. 3a. Now, the model moment center is far forward of the flexures, bridges, and the balance moment center (this situation often exists during calibration and use of a sting balance). In this case, independent calibration loads can still be applied at the model moment center. However, the measured bridge outputs may or may not be linearly independent depending on the sensitivities of the individual bridges.

The two examples discussed in the previous paragraph indicate that it would be useful to develop a quantitative test that "objectively" determines if the applied loads and the measured bridge outputs of a strain–gage balance themselves are linearly independent. Results of this test would tell the user of a balance if a unique, i.e., reversible, mapping between points described in the "load space" and points described in the "output space" can be expected in a situation when (i) a traditional balance is not used as originally intended, (ii) a balance of non–traditional design is to be evaluated, or (iii) connections between bridge output measurements and applied loads of a chosen balance are not easily understood. The proposed test must also work with both the *Iterative* and the *Non–Iterative Method* that are used in the aerospace testing community for the prediction of balance loads as the load prediction uniqueness characteristics are independent of the specific method that is used for the regression analysis of balance calibration data (see Ref. [2] for a description of the *Iterative* and *Non–Iterative Method*).

In the next section of the paper a universally applicable control volume analysis of the inputs and outputs of a strain–gage balance is introduced to better support the author's hypothesis that the total number of load components and the total number of bridge outputs of a strain–gage balance must <u>always</u> match. This hypothesis is the fundamental assumption that is used to justify the proposed uniqueness test of the load predictions of a strain–gage balance. Afterwards, the test itself is described. Finally, data from the calibration of a six–component force balance is used to illustrate benefits of the proposed new test.

## II. Control Volume Analysis

### A. General Remarks

The definition of the test of the uniqueness of strain–gage balance load predictions is based on the author's hypothesis that the number of independently applied load components must <u>always</u> match the total number of independently measured bridge outputs. It is not immediately obvious that this assumption applies to all strain–gage balance types because, for example, temperature effects or bellows pressure of an air balance also influence the electrical outputs of a strain–gage balance. Therefore, the author decided to perform a rigorous control volume analysis of the inputs and outputs of a strain–gage balance in order to support his hypothesis.

In general, a control volume describes a physical space that has precisely defined boundaries. Therefore, a control volume analysis of the inputs and outputs of a balance has the advantage that all variables influencing its behavior can clearly be identified and separated. These variables may be split into three categories: (i) input variables, (ii) output variables, and (iii) state variables. By definition, input and output variables cross the boundaries of the control volume. State variables, on the other hand, describe conditions inside the control volume that influence the bridge outputs. Figure 4 shows different elements of the control volume analysis of the inputs and outputs of a strain–gage balance. It describes the balance in very generic terms because the control volume description applies to all strain–gage balances that measure loads in fluid dynamic testing (e.g., wind tunnel test of an aircraft or automobile, towing tank test of a ship's hull).

### B. Basic Load Component and Bridge Output Vector Description

The input variables of the control volume are the loads that act on the metric support of the balance. This metric support is either a calibration body or a model structure that is rigidly attached to the metric part of the balance. These input variables can be described as follows ...

$$\textit{Load Component Vector} \implies \begin{bmatrix} \lambda_1 \\ \vdots \\ \lambda_i \\ \vdots \\ \lambda_{n'} \end{bmatrix} \quad \textit{where} \quad 1 \leq i \leq n' \tag{1}$$

where $\lambda_i$ is a load component and $n'$ is the total number of load components that independently act on the metric support. The output variables are the measured electrical outputs (or output combinations) of the bridges that exit the control volume through the wire harness (see Fig. 4). Again, the output variables can be described in vector format as follows ...

$$\textit{Output or Output Combination Vector} \implies \begin{bmatrix} \rho_1 \\ \vdots \\ \rho_j \\ \vdots \\ \rho_{n''} \end{bmatrix} \quad \textit{where} \quad 1 \leq j \leq n'' \tag{2}$$

where $\rho_j$ is an electrical output and $n''$ is the total number of bridge outputs that independently respond to loads acting on the metric support. Now, assuming for the time being that all state variables of the control volume (e.g., the uniform temperature of the balance -or- the bellows pressure of an air balance) remain constant, a first result of the control volume analysis can be summarized as follows:

**First Result of Control Volume Analysis**

*It is assumed that the state variables of the balance (e.g., temperature, bellows pressure) remain unchanged. Then, the independent "information" leaving the control volume of the balance, i.e., the total number of independent bridge output measurements that exit through the wire harness, cannot be greater than the independent "information" entering the control volume, i.e., the total number of independently applied load components that act on the metric part of the balance.*

The first result of the control volume analysis means that nothing can be gained by having more bridge output measurements than applied load components because, in principle, the bridge output set cannot have more independent variables than the load component set that acts on the metric support. In other words, the total number of independent input variables limits the maximum number of independent output variables that the balance can have as long as the state variables remain unchanged.

The first result of the control volume analysis is also supported by rules that govern transformations of systems of implicit functions. Let us assume, for example, that a strain–gage balance measures $n''$ bridge outputs ($\rho_1, \ldots, \rho_{n''}$) and that each bridge output is a function of $n'$ load components ($\lambda_1, \ldots, \lambda_{n'}$). Let us also assume that the number $n''$ of bridge outputs is greater than the number $n'$ of load components. Then, according to the *General Theorem on the Inversion of Transformations* (Ref. [3], pp. 261–277), the $n'$ load components may be inverted implicitly to give $\lambda_i = F_i(\rho_1, \ldots, \rho_{n'})$ for $1 \leq i \leq n'$. This result implies that the remaining bridge output measurements $\rho_{n'+1},\ \ldots,\ \rho_{n''}$ are functions of $\rho_1,\ \ldots,\ \rho_{n'}$ because (i) they are defined as $\rho_j = G_j(\lambda_1,\ \ldots,\ \lambda_{n'})$ for $n'+1 \leq j \leq n''$ and (ii) the load components $\lambda_1,\ \ldots,\ \lambda_{n'}$ can be expressed as $\lambda_i = F_i(\rho_1,\ \ldots,\ \rho_{n'})$ for $1 \leq i \leq n'$. This conclusion supports the author's hypothesis that the number of independently measured bridge outputs must always equal the number of independently applied load components if reliable load predictions are to be obtained from regression models of strain–gage balance calibration data. The author's hypothesis can also be expressed as a balance design requirement:

**Balance Design Requirement**

*A strain–gage balance must be designed such that the total number of independently applied load components equals the total number of independently measured bridge outputs (or output combinations).*

Statements made in the previous paragraph can easily be put into a practical context. Let us assume that a balance measures four bridge outputs $(\rho_1, \rho_2, \rho_3, \rho_4)$. Let us also assume that only the normal force $(NF)$, the axial force $(AF)$, and the pitching moment $(PM)$ were independently applied during its calibration. Consequently, the four bridge outputs can only be a function of the three applied load components. In addition, according to the *General Theorem on the Inversion of Transformations*, the three load components may be inverted implicitly to give $NF = F_1(\rho_1, \rho_2, \rho_3)$, $AF = F_2(\rho_1, \rho_2, \rho_3)$, and $PM = F_3(\rho_1, \rho_2, \rho_3)$. Therefore, the fourth bridge output $\rho_4$ must be a function of $\rho_1$, $\rho_2$, and $\rho_3$ because (i) the output $\rho_4$ can only be a function of the three load components that were independently applied during the calibration and (ii) the applied load components themselves can be expressed as functions of $\rho_1$, $\rho_2$, and $\rho_3$.

The balance design requirement above can mathematically be described by using the total number of load components and bridge output measurements that were defined in Eqs. (1) and (2). We also know that the maximum number of independent load components in 3–dimensional space equals six. Then, we get ...

$$n' \ = \ n'' \ = \ n \ \leq \ 6 \tag{3}$$

where $n$ is the total number of independent load components that are being applied to the metric support of the balance. Finally, we get the following basic load component and bridge output vector description for the special case when state variables have no influence on the electrical outputs of the bridges:

**Basic Load Component and Bridge Output Vector Description**

$$\textit{Load Component Vector} \implies \begin{bmatrix} \lambda_1 \\ \vdots \\ \lambda_i \\ \vdots \\ \lambda_n \end{bmatrix} \qquad (4a)$$

$$\textit{Output or Output Combination Vector} \implies \begin{bmatrix} \rho_1 \\ \vdots \\ \rho_i \\ \vdots \\ \rho_n \end{bmatrix} \qquad (4b)$$

where

$$1 \ \leq \ i \ \leq \ n \ \leq \ 6 \qquad (4c)$$

The load component and bridge output vectors have to be extended if state variables influence the behavior of the balance. The resulting more general definition of the vectors is discussed in the next section.

## C. Extended Load Component and Bridge Output Vector Description

Balance applications exist when state variables have a significant impact on the bridge outputs. One example is the use of a semi–span balance in a cryogenic or pressurized wind tunnel. In that situation, assuming that the balance temperature is not actively controlled, the change of the uniform balance temperature relative to a fixed reference temperature may become a state variable as heat flows across the control volume boundary. Similarly, an air balance is often used to operate propulsion simulators that are attached to a test article during a wind tunnel test. Then, high pressure air flows across the control volume boundary. By design, the air supply line of an air balance bridges the metric and non–metric part. The load path from the metric to the non–metric part may change whenever supply line pressure changes occur. Consequently, the bellows pressure change of the air balance may be used as a state variable because (i) it describes internal pressure effects on both balance geometry and bridge outputs and (ii) it can easily be measured. It has to be emphasized that the mass flow rate through the air balance is not a state variable because mass flow rate variations directly result in load changes on the test article during a wind tunnel test that are felt by the metric part of the balance (a green arrow is used in Fig. 4 to highlight this relationship between the control volume input "Mass Flow" and the control volume input "Model Loads"). This direct connection makes it possible to completely ignore the mass flow rate during the calibration of an air balance. A state variable vector may be introduced in all these balance applications that could have the following format ...

$$State\ Variable\ Vector \implies \begin{bmatrix} \sigma_1 \\ \vdots \\ \sigma_\xi \\ \vdots \\ \sigma_q \end{bmatrix} \tag{5a}$$

where $\sigma_\xi$ is a state variable and $q$ is the total number of state variables of the chosen balance. Two examples of state variables can be described as follows:

$$Change\ of\ Uniform\ Balance\ Temperature \implies \sigma_\xi = \Delta T \tag{5b}$$

$$Change\ of\ Bellows\ Pressure\ of\ an\ Air\ Balance \implies \sigma_\xi = \Delta p \tag{5c}$$

Typically, the total number $q$ of state variables is small in a real–world application. It stays below three in most cases as, for example, the complexity of an air balance does not make it practical to use more than two bellows. In addition, the inclusion of state variables greatly increases the complexity of the calibration of the balance. State variables have another important characteristic: they are neither inputs nor outputs of the control volume. Instead, they describe the physical "state" of the balance while (i) loads are being applied on the metric support and (ii) bridge outputs are being measured. This important second result of the control volume analysis can be summarized as follows:

**Second Result of Control Volume Analysis**

*State variables of the balance, e.g., temperature or pressure differences, need to accompany the description of both input and output variable sets because inputs, i.e., loads, were applied and outputs, i.e., bridge outputs, were measured while the state variables had specific values.*

It was originally suggested in Ref. [4] to extend the independent and dependent variable sets of a balance that needs, for example, a state variable like the uniform balance temperature difference for a description of its characteristics. This approach can directly be applied to the load component and bridge output vectors that are defined in Eqs. (1) and (2) above. State variables can simply be treated as both another set of load components and bridge outputs. Then, the state variable vector of Eq. (5a) can be expressed as:

$$\underbrace{\begin{bmatrix} \sigma_1 \\ \vdots \\ \sigma_q \end{bmatrix}}_{state\ variables} = \underbrace{\begin{bmatrix} \lambda_{n+1} \\ \vdots \\ \lambda_{n+q} \end{bmatrix}}_{used\ like\ loads} = \underbrace{\begin{bmatrix} \rho_{n+1} \\ \vdots \\ \rho_{n+q} \end{bmatrix}}_{used\ like\ outputs} \tag{6}$$

Equation (6) can also be summarized as follows:

**State Variable Characteristic**

$$\sigma_{i-n} \quad = \quad \lambda_i \quad = \quad \rho_i \qquad (7a)$$

where

$$n+1 \quad \leq \quad i \quad \leq \quad n+q \qquad (7b)$$

For simplicity, the sum of the total number of load components (or bridge outputs) and the total number of state variable can be written as follows:

$$\nu \quad = \quad n \; + \; q \tag{8}$$

Finally, an extended load component and bridge output vector description of a strain–gage balance can be summarized by using the following relationships:

**Extended Load Component and Bridge Output Vector Description**

$$Extended\ Load\ Component\ Vector \quad \Longrightarrow \quad \begin{bmatrix} \lambda_1 \\ \vdots \\ \lambda_i \\ \vdots \\ \lambda_\nu \end{bmatrix} \qquad (9a)$$

$$Extended\ Output\ or\ Output\ Combination\ Vector \quad \Longrightarrow \quad \begin{bmatrix} \rho_1 \\ \vdots \\ \rho_i \\ \vdots \\ \rho_\nu \end{bmatrix} \qquad (9b)$$

where

$$1 \;\leq\; i \;\leq\; \nu \qquad (9c)$$

$$\nu - q \;\leq\; 6 \qquad (9d)$$

The extended load component and bridge output vector description still has the same basic property that the total number of independently applied load components and state variables must match the total number of independently measured bridge outputs and state variables. The load component vector defined Eq. ($9a$) represents a point in the $\nu$–dimensional "load space." The output vector defined in Eq. ($9b$) represents a point in the $\nu$–dimensional "output space."

