Wind Tunnel Balance Calibration: *Are 1,000,000 Data Points Enough?*

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Measurement systems are typically calibrated based on standard practices established by a metrology standards laboratory, for example the National Institute for Standards and Technology (NIST), or dictated by an organization's metrology manual. Therefore, the calibration is designed and executed according to an established procedure.

However, for many aerodynamic research measurement systems a universally accepted standard, traceable approach does not exist. Therefore, a strategy for how to develop a calibration protocol is left to the developer or user to define based on experience and recommended practice in their respective industry. Wind tunnel balances are one such measurement system. Many different calibration systems, load schedules and procedures have been developed for balances with little consensus on a recommended approach. Especially lacking is guidance the number of calibration data points needed. Regrettably, the number of data points tends to be correlated with the perceived quality of the calibration. Often, the number of data points is associated with ones ability to generate the data rather than by a defined need in support of measurement objectives. Hence the title of the paper was conceived to challenge recent observations in the wind tunnel balance community that shows an ever increasing desire for more data points per calibration absent of guidance to determine when there are enough.

This paper presents fundamental philosophy of balance calibration to aid in the development of calibration procedures and provides a framework that is generally applicable to the characterization and calibration of other measurement systems. Questions that need to be answered are for example: What constitutes an adequate calibration?, How much data are needed in the calibration?, How good is the calibration? This paper will assist a practitioner in answering these questions determining how to evaluate a calibration based on objective measures. This will enable the developer and user to design calibrations to meet the user's objectives and a basis for comparing existing calibrations that may have been developed in an ad-hoc manner.

I. Introduction

In recent years, a portion of the balance literature reflects a persistent growth in the number of design points, reaching into the thousands of points. For example, Reference 1 proposes a design with 2082 points. In contrast, there have been impressive advancements in the use of statistical design of experiments and response surface methods for balance calibration that enables the specification of an adequate design size, which are usually less than 100 points. For example, Reference 2 proposes a design based on these techniques with 65 points. We find it fascinating that there appears to be two diverging perspectives on design size, and thus our title, which playfully anticipates the proposal of a 1,000,000 point design in the near future. Also in the literature, there is much discussion on calibration systems, whether automated or manual, and it appears that the choice of the system impacts the calibration design size. We consider it ill advised when the simple fact that one can get the data drives the decisions on what data are obtained rather than need for the data to meet clear objectives. As in many fields, this trend of faster more available data usually quickly leads to frustration over data management challenges, and most importantly a loss of interpretability that comes with the shear volume of data. In this paper, we endeavor to avoid the discussion of calibration systems and focus solely on the calibration design process that can be adapted for any hardware system.

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The design and production of a state-of-the-art balance requires structural analysis, fabrication, instrumentation, and calibration. In each of these phases there is a design or strategy developed, whether the method of structural analysis or the instrumentation configuration. Often, the word design is associated with mechanical physical systems, however our usage of design refers to the translation of abstract ideas and purposes into a plan of action. For example, initial balance design activities require the interpretation of the aerospace researcher's needs into a particular instrument's specifications, such as the overall physical dimensions of the balance and its load capacity, both individually and in combination of loads. Without a careful design, even the most accurate balance will fail to meet its intended purpose.

II. Calibration Philosophy

In this paper, we explore the concept of design as it applies to the calibration or characterization phase. Calibration represents the final stage in the production of a force balance, and it has multiple objectives. From our perspective, we have observed a disturbing trend in balance calibration that is often solely focused on the generation of calibration data, where there is a perception that more data are better. We acknowledge that more data does provide more information, however more data are not free and more of the wrong data does not produce actionable information. In this paper, we endeavor to suggest a revitalization of a broader, more holistic view of calibration as a verification and validation activity where the calibration process and data support an assessment of multiple performance metrics, as follows.

- Material properties/performance under load
- Accuracy of the structural analysis in predicting component outputs
- Proof load demonstration for structural safety
- Balance structural health after wind tunnel usage
- Calibration hardware system's ability to accurately apply the loads
- Data acquisition system performance
- Quality of the mechanical interfaces
- Workmanship of the strain gage installation and wiring
- Balance deflections for model system dynamics and positioning
- Interactions or cross-affects between components
- Development of a mathematical model (or calibration matrix)
- Thermal and/or pressure affects on the mathematical model
- Balance precision and accuracy

From this list, we see that balance calibration involves much more than simply generating data, and furthermore, it requires expertise in multiple disciplines, including structural analysis, materials and fabrication, metrology, strain gage technology, and mathematical modeling. Calibration requires an experienced observer executing the calibration and interpreting the data. A good calibration plan, or design, specifies a deliberate sequence of activities to assess all of these critical performance criteria.

During calibration, data anomalies are not simply mathematical nuisances, they are signals that allow a balance engineer to diagnose causal factors and take corrective actions before the balance is used in wind tunnel research. These corrective actions may require exploring more complex mathematical models, however they might require extensive rework in fabrication to improve the quality of mechanical interfaces or the removal and re-installation of strain gages, which are anomalies that may not be adequately corrected mathematically.

Ultimately, the deliverables from the calibration are the mathematical model, also referred to as the calibration matrix, and an estimate of balance precision and accuracy. While the calibration data are a means to generate these deliverables, we emphasize that the data are not typically supplied to the wind tunnel. Furthermore, the data produced in a calibration are usually not based on anticipated aerodynamic loads in wind tunnel testing, other than their full-scale load ranges, and usually not intended to simulate the wind tunnel conditions. However, while outside the scope of this paper, we believe that a better integration of anticipated aerodynamic loading would inform better calibration designs, as suggested by Ulbrich in reference 1 and Crooks in reference 3.

A calibration design is a schedule of conditions that specifies the balance position, loads applied, sequence of load application, and the responses to be recorded. In our terminology, we refer to a single load combination as a design point and the collection of these design points as the design matrix. We refer to the design size as the number of design points. Essentially, every calibration design must answer the following fundamental questions.

- What design points (load combinations) should be executed?
- What order should they be executed?
- How many points are required?

Whether deliberately or strategically, these questions are answered when a calibration design is specified. As previously stated, these are not solely mathematical or statistical questions as if the data were being generated in a computer simulation, rather they require a broader perspective on the purpose of calibration recognizing that a physical device is generating the data. From our observation, the perspective and domain expertise of the calibration designer drives their design philosophy. For example, a computer scientist is likely to specify a much larger design, while a calibration engineer is seeking the smallest most efficient as possible. Additionally, the execution order has practical considerations such as the calibration system constraints as well as statistical considerations such as separating error sources and defending against systematic errors that may occur during execution.

While the design points and how many are typically associated with answering most of the metrics listed previously, another important set of loadings or points need to be considered called confirmation points. These are points independent of the mathematical model development and enable independent assessment of the performance. An adequate number and location of confirmation points has been a topic of debate. However, the need for them is not disputed. Confirmation points aligned with expected use cases can be one approach.

From the perspective of a balance engineer, calibration is the final examination of their mechanical design. As young engineers, we remember very well when our first balance went into calibration, after nearly a year of fabrication and instrumentation, and it most certainly felt like a major academic final examination. However, this was not academic, since there were real consequences if the balance did not meet its anticipated performance. For most mechanical engineers that enjoy the benefit of designing for large factors of safety, they often do not receive direct feedback on their mechanical design and analysis. For balance engineers, there is direct and precise feedback on their structural analysis and assumptions. This is particularly important for the first calibration of a new balance, and it implies that the calibration design approach may be different for new balances compared to those that have a long history of usage. Alternatively, the calibration design of heavily used balances may need to be more focused on structural health rather than predicted structural performance, since it had been calibrated many times before.

III. Calibration Philosophy at NASA Langley

At Langley Research Center, traditional calibration methods are derived from reference 4, Guarino in 1964 and depicted in the Appendix under the NASA LaRC 9-Point Design and NASA LaRC 5-Point Design. The design philosophy was developed by experienced balance engineers, and it assesses balance health and supports statistical modeling of the calibration matrix. It features a systematic approach to apply up to three loads simultaneously to estimate the coefficients in a complete second-order model (27 terms), and it specifies up to six components simultaneously in confirmation sequences, not shown here, to test the performance of the calibration model. The loading sequences were designed to graphically estimate all of the calibration coefficients and included algorithms to directly compensate for pre-load effects on specific non-linear coefficients. In all sequences, a single force or moment is incremented sequentially in ascending and descending order, either in four or two equal increments, and may include additional forces and moments that are held constant. The incremental loading supports an assessment of hysteresis and zero shift attributed to specific load combinations. A clever feature of the design is to transfer small forces at long distances to generate the moments. This approach is known as long-arm calibration methodology, and it seeks to physically isolate the effects of moments from forces, as if a pure moment, or pure couple, is being applied. The sequence begins with a primary calibration that involves 18 sequences to load each of the six components individually to their full-scale load level. For a new balance or one returning from wind tunnel usage, this primary sequence quickly identifies issues in fabrication, strain gage installation, and/or potential operational damage before proceeding to more complex load combinations.

