Comparison of Turbulent Thermal Diffusivity and Scalar Variance Models

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- Model Formulations
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Introduction

- Turbulent heat flux
  - Defined by velocity-temperature (or enthalpy) correlation terms that appear in the RANS energy equation.

$$q_j^T \equiv \overline{\rho u_j'' h''}$$

  - Often represented using an eddy diffusivity approach:

$$q_j^T \approx -\bar{\rho}\alpha_T \frac{\partial \tilde{h}}{\partial x_j} = -\frac{\mu_T}{Pr_T}\frac{\partial \tilde{h}}{\partial x_j}$$

- Turbulent Prandtl number
  - Defined as the ratio of the turbulent eddy viscosity to thermal diffusivity:

$$Pr_T \equiv \frac{\mu_T}{\bar{\rho}\alpha_T}$$

Motivation

- Turbulent Prandtl number variation
  - Nominally 0.9 in boundary layers, higher near the wall.
  - As low as 0.5 in free shear layers.
  - In general, it varies throughout the flow field.
- Simulation sensitivity to the choice of constant Pr_t
  - Wall heat transfer problems.
  - Combustion applications.
  - Thermal decay of heated jets.
- Jet noise
  - Differences observed in noise levels between cold and hot jets.
  - Additional acoustic analogy model source term related to the thermal variance.

Model Formulations

- Constant Pr_t.
- 0-Eq (algebraic) expression for $\bar{\rho}\alpha_T$.
- 2-Eq thermal variance ($\overline{\theta^2}$) & dissipation rate (ϵ_θ) transport models, which then provide $\bar{\rho}\alpha_T$.
- Scalar flux ($\overline{\rho u_j'' h''}$) transport models.
- Direct statistical output from LES/DNS.

0-Eq Models

- WC: Wassel & Catton (1973)

$$Pr_T = \frac{C_3}{C_1 Pr}\left[1 - \exp\left(\frac{-C_4}{(\mu_T/\mu_L)}\right)\right]\left[1 - \exp\left(\frac{-C_2}{(\mu_T/\mu_L)Pr}\right)\right]^{-1}$$

- KC: Kays & Crawford (1993)

$$Pr_T = \left\{\frac{1}{2Pr_{T_\infty}} + \frac{CPe_T}{\sqrt{Pr_{T_\infty}}} - (CPe_T)^2\left[1 - \exp\left(\frac{-1}{CPe_T\sqrt{Pr_{T_\infty}}}\right)\right]\right\}^{-1}$$

$$Pe_T = \frac{\mu_T}{\mu_L}Pr$$

2-Eq Models

$$\alpha_T = C_\lambda f_\lambda k \, \tau_m$$

$$\bar{\rho}\frac{D\overline{\theta^2}}{Dt} = \frac{\partial}{\partial x_j}\left[\bar{\rho}\left(\alpha_L + \frac{\alpha_T}{\sigma_\theta}\right)\frac{\partial k_t}{\partial x_j}\right] + 2\mathcal{P}_\theta - 2\rho\epsilon_\theta$$

$$\bar{\rho}\frac{D\epsilon_\theta}{Dt} = \frac{\partial}{\partial x_j}\left[\bar{\rho}\left(\alpha_L + \frac{\alpha_T}{\sigma_{\epsilon_\theta}}\right)\frac{\partial \epsilon_\theta}{\partial x_j}\right] + \left[C_{d1}f_{d1}\frac{\epsilon_\theta}{\overline{\theta^2}} + C_{d2}f_{d2}\frac{\epsilon}{k}\right]\mathcal{P}_\theta + C_{d3}f_{d3}\frac{\epsilon}{k}\left(\frac{\mathcal{P}}{\bar{\rho}\epsilon}\right)\bar{\rho}\epsilon_\theta$$
$$- \left[C_{d4}f_{d4}\frac{\hat{\epsilon}_\theta}{\overline{\theta^2}} + C_{d5}f_{d5}\frac{\hat{\epsilon}}{k}\right]\bar{\rho}\epsilon_\theta + L_{\epsilon_\theta}$$