## D. Examples

Three examples may be used to illustrate the above description of the load component and bridge output vectors for typical balances. The first example is a six–component force balance in force balance format. It is assumed that the balance will be used in the wind tunnel near its constant uniform calibration temperature (i.e., no temperature dependent state variable is needed for the description of the balance behavior). Then, data points in the load and output spaces can be described as follows ($\nu = n = 6$):

$$\underbrace{\begin{bmatrix} N1 \\ N2 \\ S1 \\ S2 \\ AF \\ RM \end{bmatrix}}_{loads} \iff \underbrace{\begin{bmatrix} rN1 \\ rN2 \\ rS1 \\ rS2 \\ rAF \\ rRM \end{bmatrix}}_{outputs} \tag{10}$$

The second example is a six–component air balance with a single bellows assembly that was designed and gaged like a force balance. Again, the balance will be tested near its constant uniform calibration temperature. Therefore, no temperature dependent state variable is needed. However, pressure effects on the bridge output measurements are expected to be significant. Therefore, the bellows pressure difference $\Delta p$ is introduced as one additional state variable in order to better describe the expected balance behavior. The resulting data points in the load and output spaces can be described as follows ($\nu = n = 7$):

$$\underbrace{\begin{bmatrix} N1 \\ N2 \\ S1 \\ S2 \\ AF \\ RM \\ \Delta p \end{bmatrix}}_{loads} \iff \underbrace{\begin{bmatrix} rN1 \\ rN2 \\ rS1 \\ rS2 \\ rAF \\ rRM \\ \Delta p \end{bmatrix}}_{outputs} \tag{11}$$

The third example is a three–component canard balance. Again, it will be used near its constant uniform calibration temperature (no state variables are needed to describe its behavior). The resulting data points in the load and output spaces can be described as follows ($\nu = n = 3$):

$$\underbrace{\begin{bmatrix} NF \\ M1 \\ M2 \end{bmatrix}}_{loads} \iff \underbrace{\begin{bmatrix} \rho_1 \\ \rho_2 \\ \rho_3 \end{bmatrix}}_{outputs} \tag{12}$$

The assumed existence of a unique and reversible mapping between the "load space" and the "output space" has additional consequences as far as calibration and use of the balance is concerned. They will be discussed in the next section of the paper.

## III. From Calibration To Wind Tunnel Test

It was pointed out in the previous section that the ability to precisely predict loads from the measured bridge outputs of a strain–gage balance depends on the fundamental assumption that, in theory, some unique, i.e., reversible, relationship between loads and bridge outputs should exist. This assumption is illustrated in this section by revisiting some basic situations that are associated with both calibration and use of a balance.

The correct interpretation of the reversible nature of the relationship between balance loads and bridge outputs requires that a condition must exist when the balance does not experience any load. This absolute load datum of "zero load" guarantees that any load acting on the metric part will be interpreted relative to a global datum that is completely independent of the way the balance is used. The absolute load datum defines the "origin" of the "load space." It is "mapped" to the corresponding global datum in the "output space." This second global datum is defined by the raw electrical outputs of each bridge that would be recorded in an assumed "weightless" condition of the balance (see Fig. 5a). These outputs can be obtained in a laboratory environment by averaging absolute voltage measurements of the bridges for different orientations of the balance axis system relative to the gravitational acceleration. The electrical outputs of the second global datum are the exact representation of the "zero load" condition in the "output space." It is important to point out that the second global datum does not coincide with the "origin" of the "output space" unless bridge outputs are reported as output differences relative to the raw electrical outputs that would be recorded in "weightless" condition.

Another important characteristic of balance load measurements is the fact that the total balance loads associated with a specific set of bridge output measurements are the sum of different contributors. This statement holds true for both the calibration and wind tunnel test situation. Figure 5b, for example, shows the situation during a balance calibration. In this case, the total absolute load is the sum of loads caused by pure calibration loads (e.g., gravity weights or actuator loads) and loads that are caused by the physical weight of the calibration equipment. This observation can be summarized by the following relationship:

$$\text{Calibration} \quad \Longrightarrow \quad \underbrace{\lambda_i}_{balance} \quad = \quad \underbrace{\lambda_{i,E}}_{equipment\ weight} \quad + \quad \underbrace{\lambda_{i,G}}_{calibration\ load} \tag{13}$$

The load contribution associated with the weight of the calibration equipment ($\lambda_{i,E}$) is usually obtained by performing a tare load iteration (see Ref. [1] and Ref. [5] for a discussion of the tare load iteration).

Similarly, the absolute load acting on the balance during a wind tunnel test is the sum of loads caused by (i) the model weight, (ii) aerodynamic effects, and, if applicable, (iii) operating propulsion simulators that are attached to the model structure. In general, wind tunnel customers are only interested in loads that are caused by aerodynamic effects on the model structure and/or loads that are associated with operating propulsion simulators. Therefore, results from "wind–off" and "wind–on" tests need to be combined so that the unwanted weight contribution of the model to the total balance load can be identified and removed. It must also be pointed out that the use of a propulsion simulator requires "power–off" and "power–on" tests in addition to the "wind–off" and "wind–on" tests in order to separate aerodynamic loads from loads that are caused by the operating propulsion simulators. These four fundamental wind tunnel test cases can be summarized as follows (see also Fig. 6a to Fig. 6d):

$$\text{Case 1 (wind–off, power–off)} \quad \Longrightarrow \quad \underbrace{\lambda_i}_{balance} \quad = \quad \underbrace{\lambda_{i,W}}_{model\ weight} \tag{14$a$}$$

$$\text{Case 2 (wind–on, power–off)} \quad \Longrightarrow \quad \underbrace{\lambda_i}_{balance} \quad = \quad \underbrace{\lambda_{i,W}}_{model\ weight} \quad + \quad \underbrace{\lambda_{i,A}}_{aerodynamics} \tag{14$b$}$$

$$\text{Case 3 (wind–off, power–on)} \quad \Longrightarrow \quad \underbrace{\lambda_i}_{balance} \quad = \quad \underbrace{\lambda_{i,W}}_{model\ weight} \quad + \quad \underbrace{\lambda_{i,P}}_{propulsion} \tag{14$c$}$$

$$\text{Case 4 (wind–on, power–on)} \quad \Longrightarrow \quad \underbrace{\lambda_i}_{balance} \quad = \quad \underbrace{\lambda_{i,W}}_{model\ weight} \quad + \quad \underbrace{\lambda_{i,A}}_{aerodynamics} \quad + \quad \underbrace{\lambda_{i,P}}_{propulsion} \tag{14$d$}$$

The above discussion of the balance calibration and the four wind tunnel test cases highlights another important fact: the balance always experiences the resultant total loads that (i) cross the control volume boundaries and (ii) act on the metric part of the balance. In other words, the balance does not "know" the exact magnitude and sign of each individual load contributor that acts on the metric part. It also does not "know" what type of metric support, i.e., a model structure or a calibration body, is rigidly attached to the metric part. Therefore, loads acting on the model structure, i.e., the test article, must be identical to loads acting on a calibration body as long as the electrical outputs and state variables of both situations match. This important conclusion can be summarized as follows:

**Relationship between Loads, Balance Axis System, Metric Part, Model Structure, and Calibration Body**

*The balance axis system helps describe magnitude, location, and orientation of loads that act on a strain–gage balance. This Cartesian coordinate system is assumed to be fixed to the metric part (metric end) of the balance. Loads act either on a model structure or on a calibration body that is fixed to the metric part. These loads are exclusively transmitted via the metric part to the flexures and gages of the balance. Therefore, loads acting on a model structure are identical with loads acting on a calibration body whenever matching electrical outputs and state variable values are observed for both situations.*

It can also be concluded from the previous paragraph that the calibration body is a "generic" model structure that makes the precise application of loads in the balance axis system possible. This alternate interpretation of a calibration body can be described as follows:

**Calibration Body ≡ Generic Model Structure**

*The calibration body of a strain–gage balance is a highly specialized "generic" model structure, i.e., test article. It is primarily used in a laboratory environment for the application of a set of "calibration" loads that can precisely be described in the balance axis system. These loads and related electrical outputs of the bridges are used to develop a regression model of the balance behavior under load. This regression model makes it possible to predict loads from measured bridge outputs during a wind tunnel test.*

Another conclusion can be drawn from the author's hypothesis that a unique mapping between "load space" and "output space" must exist. Two different approaches are used in the aerospace testing community for the prediction of balance loads. The first approach, i.e., the *Non–Iterative Method*, directly fits calibration loads as a function of the measured bridge outputs and uses the resulting regression models for the load prediction (see App. 1). The second approach, i.e., the *Iterative Method*, first fits bridge outputs as a function of the loads (see App. 2). Afterwards, it constructs an iteration scheme from the result so that loads can be predicted from outputs during the wind tunnel test. Therefore, because of the "reversible" nature of the relationship between "load space" and "output space," it should be possible under certain conditions to directly obtain the regression coefficients of the load components from the regression coefficients of the bridge outputs. This conclusion is rigorously proven in App. 3 for the special case when the regression models of the loads and bridge outputs only consist of linear terms. The resulting analytic relationships between the two regression coefficient sets are given in Eqs. (33) and (36).

The next section of the paper describes the proposed test of the uniqueness of the relationship between applied loads and measured bridge outputs of a strain–gage balance.

## IV. Description of Uniqueness Test

The proposed uniqueness test evaluates the linear independence of the "load space" and the "output space" separately by computing the maximum variance inflation factor (VIF) of simple linear math models. First, the load component set of the balance calibration data is tested for linear independence by computing the maximum VIF of a math model that is constructed from the linear terms associated with each load component. Then, the bridge output set of the balance calibration data is tested for linear independence by computing the maximum VIF of a math model that is constructed from the linear terms associated with each bridge output. Now, the maximum variance inflation factor of each set is compared with recommended thresholds from the literature in order to decide if the set is linearly independent or dependent. The test considers a load or bridge output set to be linearly independent if its variance inflation factor maximum is less than the conservative threshold of 5 (threshold is taken from Ref. [6], p. 658). Similarly, the test considers a load or bridge output set to be linearly dependent if its variance inflation factor maximum exceeds the threshold of 50. The uniqueness test uses the threshold values of 5 and 50 instead of the often recommended value of 10. These choices were made because the use of 5 instead of 10 makes the test for linear independence more reliable. Likewise, the use of 50 instead of 10 makes the test for linear dependence more reliable.

Table 1 below lists four typical cases that are of special interest as far as the interpretation of the uniqueness test results are concerned. *Case 1* describes the situation when the maximum VIF of both the load component and bridge output set is less than the literature recommended threshold of 5. This result means that a highly unique mapping between the load component set and the bridge output set can be constructed from the calibration data. On the other hand, it is observed for *Case 2* that the maximum VIF of both sets is greater than 50. This observation may indicate a potential experimental design issue that should be addressed because the linear dependencies of the load component set are directly reflected in the linear dependencies of the bridge output set. *Case 3* describes a situation when the maximum VIF of the load component set exceeds 50 while, at the same time, the maximum VIF of the bridge output set is smaller than 5. This situation is theoretically impossible because, based on the control volume analysis of the inputs and outputs of a strain–gage balance, the measurements recorded by the bridge output set cannot have more independent "information" than the applied load component set. Finally, *Case 4* describes a situation when the maximum VIF of the load component set is smaller than 5 while, at the same time, the maximum VIF of the bridge output set is greater than 50. This observation is an indication that the bridge output set may be linearly dependent. Therefore, the balance load predictions may not be very reliable because a unique and reversible mapping between "load space" and "output space" may not exist.

**Table 1:** List of four typical uniqueness test results.

| *Case* | *Load Component Set* $\Longrightarrow \lambda_1, \lambda_2, \ldots, \lambda_\nu$ | *Bridge Output Set* $\Longrightarrow \rho_1, \rho_2, \ldots, \rho_\nu$ | *Comments* |
|---|---|---|---|
| 1 | $VIF_{max}(\lambda_i) < 5$ | $VIF_{max}(\rho_i) < 5$ | *linearly independent loads & outputs* |
| 2 | $VIF_{max}(\lambda_i) > 50$ | $VIF_{max}(\rho_i) > 50$ | *experimental design issues* |
| 3 | $VIF_{max}(\lambda_i) > 50$ | $VIF_{max}(\rho_i) < 5$ | *theoretically impossible result* |
| 4 | $VIF_{max}(\lambda_i) < 5$ | $VIF_{max}(\rho_i) > 50$ | *linearly dependent bridge outputs* |

It is important to emphasized that the proposed uniqueness test does not require a least squares fit of the calibration data. Only variance inflation factors are computed by using either loads or bridge outputs to define regressors of a simple linear math model. The variance inflation factor was selected for the uniqueness test because it has the ability to capture both linear and near–linear dependencies between regressors.

It has to be mentioned for completeness that an alternate test of the uniqueness of the mapping between "load space" and "output space" can be defined that is based on the *General Theorem on the Inversion of Transformations* (see Ref. [3], pp. 261–277). The alternate test uses the *Jacobian* that is obtained from the balance calibration data. The *Jacobian* is the determinant of the *Jacobian matrix* that consists of the first order partial derivatives of the bridge outputs with respect to each load component. A unique mapping exists whenever the *Jacobian* is not equal to zero. A numerical approximation of the *Jacobian matrix* is easily obtained if the *Iterative Method* is used for the prediction of balance loads. It equals the matrix **C1** that results from the least squares fit of the bridge outputs (see Ref. [1] for a description of the *Iterative Method*). The alternate test will easily identify *Case 1* that is listed in Table 1 for the original test. However, the alternate test cannot distinguish between *Case 2* and *Case 4* as the *Jacobian* of both cases will be close to zero. In addition, the alternate test requires some reasonable threshold for zero because the *Jacobian* of linearly related real–world data will never exactly be zero. The original test, on the other hand, has the advantage that it is more universally applicable. It does not require a least squares fit of the calibration data and its use is not restricted to a situation when the number of load components matches the number of bridge output measurements.

## V. Discussion of Example

Calibration data of a six–component balance was selected to demonstrate the application of the proposed load prediction uniqueness test. The chosen balance is NASA's MC60E balance. The MC60E is a 2.0 inch diameter force balance. Table 2 below lists load capacities of the balance in force balance format. The balance was calibrated in 2008 using Triumph Aerospace's Automatic Balance Calibration System (ABCS). The final calibration data set consisted of 1904 loadings.

**Table 2:** Load capacities of NASA's MC60E six–component force balance.

| $N1$, lbs | $N2$, lbs | $S1$, lbs | $S2$, lbs | $RM$, in–lbs | $AF$, lbs |
|---|---|---|---|---|---|
| 2,500 | 2,500 | 1,250 | 1,250 | 5,000 | 700 |

The analysis of the selected balance calibration data set was performed by using NASA's BALFIT regression analysis tool (cf. Ref. [7]). The data was analyzed by using the *Iterative Method* so that uniqueness test results for both the load component set and the bridge output set could directly be compared.

First, the load component set and the bridge output set of the original calibration data was tested. Figure 7a shows the uniqueness test results for the load component set. Figure 7b shows the corresponding uniqueness test results for the bridge output set. The largest variance inflation factor of both tests is $\approx 1.45$. This value is well below the variance inflation factor threshold of 5. Therefore, load predictions obtained from the original data set should have a high degree of uniqueness because no significant near–linear or linear dependencies among members of both the load component set and the bridge output set were found.