While this approach has been the subject of much criticism in the balance community as being outdated and inefficient, we feel that those criticisms fail to recognize all of the objectives of balance calibration. For example, from a balance engineer's perspective, it allows for an incremental structural test by building in increments rather than immediately applying full-scale loads, which could reveal catastrophic discrepancies in either the balance structural analysis or calibration hardware. Also note that after each increment is applied, particularly in the early primary loads, the engineer and calibrator are afforded a decision point before proceeding to the next increment. From a safety perspective of both the calibration personnel and the instrument asset, this systematic, methodical process builds the calibrators situational awareness as he learns the balance characteristics moving from simple to more complex load combinations. From a model estimation perspective, graphical analyses of the raw outputs as a function of load applied, often overlaying multiple load sequences, are essential in understanding balance performance by an experienced balance engineer. In addition, the partitioned assignment of sequences to estimate specific calibration coefficients easily lends itself to developing customized calibrations to estimate or verify a subset of individual coefficients. As an example, the primary calibration enables the estimation of 12 of the 27 coefficients in the full model. We suggest that after a balance undergoes its first full calibration and the active or important coefficients are identified, subsequent calibrations could focus on verifying those specific coefficients and assess overall performance with multi-component confirmation sequences.

IV. Calibration Model Development – How many points are needed?

From a purely mathematical view of calibration, we can derive the minimum number of calibration points based as function of the number of model terms to be estimated. As an illustration, consider a simple one-component load cell where we estimate the bias, sensitivity, and residual error. If we let the load cell output or signal be y, the applied load x, and the error term ε, then our calibration equation is as follows:

$$y = \beta_0 + \beta_1 x + \varepsilon$$

where, β_0, β_1 are the intercept (bias or electrical zero) and the slope (sensitivity coefficient), respectively. There are two terms that need to be estimated β_0 and β_1 from the calibration experiment. From a linear algebra perspective, we need two unique equations to estimate two unknowns, or from a practical perspective we need two calibration design points. For example, an applied load of x=0 and 100% of full-scale (FS) instrument capacity could be performed. Clearly, in practice, this is insufficient because this represents a direct solution and does provide a means to estimate the residual error. To estimate error, we could include an intermediate level of x, such as at 50% or we could replicate one or more the design points at 0 and/or 100%. In the first case, we can test robustness to our assumption of a purely linear model, and the second approach we can assess experimental pure error by replicating conditions. Furthermore, in practice we might consider adding one or two additional increments of load. However, it would not seem justified or rational to add 20 increments of load, in 5% of FS increments unless we believed that a very high-order polynomial model is required or that the function is not continuous. In most transducer applications, we do not believe that the relationship is high-order nor discontinuous.

Extending this to a six-component force balance there are 27 coefficients in a full second-order model plus an intercept term, known as the unloaded zero, which results in 28 coefficients to estimate, plus an error term.

$$y = \beta_0 + \sum_{i=1}^{6} \beta_i x_i + \sum_{j=1}^{6} \sum_{i<j} \beta_{ij} x_i x_j + \sum_{i=1}^{6} \beta_{ii} x_i^2 + \varepsilon$$

Therefore, it naturally follows that a full calibration could be performed in 28 unique design points. We note here without further explanation that it can be shown in a liner algebra perspective the equations in the projected model space, not the design space, need to be unique to support the estimation in 28 design points. As previously discussed in the simple load cell example, a strategy is required to estimate the residual error. For argument sake, we will specify one replicate point. This leads to a minimum design size of 29 points. Our goal here is not to belabor the details of the mathematics, which can be shown, rather to establish a rationale for the minimum number of design points. Note that in this calculation of minimum design size, we assume that all of the 27 model coefficients will be active or significant and need to be estimated. A cursory review of typical monolithic balances reveals that only about 15 coefficients need to be estimated, which means that even with only 29 points, there are more points than necessary to estimate the significant effects. In practice, we would not recommend a calibration design this small for several reasons that have been previously cited based on broader calibration objectives that extend beyond model estimation. However, it is obvious that hundreds or thousands of design points for the purpose of second-order model estimation are not justified.

The deliverable to the wind tunnel from a calibration is typically a matrix of 28 coefficients for each component and an estimate of accuracy, regardless of whether the calibration design was based on 29 points, 100 points, 1,000 points, or 1,000,000 points. Certainly there are benefits to additional design points that may improve the distribution or coverage throughout the design space, allow for additional replication, provide better estimates of error, and support the testing of model sufficiency. However, there is a point of diminishing returns in blindly increasing the design size without assessing additional benefit. Said another way, 1,000,001 points is just a little bit better than a 1,000,000 points, but its likely not of any practical significance. We suggest that a good, adequate calibration design that has clear objectives should specify more than the minimum number of design points, but certainly not an excessive number of points.

In the next section, we apply the concepts presented on calibration design objectives and consider multiple discipline perspectives by comparing several common designs

V. Calibration Design Evaluations

A total of seven designs are compared that include two from response surface methodologies (RSM) a Central Composite Design (CCD) (reference 6) and a Box-Behnken Design (BBD), two utilized at NASA Langley a 5-Point and a 9-Point design, one from Triumph Force Measurement System's Automatic Balance Calibration System - ABCS (reference 5), one from the European Transonic Wind Tunnel (ETW) (reference 5), and one from NASA Ames (reference 1).

Each design is evaluated against the criteria discussed in the previous sections and restated hear for completeness with exceptions being noted where applicable. The graphs in figures cc-gg, in the appendix, show each design with the number and types of load sequences. The authors can be contacted to obtain each design in Excel format. Also, the figures cc.cc show the two-component load combinations for each of the 15 two-factor combinations for each design. These plots are very revealing in the number of points, density, as well as location of the points. This is followed by evaluation and critique against the primary topic of this paper, how many points are enough.

All of the criteria listed in Calibration Philosophy section (listed below) except for thermal and pressure affects are captured by all of the designs described in this paper. The execution protocol, such as timing and run order can impact the balance engineer's ability to perform the assessments listed and needs to be considered. For example, instead of executing the calibration as quickly as possible, time between some of the initial loadings may be needed to ensure items such as material properties or structural health before moving through the remaining design points. However, the design points provide the opportunity and information necessary.

Balance Calibration Philosophy Criteria (BCPC)

- Material properties/performance under load
- Accuracy of the structural analysis in predicting component outputs
- Assessment of balance structural health after wind tunnel usage
- Calibration hardware system's ability to accurately apply the loads?
- Data acquisition system performance (in the case of on-board electronics)
- Quality of the mechanical interfaces
- Workmanship of the strain gage installation and wiring
- Overall balance deflections for model system dynamics and positioning
- Characterization of interactions or cross-affects between components?
- Development of a mathematical model (or calibration matrix)?
- *Thermal affects on the mathematical model
- Estimate of balance precision and accuracy

*Not addressed in this paper

The one-exception on the list is: Proof load demonstration for structural safety. This requires the loading of all six-components (or a subset applicable to the wind tunnel test where the balance will be utilized) to satisfy. This loading can be augmented to the existing designs, and is for the NASA LaRC designs as a matter of practice, and hence is not considered a major weakness for any of the designs.

The remaining evaluation is centered around the quantity and location of points in the designs to provide the information needed to address the items in the balance calibration philosophy criteria (BCPC). Tables 1 and 2 list each design and some of the quantitative criteria that will be discussed. Following the tables, the designs have been paired, where similarities were observed, then described and critiqued.