$$L_{\epsilon_\theta} = f_{\epsilon_\theta}\bar{\rho}\left[(C_{d4} - 4)\frac{\hat{\epsilon}_\theta}{\overline{\theta^2}}\epsilon_\theta + C_{d5}\frac{\hat{\epsilon}}{k}\epsilon_\theta - \frac{\left(\epsilon_\theta^*\right)^2}{\overline{\theta^2}} + (2 - C_{d1} - C_{d2}Pr)\frac{\epsilon_\theta}{\overline{\theta^2}}\frac{\mathcal{P}_{k_\theta}^*}{\bar{\rho}}\right]$$

$$\mathcal{P} = \tau_{ij}^T\frac{\partial \widetilde{u}_i}{\partial x_j} \approx \mu_T\frac{\partial \widetilde{u}_i}{\partial x_j}\frac{\partial \widetilde{u}_i}{\partial x_j}$$

$$\mathcal{P}_\theta = \bar{\rho}\alpha_T\frac{\partial \widetilde{\Theta}}{\partial x_j}\frac{\partial \widetilde{\Theta}}{\partial x_j}$$

$$\mathcal{P}_\theta^* = \bar{\rho}\alpha_T\frac{\partial \widetilde{\Theta}}{\partial x}\frac{\partial \widetilde{\Theta}}{\partial x}$$

$$\hat{\epsilon} = \epsilon - 2\nu_L\left(\frac{\partial k}{\partial y}\right)^2 f_{\hat{\epsilon}}$$

$$\hat{\epsilon}_\theta = \epsilon_\theta - \alpha_L\left(\frac{\partial \overline{\theta^2}}{\partial y}\right)^2 f_{\hat{\epsilon}_\theta}$$

$$\hat{\epsilon}_\theta^* = \epsilon_\theta - \frac{\alpha_L\overline{\theta^2}}{y^2}f_{\hat{\epsilon}_\theta^*}$$

2-Eq Models

- Turbulent velocity timescale $\tau_u = k/\epsilon$
- Turbulent thermal timescale $\tau_\theta = \overline{\theta^2}/2\epsilon_\theta$
- Turbulent timescale ratio $R = \tau_\theta/\tau_u$
- Mixed timescale $\tau_m = \tau_u^l \tau_\theta^m$
  - Geometric Average

    $$\tau_m = \sqrt{\tau_u \tau_\theta} = \tau_u\sqrt{2R}$$
  - Harmonic Average

    $$\tau_m = \frac{2}{1/\tau_u + 0.5/\tau_\theta} = \frac{\tau_u 2R}{R+0.5}$$
  - Nagano, Tagawa, Tsuji (1991)

    $$\tau_m = \frac{\tau_\theta^2}{\tau_u} = \tau_u(2R)^2$$

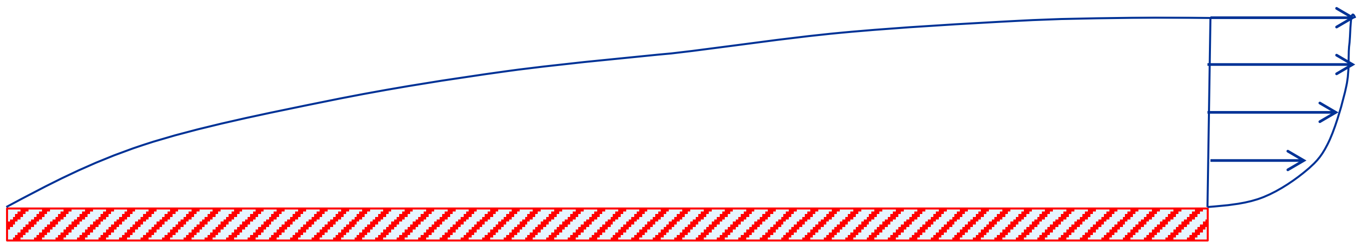
2-Eq Models

- Four models examined:
  - AKN: Abe, Kondoh, & Nagano (1995)
  - DWX: Deng, Wu, & Xi (2000)
  - SSZ: Sommer, So, & Zhang (1993)
  - BCD: Brinckman, Calhoon, & Dash (2007)