In the next part of the investigations, the original 1904–point calibration data set was modified so that *Case 4* in Table 1 could be demonstrated. Therefore, the axial force bridge output of the original data set was replaced by the sum of the outputs of the two normal force bridges plus random noise between $-20$ and $+20$ microV/V. Figure 8a shows the result after the test was applied to the load component set of the modified data. The variance inflation factors show exact agreement with values that were reported in Fig. 7a for the original data set. This result is expected as both the original and modified data sets use exactly the same loads. Figure 8b shows the result after the test was applied to the bridge output set of the modified data. This time, the maximum variance inflation factor equals 2458. This value significantly exceeds the limit of 50 that the author suggests as a threshold for linear dependence. Therefore, the test result confirms that the bridge output set has an unwanted linear dependency.

## VI. Summary and Conclusions

A new test was presented that assesses the uniqueness of strain–gage balance load predictions. First, basic assumptions were discussed that justify the new test. Therefore, a control volume analysis of the inputs and outputs of a strain–gage balance was presented in support of the author's hypothesis that the number of load components of the balance must always match the number of measured bridge outputs (or measured bridge output combinations) if reliable load predictions are to be achieved during a wind tunnel test.

The suggested control volume analysis has another benefit: it clearly separates acting load components and measured bridge outputs from state variables. State variables need to be used to describe the physical condition of the balance while loads are being applied and bridge outputs are being measured. In addition, based on results of the control volume analysis, it is rigorously shown in App. 3 that the regression coefficients of the balance load components, i.e., the result of the application of the *Non–Iterative Method*, can directly be obtained from the regression coefficients of the bridge outputs, i.e., from the result of the application of the *Iterative Method*, whenever linear terms are exclusively used in the regression models.

Details of the new test were defined. The new test checks the linear independence of load components and bridge outputs of a balance calibration data set separately by determining the maximum variance inflation factor of related linear math models. The regressors used in the math models are constructed from either the load component set or the bridge output set. The maximum variance inflation factor of each math model is compared with the literature recommended threshold of 5 in order to decide if the investigated variable set, i.e., either the load component set or the bridge output set, is linearly independent. Highly unique balance load predictions can be expected if the maximum variance inflation factor of both sets is less than the threshold of 5. Unwanted linear dependencies between either members of the load component set or members of the bridge output set may exist and should be investigated if the maximum variance inflation factor exceeds the threshold of 50. Data from the machine calibration of a six–component force balance was used to illustrate the application of the uniqueness test.

Most balance calibration experiments are designed from the start such that the applied load component set is linearly independent. However, for example, users of the *Iterative Method* may have never checked if the measured bridge output set of the calibration data is also linearly independent. Similarly, users of the *Non–Iterative Method* are in danger of potentially using a linearly dependent bridge output set for the fit of the balance loads unless the linear independence of the bridge output set is objectively confirmed. The total number of independent bridge output measurements of a real–world strain-gage balance calibration data set is always "equal to" or "less than" the total number of independently applied load components. Consequently, the bridge output set must be tested for linear dependencies because a linearly independent load set will not automatically lead to linearly independent bridge output measurements. In other words, the check of the linear independence of the bridge output set is as important as the check of the linear independence of the load component set whenever an analyst wants to be confident about the reliability of load predictions that are obtained from regression models of strain–gage balance calibration data.


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## References


[1]American Institute of Aeronautics and Astronautics, *Recommended Practice – Calibration and Use of Internal Strain Gage Balances with Application to Wind Tunnel Testing*, AIAA R–091–2003, American Institute of Aeronautics and Astronautics, Reston, Virginia, 2003; pp. 4–5.

[2]Ulbrich, N., "Comparison of Iterative and Non–Iterative Strain–Gage Balance Load Calculation Methods," AIAA 2010–4202, paper presented at the 27th AIAA Aerodynamic Measurement Technology and Ground Testing Conference, Chicago, Illinois, June/July 2010.

[3]Courant, R., and John, F., *Introduction to Calculus and Analysis – Vol. 2*, 1st. ed., Interscience Publishers, John Wiley & Sons, New York, 1974; pp. 261–277.

[4]Ulbrich, N., "Iterative Strain–Gage Balance Calibration Data Analysis for Extended Independent Variable Sets," AIAA 2011–0949, paper presented at the 49th AIAA Aerospace Sciences Meeting, Orlando, Florida, January 2011.

[5]Ulbrich, N., "A Universal Tare Load Prediction Algorithm for Strain–Gage Balance Calibration Data Analysis," AIAA 2011–6090, paper presented at the 47th AIAA/ASME/SAE/ASEE Joint Propulsion Conference, San Diego, California, July/August 2011.

[6]Myers, R. H., and Montgomery, D. C., *Response Surface Methodology – Process and Product Optimization Using Designed Experiments*, 1st. ed., John Wiley & Sons, Inc., New York, 1995; p. 658.

[7]*BALFIT – Software Tool for the Regression Analysis of Multivariate Data – Version 3.01.21*, distributed by the Software Release Authority, Technology Partnerships Office, NASA Ames Research Center, Moffett Field, California, December 2014.

[8]DeLoach, R., and Ulbrich, N., "A Statistical Theory of Bi–directionality," AIAA 2013–2995, paper presented at the AIAA Ground Testing Conference, San Diego, California, June 2013.

[9]Kammeyer, M. E., and Ulbrich, N., "The Use of Absolute-Value Terms in Regression Modeling of Multi–Piece Force Balances," AIAA 2015–3382, paper presented at the 31st AIAA Aerodynamic Measurement Technology and Ground Testing Conference, Dallas, Texas, June 2015.

[10]Strang, G., *Introduction to Applied Mathematics*, Wellesley–Cambridge Press, Wellesley, Massachusetts, 1986; pp. 35–39.

[11]Montgomery, D. C., Peck, E. A., and Vining, G. G., *Introduction to Linear Regression Analysis*, 4th ed., John Wiley & Sons, Inc., New York, 2006; pp. 63–75.

[12]Burden, R. L. and Faires, J. D., *Numerical Analysis*, 3rd edition, PWS–Kent Publishing Company, Boston, Massachusetts, 1985; p. 334.

## Appendix 1: Least Squares Fit of a Load Component

In general, a strain–gage balance calibration data set consists of two subsets: (i) the applied calibration loads, i.e., the forces and moments; (ii) the electrical outputs of the bridges that were measured when the calibration loads were applied. In addition, state variables like, for example, temperature or bellows pressure differences, may also have to be taken into account during the balance calibration if they significantly influence the load path from the metric to the non–metric part of the balance. Ultimately, a regression analysis of the balance calibration data needs to be performed so that loads can be predicted during a wind tunnel test from the measured bridge outputs and state variables.

Different methods are used in the aerospace testing community for the regression analysis of strain–gage balance data. The *Non–Iterative Method*, for example, fits each load directly as a function of the bridge outputs and state variables. State variables are treated exactly like the bridge outputs during the regression analysis. In other words, they are used as just another set of independent variables that define regressors. The resulting regression model of a load component $\lambda_i$ has the following structure . . .

**Regression Model $F_i$ of a Load Component $\lambda_i$**

$$\lambda_i(k) \quad = \quad \underbrace{F_i(\rho_1, \; \ldots \; , \rho_\nu)}_{regression\ model} \quad = \quad \sum_{\xi=1}^{\eta} \underbrace{c_{\xi,i}}_{coeff.} \cdot \underbrace{f_\xi\{\rho_1(k), \; \ldots \; , \rho_\nu(k)\}}_{regressors\ (see\ Table\ 3)} \qquad (15)$$

where

$$1 \; \leq \; i \; \leq \; \nu \quad ; \quad 1 \; \leq \; k \; \leq \; p$$

where $f_\xi$ is a regressor, $c_{\xi,i}$ is the corresponding regression coefficient, and $\rho_1(k), \ldots, \rho_\nu(k)$ are the independent variables, i.e., the bridge outputs and state variables, that define the regressors. The regression model of a load is constructed from different types of math terms (regressors). Table 3 below lists regressors that are often used for the analysis of strain–gage balance data (term definitions were taken from Ref. [1]).

**Table 3:** Regressor choices for the analysis of a balance load component.

| *Type* | $f_\xi\{\rho_1(k), \ldots, \rho_\nu(k)\}$ | $\gamma$ | $\delta$ ; $\delta \neq \gamma$ | *Comments* |
|---|---|---|---|---|
| *1* | 1 | – | – | intercept |
| *2* | $\rho_\gamma(k)$ | $1 \leq \gamma \leq \nu$ | – | linear |
| *3* | $\rho_\gamma^2(k)$ | $1 \leq \gamma \leq \nu$ | – | quadratic |
| *4* | $\rho_\gamma(k) \cdot \rho_\delta(k)$ | $1 \leq \gamma \leq \nu$ | $1 \leq \delta \leq \nu$ | cross–product |
| *5* | $\rho_\gamma^3(k)$ | $1 \leq \gamma \leq \nu$ | – | cubic |
| *6* | $\lvert\rho_\gamma(k)\rvert$ | $1 \leq \gamma \leq \nu$ | – | bi–directional data |
| *7* | $\rho_\gamma(k) \cdot \lvert\rho_\gamma(k)\rvert$ | $1 \leq \gamma \leq \nu$ | – | bi–directional data |
| *8* | $\lvert\rho_\gamma^3(k)\rvert$ | $1 \leq \gamma \leq \nu$ | – | bi–directional data |
| *9* | $\lvert\rho_\gamma(k) \cdot \rho_\delta(k)\rvert$ | $1 \leq \gamma \leq \nu$ | $1 \leq \delta \leq \nu$ | bi–directional data |
| *10* | $\rho_\gamma(k) \cdot \lvert\rho_\delta(k)\rvert$ | $1 \leq \gamma \leq \nu$ | $1 \leq \delta \leq \nu$ | bi–directional data |
| *11* | $\lvert\rho_\gamma(k)\rvert \cdot \rho_\delta(k)$ | $1 \leq \gamma \leq \nu$ | $1 \leq \delta \leq \nu$ | bi–directional data |

The regressors for the least squares analysis of a load component are the intercept (*Type 1*), linear terms (*Type 2*), and a variety of additional math term types (*Type 3* to *Type 11*). They are briefly reviewed in this appendix. First, the intercept (*Type 1*) is discussed. It is needed in the regression model of Eq. (15) so that constant zero–offsets of the load can numerically be included in the analysis. Now, regressors are discussed that are (i) constructed from the electrical outputs and (ii) are suitable for all balance types (single–piece balances, multi–piece balances, canard balances, sting balances, etc.). The linear terms (*Type 2*) fall into this category. In addition, quadratic terms (*Type 3*) and cross–product terms (*Type 4*) may also be applied to calibration data of all balance types. Some users of strain–gage balances include third order terms (*Type 5*) in the regression models of the loads. It is the author's experience that the potential use of third order terms in the regression models of balance loads should always carefully be evaluated because their successful application highly depends on design characteristics of the given balance.

It is known in the aerospace testing community that the normal and side force bridge outputs of one widely used family of multi–piece balances show "bi–directional" behavior. This balance design specific characteristic can be described as a dependency of the primary sensitivities of the normal and side force bridges on the sign of the related primary gage loads (see Refs. [1], [8], and [9] for a discussion of this issue). Consequently, assuming that a primary gage output of a force balance is plotted versus the related primary gage load (or vice versa), the resulting data point set may be approximated by two straight lines of different slopes that are joined together near zero load.

Reference [1] recommends a variety of absolute value terms of the loads for regression models of balance outputs that show "bi–directional" behavior. Similarly, because the plots of the primary gage output versus the primary gage load are invertible, one can also use different types of absolute value terms of the outputs in regression models of the loads if an analyst prefers to use the *Non–Iterative Method* for the data analysis. Then, after replacing loads by outputs in the traditional definition of the absolute value terms, we get six additional types of possible regressors that are listed in Table 3 as *Type 6* to *Type 11*.

It is important to point out that not every multi–piece balance design has "bi–directional" behavior. Therefore, absolute value terms should <u>only</u> be used in the regression model of a load if the chosen balance is known to have "bi–directional" bridge outputs. In addition, the absolute value function classes defined in Table 3 cannot represent a constant shift $\rho_\circ$ of the outputs (for that purpose a function like $|\rho_\gamma(k) - \rho_\circ|$ would have to be used instead of $|\rho_\gamma(k)|$). Therefore, output differences relative to the natural zeros of the bridge outputs instead of absolute voltages <u>must</u> <u>always</u> be used to define the calibration data of a balance with "bi–directional" characteristics whenever the data is to be analyzed using the *Non–Iterative Method*.

The definition of regressors associated with state variables is more restrictive. The author suggests to <u>only</u> use state variables as linear or quadratic terms (*Type 2* and *Type 3*). Fortunately, many balances are used near their calibration temperature, have negligible temperature sensitivities, or no bellows. Then, state variables can completely be omitted in the regression model of a load component. Table 4 below summarizes the recommended use of regressors that can be obtained from balance outputs and state variables.

**Table 4:** Recommended use of the regressors for the fit of a load component.

| *Type* | $f_\xi\{\rho_1(k),\ldots,\rho_\nu(k)\}$ | All Balance Types | Bi–directional Balance | State Variables |
|---|---|---|---|---|
| *1* | $1$ | × | × | – |
| *2* | $\rho_\gamma(k)$ | × | × | × |
| *3* | $\rho_\gamma^2(k)$ | × | × | × |
| *4* | $\rho_\gamma(k)\cdot\rho_\delta(k)$ | × | × | – |
| *5* | $\rho_\gamma^3(k)$ | use with caution | use with caution | – |
| *6* | $\lvert\rho_\gamma(k)\rvert$ | – | × | – |
| *7* | $\rho_\gamma(k)\cdot\lvert\rho_\gamma(k)\rvert$ | – | × | – |
| *8* | $\lvert\rho_\gamma^3(k)\rvert$ | – | × | – |
| *9* | $\lvert\rho_\gamma(k)\cdot\rho_\delta(k)\rvert$ | – | × | – |
| *10* | $\rho_\gamma(k)\cdot\lvert\rho_\delta(k)\rvert$ | – | × | – |
| *11* | $\lvert\rho_\gamma(k)\rvert\cdot\rho_\delta(k)$ | – | × | – |

It is important to emphasize that Tables 3 and 4 above list all regressors types that are currently being used in the aerospace testing community for the analysis of strain–gage balance data. Real–world calibration data will often only allow for the use of a small subset of all possible regressors. The final set of regressors has to be selected such that (i) the calibration data itself supports the chosen regressors, (ii) no unwanted linear or near–linear dependencies exist between the regressors, and (iii) the regression model is hierarchical. It is the author's experience that only in that case a robust and reliable regression model of a load component can be obtained that will not suffer from load prediction problems during use in the wind tunnel.