	CCD	BBD	NASA LaRC 5-Point	NASA LaRC 9-Point	ABCS	ETW	NASA ARC
Number of Points	49	65	410	738	1063	1631	2082
Number of Levels	5	3	5	9	13	31	21

Table 1. Each Design with the number of points and factor levels. (Number of levels for Normal Force)

Components Loaded	CCD	BBD	NASA LaRC 5-Point	NASA LaRC 9-Point	ABCS	ETW	NASA ARC
	Number of Points						
0	5	5	44	44	29	127	48
1	12	0	174	262	317	379	186
2	0	60	174	390	717	979	636
3	0	0	18	42	0	146	620
4	0	0	0	0	0	0	528
5	0	0	0	0	0	0	64
6	32	0	0	0	0	0	0

Table 2. Each design with the number of points from 0 to 6 components. (Component considered loaded if > 0.15)

Central Composite Design (CCD, 49 points), Box-Benken Design (BBD, 65 points)
References 6 and 2 respectively.
Descriptions:

- CCD: Load sequences or conditions as depicted in table 3. Two-factor plots in figure 1.
- BBD: Load sequences or conditions as depicted in table 4. Two-factor plots in figure 2.
- Pure moments are specified

Critique:

- Most efficient designs (fewest points) for developing a 2nd order model
- Separating effects due to the complex loading scenarios during execution by inspection can be challenging for analyst who are unfamiliar with the design structure, however separating effects can be done quite easily (see reference 6).
- CCD: Multiple combinations of six-component loads are performed to maximize the information contained in each point, making the design very efficient; however there is not an assumption that six-factor interactions exist.
- .BBD: All combinations of 2-component loadings are performed to maximize information for a 2nd order model development per point.

- CCD: 5 unique levels of each component are loaded providing sufficient information for a 2^{nd} order model (only 3 are needed) along with additional levels for checking lack of fit and supporting mixed cubic terms.
- BBD: 3-levels of each component are loaded providing sufficient information for a 2^{nd} order model (only 3 are needed). However, this design is restricted to a second-order model, without the ability to assess that model assumption
- CCD: Single component loads are applied that support balance health and performance diagnostics
- BBD: No single component loads are applied, therefore does not easily lend itself to some of the BCPC
- To aid in the balance evaluation, additional increments of the single component points may be warranted on the order of 25% or 50%.

NASA Langley 5-Point Design (410 points), NASA Langley 9-Point Design (738 points)

The NASA Langley 9-Point design totaling 738 runs was discussed in the Calibration Philosophy at Langley section previously and reference 5 provides additional information. The NASA Langley 5-Point design mimics the 9-Point Design while removing the 25% and 75% increments and decrements.

Descriptions:

- Load sequences or conditions as depicted in table 5. Two-factor plots in figures 3 and 4.
- Pseudo moments specified.
- Combined load sequences are executed with one load held constant at full scale while the second component is incremented and decremented

Critique:

- Provides very simple and intuitive points for evaluating the BCPC.
- It is dominated by single and two-component combinations.
- It is a combination (combinatorial) of the first half of the CCD single component loadings, incremented, and a subset of the BBD two component loadings, if this were a LaRC 3-pt design
- Each coefficient in the math model has specific loadings to compute and therefore lends itself well to evaluation as the calibration is performed. It does not require any extensive analysis as the points were developed around a graphical analysis technique that provides insight to performance.
- For multi-axis loadings such as Normal and axial, only the positive loading of one of these components is used instead of all possible combinations (Axial and Side combined loads with other components in another plane). There is an assumption of symmetry in balance behavior to support this approach. However, it is a limitation in the symmetry of the load schedule.
- While the incrementing and decrementing of two-factor loads provides some additional information on the BCPC, the number of increments increases the number of points in the design.
- 5 and 9-levels respectively of each component are loaded providing sufficient information for a 2^{nd} order model (only 3 are needed) along with additional levels for checking lack of fit. The 9-Point design has 3 times the required levels and clear justification for this additional number should be considered.

ABCS Design (1063 points), ETW Design (1631 points)

Reference 5.

Descriptions:

- ABCS: Load sequences or conditions as depicted in table 6. Two-factor plots in figure 5.
- ETW: Load sequences or conditions as depicted in table 7. Two-factor plots in figure 6.
- Pure moments are specified.
- Dominated by similar single and two-component component loadings with finer increments and different levels of the held constant loads. ETW contains some 3-component combinations.

Critique:

- The number of points needed to evaluate the BCPC is well exceeded. The information gained by additional points with differing levels of combined loads beyond those needed for the math model or balance engineering characteristics, seem to be of little value. This assumes the balance in monolithic and sufficiently modeled with a 2^{nd} order model. Therefore, a question for the design intent of the additional points needs to be addressed.
- 13 and 31 levels of each component are loaded respectively for each design, providing sufficient information for a 2^{nd} order model (only 3 are needed) along with additional levels mainly for estimating

higher-order pure terms, like N^3. However, without a clear justification, this high number of levels appears excessive since 13 is almost 4 times the amount needed for the model development and 31 is over 10 times the needed amount.

NASA Ames Design (2082 points)
This design is documented in reference 1 as a potential design for certain types of testing. It was executed on the Triumph ABCS as discussed in the reference.
Description:

- Load sequences or conditions as depicted in table 8. Two-factor plots in figure 7.
- Contains up to 4-component with finer increments and different levels of the held constant loads.
- Pure moments are specified.
- Combined loadings are two, three, four and five components with a similar scenario of holding one constant and varying the other in increments and decrements.

Critique:

- Three, four and five component loadings do not provide any direct value in developing a 2nd order model. If 3rd order or higher interactions are suspected, including 3-way interactions in the model, then these may be warranted at some level for determining those terms.
- If the higher combinations are to be used for confirmation points, that would be an excellent use. However, the shear volume again is excessive since balance behavior is repeatable and continuous throughout the operating envelope as assumed by the models that are developed. The information or value of so many design points must be questioned and justified both for the BCPC and the math model development.
- 21 levels of each component are loaded providing sufficient information for a 2nd order model (only 3 are needed) along with additional levels for checking lack of fit. However, one could consider this extremely excessive since 21 is 7 times the amount needed for the model development.

VI. Summary

A review of balance calibration objectives was discussed and summarized in the BCPC to be used a guide when developing a calibration design. All calibration designs should strategically consider these criteria and develop a rationale for how many points are needed to answer these questions. After sufficient points are chosen to answer these questions, then the calibration points should be considered sufficient. While there can always be *what if?* scenarios of whether the calibration has captured the balance behavior, the fundamental impact would be to change the math model and then adjust the design to capture those additional coefficients. However, in this scenario every point should be questioned as to its value in answering a question because each additional point has an associated cost in time to design, analyze and evaluate.

While the focus of this paper is on designs to support a 2nd order math model, the authors realize that some of the designs may have been developed with non-monolithic balances in mind and assumed a more complex model, reference AIAA RP here. However, each of these designs are proposed for monolithic balance (except the ARC design). A similar evaluation of the designs for a more complex math model that is typically associated with non-monolithic balances could be performed based on the concepts presented in this paper. Regardless of the mathematical model, we suggest that any calibration design should be based on objective criteria and its design size justified.

From the designs we reviewed, the CCD and BBD, with less than 100 design points, meet the BCPC. All other designs routinely used for balance calibration are essentially augmentations and/or subsets of these basic design structures. Comparing the majority of the designs with the minimum from the CCD and BBD, one can imagine a design that incorporates these minimum designs for efficiency with the remaining insights needed from the BCPC (other than the model estimation). While these designs could be augmented, as discussed, it is clear that designs on the order of 100 to 150 points are reasonable. Therefore, ever increasing the design size toward a 1,000,000, as the paper title indicates, is not justified based on the guidance provided in this paper. Excessively larger designs are contrary to the smaller designs that are capable of meeting the calibration objectives. In addition, there are analytical techniques that can be found in reference 6 for analyzing design prior to execution to ensure they meet the quantitative objectives of the calibration.

