National Aeronautics and Space Administration

Confidence intervals for laboratory sonic boom annoyance tests

Jonathan Rathsam
Andrew Christian

Statistical Engineering Knowledge Exchange Workshop
Crystal City, Virginia
April 12, 2016

Acknowledgments

- Commercial Supersonic Technology Project
  - Jacob Klos, Alexandra Loubeau, Jerry Rouse, Kevin Shepherd
- Design Environment for Novel Vertical Lift Vehicles Subproject
  - Ran Cabell and Colin Theodore

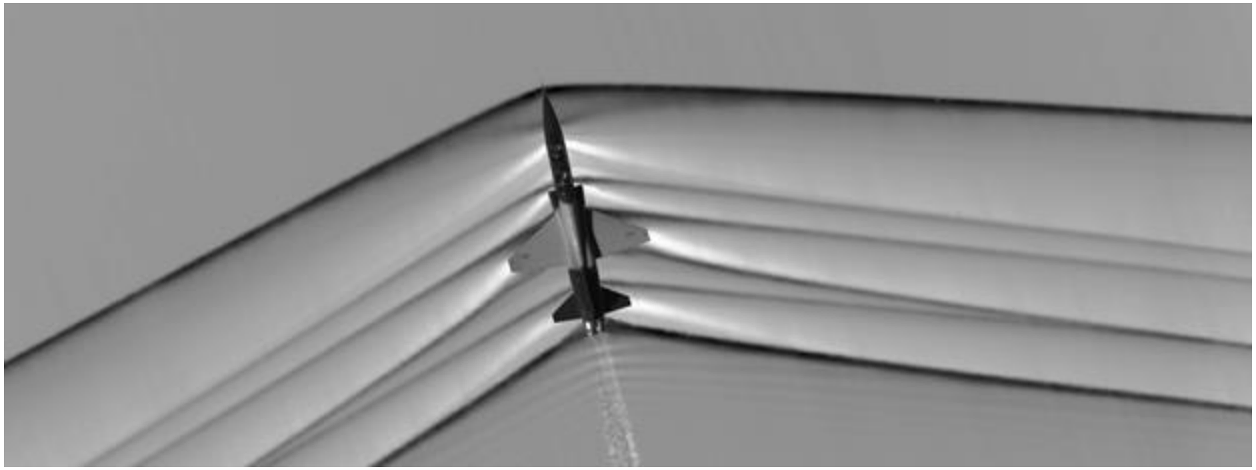
Outline

1. Sonic boom research
2. Annoyance caused by sonic boom vibrations
3. Confidence interval estimation methods
   a. Delta Method
   b. Bootstrap (Parametric and Non-Parametric)
   c. Bayesian Posterior Estimation
4. Results

Supersonic Flight

-Flying above speed of sound continuously produces shock wave

-Sound of shock wave is a sonic boom



-Business travelers, cargo shippers, and traveling public

-Market potential validated in numerous studies [Henne 2005]

-New US-led aircraft manufacturing sector

Historic sonic boom highlights

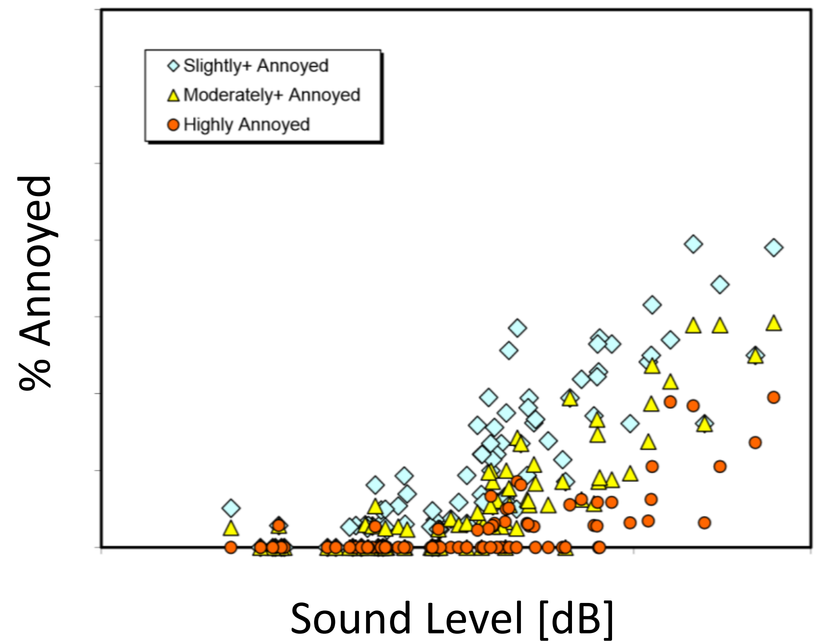
• 1947	Chuck Yeager first flies supersonically
• 1964	Sonic boom tests end early due to public complaints
• 1973	Supersonic flight forbidden over land
• 2003	Shaped Sonic Boom Demonstration
• 2016	NASA announces preliminary design of supersonic X-plane

Supersonic X-plane

Motivation

- Aircraft noise regulators (FAA, ICAO) considering allowing commercial supersonic flight

- Community annoyance prediction model
  -Link predicted booms to community annoyance
  -Support new regulations
  -Support aircraft designers

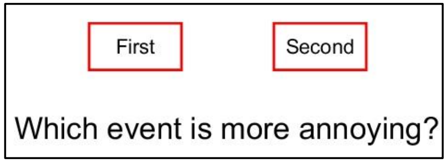


[Fidell, et al. 2012]