Now, assuming that a suitable subset of all possible regressors for the analysis of the balance calibration data was found, the coefficients $c_{1,i},\ \ldots, c_{\eta,i}$ of the regression model of the load component given Eq. (15) need to be computed. Therefore, a global regression analysis of the calibration data is performed (see Ref. [10] or Ref. [11] for a detailed description of the global regression analysis approach). First, the vector containing the dependent variables, i.e., the loads of the load component with index $i$, needs to be defined. It can be written as follows ...

$$[\,\mathbf{s}_i\,]_{p\times 1} \quad = \quad \begin{bmatrix} \lambda_i(1) \\ \lambda_i(2) \\ \vdots \\ \lambda_i(p) \end{bmatrix} \tag{16a}$$

assuming that the balance calibration data set has a total number of $p$ data points. In the next step, a rectangular matrix containing the regressor values of the chosen math terms has to be assembled. This matrix has $p$ rows and $\eta$ columns. It can be summarized as follows ...

$$[\,\mathbf{R}\,]_{p\times\eta} \quad = \quad \begin{bmatrix} 1 & \rho_1(1) & \rho_2(1) & \ldots & \rho_\nu(1) & \ldots & f_\eta\{\ldots\} \\ 1 & \rho_1(2) & \rho_2(2) & \ldots & \rho_\nu(2) & \ldots & f_\eta\{\ldots\} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & \rho_1(p) & \rho_2(p) & \ldots & \rho_\nu(p) & \ldots & f_\eta\{\ldots\} \end{bmatrix} \tag{16b}$$

assuming that the intercept, linear terms, and a few other terms were chosen for the regression model. It is required that the column vectors of matrix $\mathbf{R}$ are linearly independent or have only moderate near–linear dependencies. Finally, the unknown regression coefficients can be assembled in vector format. Then, we get:

$$[\,\mathbf{C}_i\,]_{\eta\times 1} \quad = \quad \begin{bmatrix} c_{1,i} \\ c_{2,i} \\ \vdots \\ c_{\eta,i} \end{bmatrix} \tag{16c}$$

At this point, the least squares problem associated with the fit of the balance loads as a function of the measured bridge outputs and state variables can be described. We get the following relationship ...

$$[\,\mathbf{R}\,]_{p\times\eta} \;\cdot\; [\,\mathbf{C}_i\,]_{\eta\times 1} \quad = \quad [\,\mathbf{s}_i\,]_{p\times 1} \tag{17}$$

where the elements of the rectangular matrix $\mathbf{R}$ and of the column vector $\mathbf{s}_i$ are known because they were obtained from the loads, outputs, and state variables that were recorded during the balance calibration.

It is shown in the literature that the coefficients of the regression model of the load component with index "$i$" are the solution of the related *Normal Equations* (see Ref. [10] or Ref. [11]). They are defined as follows:

**Normal Equations of a Load Component with Index $i$**

$$[\,\mathbf{R}^{\mathbf{T}}\mathbf{R}\,]_{\eta\times\eta} \;\cdot\; [\,\mathbf{C}_i\,]_{\eta\times 1} \quad = \quad [\,\mathbf{R}^{\mathbf{T}}\,]_{\eta\times p} \;\cdot\; [\,\mathbf{s}_i\,]_{p\times 1} \tag{18}$$

Finally, after multiplying both sides of Eq. (18) with the inverse of the square matrix $\mathbf{R^T R}$, we get the desired least squares solution of the coefficients of the regression model of the load component with index $i$:

**Regression Coefficients of the fitted Load Component with Index $i$**

$$[\,\mathbf{C}_i\,]_{\eta\times 1} \quad = \quad [\,(\mathbf{R^T R})^{-1}\,]_{\eta\times\eta} \;\cdot\; [\,\mathbf{R^T}\,]_{\eta\times p} \;\cdot\; [\,\mathbf{s}_i\,]_{p\times 1} \qquad (19)$$

It is useful to briefly review the solution for the total number of $\eta$ coefficients of the load component with index $i$ that is given in Eq. (19). First, of course, the given solution is only valid for the load component with index $i$ as only its calibration load values are contained in the vector $\mathbf{s}_i$. In addition, the chosen regression model of the load component is hidden within the matrices $\mathbf{R^T}$ and $(\mathbf{R^T R})^{-1}$ which were constructed by using the bridge outputs and state variable values of the calibration as input. Regression coefficients of the other load components are simply obtained by updating (i) the regression model that is hidden within matrices $\mathbf{R^T}$ and $(\mathbf{R^T R})^{-1}$ and (ii) the loads contained in vector $\mathbf{s}_i$.

# Appendix 2: Least Squares Fit of a Bridge Output

Typical strain–gage balance calibration data consists of calibration loads, i.e., forces and moments, and measured bridge outputs that define regression models so that loads can be predicted from measured outputs during a wind tunnel test. In addition, state variables like, for example, temperature or bellows pressure differences, may also have to be taken into account during the balance calibration and regression analysis if they significantly influence load predictions during the wind tunnel test. In Appendix 1 it was illustrated how the *Non–Iterative Method* can be used to develop a regression model for the prediction of balance loads. An alternate analysis approach exists that is called the *Iterative Method* (see Ref. [1]). This approach first fits each bridge output as a function of the loads and state variables. Afterwards, it constructs a load iteration scheme from the regression coefficients of the outputs so that loads can be predicted from outputs during a wind tunnel test. State variables are treated exactly like the loads during the regression analysis, i.e., they are simply used as another set of independent variables. The resulting regression model of an output $\rho_i$ has the following basic structure . . .

**Regression Model $G_i$ of a Bridge Output $\rho_i$**

$$\rho_i(k) \quad = \quad \underbrace{G_i(\lambda_1, \ \ldots \ , \lambda_\nu)}_{regression\ model} \quad = \quad \sum_{\xi=1}^{\mu} \underbrace{d_{\xi,i}}_{coeff.} \cdot \underbrace{g_\xi\{\lambda_1(k), \ \ldots \ , \lambda_\nu(k)\}}_{regressors\ (see\ Table\ 5)} \qquad (20)$$

where

$$1 \ \leq \ i \ \leq \ \nu \quad ; \quad 1 \ \leq \ k \ \leq \ p$$

where $g_\xi$ is a regressor, $d_{\xi,i}$ is the corresponding regression coefficient, and $\lambda_1(k), \ldots, \lambda_\nu(k)$ are the independent variables, i.e., the loads and state variables that define the regressors. The regression model of a bridge output is constructed from different types of math terms (regressors). Table 5 below lists regressors that are often used for the analysis of strain–gage balance data (term definitions were taken from Ref. [1]).

**Table 5:** Regressor choices for the regression analysis of a bridge output.

| *Type* | $g_\xi\{\lambda_1(k), \ldots, \lambda_\nu(k)\}$ | $\gamma$ | $\delta\ ;\ \delta \neq \gamma$ | *Comments* |
|---|---|---|---|---|
| *1* | $1$ | $-$ | $-$ | intercept |
| *2* | $\lambda_\gamma(k)$ | $1 \leq \gamma \leq \nu$ | $-$ | linear |
| *3* | $\lambda_\gamma^2(k)$ | $1 \leq \gamma \leq \nu$ | $-$ | quadratic |
| *4* | $\lambda_\gamma(k) \cdot \lambda_\delta(k)$ | $1 \leq \gamma \leq \nu$ | $1 \leq \delta \leq \nu$ | cross–product |
| *5* | $\lambda_\gamma^3(k)$ | $1 \leq \gamma \leq \nu$ | $-$ | cubic |
| *6* | $\lvert\lambda_\gamma(k)\rvert$ | $1 \leq \gamma \leq \nu$ | $-$ | bi–directional data |
| *7* | $\lambda_\gamma(k) \cdot \lvert\lambda_\gamma(k)\rvert$ | $1 \leq \gamma \leq \nu$ | $-$ | bi–directional data |
| *8* | $\lvert\lambda_\gamma^3(k)\rvert$ | $1 \leq \gamma \leq \nu$ | $-$ | bi–directional data |
| *9* | $\lvert\lambda_\gamma(k) \cdot \lambda_\delta(k)\rvert$ | $1 \leq \gamma \leq \nu$ | $1 \leq \delta \leq \nu$ | bi–directional data |
| *10* | $\lambda_\gamma(k) \cdot \lvert\lambda_\delta(k)\rvert$ | $1 \leq \gamma \leq \nu$ | $1 \leq \delta \leq \nu$ | bi–directional data |
| *11* | $\lvert\lambda_\gamma(k)\rvert \cdot \lambda_\delta(k)$ | $1 \leq \gamma \leq \nu$ | $1 \leq \delta \leq \nu$ | bi–directional data |

In general, the available regressors for the least squares analysis of a bridge output are the intercept

(*Type 1*), linear terms (*Type 2*), and a variety of additional math term types (*Type 3* to *Type 11*). They are briefly reviewed in this appendix.

First, the regressor associated with the intercept (*Type 1*) is discussed. This term makes it possible to numerically represent a non–zero offset in the bridge output. Its regression coefficient equals the least squares approximation of the natural zero, i.e., of the output at zero load, if absolute voltage measurements are used in the balance calibration data set. Now, regressors are discussed that are (i) constructed from the loads and (ii) are suitable for all balance types (single–piece balances, multi–piece balances, canard balances, sting balances, etc.). The linear terms (*Type 2*) fall into this category. In addition, quadratic terms (*Type 3*) and cross–product terms (*Type 4*) may be applied to calibration data of all balance types. Some users of strain–gage balances include third order terms (*Type 5*) in the regression models of the bridge outputs. It is the author's experience that the potential use of third order terms in the regression models of outputs should always carefully be evaluated because their successful application highly depends on design characteristics of the given balance.

It is known in the aerospace testing community that the normal and side force bridge outputs of one widely used family of multi–piece balances show "bi–directional" behavior. This balance design specific characteristic can be described as a dependency of the primary sensitivities of the normal and side force bridges on the sign of the related primary gage loads (see Refs. [1], [8], and [9] for a detailed discussion of this issue). Consequently, assuming that a primary gage output of a force balance is plotted versus the related primary gage load (or vice versa), the resulting data point set may be approximated by two straight lines of different slopes that are joined together near zero load.

Reference [1] recommends to use absolute value terms of the loads for the regression models of bridge outputs that show "bi–directional" behavior. Then, we get six additional types of possible regressors that are listed in Table 5 as *Type 6* to *Type 11.* It is important to point out that not every multi–piece balance design has "bi–directional" behavior. Therefore, absolute value terms should <u>only</u> be used in the regression model of a bridge output if the chosen output is known to have "bi–directional" behavior.

The definition of regressors associated with state variables is more restrictive. The author recommends to <u>only</u> use state variables as linear or quadratic terms (*Type 2* and *Type 3*). Fortunately, many balances are used near their calibration temperature, have negligible temperature sensitivities, or no bellows. Then, state variables can completely be omitted in the regression model of a load component. Table 6 below summarizes the recommended use of the regressors that can be constructed from loads and state variables.

**Table 6:** Recommended use of regressors for the fit of a bridge output.

| *Type* | $g_\xi\{\lambda_1(k), \ldots, \lambda_\nu(k)\}$ | All Balance Types | Bi–directional Balance | State Variables |
|---|---|---|---|---|
| *1* | $1$ | × | × | – |
| *2* | $\lambda_\gamma(k)$ | × | × | × |
| *3* | $\lambda_\gamma^2(k)$ | × | × | × |
| *4* | $\lambda_\gamma(k) \cdot \lambda_\delta(k)$ | × | × | – |
| *5* | $\lambda_\gamma^3(k)$ | use with caution | use with caution | – |
| *6* | $\lvert\lambda_\gamma(k)\rvert$ | – | × | – |
| *7* | $\lambda_\gamma(k) \cdot \lvert\lambda_\gamma(k)\rvert$ | – | × | – |
| *8* | $\lvert\lambda_\gamma^3(k)\rvert$ | – | × | – |
| *9* | $\lvert\lambda_\gamma(k) \cdot \lambda_\delta(k)\rvert$ | – | × | – |
| *10* | $\lambda_\gamma(k) \cdot \lvert\lambda_\delta(k)\rvert$ | – | × | – |
| *11* | $\lvert\lambda_\gamma(k)\rvert \cdot \lambda_\delta(k)$ | – | × | – |

It is important to remember that Tables 5 and 6 above list all regressors types that are currently being used in the aerospace testing community for the analysis of strain–gage balance data. Real–world calibration data will often only allow for the use of a small subset of all possible regressors. The final set of regressors has to be selected such that (i) the calibration data itself supports the chosen regressors, (ii) no unwanted linear or near–linear dependencies exist between the regressors, and (iii) the regression model is hierarchical. It is the author's experience that only in that case a reliable regression model of the bridge output can be obtained that (i) will not suffer from output prediction problems and (ii) can successfully be used to construct the load iteration scheme that the *Iterative Method* uses to predict load from outputs during a wind tunnel test.