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Appendix

Table 3. Central Composite Design (CCD) Loading Sequence

Sequence	Normal	Axial	Pitch	Roll	Yaw	Side	Points
1	**±1**	**0**	**0**	**0**	**0**	**0**	**2**
2	**0**	**±1**	**0**	**0**	**0**	**0**	**2**
3	**0**	**0**	**±1**	**0**	**0**	**0**	**2**
4	**0**	**0**	**0**	**±1**	**0**	**0**	**2**
5	**0**	**0**	**0**	**0**	**±1**	**0**	**2**
6	**0**	**0**	**0**	**0**	**0**	**±1**	**2**
7	-0.50	-0.50	-0.50	-0.50	-0.50	-0.50	**1**
8	0.50	-0.50	-0.50	-0.50	-0.50	0.50	**1**
9	-0.50	0.50	-0.50	-0.50	-0.50	0.50	**1**
10	0.50	0.50	-0.50	-0.50	-0.50	-0.50	**1**
11	-0.50	-0.50	0.50	-0.50	-0.50	0.50	**1**
12	0.50	-0.50	0.50	-0.50	-0.50	-0.50	**1**
13	-0.50	0.50	0.50	-0.50	-0.50	-0.50	**1**
14	0.50	0.50	0.50	-0.50	-0.50	0.50	**1**
15	-0.50	-0.50	-0.50	0.50	-0.50	0.50	**1**
16	0.50	-0.50	-0.50	0.50	-0.50	-0.50	**1**
17	-0.50	0.50	-0.50	0.50	-0.50	-0.50	**1**
18	0.50	0.50	-0.50	0.50	-0.50	0.50	**1**
19	-0.50	-0.50	0.50	0.50	-0.50	-0.50	**1**
20	0.50	-0.50	0.50	0.50	-0.50	0.50	**1**
21	-0.50	0.50	0.50	0.50	-0.50	0.50	**1**
22	0.50	0.50	0.50	0.50	-0.50	-0.50	**1**
23	-0.50	-0.50	-0.50	-0.50	0.50	0.50	**1**
24	0.50	-0.50	-0.50	-0.50	0.50	-0.50	**1**
25	-0.50	0.50	-0.50	-0.50	0.50	-0.50	**1**
26	0.50	0.50	-0.50	-0.50	0.50	0.50	**1**
27	-0.50	-0.50	0.50	-0.50	0.50	-0.50	**1**
28	0.50	-0.50	0.50	-0.50	0.50	0.50	**1**
29	-0.50	0.50	0.50	-0.50	0.50	0.50	**1**
30	0.50	0.50	0.50	-0.50	0.50	-0.50	**1**
31	-0.50	-0.50	-0.50	0.50	0.50	-0.50	**1**
32	0.50	-0.50	-0.50	0.50	0.50	0.50	**1**
33	-0.50	0.50	-0.50	0.50	0.50	0.50	**1**
34	0.50	0.50	-0.50	0.50	0.50	-0.50	**1**
35	-0.50	-0.50	0.50	0.50	0.50	0.50	**1**
36	0.50	-0.50	0.50	0.50	0.50	-0.50	**1**
37	-0.50	0.50	0.50	0.50	0.50	-0.50	**1**
38	**0**	**0**	**0**	**0**	**0**	**0**	**5**

Table 4. Box-Behnken Design (BBD) Loading Sequence

Sequence	Normal	Axial	Pitch	Roll	Yaw	Side	Points
1	±1	±1	0	0	0	0	4
2	±1	0	±1	0	0	0	4
3	±1	0	0	±1	0	0	4
4	±1	0	0	0	±1	0	4
5	±1	0	0	0	0	±1	4
6	0	±1	±1	0	0	0	4
7	0	±1	0	±1	0	0	4
8	0	±1	0	0	±1	0	4
9	0	±1	0	0	0	±1	4
10	0	0	±1	±1	0	0	4
11	0	0	±1	0	±1	0	4
12	0	0	±1	0	0	±1	4
13	0	0	0	±1	±1	0	4
14	0	0	0	±1	0	±1	4
15	0	0	0	0	±1	±1	4
16	0	0	0	0	0	0	5

Table 5. NASA Langley 5 and 9-Point Design Loading Sequence

Sequence	Normal	Axial	Pitch	Roll	Yaw	Side
1	0.0	**1.0**	0.0	0.0	0.0	0.0
2	0.0	**-1.0**	0.0	0.0	0.0	0.0
3	**1.0**	0.0	0.0	0.0	0.0	0.0
4	**-1.0**	0.0	0.0	0.0	0.0	0.0
5	0.1	0.0	**1.0**	0.0	0.0	0.0
6	-0.1	0.0	**1.0**	0.0	0.0	0.0
7	0.1	0.0	**-1.0**	0.0	0.0	0.0
8	-0.1	0.0	**-1.0**	0.0	0.0	0.0
9	0.0	0.0	0.0	**1.0**	0.0	0.0
10	0.0	0.0	0.0	**1.0**	0.0	0.0
11	0.0	0.0	0.0	**-1.0**	0.0	0.0
12	0.0	0.0	0.0	**-1.0**	0.0	0.0
13	0.0	0.0	0.0	**1.0**	0.0	0.1
14	0.0	0.0	0.0	**1.0**	0.0	-0.1
15	0.0	0.0	0.0	**-1.0**	0.0	-0.1
16	0.0	0.0	0.0	**-1.0**	0.0	0.1
17	0.0	0.0	0.0	0.0	**1.0**	0.0
18	0.0	0.0	0.0	0.0	**1.0**	0.0
19	0.0	0.0	0.0	0.0	**-1.0**	0.0
20	0.0	0.0	0.0	0.0	**-1.0**	0.0
21	0.0	0.0	0.0	0.0	0.0	**1.0**
22	0.0	0.0	0.0	0.0	0.0	**-1.0**
23	**1.0**	**1.0**	0.0	0.0	0.0	0.0
24	**-1.0**	**1.0**	0.0	0.0	0.0	0.0
25	**1.0**	0.0	**1.0**	0.0	0.0	0.0
26	**-1.0**	0.0	**1.0**	0.0	0.0	0.0
27	**-1.0**	0.0	**-1.0**	0.0	0.0	0.0
28	**1.0**	0.0	**-1.0**	0.0	0.0	0.0
29	**1.0**	0.0	0.0	**1.0**	0.0	0.0
30	**-1.0**	0.0	0.0	**1.0**	0.0	0.0
31	**-1.0**	0.0	0.0	**-1.0**	0.0	0.0
32	**1.0**	0.0	0.0	**-1.0**	0.0	0.0

33	**1.0**	0.0	0.0	0.0	**1.0**	0.5
34	**1.0**	0.0	0.0	0.0	0.0	0.5
35	**-1.0**	0.0	0.0	0.0	0.0	0.5
36	**-1.0**	0.0	0.0	0.0	**1.0**	0.5
37	0.1	**1.0**	**1.0**	0.0	0.0	0.0
38	-0.1	**1.0**	**1.0**	0.0	0.0	0.0
39	-0.1	**1.0**	**-1.0**	0.0	0.0	0.0
40	0.1	**1.0**	**-1.0**	0.0	0.0	0.0
41	0.0	**1.0**	0.0	**1.0**	0.0	0.0
42	0.0	**1.0**	0.0	**1.0**	0.0	0.0
43	0.0	**1.0**	0.0	**-1.0**	0.0	0.0
44	0.0	**1.0**	0.0	**-1.0**	0.0	0.0
45	0.0	**1.0**	0.0	0.0	**1.0**	0.0
46	0.0	**1.0**	0.0	0.0	**1.0**	0.0
47	0.0	**1.0**	0.0	0.0	**-1.0**	0.0
48	0.0	**1.0**	0.0	0.0	**-1.0**	0.0
49	0.0	**1.0**	0.0	0.0	0.0	**1.0**
50	0.0	**1.0**	0.0	0.0	0.0	**-1.0**
51	0.1	0.0	**1.0**	**1.0**	0.0	0.0
52	-0.1	0.0	**1.0**	**1.0**	0.0	0.0
53	0.1	0.0	**1.0**	**-1.0**	0.0	0.0
54	-0.1	0.0	**1.0**	**-1.0**	0.0	0.0
55	-0.1	0.0	**-1.0**	**1.0**	0.0	0.0
56	0.1	0.0	**-1.0**	**1.0**	0.0	0.0
57	-0.1	0.0	**-1.0**	**-1.0**	0.0	0.0
58	0.1	0.0	**-1.0**	**-1.0**	0.0	0.0
59	0.1	0.0	**1.0**	0.0	**1.0**	0.5
60	0.1	0.0	**1.0**	0.0	0.0	0.5
61	-0.1	0.0	**1.0**	0.0	**1.0**	0.5
62	-0.1	0.0	**1.0**	0.0	0.0	0.5
63	0.1	0.0	**-1.0**	0.0	0.0	0.5
64	0.1	0.0	**-1.0**	0.0	**1.0**	0.5
65	-0.1	0.0	**-1.0**	0.0	0.0	0.5
66	-0.1	0.0	**-1.0**	0.0	**1.0**	0.5
67	0.0	0.0	0.0	**1.0**	**1.0**	0.1
68	0.0	0.0	0.0	**1.0**	**1.0**	-0.1
69	0.0	0.0	0.0	**-1.0**	**1.0**	0.1
70	0.0	0.0	0.0	**-1.0**	**1.0**	-0.1
71	0.0	0.0	0.0	**-1.0**	**-1.0**	-0.1
72	0.0	0.0	0.0	**-1.0**	**-1.0**	0.1
73	0.0	0.0	0.0	**1.0**	**-1.0**	-0.1
74	0.0	0.0	0.0	**1.0**	**-1.0**	0.1
75	0.0	0.0	0.0	**1.0**	0.0	**1.0**
76	0.0	0.0	0.0	**1.0**	0.0	**-1.0**
77	0.0	0.0	0.0	**-1.0**	0.0	**1.0**
78	0.0	0.0	0.0	**-1.0**	0.0	**-1.0**
79	0.0	0.0	0.0	0.0	**1.0**	**1.0**
80	0.0	0.0	0.0	0.0	**1.0**	**-1.0**
81	0.0	0.0	0.0	0.0	**-1.0**	**1.0**
82	0.0	0.0	0.0	0.0	**-1.0**	**-1.0**