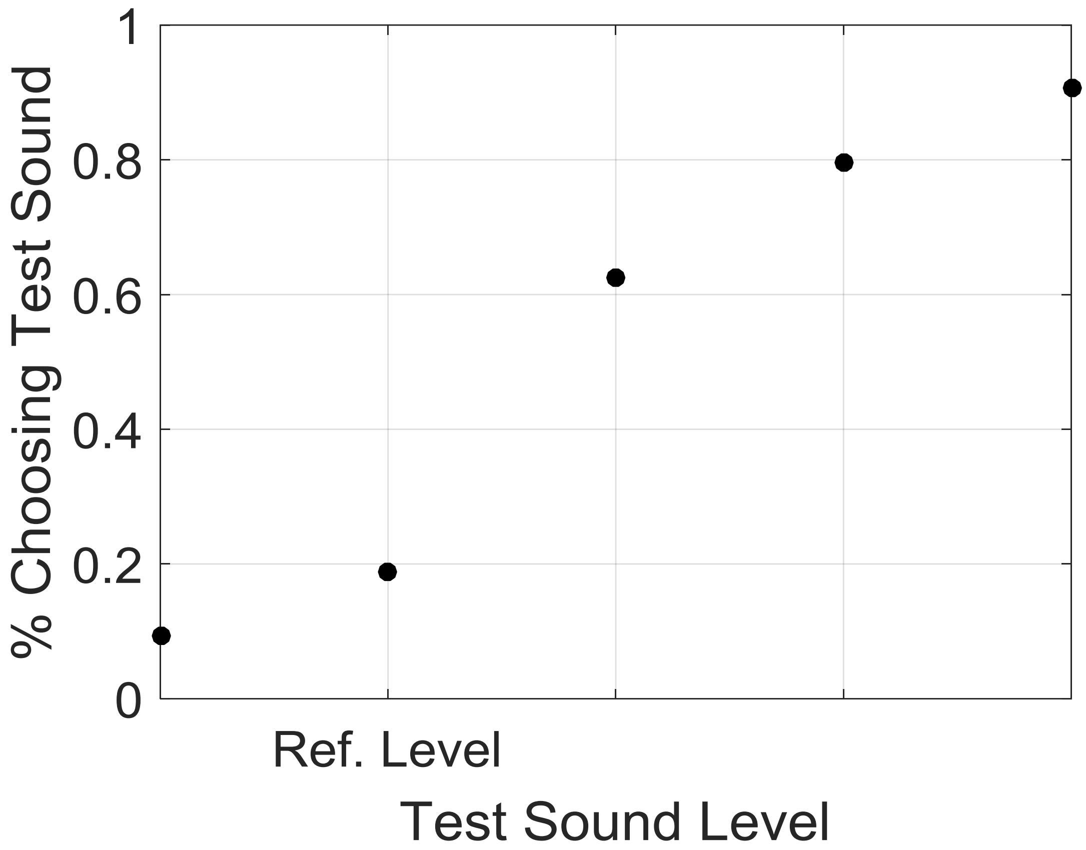
Laboratory study

- Is there a vibration penalty?
  - increment in sound level that yields same annoyance increment as realistic vibration
- If so, how great? (high and low vibration)

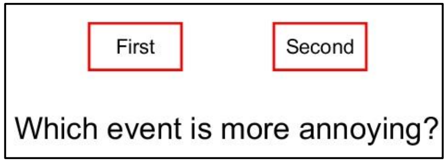
Test Method



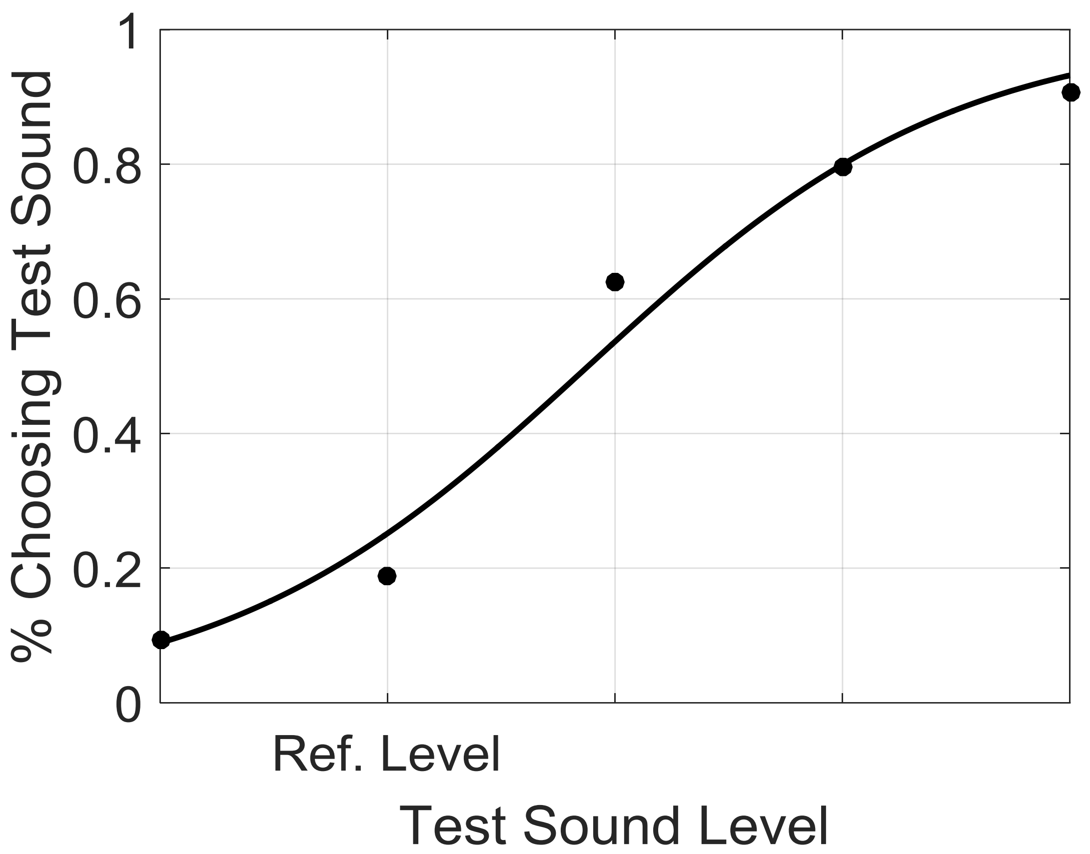
Reference contains sound *and* vibration



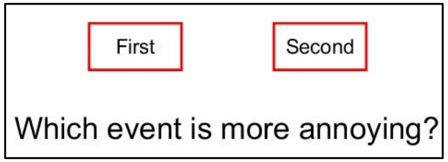
Test Method



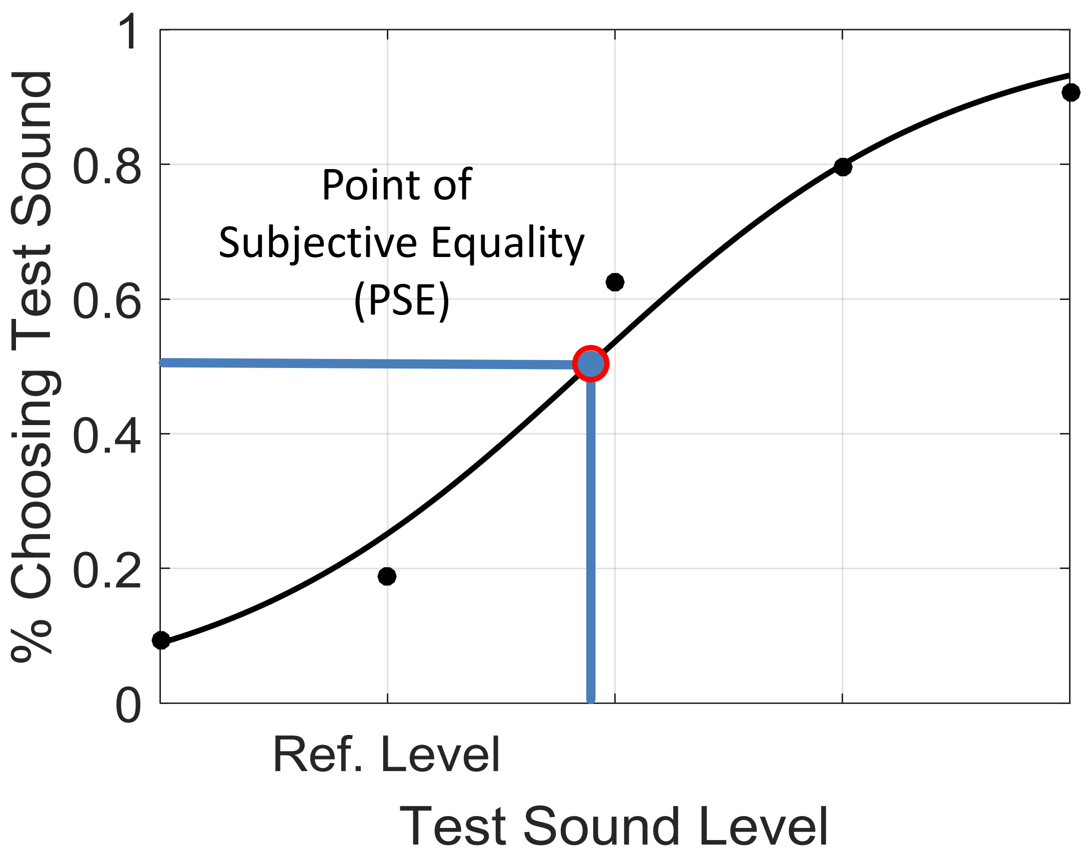
Reference contains sound *and* vibration