Now, assuming that a suitable subset of regressors for the analysis of the balance calibration data was found, the coefficients $d_{1,i},\ \ldots, d_{\mu,i}$ of the regression model of the bridge output given Eq. (20) need to be computed. Therefore, a global regression analysis of the balance calibration data is performed (see Ref. [10] or Ref. [11] for a detailed description of the global regression analysis approach). First, the vector containing the dependent variables, i.e., the electrical outputs of the bridge with index $i$, needs to be defined. It can be written as follows ...

$$[\,\mathbf{r}_i\,]_{p\times 1} \quad = \quad \begin{bmatrix} \rho_i(1) \\ \rho_i(2) \\ \vdots \\ \rho_i(p) \end{bmatrix} \tag{21a}$$

assuming that the balance calibration data set has a total number of $p$ data points. In the next step, a rectangular matrix containing the regressor values of all math terms has to be assembled. This matrix has $p$ rows and $\mu$ columns. It can be summarized as follows ...

$$[\,\mathbf{S}\,]_{p\times\mu} \quad = \quad \begin{bmatrix} 1 & \lambda_1(1) & \lambda_2(1) & \ldots & \lambda_\nu(1) & \ldots & g_\mu\{\ldots\} \\ 1 & \lambda_1(2) & \lambda_2(2) & \ldots & \lambda_\nu(2) & \ldots & g_\mu\{\ldots\} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & \lambda_1(p) & \lambda_2(p) & \ldots & \lambda_\nu(p) & \ldots & g_\mu\{\ldots\} \end{bmatrix} \tag{21b}$$

assuming that, for example, the intercept, linear terms, and a few other terms are supported by the calibration data. It is required that the column vectors of matrix $\mathbf{S}$ are linearly independent or have only moderate near–linear dependencies. Finally, it remains to assemble the unknown regression coefficients in vector format. Then, we get:

$$[\,\mathbf{D}_i\,]_{\mu\times 1} \quad = \quad \begin{bmatrix} d_{1,i} \\ d_{2,i} \\ \vdots \\ d_{\mu,i} \end{bmatrix} \tag{21c}$$

Now, the least squares problem associated with the fit of the bridge outputs as a function of loads and state variables can be described in matrix format. We get the following relationship ...

$$[\,\mathbf{S}\,]_{p\times\mu} \;\cdot\; [\,\mathbf{D}_i\,]_{\mu\times 1} \quad = \quad [\,\mathbf{r}_i\,]_{p\times 1} \tag{22}$$

where the elements of the rectangular matrix $\mathbf{S}$ and the column vector $\mathbf{r}_i$ are known because they were obtained from the loads, electrical outputs, and state variables that were recorded during the calibration.

It is shown in the literature that the regression coefficients of the bridge output with index "$i$" are the solution of the related *Normal Equations* (see Ref. [10] or Ref. [11]). These equations are defined as follows:

**Normal Equations of a <u>Bridge Output</u> with Index $i$**

$$[\,\mathbf{S^T S}\,]_{\mu\times\mu} \;\cdot\; [\,\mathbf{D}_i\,]_{\mu\times 1} \quad = \quad [\,\mathbf{S^T}\,]_{\mu\times p} \;\cdot\; [\,\mathbf{r}_i\,]_{p\times 1} \tag{23}$$

Finally, after multiplying both sides of Eq. (23) with the inverse of the square matrix $\mathbf{S^T S}$, we get the desired least squares solution of the coefficients of the regression model of the bridge output with index $i$:

**Regression Coefficients of the fitted Bridge Output with Index $i$**

$$[\,\mathbf{D}_i\,]_{\mu\times 1} \quad = \quad [\,(\mathbf{S^T S})^{\mathbf{-1}}\,]_{\mu\times\mu} \;\cdot\; [\,\mathbf{S^T}\,]_{\mu\times p} \;\cdot\; [\,\mathbf{r}_i\,]_{p\times 1} \qquad (24)$$

It is useful to review the solution for the total number of $\mu$ coefficients of the bridge output with index $i$ that is given in Eq. (24). First, of course, the solution is only valid for the bridge output with index $i$ as only its calibration outputs are contained in the vector $\mathbf{r}_i$. In addition, the selected regression model of the bridge output is hidden within the matrices $\mathbf{S^T}$ and $(\mathbf{S^T S})^{\mathbf{-1}}$ which were constructed by using the loads and state variable values of the calibration as input. Regression coefficients of other bridge outputs are simply obtained by updating (i) the regression model that is hidden within matrices $\mathbf{S^T}$ and $(\mathbf{S^T S})^{\mathbf{-1}}$ and (ii) the outputs contained in vector $\mathbf{r}_i$.

## Appendix 3: Relationship between Regression Coefficient Sets

The calibration data set of a balance consists of loads and outputs that were obtained in a calibration laboratory. This data set contains a certain amount of information about the physical characteristics of the balance that may be used to construct regression models for the prediction of balance loads from measured outputs during a wind tunnel test. Two fundamentally different methods, i.e., the *Non–Iterative* and the *Iterative Method*, are used in the aerospace testing community to get regression models that may be used for the balance load prediction during a wind tunnel test. The first approach directly fits loads as a function of bridge outputs (see App. 1). The second approach, on the other hand, switches the independent and dependent variables of the first approach. Therefore, it fits the bridge outputs as a function of the loads and afterwards constructs a load iteration scheme from the result so that loads can be predicted from outputs during a wind tunnel test (see App. 2).

Superficially viewed, no direct relationship between the regression coefficients of the *Non–Iterative* and the *Iterative Method* seems to exist. However, the required "unique" mapping between the "load space" and the "output space" suggests that, under certain circumstances, a direct analytic relationship between the regression models of both methods must exist. In other words, it must be possible for some special cases to derive a relationship that relates the regression coefficients of the load component $\lambda_i$ that are defined in Eq. (19) to the regression coefficients of the bridge output $\rho_i$ that are defined in Eq. (24). This relationship can be obtained, for example, if the following two assumptions are made:

**Assumption 1:** Only the "$n$" linear terms associated with the bridge outputs (i.e., $\rho_1, \ldots, \rho_n$) are a part of the regression model of the selected load component $\lambda_i$ with index $i$ (the intercept, quadratic, cubic, absolute value, cross–product terms, and state variables are omitted in the regression model).

**Assumption 2:** Only the "$n$" linear terms associated with the loads (i.e., $\lambda_1, \ldots, \lambda_n$) are a part of the regression model of the selected bridge output $\rho_i$ with index $i$ (the intercept, quadratic, cubic, absolute value, cross–product terms, and state variables are omitted in the regression model).

The two assumptions above result in a significant simplification of the indices that are used in Eq. (15) and Eq. (20). We get the following relationship:

$$n \quad = \quad \nu \quad = \quad \eta \quad = \quad \mu \tag{25}$$

The starting point of the derivation of a relationship between the coefficients sets of the *Non–Iterative* and *Iterative Method* are the *Normal Equations* that are given Eqs. (18) and (23). Then, after replacing the indices $\eta$ and $\mu$ in Eqs. (18) and (23) by index $n$, we get the following two matrix equations:

$$Eq.\ (18) \implies [\,\mathbf{R^T R}\,]_{n\times n} \cdot [\,\mathbf{C}_i\,]_{n\times 1} = [\,\mathbf{R^T}\,]_{n\times p} \cdot [\,\mathbf{s}_i\,]_{p\times 1} \quad ; \quad 1 \leq i \leq n \tag{26a}$$

$$Eq.\ (23) \implies [\,\mathbf{S^T S}\,]_{n\times n} \cdot [\,\mathbf{D}_i\,]_{n\times 1} = [\,\mathbf{S^T}\,]_{n\times p} \cdot [\,\mathbf{r}_i\,]_{p\times 1} \quad ; \quad 1 \leq i \leq n \tag{26b}$$

The two matrix equations define the *Normal Equations* of an applied load component or a measured bridge output with index $i$. The related regression coefficients are stored in column vectors $\mathbf{C}_i$ and $\mathbf{D}_i$ that have $n$ rows each. It is possible to express the normal equations in a more general format if the regression coefficients of all load components and bridge outputs are assembled in matrix format. Then, after considering the definition of column vector $\mathbf{C}_i$ given in Eq. (16$c$), the regression coefficients of all load components can be described in matrix format as follows:

$$[\,\mathbf{C}\,]_{n\times n} = [\ \mathbf{C}_1\ \ \mathbf{C}_2\ \ \ldots\ \ \mathbf{C}_i\ \ \ldots\ \ \mathbf{C}_n\ ]_{n\times n} = \begin{bmatrix} c_{1,1} & c_{1,2} & \ldots & c_{1,i} & \ldots & c_{1,n} \\ c_{2,1} & c_{2,2} & \ldots & c_{2,i} & \ldots & c_{1,n} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ c_{n,1} & c_{n,2} & \ldots & c_{n,i} & \ldots & c_{1,n} \end{bmatrix} \tag{27a}$$

Similarly, after considering the definition of column vector $\mathbf{D}_i$ given in Eq. (21$c$), the regression coefficients of all measured bridge outputs can be described in matrix format as follows:

$$[\,\mathbf{D}\,]_{n\times n} = [\ \mathbf{D}_1\ \ \mathbf{D}_2\ \ \ldots\ \ \mathbf{D}_i\ \ \ldots\ \ \mathbf{D}_n\ ]_{n\times n} = \begin{bmatrix} d_{1,1} & d_{1,2} & \ldots & d_{1,i} & \ldots & d_{1,n} \\ d_{2,1} & d_{2,2} & \ldots & d_{2,i} & \ldots & d_{1,n} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ d_{n,1} & d_{n,2} & \ldots & d_{n,i} & \ldots & d_{1,n} \end{bmatrix} \tag{27b}$$

It is also necessary to extend the column vectors $\mathbf{s}_i$ and $\mathbf{r}_i$ on the right–hand sides of Eqs. (26$a$) and (26$b$) to matrix format. Then, using the definition of vector $\mathbf{s}_i$ given in Eq. (16$a$) in combination with the definition of matrix $\mathbf{S}$ given in Eq. (21$b$) and knowing that only linear terms are used in the regression model of the bridge outputs, we get the following result:

$$[\,\mathbf{S}\,]_{p\times n} \quad = \quad [\ \mathbf{s}_1 \ \ \mathbf{s}_2 \ \ \ldots \ \ \mathbf{s}_i \ \ \ldots \ \ \mathbf{s}_n\ ]_{p\times n} \quad = \quad \begin{bmatrix} \lambda_1(1) & \lambda_2(1) & \ldots & \lambda_i(1) & \ldots & \lambda_n(1) \\ \lambda_1(2) & \lambda_2(2) & \ldots & \lambda_i(2) & \ldots & \lambda_n(2) \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \lambda_1(p) & \lambda_2(p) & \ldots & \lambda_i(p) & \ldots & \lambda_n(p) \end{bmatrix} \tag{28$a$}$$

Similarly, using the definition of vector $\mathbf{r}_i$ given in Eq. (21$a$) in combination with the definition of matrix $\mathbf{R}$ given in Eq. (16$b$) and knowing that only linear terms are used in the regression model of the load components, we get the following result:

$$[\,\mathbf{R}\,]_{p\times n} \quad = \quad [\ \mathbf{r}_1 \ \ \mathbf{r}_2 \ \ \ldots \ \ \mathbf{r}_i \ \ \ldots \ \ \mathbf{r}_n\ ]_{p\times n} \quad = \quad \begin{bmatrix} \rho_1(1) & \rho_2(1) & \ldots & \rho_i(1) & \ldots & \rho_n(1) \\ \rho_1(2) & \rho_2(2) & \ldots & \rho_i(2) & \ldots & \rho_n(2) \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \rho_1(p) & \rho_2(p) & \ldots & \rho_i(p) & \ldots & \rho_n(p) \end{bmatrix} \tag{28$b$}$$

Finally, after replacing the four column vectors $\mathbf{C}_i$, $\mathbf{D}_i$, $\mathbf{s}_i$ and $\mathbf{r}_i$ by the matrices $\mathbf{C}$, $\mathbf{D}$, $\mathbf{S}$ and $\mathbf{R}$, the *Normal Equations* given in Eqs. (26$a$) and (26$b$) can be written in an expanded format as follows:

$$[\,\mathbf{R^T R}\,]_{n\times n} \cdot [\,\mathbf{C}\,]_{n\times n} \quad = \quad [\,\mathbf{R^T}\,]_{n\times p} \cdot [\,\mathbf{S}\,]_{p\times n} \tag{29$a$}$$

$$[\,\mathbf{S^T S}\,]_{n\times n} \cdot [\,\mathbf{D}\,]_{n\times n} \quad = \quad [\,\mathbf{S^T}\,]_{n\times p} \cdot [\,\mathbf{R}\,]_{p\times n} \tag{29$b$}$$

Now, it is possible to directly connect the regression coefficients of the load components to the regression coefficients of the measured bridge outputs. This goal can be achieved in several steps. First, both sides of Eq. (29$b$) are transposed. Then, we get:

$$\Big\{\, [\,\mathbf{S^T S}\,]_{n\times n} \cdot [\,\mathbf{D}\,]_{n\times n} \,\Big\}^{\mathbf{T}} \quad = \quad \Big\{\, [\,\mathbf{S^T}\,]_{n\times p} \cdot [\,\mathbf{R}\,]_{p\times n} \,\Big\}^{\mathbf{T}} \tag{30}$$

In the next step, Eq. (30) is simplified by applying two fundamental matrix operator rules that are listed in the literature (Ref. [12], p. 334, $(\mathbf{AB})^{\mathbf{T}} = \mathbf{B^T A^T}$ and $(\mathbf{A^T})^{\mathbf{T}} = \mathbf{A}$):

$$[\,\mathbf{D^T}\,]_{n\times n} \cdot [\,\mathbf{S^T S}\,]_{n\times n} \quad = \quad [\,\mathbf{R^T}\,]_{n\times p} \cdot [\,\mathbf{S}\,]_{p\times n} \tag{31}$$

It can be concluded by visual inspection that the right–hand side of Eq. (31) equals the left–hand side of Eq. (29$a$). Then, after replacing the right–hand side of Eq. (31) by the left–hand side of Eq. (29$a$), we get the following result:

$$[\,\mathbf{D^T}\,]_{n\times n} \cdot [\,\mathbf{S^T S}\,]_{n\times n} \quad = \quad [\,\mathbf{R^T R}\,]_{n\times n} \cdot [\,\mathbf{C}\,]_{n\times n} \tag{32}$$

Now, it is finally possible to solve Eq. (32) for the regression coefficients of all fitted load components $\lambda_1, \ldots, \lambda_n$ that are contained in matrix $\mathbf{C}$. It is simply required to multiply both sides of Eq. (32) with the inverse of the square matrix $\mathbf{R^T R}$. Then, we get:

**Linear Transformation between Regression Coefficient Sets (Version 1)**

$$\underbrace{[\,\mathbf{C}\,]_{n\times n}}_{\textit{Set 1}} \quad = \quad \underbrace{[\,(\mathbf{R^T R})^{-1}\,]_{n\times n}}_{\textit{output dependent}} \cdot \underbrace{[\,\mathbf{D^T}\,]_{n\times n}}_{\textit{Set 2}} \cdot \underbrace{[\,\mathbf{S^T S}\,]_{n\times n}}_{\textit{load dependent}} \tag{33}$$

*Set 1* $\Longrightarrow$ regression coefficients of all fitted load components (used by *Non–Iterative Method*)
*Set 2* $\Longrightarrow$ regression coefficients of all fitted bridge outputs (used by *Iterative Method*)

Equation (33) is the first version of the linear transformation between the regression coefficients of the fitted loads (Set 1) and the regression coefficients of the fitted bridge outputs (Set 2). It is valid for the special case when only linear terms of the load components and bridge outputs are used for the least squares fit of the balance calibration data.