Table 6. Triumph ABCS Design Loading Sequence

A loading sequence for this design can be supplied upon request.

Table 7. ETW Design Loading Sequence

This table is from reference 5. The number of points does not match the design evaluated for this paper. However, it was derived from this sequence.

Data File	Load Combination	Loading Sequence	Load Increment (% of 2 FS)	Number of Calibration Data Points
B113##00.DAT	Fx=Fy=Fz=Mx=My=Mz=0			5
B113##01.DAT	Fx	1	2.5	41
B113##02.DAT	Fx	2	5	41
B113##03.DAT	Fy	1	2.5	41
B113##04.DAT	Fy	2	5	41
B113##05.DAT	Mx	1	2.5	41
B113##06.DAT	Mx	2	5	41
B113##07.DAT	My	1	2.5	41
B113##08.DAT	My	2	5	41
B113##09.DAT	Mz	1	2.5	41
B113##10.DAT	Mz	2	5	41
B113##11.DAT	Fx , Fy	2 , 3	10 (20) , 25 (50)	76
B113##12.DAT	Fx , Mx	2 , 3	10 (20) , 25 (50)	76
B113##13.DAT	Fx , My	2 , 3	10 (20) , 25 (50)	76
B113##14.DAT	Fx , Mz	2 , 3	10 (20) , 25 (50)	76
B113##15.DAT	Fy , Mx	2 , 3	10 (20) , 25 (50)	76
B113##16.DAT	Fy , My	2 , 3	10 (20) , 25 (50)	76
B113##17.DAT	Fy , Mz	2 , 3	10 (20) , 25 (50)	76
B113##18.DAT	Mx , My	3 , 2	25 (50) , 10 (20)	76
B113##19.DAT	Mx , Mz	2 , 3	10 (20) , 25 (50)	84
B113##20.DAT	My , Mz	3 , 2	25 (50) , 10 (20)	76
B113##21.DAT	Fz	1	2.5	41
B113##22.DAT	Fz	2	5	41
B113##23.DAT	Fx , Fz	2 , 3	10 (20) , 25 (50)	76
B113##24.DAT	Fy , Fz	2 , 3	10 (20) , 25 (50)	76
B113##25.DAT	Mx , Fz	4 , 3	10 (20) , 25 (50)	80
B113##26.DAT	My , Fz	4 , 3	10 (20) , 25 (50)	80
B113##27.DAT	Mz , Fz	2 , 3	10 (20) , 25 (50)	76
B113##28.DAT	Fx=Fy=Fz=Mx=My=Mz=0			5
				1658
				Total

Table 8. NASA Ames Design Loading Sequence

(reference 1)

SERIES	POINTS	N1	N2	S1	S2	RM	AF
1	17	±N1 −100.0% to +100.0%	< ±2%	< ±2%	< ±2%	< ±2%	< ±2%
2	17	< ±2%	±N2 −100.0% to +100.0%	< ±2%	< ±2%	< ±2%	< ±2%
3	17	< ±2%	< ±2%	±S1 −100.0% to +100.0%	< ±2%	< ±2%	< ±2%
4	17	< ±2%	< ±2%	< ±2%	±S2 −100.0% to +100.0%	< ±2%	< ±2%
5	17	< ±2%	< ±2%	< ±2%	< ±2%	±RM −100.0% to +100.0%	< ±2%
6	17	< ±2%	< ±2%	< ±2%	< ±2%	< ±2%	±AF −100.0% to +100.0%
7	105	±N1 −100.0% to +100.0%	±N2 −100.0% to +100.0%	< ±2%	< ±2%	< ±2%	< ±2%
8	105	< ±2%	< ±2%	±S1 −100.0% to +100.0%	±S2 −100.0% to +100.0%	< ±2%	< ±2%
9	105	< ±2%	< ±2%	< ±2%	< ±2%	±RM −100.0% to +100.0%	±AF −100.0% to +100.0%
10	180	±N1 −66.0% to +66.0%	±N2 −66.0% to +66.0%	< ±2%	< ±2%	< ±2%	±AF −66.0% to +66.0%
11	180	< ±2%	< ±2%	±S1 −66.0% to +66.0%	±S2 −66.0% to +66.0%	< ±2%	±AF −66.0% to +66.0%
12	180	±N1 −66.0% to +66.0%	±N2 −66.0% to +66.0%	< ±2%	< ±2%	±RM −66.0% to +66.0%	< ±2%
13	180	< ±2%	< ±2%	±S1 −66.0% to +66.0%	±S2 −66.0% to +66.0%	±RM −66.0% to +66.0%	< ±2%
14	513	±N1 −66.0% to +66.0%	±N2 −66.0% to +66.0%	±S1 −66.0% to +66.0%	±S2 −66.0% to +66.0%	< ±2%	< ±2%
15	216	±N1 −33.0% to +33.0%	±N2 −33.0% to +33.0%	±S1 −33.0% to +33.0%	±S2 −33.0% to +33.0%	< ±2%	±AF −33.0% to +33.0%
16	216	±N1 −33.0% to +33.0%	±N2 −33.0% to +33.0%	±S1 −33.0% to +33.0%	±S2 −33.0% to +33.0%	±RM −33.0% to +33.0%	< ±2%