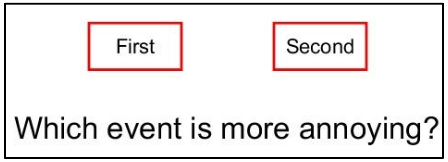
Test Method



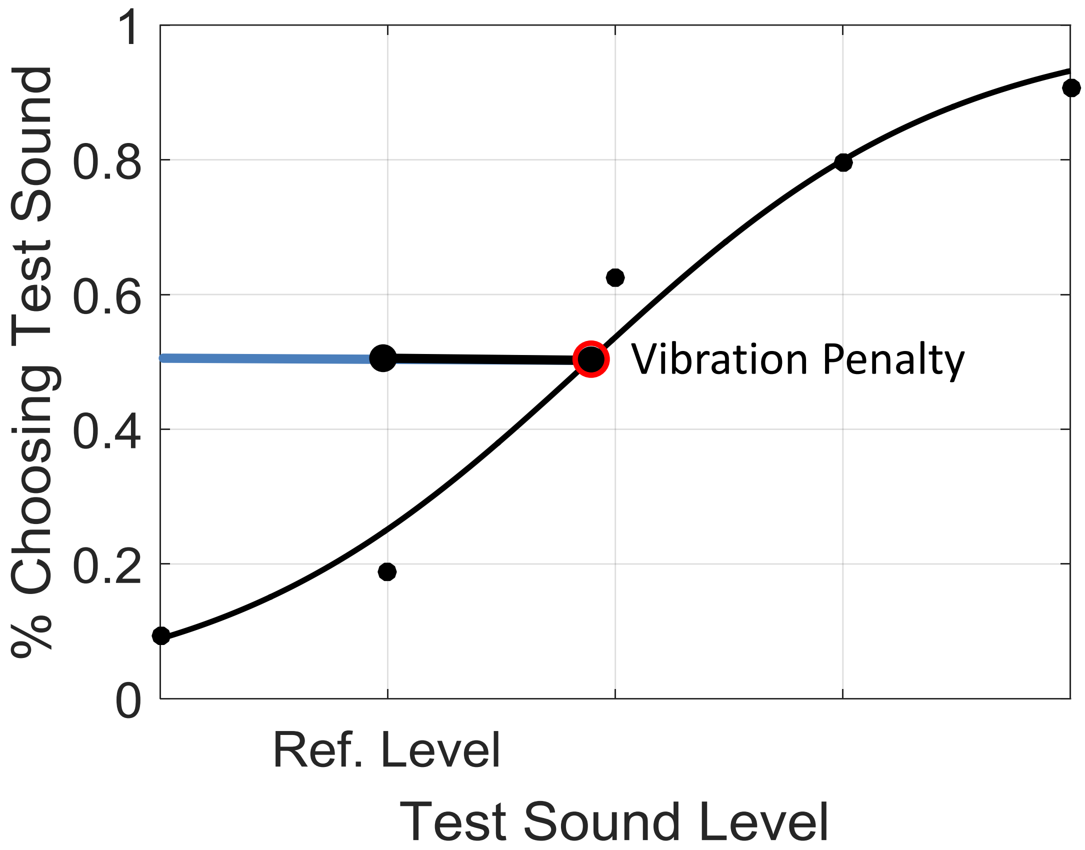
Reference contains sound *and* vibration



Test Method

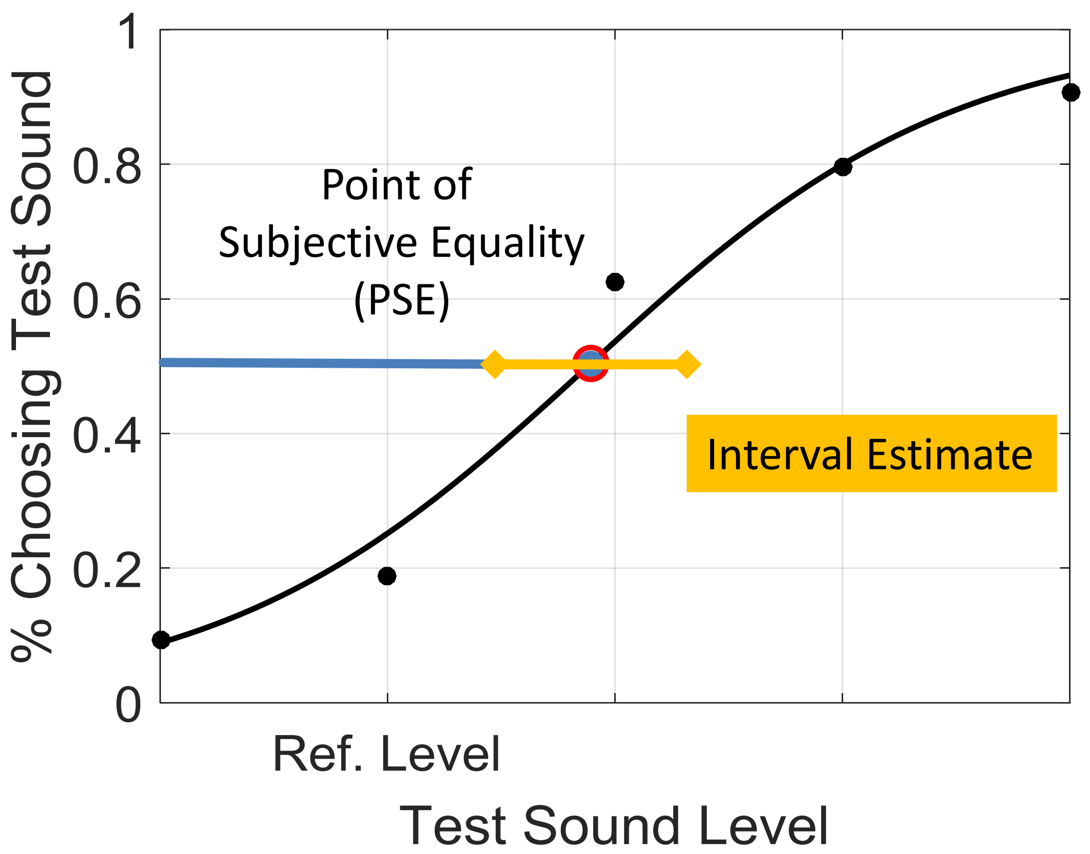


Reference contains sound *and* vibration



Test Method

Reference contains sound *and* vibration



Research Question

- What is most appropriate interval estimation technique?
  a. Delta Method
  b. Bootstrap: parametric
  c. Bootstrap: non-parametric
  d. Bayesian Posterior Estimation