It remains to derive the alternate relationship to Eq. (33) that transforms the regression coefficients associated with the *Non–Iterative Method* to the regression coefficients that are associated with the *Iterative Method*. The derivation of this alternate transformation starts by transposing both sides of Eq. (32). Then, Eq. (32) becomes ...

$$\left\{ \, [\,\mathbf{D^T}\,]_{n \times n} \, \cdot \, [\,\mathbf{S^T S}\,]_{n \times n} \, \right\}^{\mathbf{T}} \; = \; \left\{ \, [\,\mathbf{R^T R}\,]_{n \times n} \, \cdot \, [\,\mathbf{C}\,]_{n \times n} \, \right\}^{\mathbf{T}} \tag{34}$$

In the next step, Eq. (34) is simplified by applying two fundamental matrix operator rules that are listed in the literature (Ref. [12], p. 334, $(\mathbf{AB})^{\mathbf{T}} = \mathbf{B^T A^T}$ and $(\mathbf{A^T})^{\mathbf{T}} = \mathbf{A}$):

$$[\,\mathbf{S^T S}\,]_{n \times n} \, \cdot \, [\,\mathbf{D}\,]_{n \times n} \; = \; [\,\mathbf{C^T}\,]_{n \times n} \, \cdot \, [\,\mathbf{R^T R}\,]_{n \times n} \tag{35}$$

Now, it is possible to solve Eq. (35) for the regression coefficients of all fitted bridge outputs $\rho_1, \ldots, \rho_n$ that are contained in matrix $\mathbf{D}$. It is simply required to multiply both sides of Eq. (35) with the inverse of the square matrix $\mathbf{S^T S}$. Then, we get:

**Linear Transformation between Regression Coefficient Sets (Version 2)**

$$\underbrace{[\,\mathbf{D}\,]_{n \times n}}_{Set\ 2} \; = \; \underbrace{[\,(\mathbf{S^T S})^{-\mathbf{1}}\,]_{n \times n}}_{load\ dependent} \cdot \underbrace{[\,\mathbf{C^T}\,]_{n \times n}}_{Set\ 1} \cdot \underbrace{[\,\mathbf{R^T R}\,]_{n \times n}}_{output\ dependent} \tag{36}$$

*Set 1* $\Longrightarrow$ regression coefficients of all fitted load components (used by *Non–Iterative Method*)
*Set 2* $\Longrightarrow$ regression coefficients of all fitted bridge outputs (used by *Iterative Method*)

Equation (36) is the second version of the linear transformation between the regression coefficients of the fitted loads (Set 1) and the regression coefficients of the fitted bridge outputs (Set 2). It is valid for the special case when only linear terms of the load components and bridge outputs are used for the least squares fit of the balance calibration data.

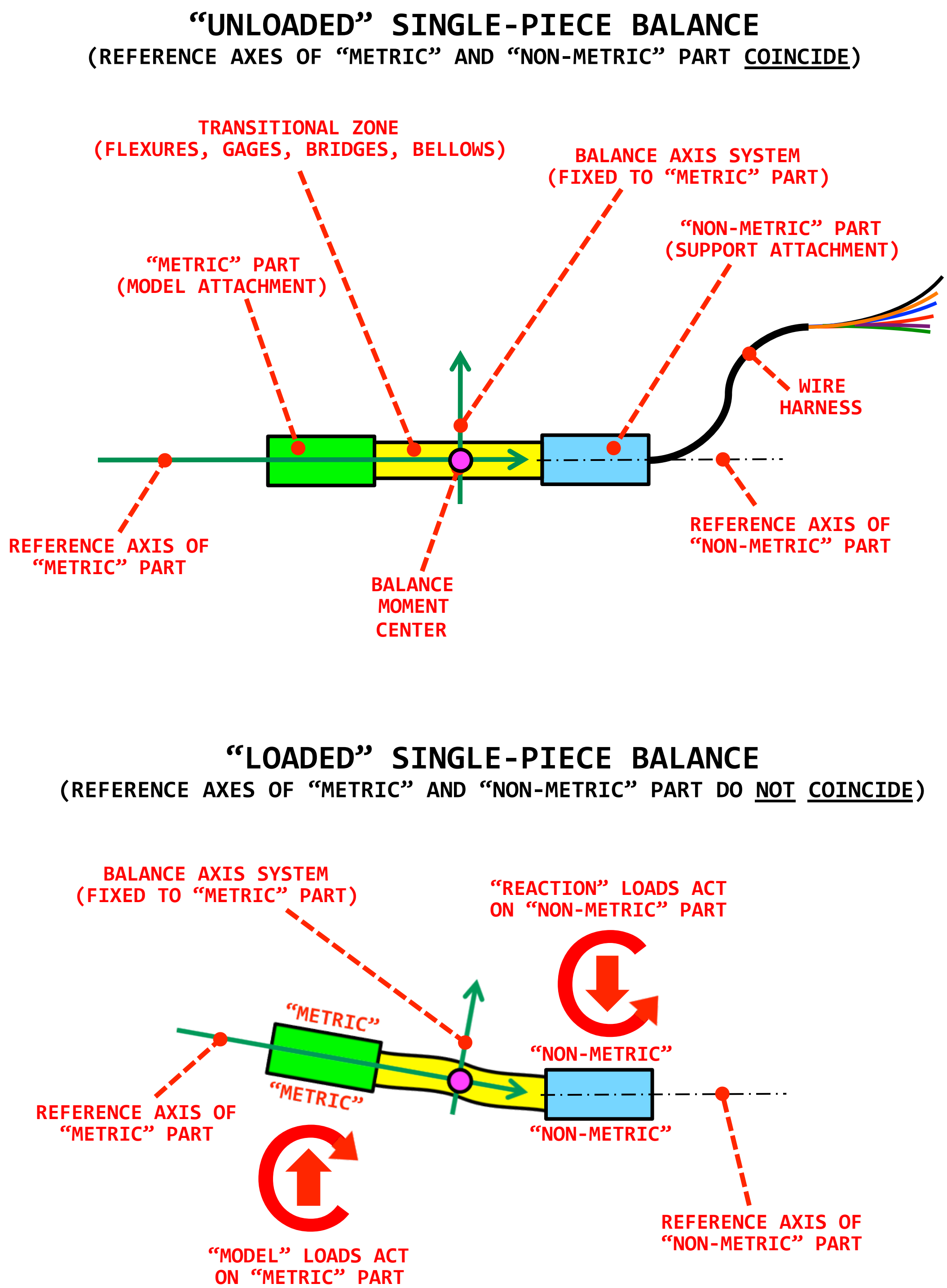


**Fig. 1** Simplified description of an “unloaded” and “loaded” single–piece balance.

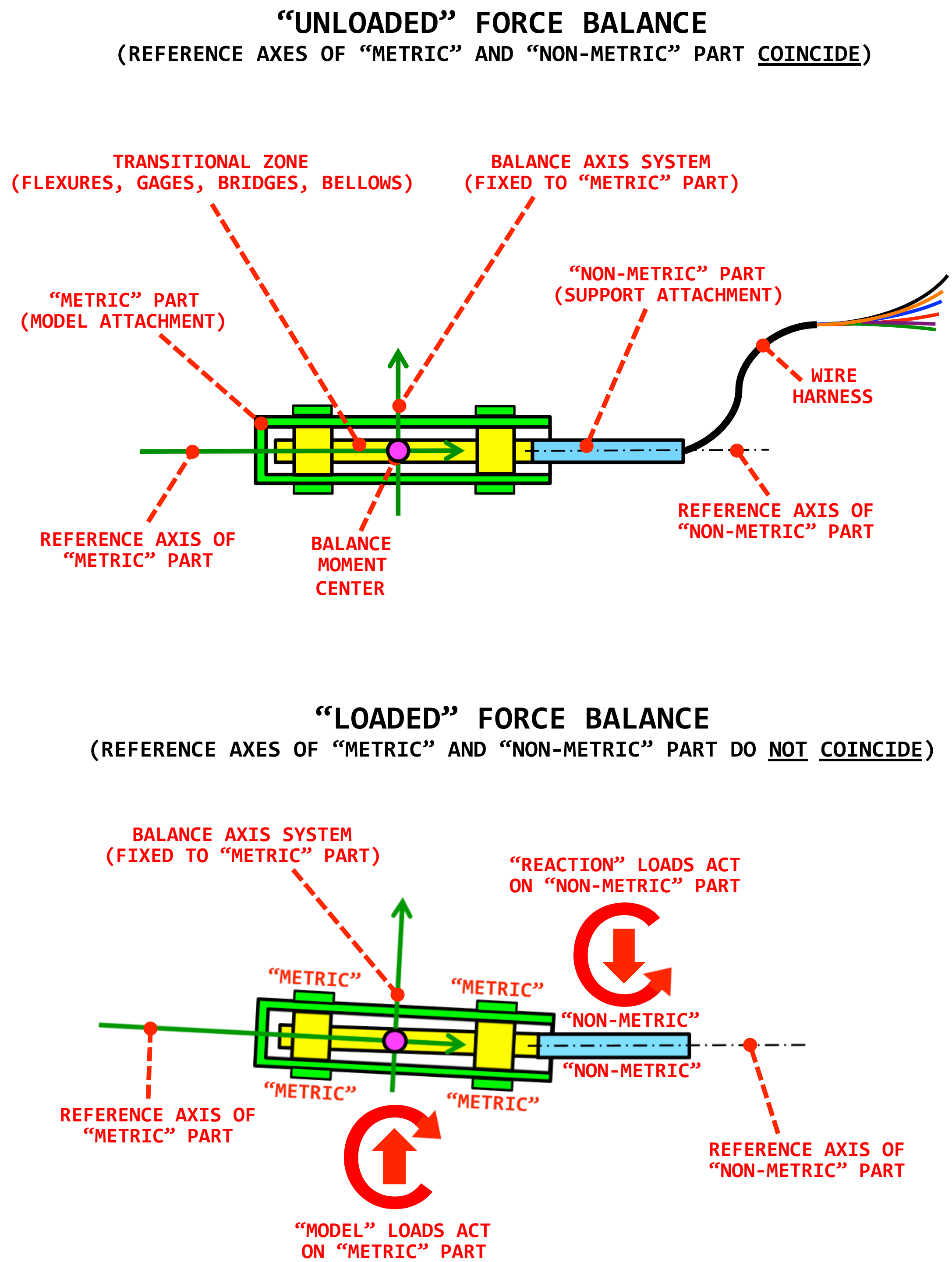


**Fig. 2** Simplified description of an "unloaded" and "loaded" multi–piece force balance.

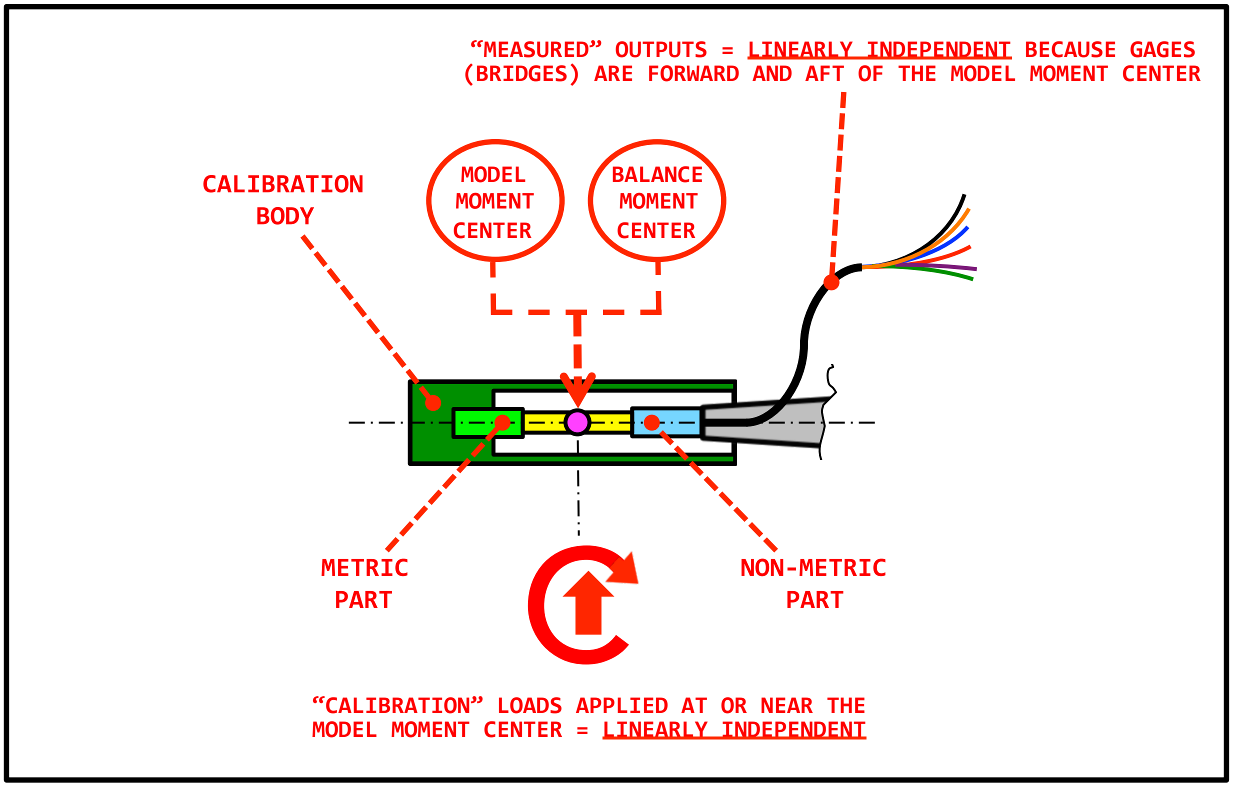


**Fig. 3a** Situation 1: All gages (bridges) of the balance are “near” and both forward & aft of the model moment center.