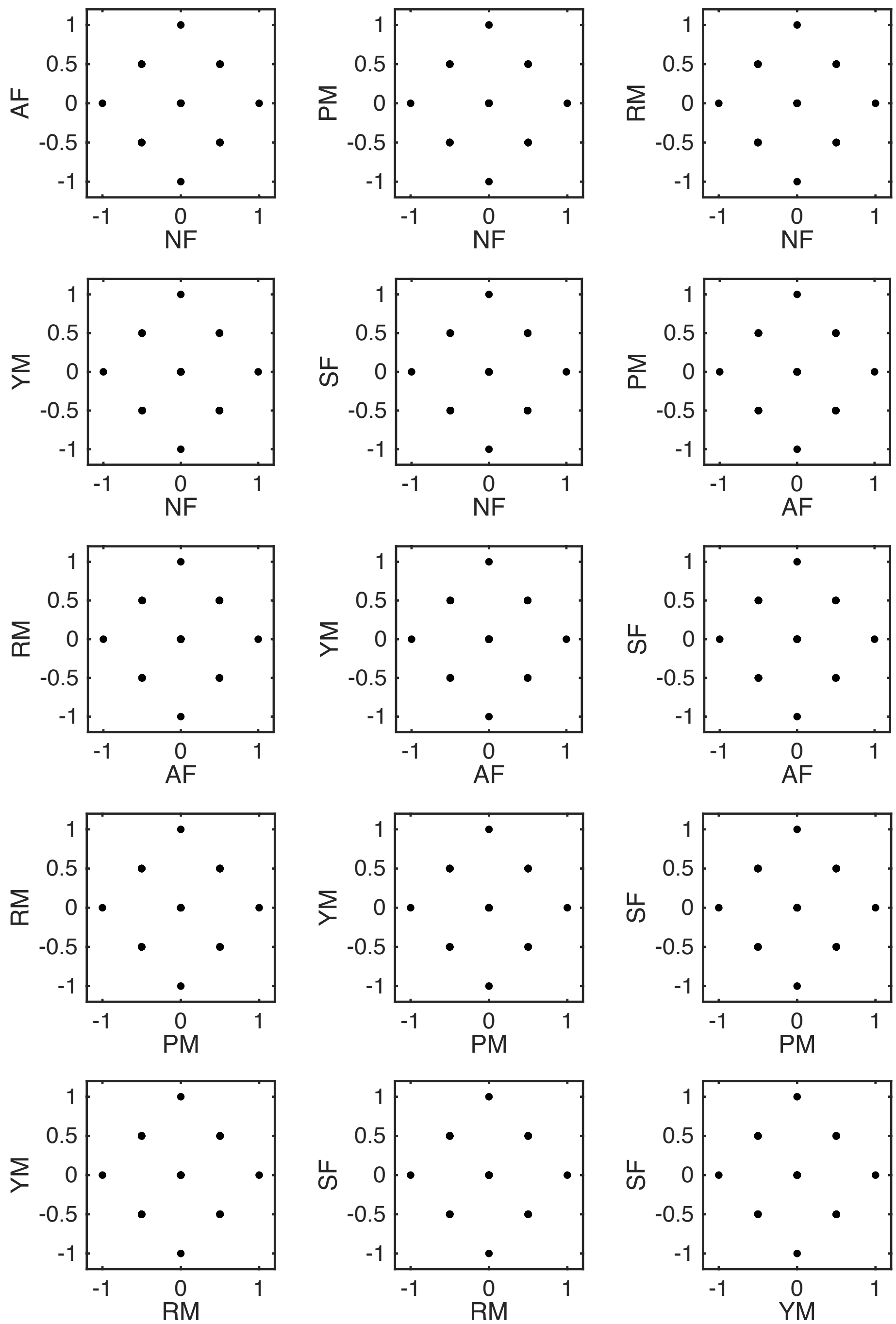

Figure 1. Central Composite Design (CCD) 2-Factor Load Plots

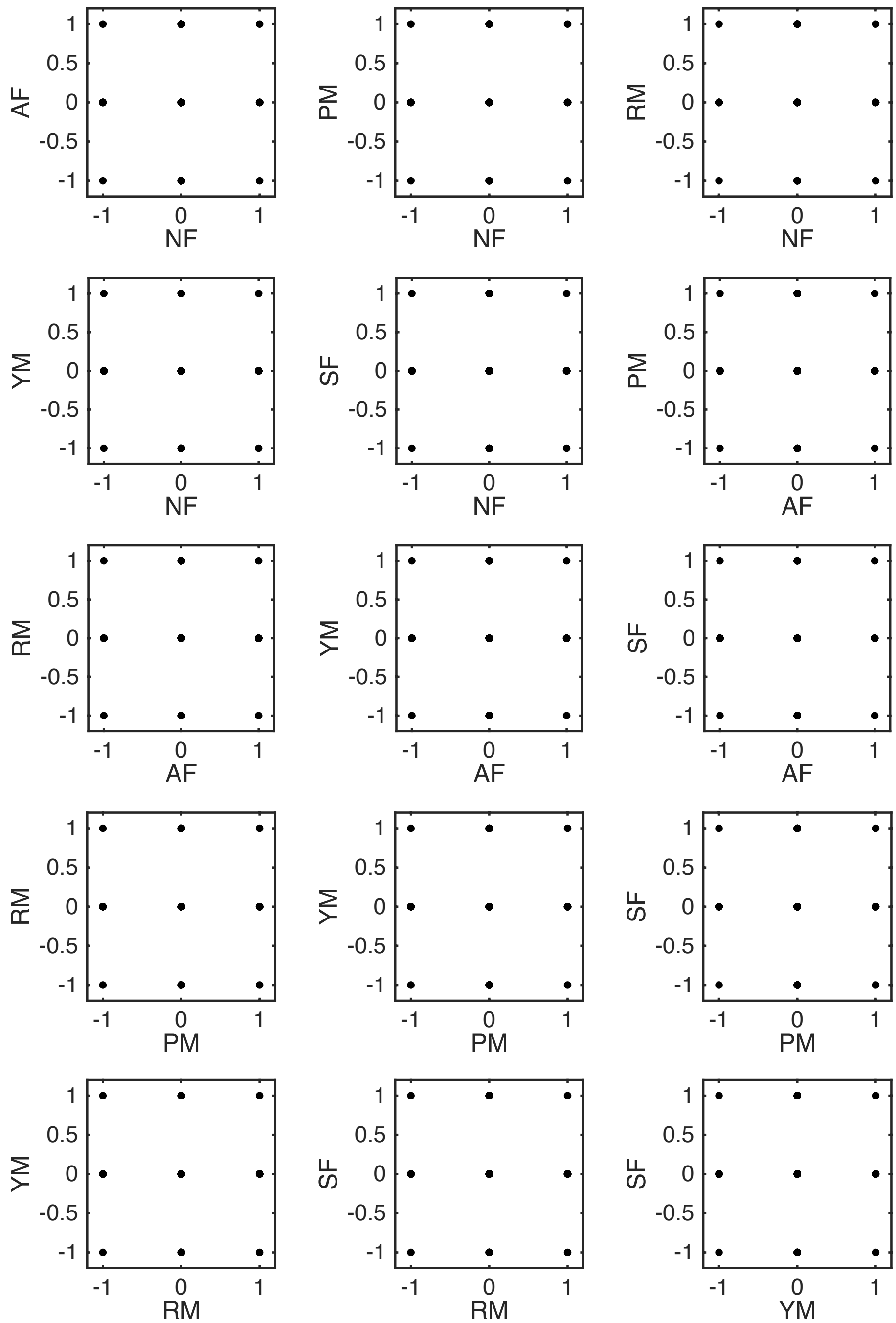

Figure 2. Box-Behnken Design (BBD) 2-Factor Load Plots