  – Two research groups had same question

Delta Method: Theory

Logistic Regression Equation

$$\Pr(y_i = 1) = \frac{e^{\beta_0+\beta_1 x}}{1 + e^{\beta_0+\beta_1 x}}$$

Point of Subjective Equality (PSE)

$$\text{PSE} = \frac{-\beta_0}{\beta_1}$$

Taylor Series Approximation to Variance of PSE [Morgan 1992]

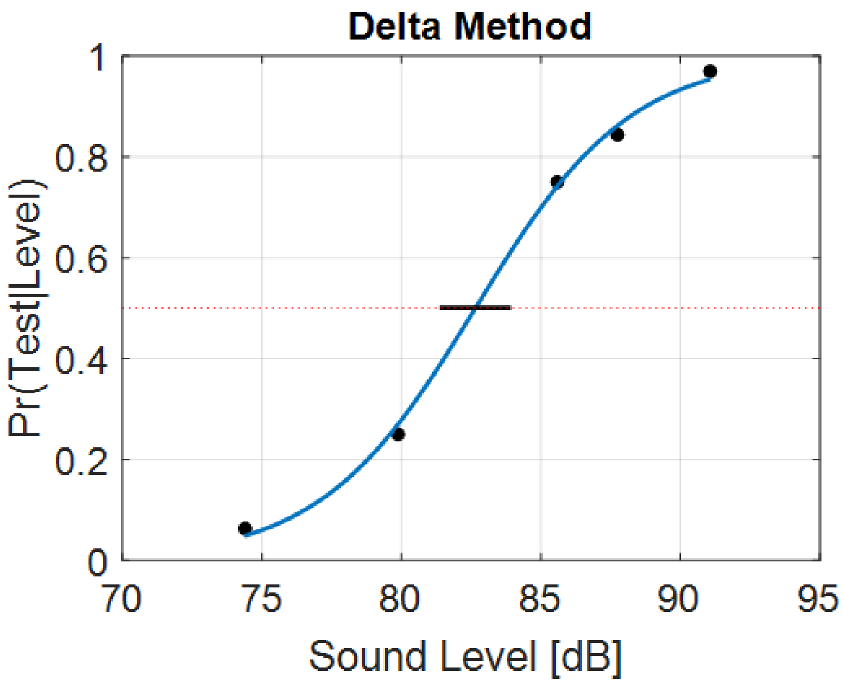
$$\text{Var (PSE)} = \frac{1}{\beta_1^2}[\text{Var}(\beta_0) + \text{PSE}^2 * \text{Var}(\beta_1) + 2 * \text{PSE} * \text{Cov}(\beta_0, \beta_1)]$$

Delta Method Confidence Interval

$$\text{PSE} \pm z_{\left(1-\frac{\alpha}{2}\right)}\sqrt{\text{Var}(PSE)}$$

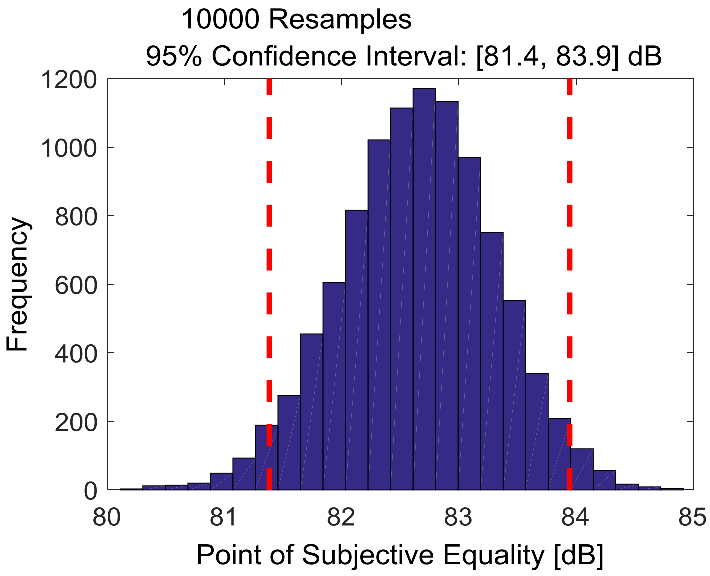
Delta Method: Application

- PSE = 82.6 dB
- 95% Conf. Interval = 81.3—83.9 dB
- Speed: 1 GLM fit
- Notes:
  - Closed form
  - Unknown failure modes



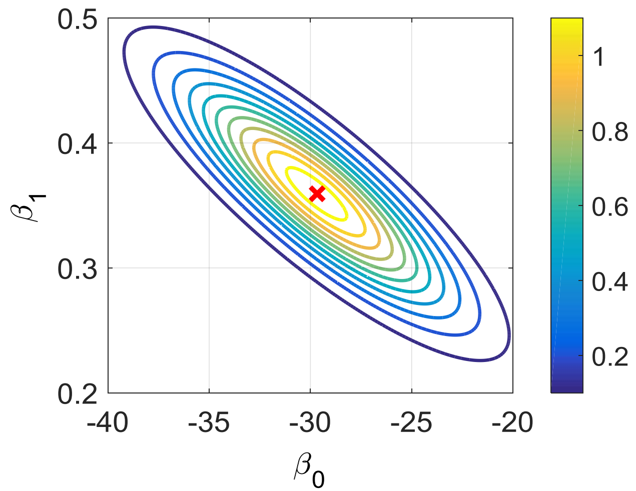
Bootstrap Analysis: Background

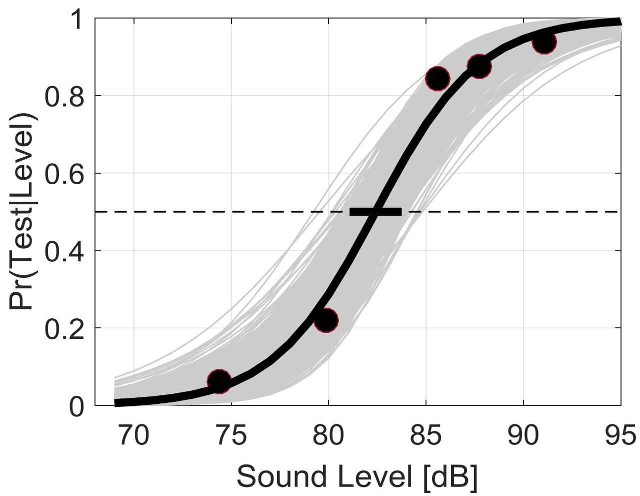
- Suppose we ran this test many times…
- Each subject of our test represents many similar subjects in the population
- Resample to simulate many experiments