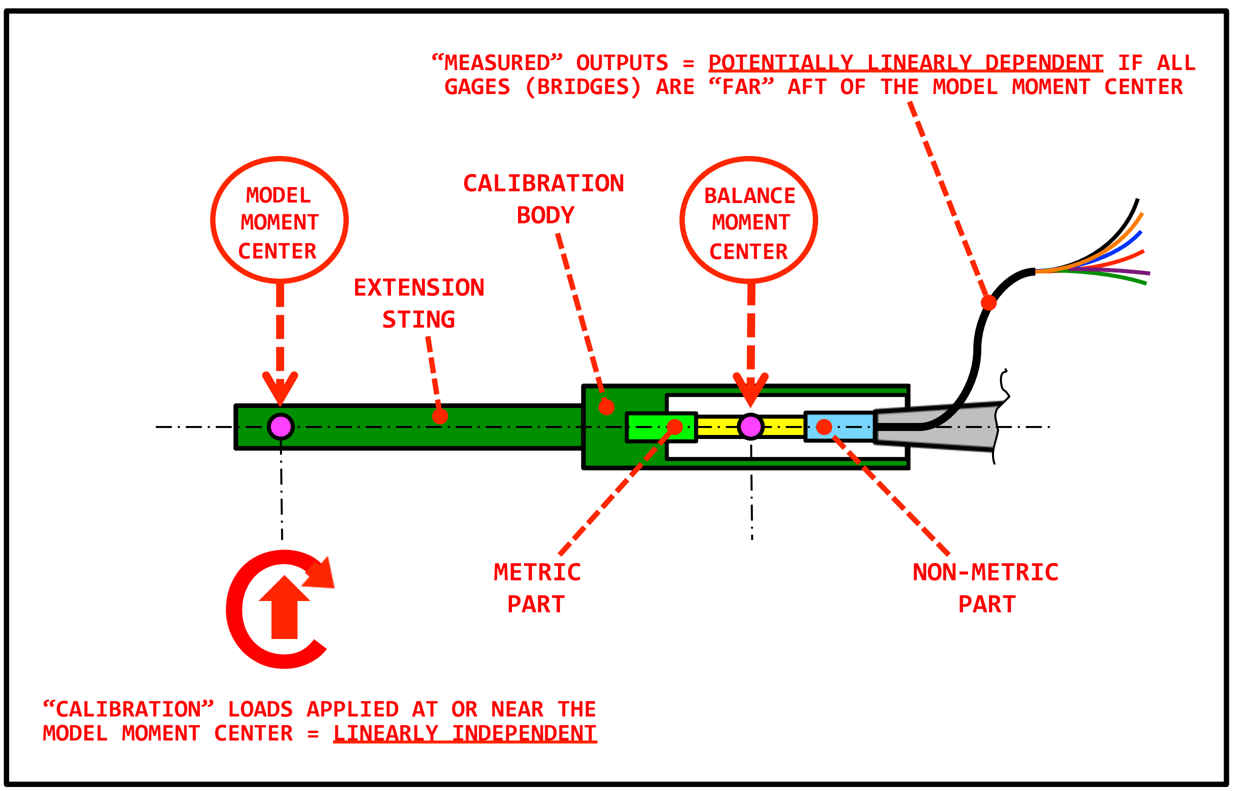


**Fig. 3b** Situation 2: All gages (bridges) of the balance are “far” aft from the model moment center.

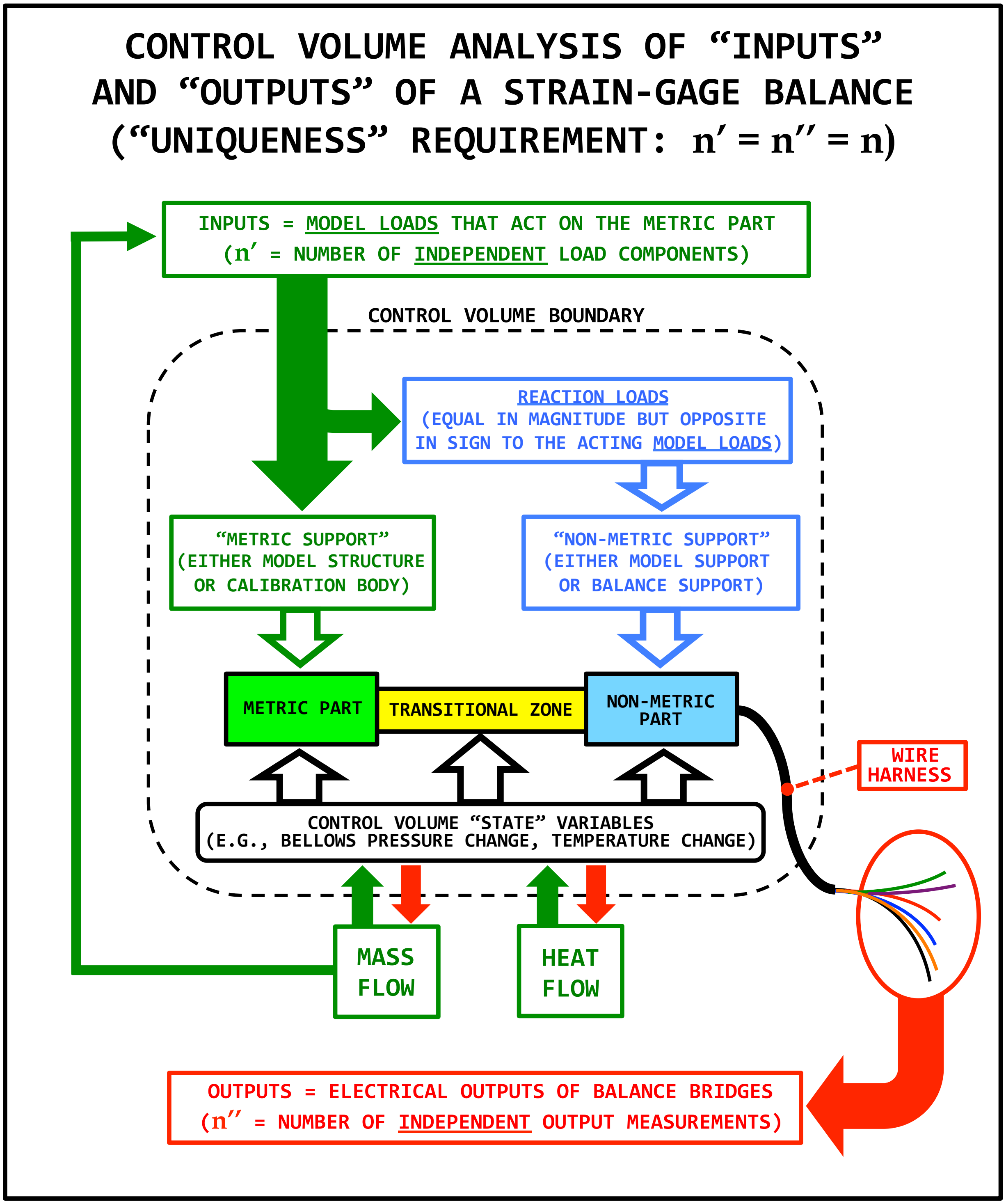


**Fig. 4** Control volume analysis of “inputs” and “outputs” of a strain–gage balance.

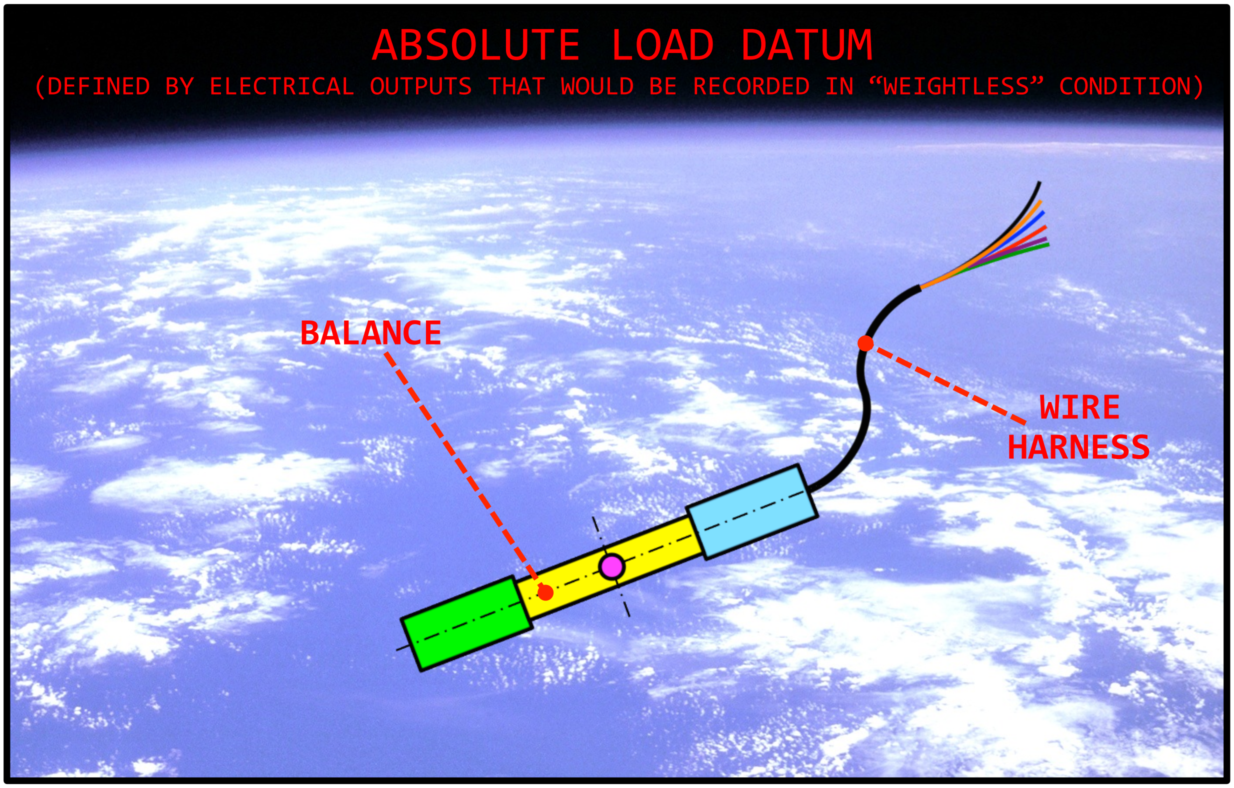


**Fig. 5a** Absolute load datum of a strain–gage balance.

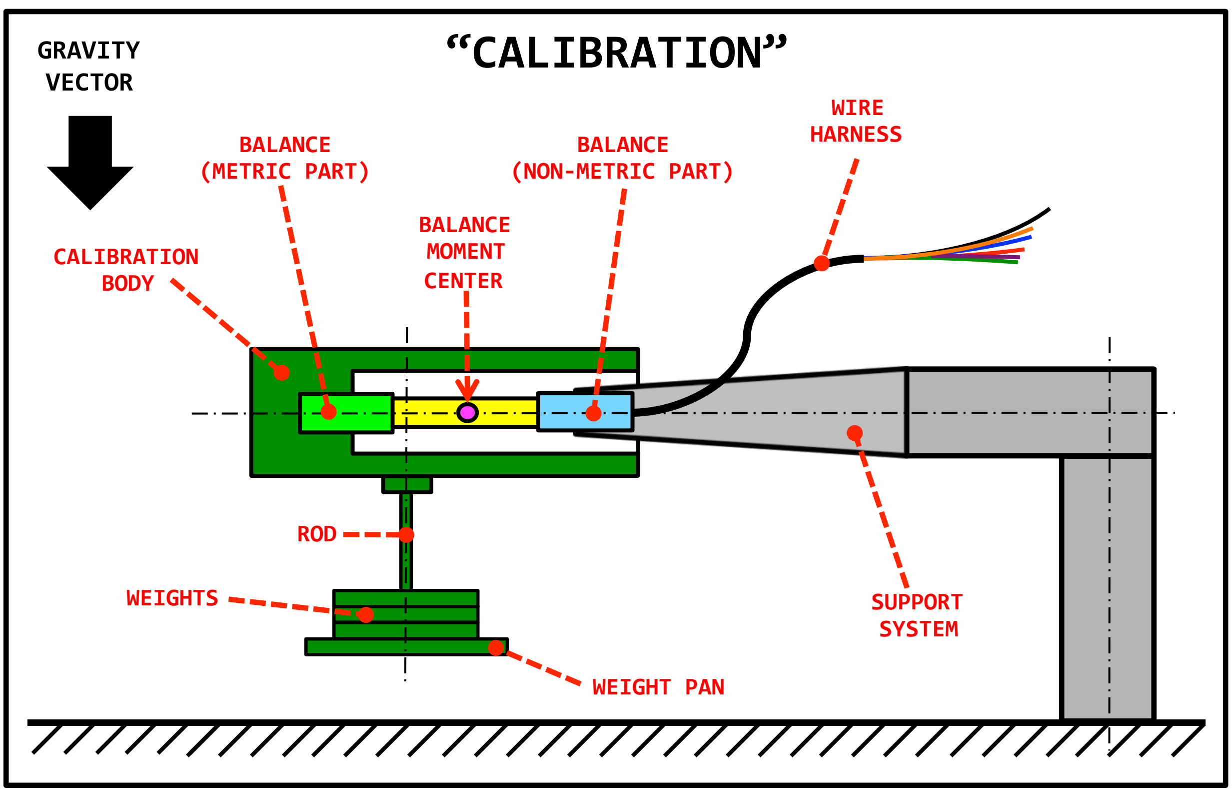


**Fig. 5b** Calibration of a strain–gage balance (model structure = calibration body, rod, weight pan, weights).

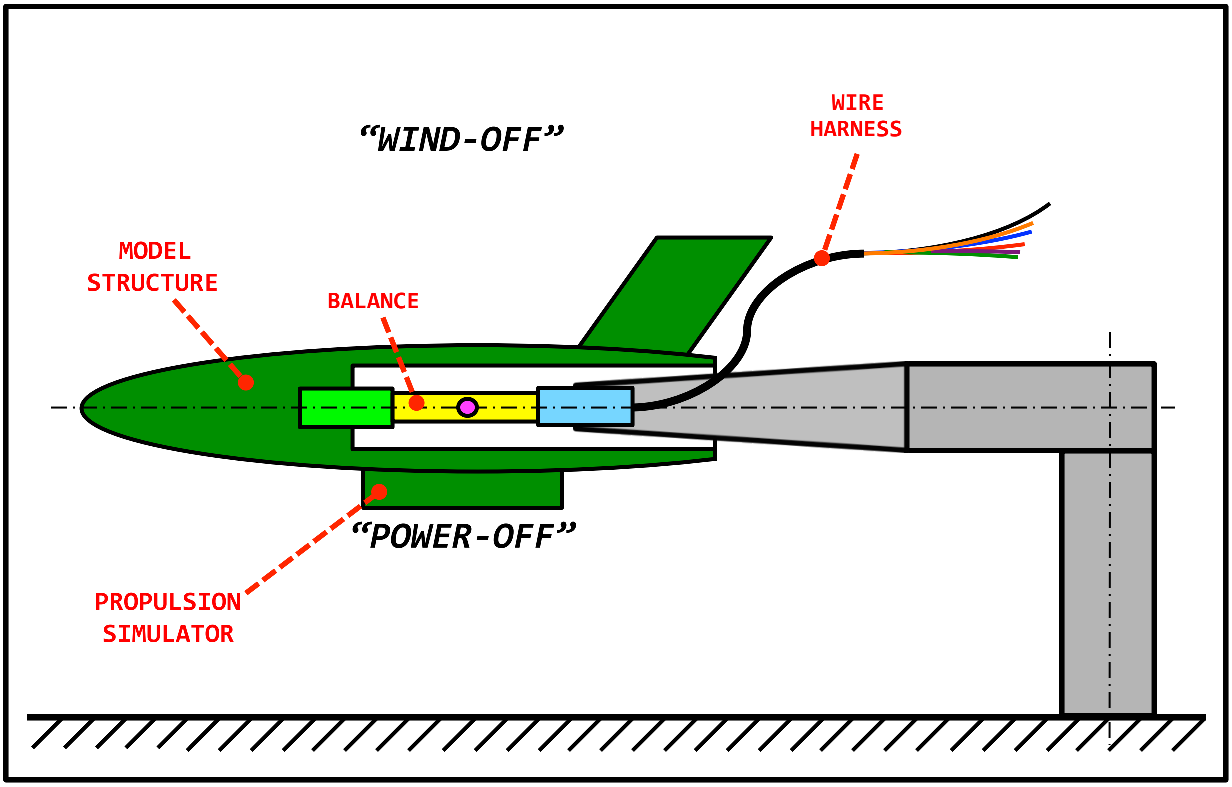


**Fig. 6a** Wind tunnel test case 1 (configuration = wind–off, power–off).

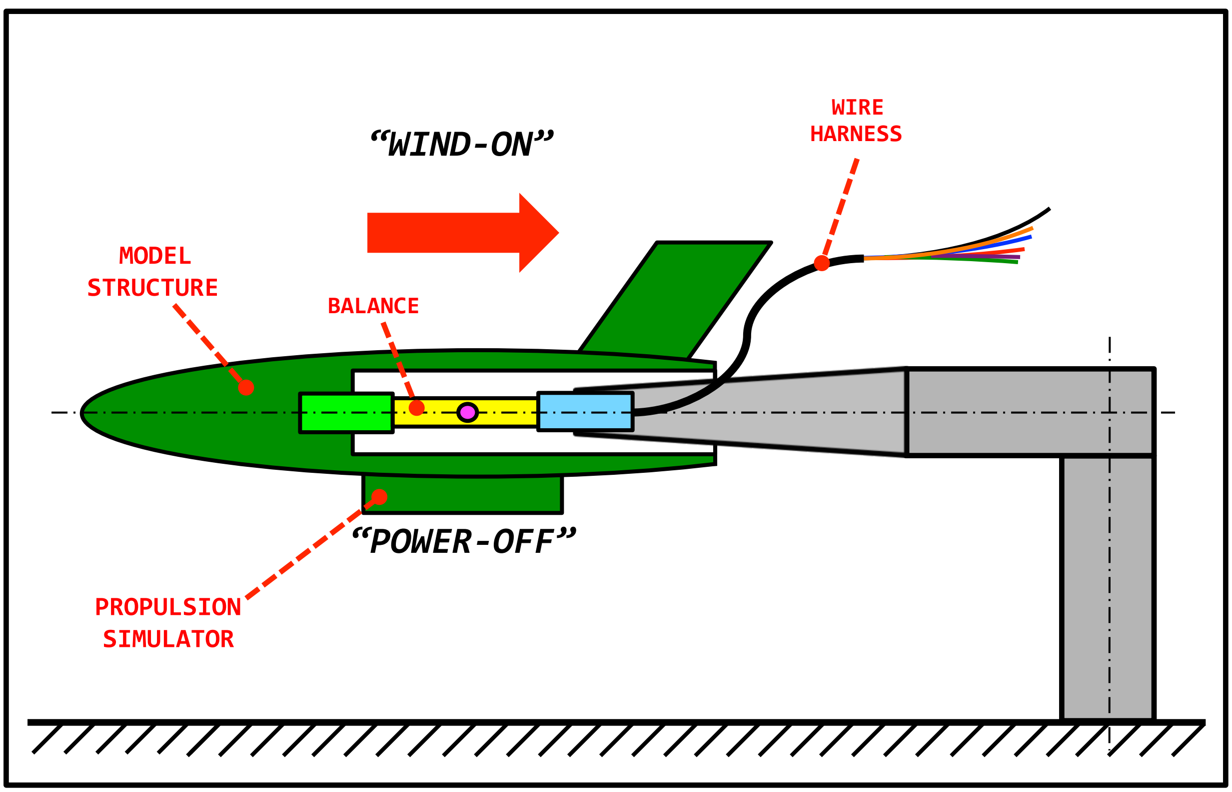


**Fig. 6b** Wind tunnel test case 2 (configuration = wind–on, power–off).

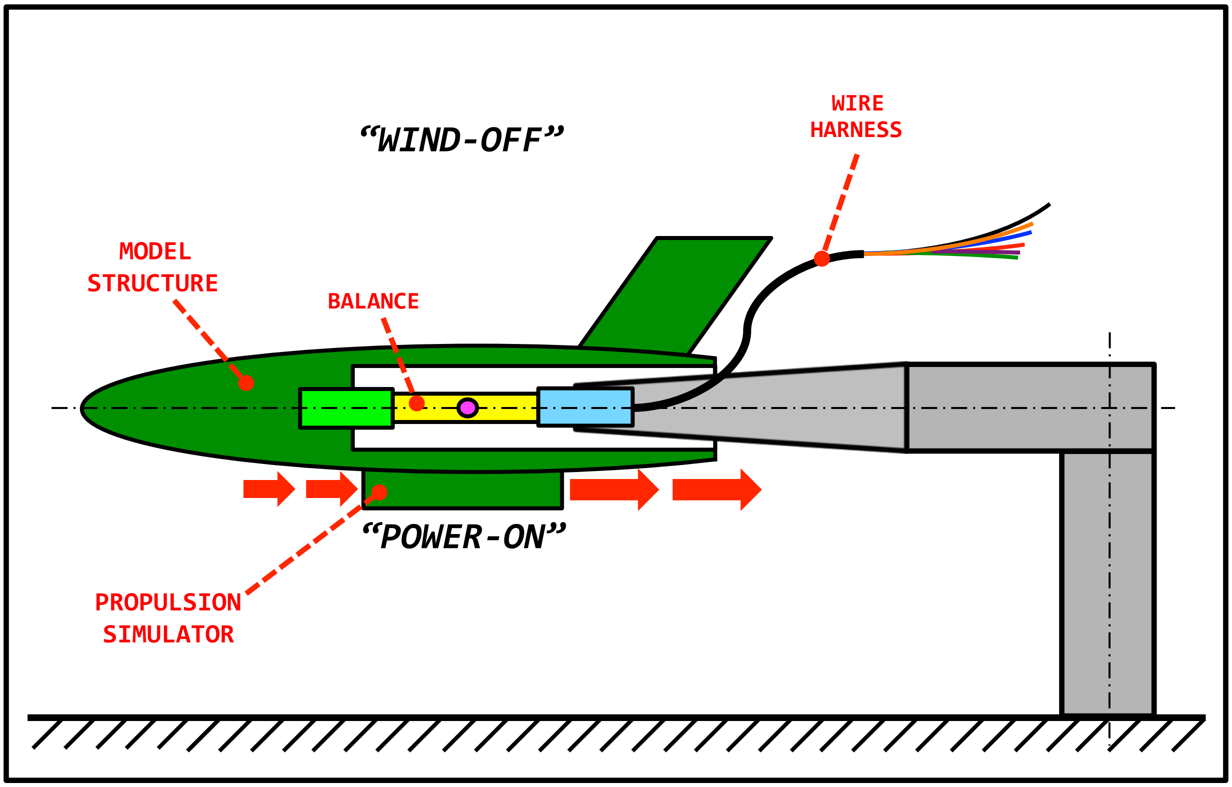


**Fig. 6c** Wind tunnel test case 3 (configuration = wind–off, power–on).

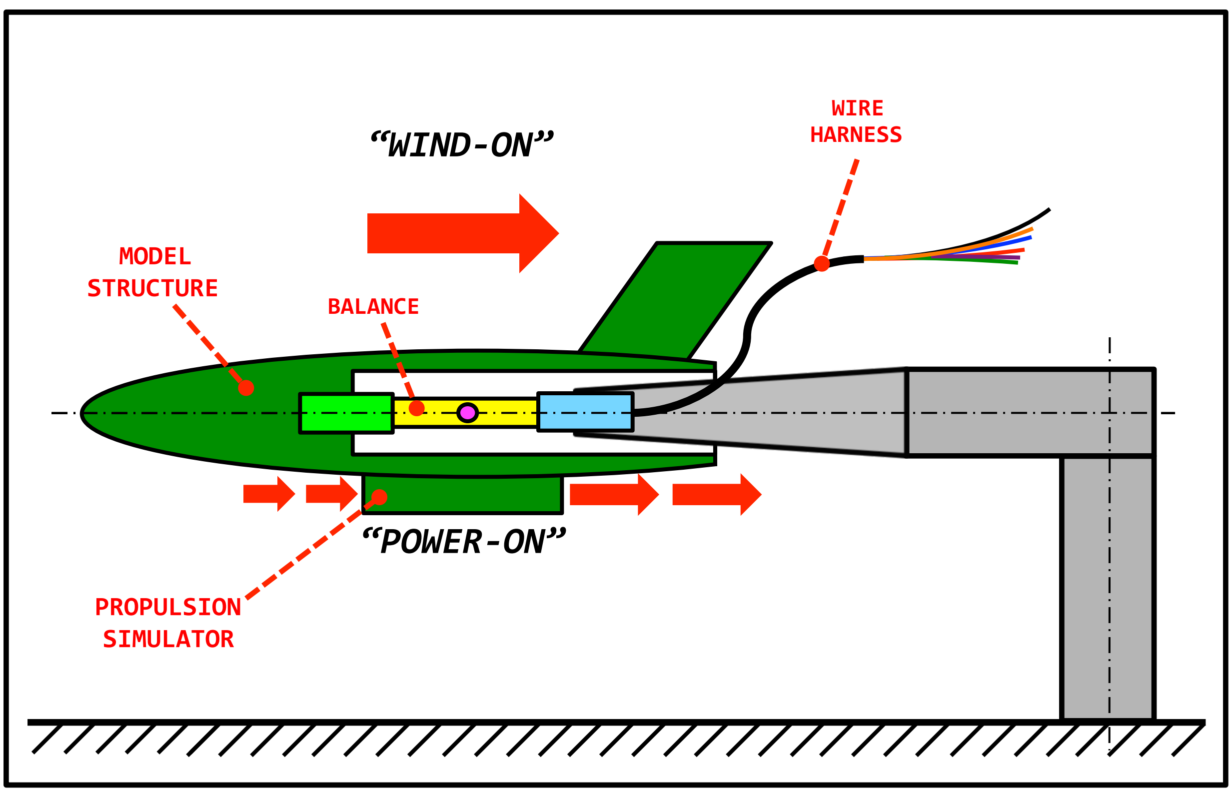


**Fig. 6d** Wind tunnel test case 4 (configuration = wind–on, power–on).

$$\text{MATH MODEL} = \eta_1\ N1 + \eta_2\ N2 + \eta_3\ S1 + \eta_4\ S2 + \eta_5\ RM + \eta_6\ AF$$

| VARIABLE INDEX | VARIABLE NAME | VARIANCE INFLATION FACTOR |
|---|---|---|
| 1 | N1 | +1.0049 |
| 2 | N2 | +1.0050 |
| 3 | S1 | +1.2542 |
| 4 | S2 | +1.2542 |
| 5 | RM | +1.0001 |
| 6 | AF | +1.0000 |