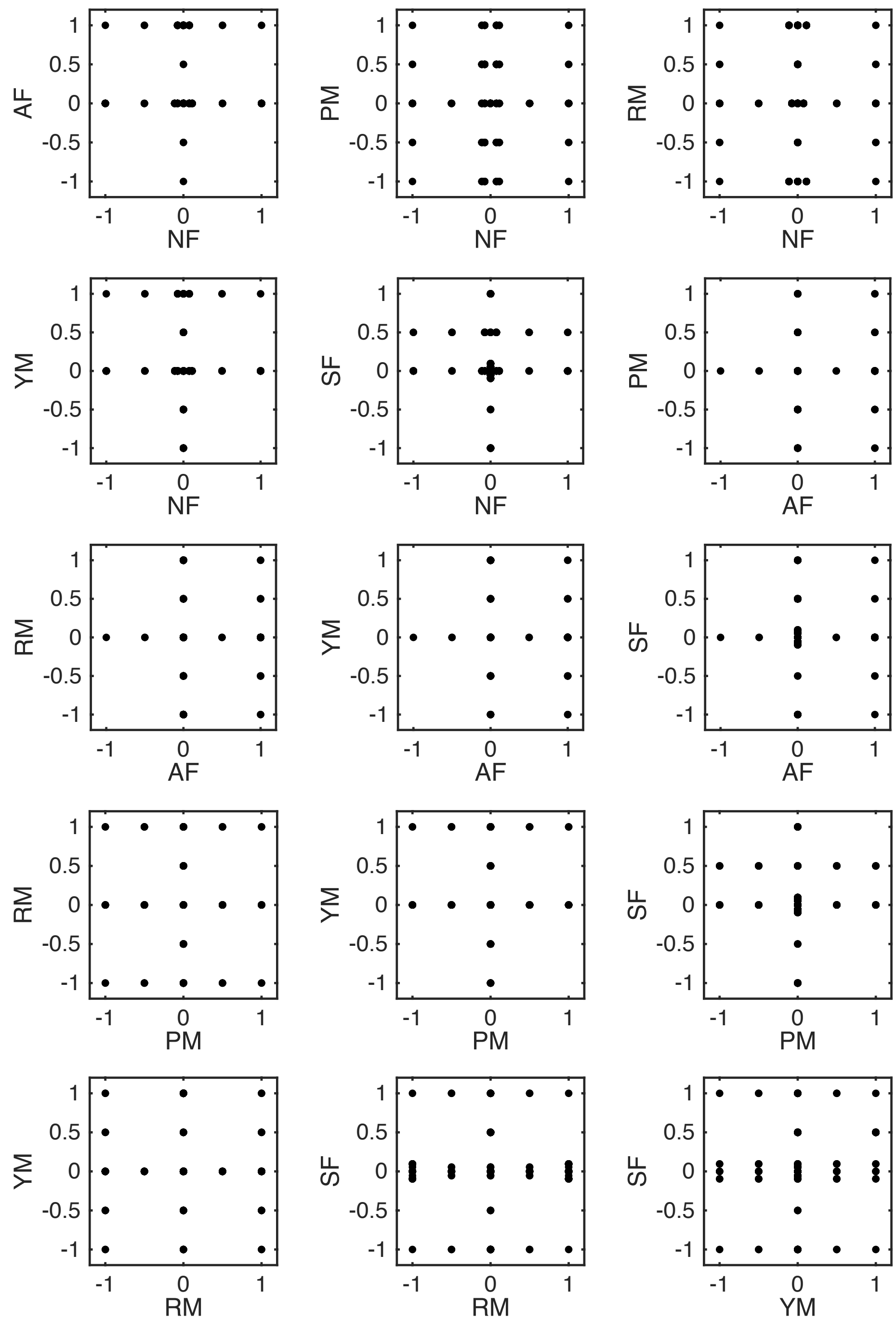

Figure 3. NASA LaRC 5-Point Design 2-Factor Load Plots

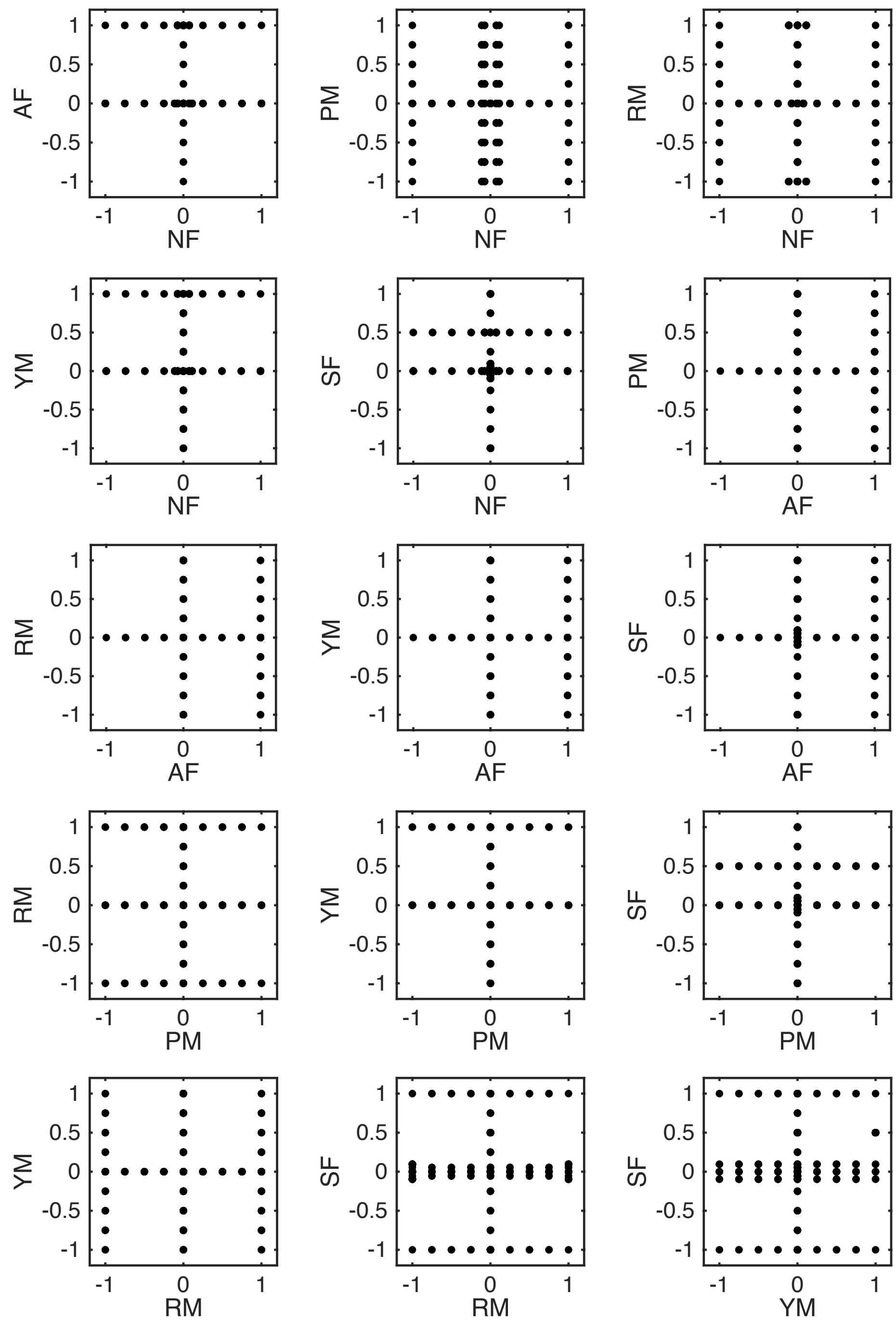

Figure 4. NASA LaRC 9-Point Design 2-Factor Load Plots

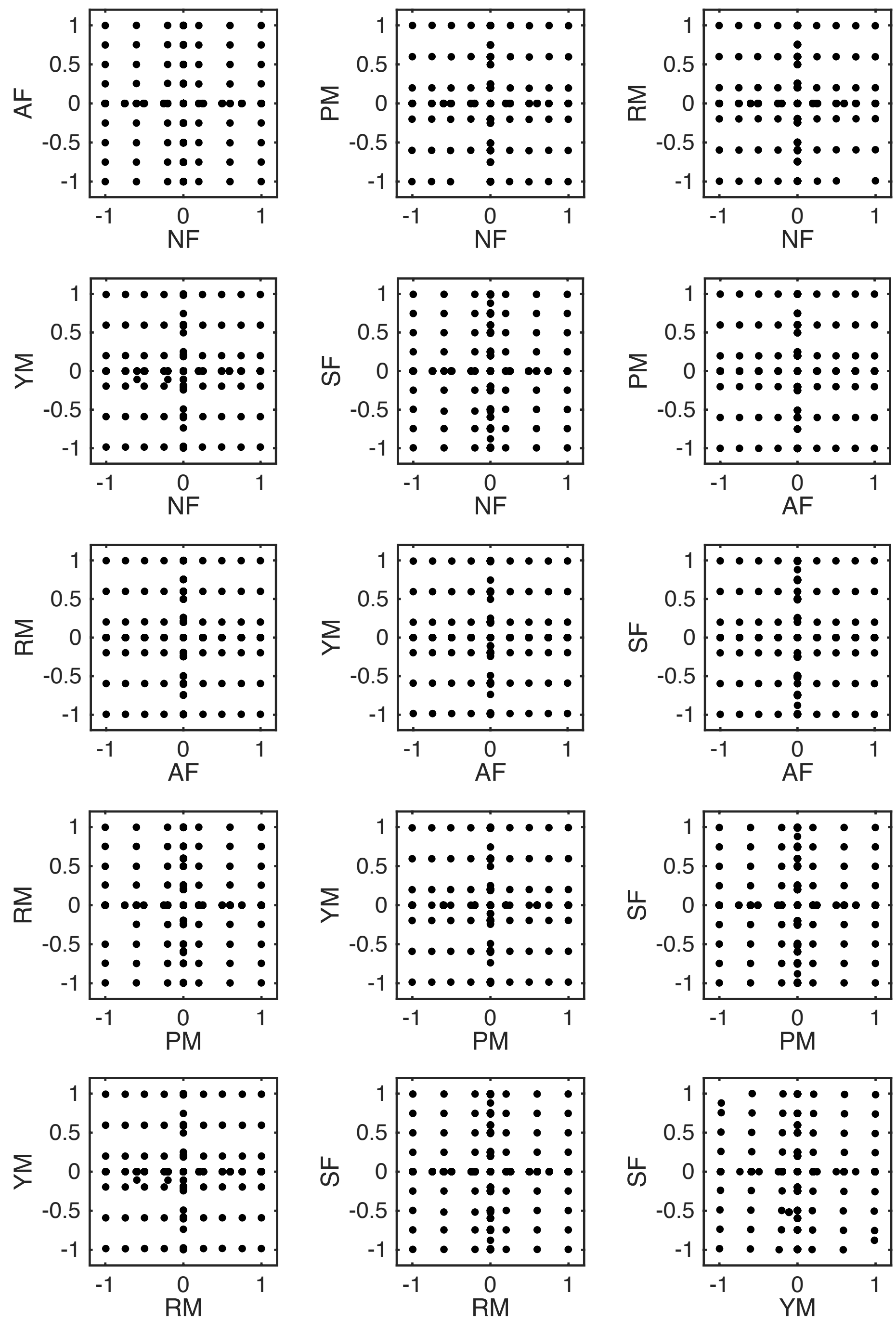