Bootstrap Analysis: Parametric

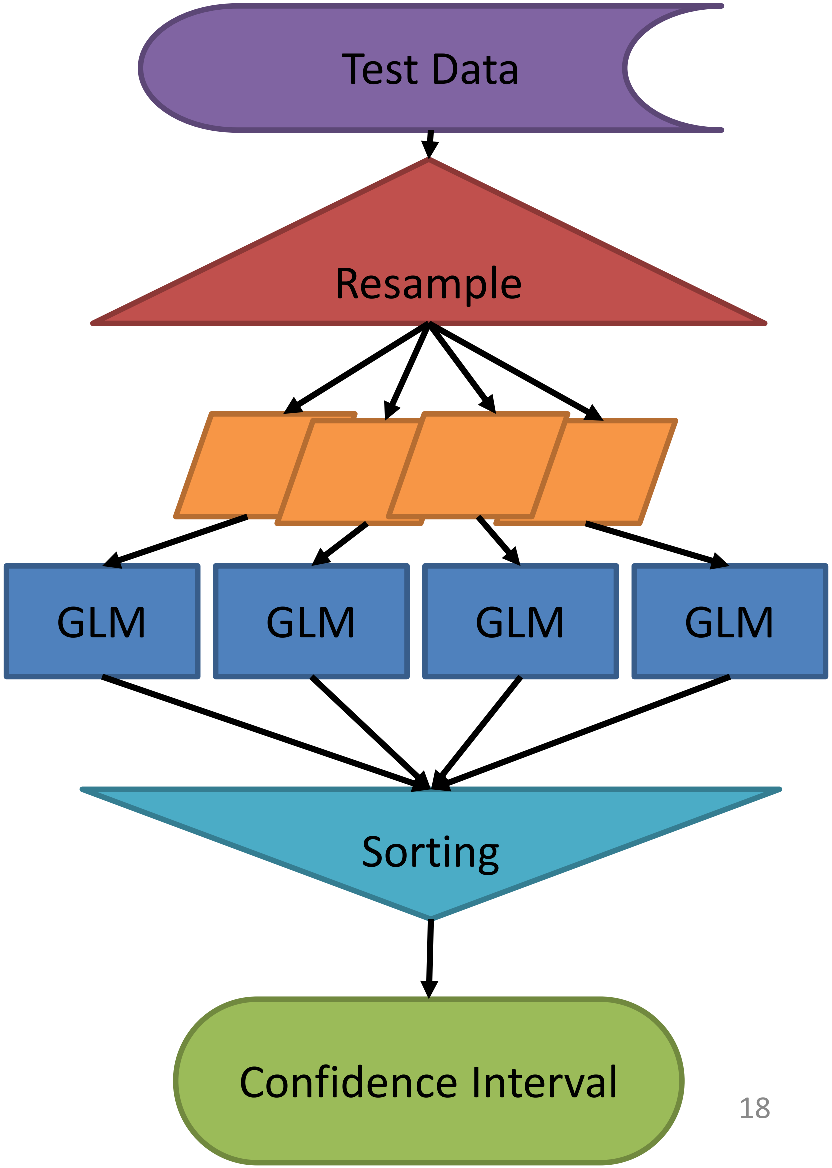
- Use GLM to fit data from single experiment
  - $<\beta_{0,}\beta_1>$
  - $Cov(\beta_{0,}\beta_1)$
- Resample from multivariate distribution





Bootstrap Analysis: Non-parametric

- Create new datasets by sampling with replacement from raw data
- For each new dataset, generate a PSE



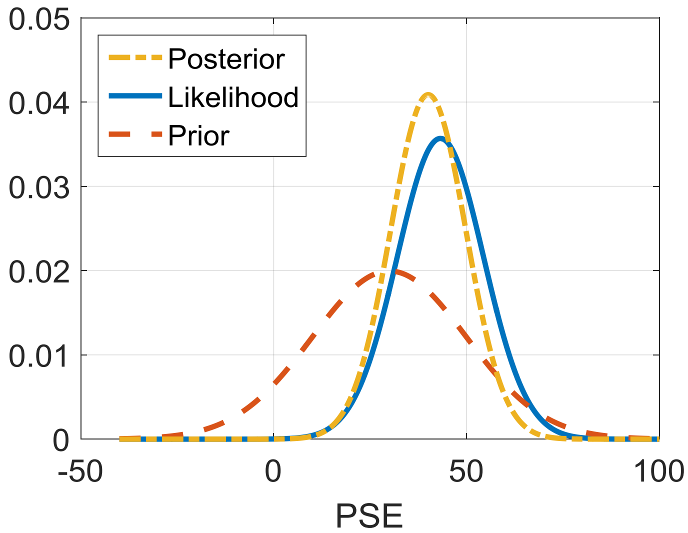
Results: Guidance Table

Method	*PSE*	*PSE Interval min—max*	*Longest Operation*	*Notes*
Delta	82.6	81.3—83.9	1 GLM fit (fastest)	•Closed form •Unknown failure modes
Bootstrap: Parametric	82.6	81.2—83.9	Sorting N resampled PSEs (2nd fastest)	•Resamples are normally distributed •Observable failure models (e.g. negative slope)
Bootstrap: Nonparametric	82.6	81.3—83.9	N GLM fits (slowest)	•Fewest assumptions •Not suitable for low-n binomial data

Bayesian Posterior Estimation

$$p(\beta_0, \beta_1 | Data) \propto L(Data | \beta_0, \beta_1) * p(\beta_0, \beta_1)$$

Posterior Likelihood Prior



- Uses all data in each calculation
- Previously analytical only
- Markov Chain Monte Carlo sampling methods evaluate posterior for arbitrary likelihoods and priors
- Evaluated in R [Kruschke 2014]

Results: Guidance Table

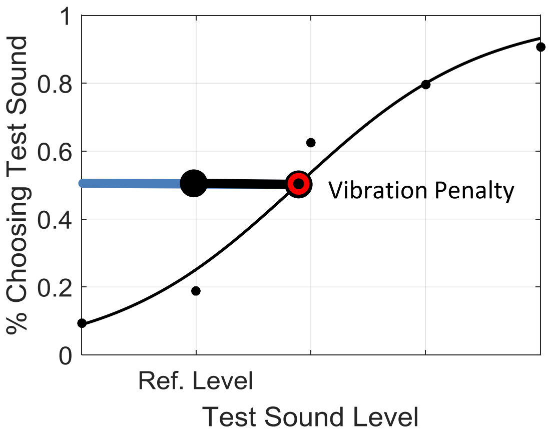
Method	*PSE*	*PSE Interval min—max*	*Longest Operation*	*Notes*
Delta	82.6	81.3—83.9	1 GLM fit (fastest)	•Closed form •Unknown failure modes
Bootstrap: Parametric	82.6	81.2—83.9	Sorting N resampled PSEs (2nd fastest)	•Resamples are normally distributed •Observable failure models (e.g. negative slope)
Bootstrap: Nonparametric	82.6	81.3—83.9	N GLM fits (slowest)	•Fewest assumptions •Not suitable for low-n binomial data
Bayesian Posterior Estimation	82.6	81.4—83.9	N likelihood evaluations (2nd slowest)	•Most flexible (can include prior information) •Diagnostics needed to ensure proper MCMC performance