**Fig. 7a** Test result for the load component set of the original MC60E calibration data.

$$\text{MATH MODEL} = \mu_1\ rN1 + \mu_2\ rN2 + \mu_3\ rS1 + \mu_4\ rS2 + \mu_5\ rRM + \mu_6\ rAF$$

| VARIABLE INDEX | VARIABLE NAME | VARIANCE INFLATION FACTOR |
|---|---|---|
| 1 | rN1 | +1.1091 |
| 2 | rN2 | +1.0672 |
| 3 | rS1 | +1.3358 |
| 4 | rS2 | +1.4469 |
| 5 | rRM | +1.1145 |
| 6 | rAF | +1.0105 |

**Fig. 7b** Test results for the bridge output set of the original MC60E calibration data.

$$\text{MATH MODEL} = \eta_1\ N1 + \eta_2\ N2 + \eta_3\ S1 + \eta_4\ S2 + \eta_5\ RM + \eta_6\ AF$$

| VARIABLE INDEX | VARIABLE NAME | VARIANCE INFLATION FACTOR |
|---|---|---|
| 1 | N1 | +1.0049 |
| 2 | N2 | +1.0050 |
| 3 | S1 | +1.2542 |
| 4 | S2 | +1.2542 |
| 5 | RM | +1.0001 |
| 6 | AF | +1.0000 |

**Fig. 8a** Test result for the load component set of the modified MC60E calibration data.

$$\text{MATH MODEL} = \mu_1\ rN1 + \mu_2\ rN2 + \mu_3\ rS1 + \mu_4\ rS2 + \mu_5\ rRM + \mu_6\ rAF$$

| VARIABLE INDEX | VARIABLE NAME | VARIANCE INFLATION FACTOR |
|---|---|---|
| 1 | rN1 | +986.9050 |
| 2 | rN2 | +1201.8668 |
| 3 | rS1 | +1.3363 |
| 4 | rS2 | +1.4442 |
| 5 | rRM | +1.1129 |
| 6 | rAF | +2458.4860 |

**Fig. 8b** Test results for the bridge output set of the modified MC60E calibration data.