Figure 5. Triumph ABCS Design 2-Factor Load Plots

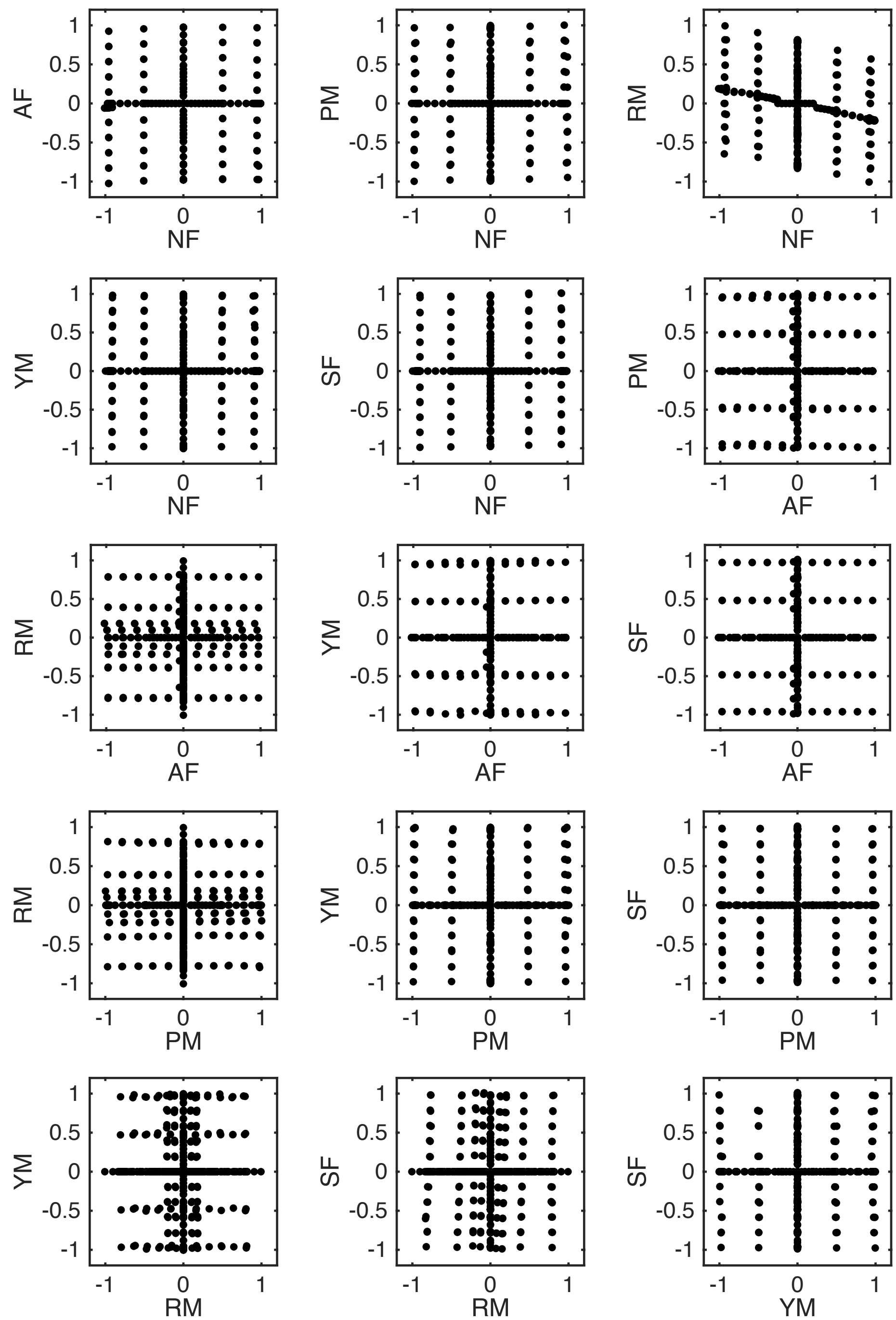

Figure 6. ETW Design 2-Factor Load Plots

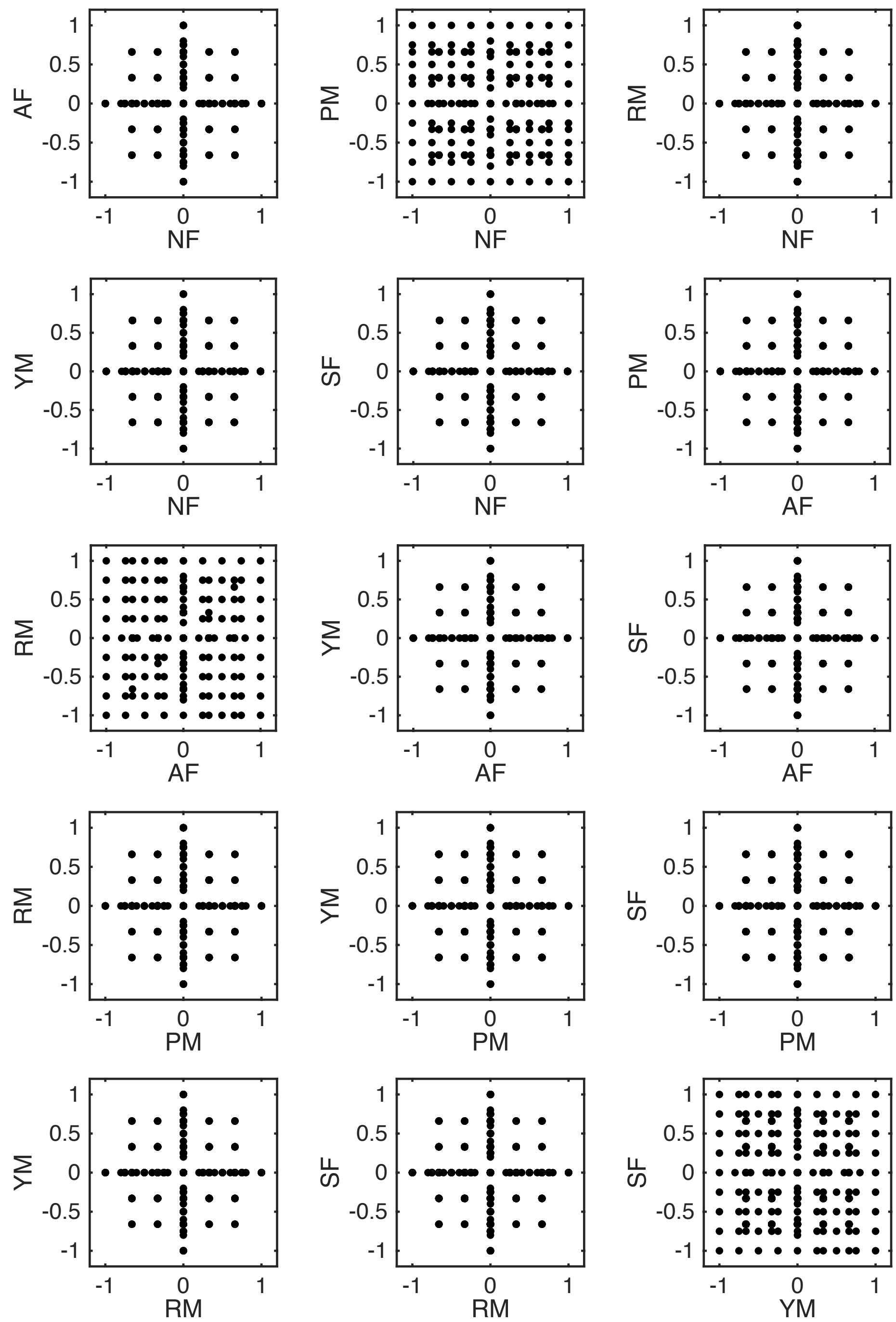

Figure 7. NASA Ames Design 2-Factor Load Plots