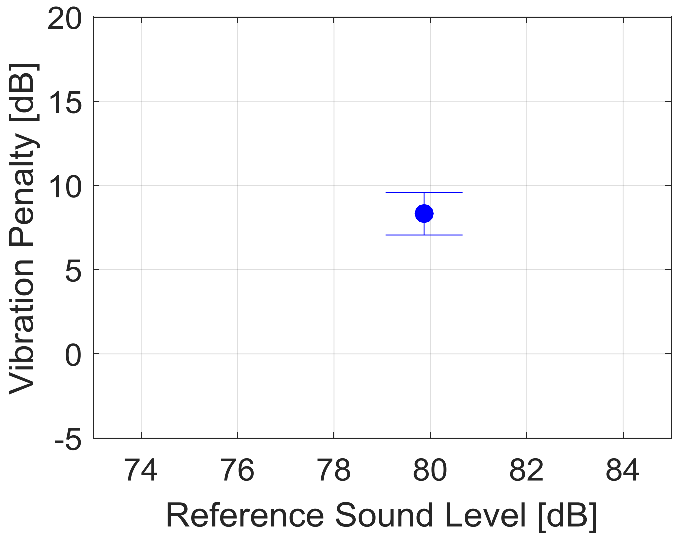
Research questions revisited (1)

- What is most appropriate interval estimation technique among four standard solutions?
  - -Results from all methods are functionally equivalent
  - -Delta Method used because fastest to calculate
  - -BPE is recommended because it has fewest assumptions

- Return to sonic boom annoyance

Research Questions Revisited (2)

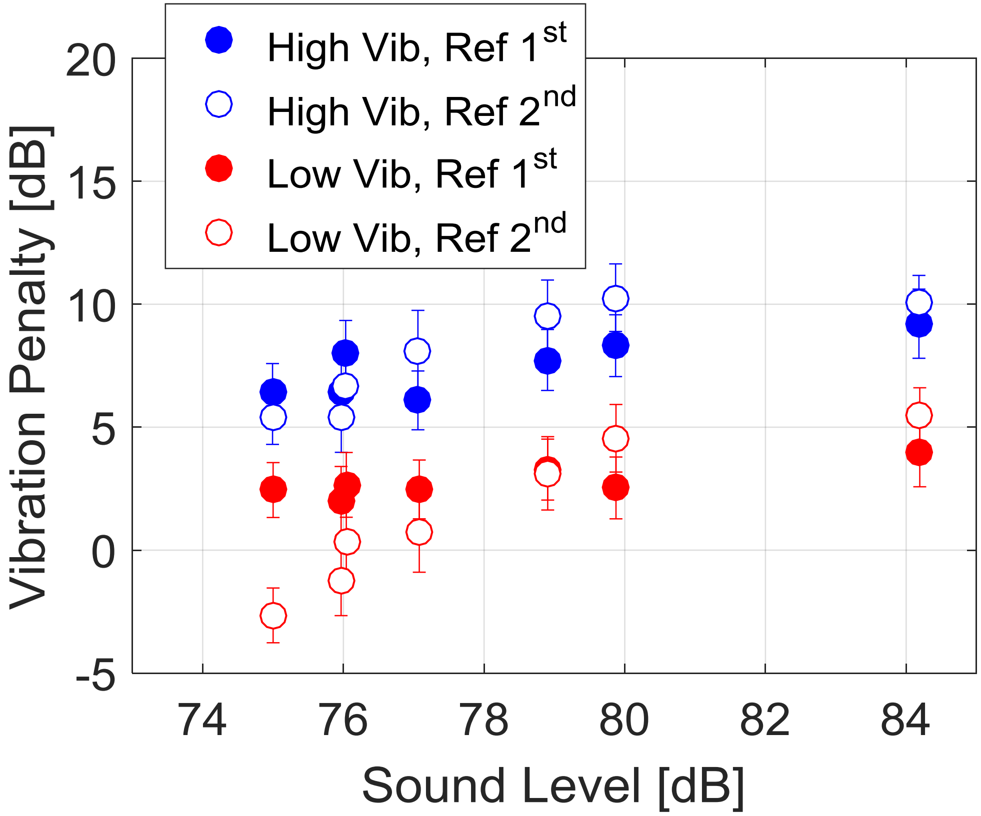




Research questions revisited (2)

- Is there a vibration penalty? Yes

0 – 5 dB for low vibration and 5 – 10 dB for high vibration



Thank You

References:

- Fidell, S. et al. "Pilot Test of a Novel Method for Assessing Community Response to Low-Amplitude Sonic Booms" NASA/CR-2012-217767 (2012).
- Henne, P.A. "Case for Small Supersonic Civil Aircraft" *Journal of Aircraft* 42 (3) 765-774 (2005).
- Kruschke, J. <u>Doing Bayesian Data Analysis: A Tutorial with R, JAGS, and Stan</u> Cambridge: Academic Press (2014).
- Morgan, B.J.T. <u>Analysis of Quantal Response Data</u> London: Chapman & Hall (1992).

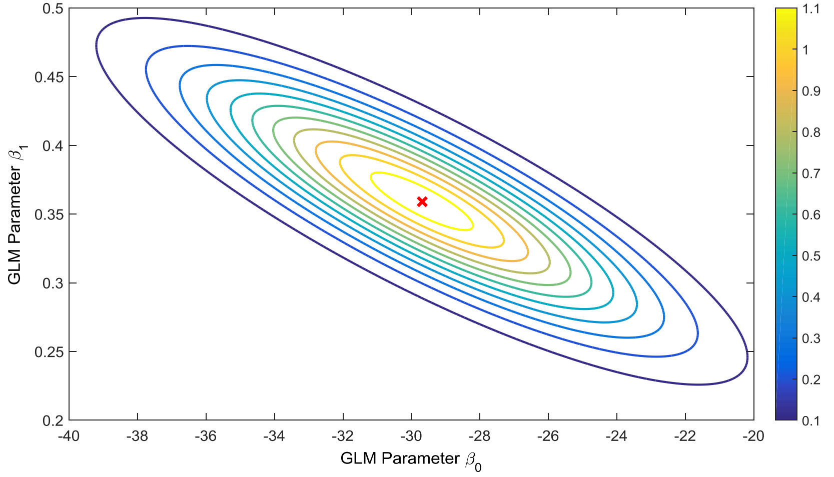
Backup Slides

Bootstrap: Paramteric

The GLM-Logit model returns two parameters:

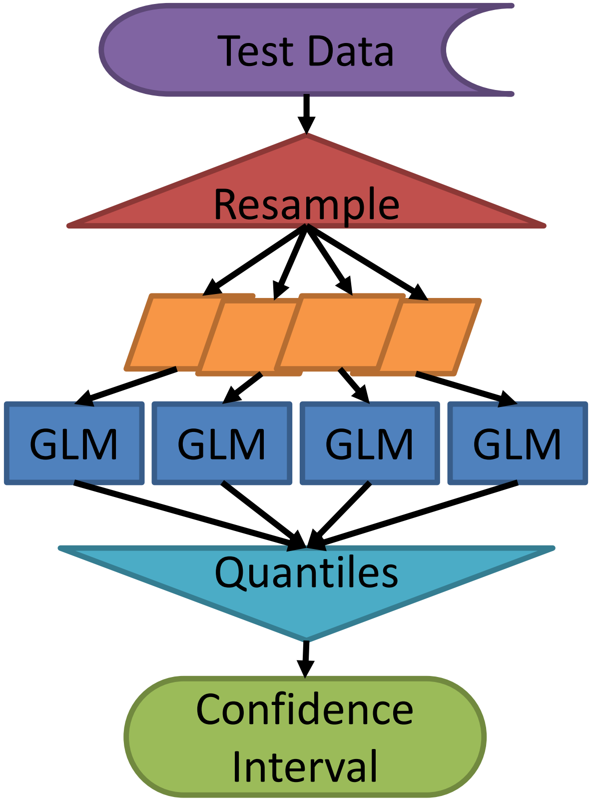
- $<\beta_{0,}\beta_1>$ -- ML estimators of the logit <u>parameters</u>
- Cov(β) -- Covariance of these <u>parameters</u>:

Resample from resulting multivariate normal distribution

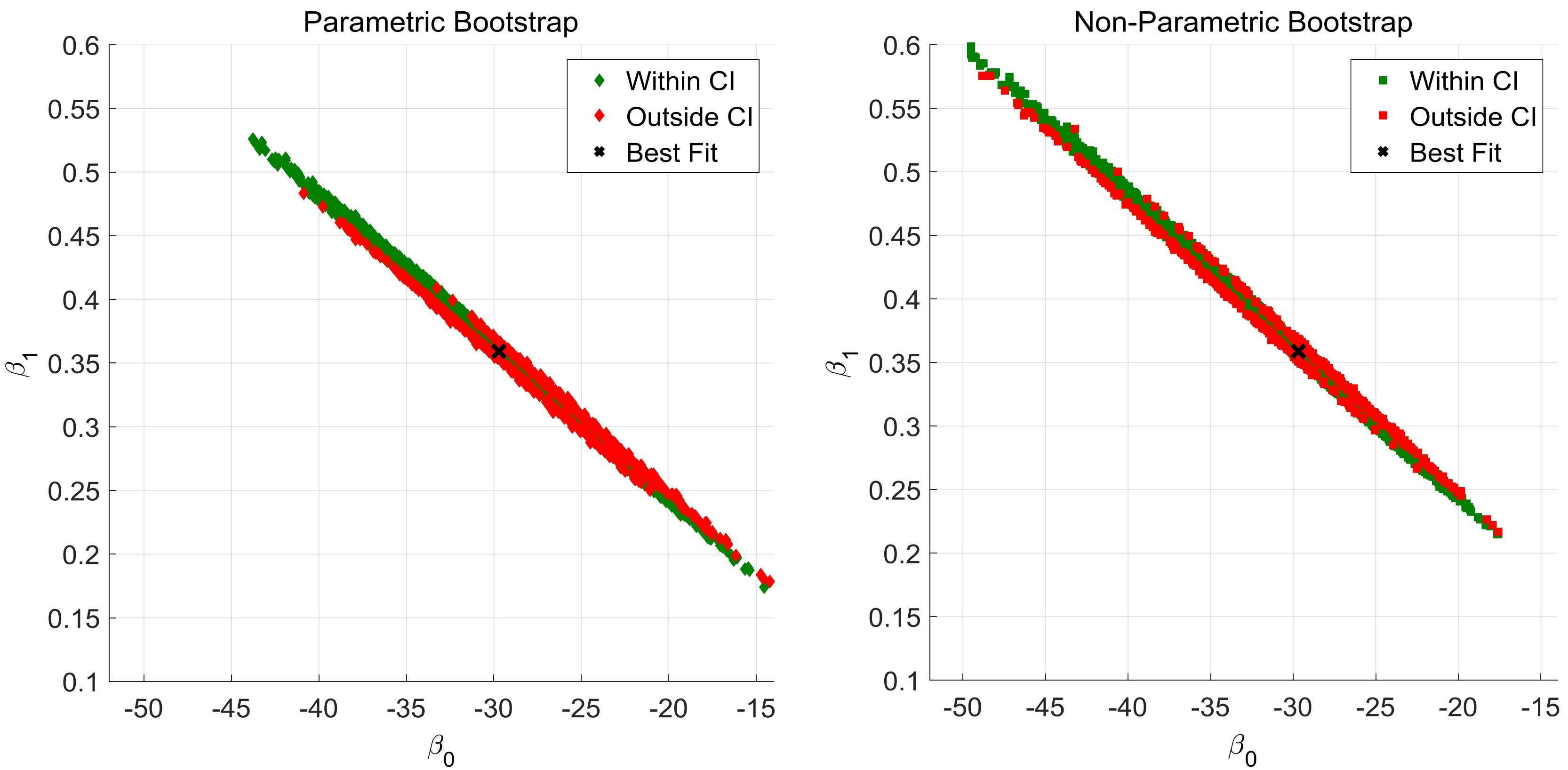


Bootstrap: Non-parametric

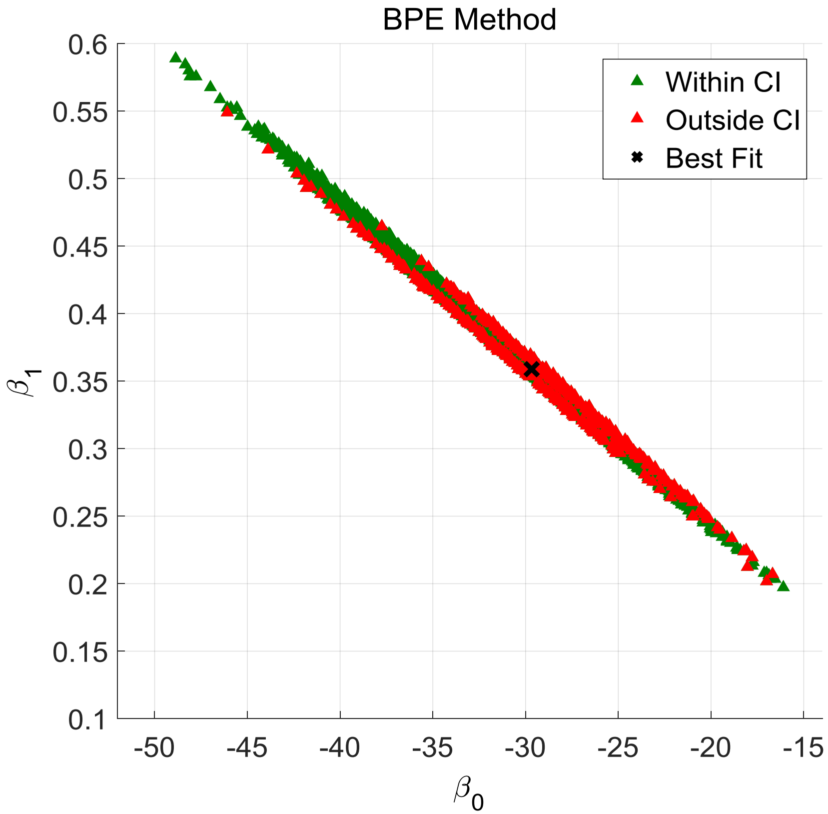
- Create resampled data sets by drawing from the initial raw data (with replacement).
- Run the GLM on each resampled set to produce the ML Logit fit for that set (discard the covariance).
- Use these fits to generate the resampled PSEs.

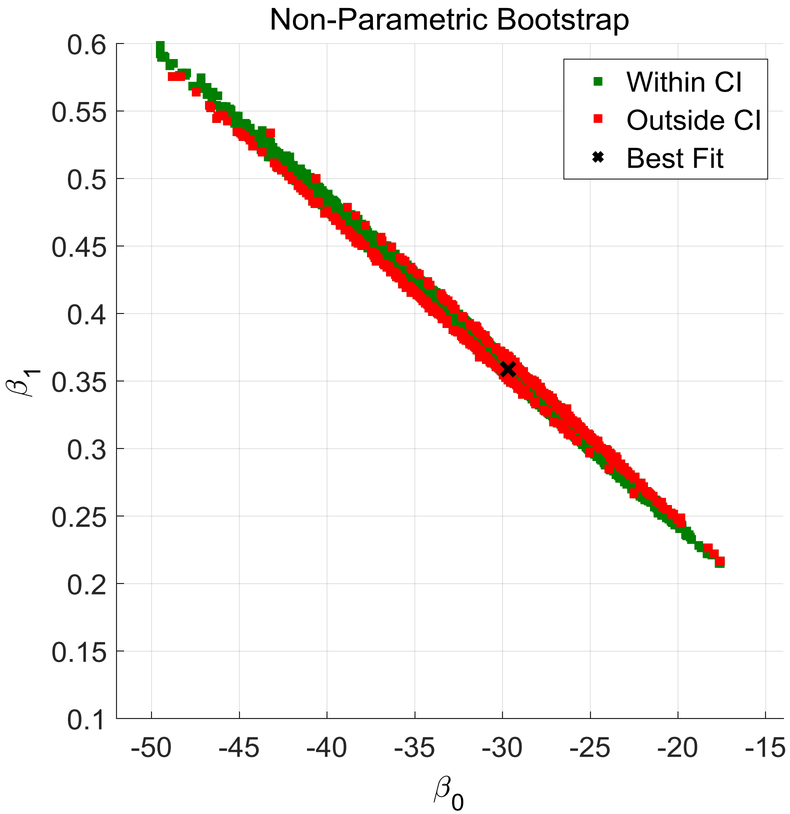


Point Clouds



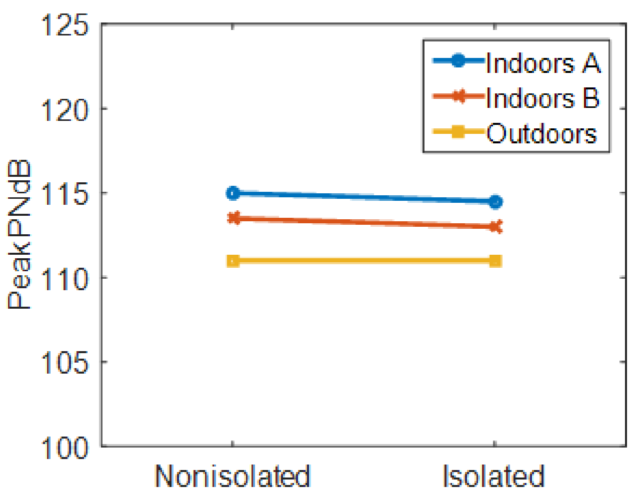
Point Clouds



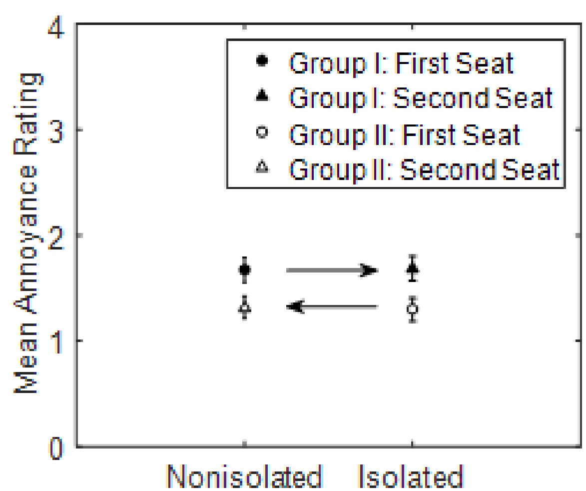


Are vibrations from a sonic boom annoying?

- "…sonic booms experienced inside were less acceptable than those experienced outside presumably because of …the rattling and shaking of items within the structure, and the *actual vibration of the structure itself.*" [Nixon and Borsky 1966]



Kryter, et al. 1968

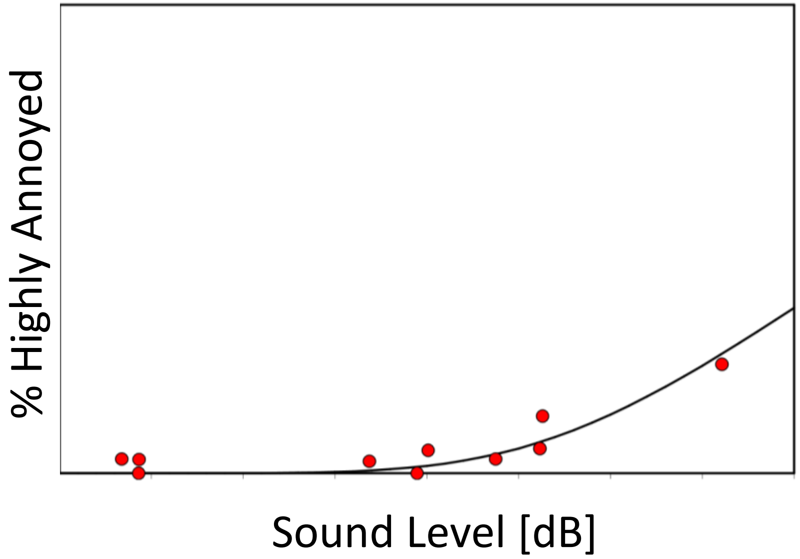


Rathsam, et al. 2014

Research Motivation

- Aircraft noise regulators (FAA, ICAO) considering allowing commercial supersonic flight

- Community annoyance prediction model
  -**Link** predicted booms to community annoyance
  -**Support** new regulations
  -**Support** aircraft designers



Fidell, et al. 2012