Results

Heated Boundary Layer

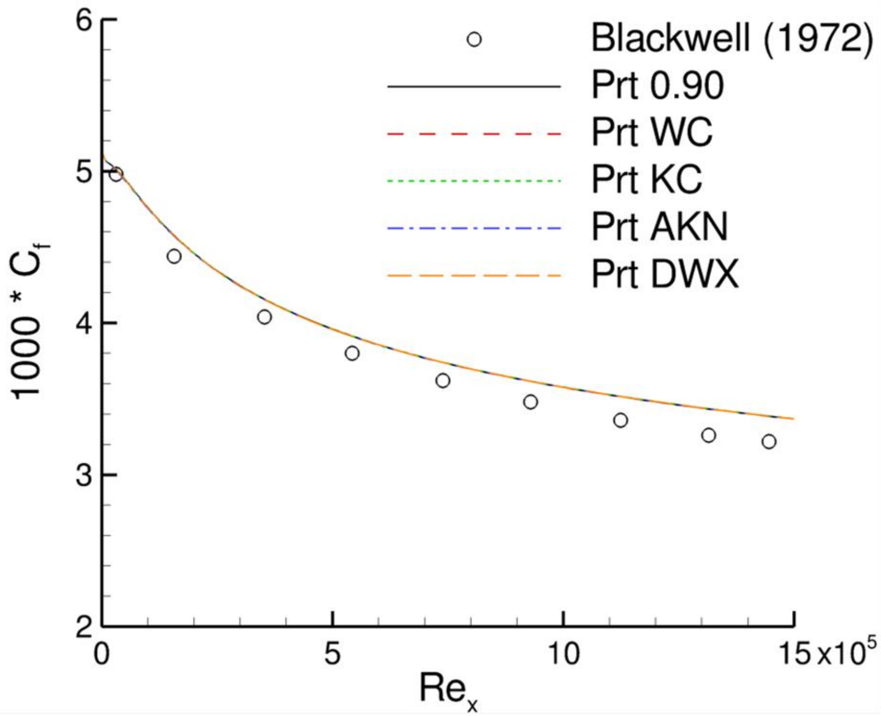
- Constant wall temperature applied from leading edge.
  - Thermal & momentum boundary layers develop together.



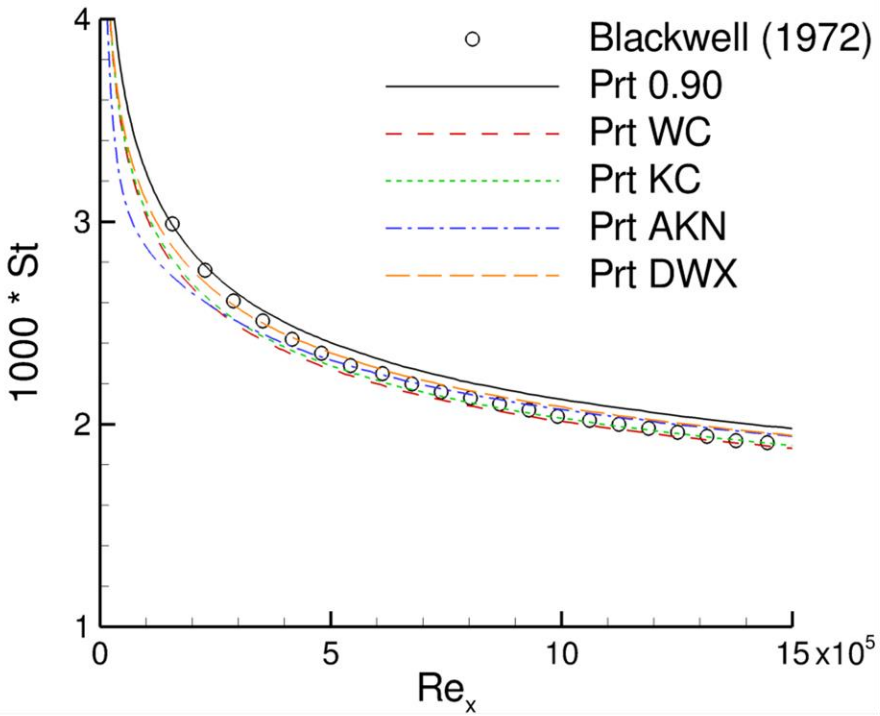
- Blackwell, Kays, & Moffat (1972)
  - U=9.65 m/s, ΔT=14 K
- Gibson, et al (1982,1984)
  - U=22.3 m/s, ΔT=14 K
- Subramanian & Antonia (1981)
  - U=8.44 m/s, ΔT=14 K

Heated Boundary Layer

Skin Friction

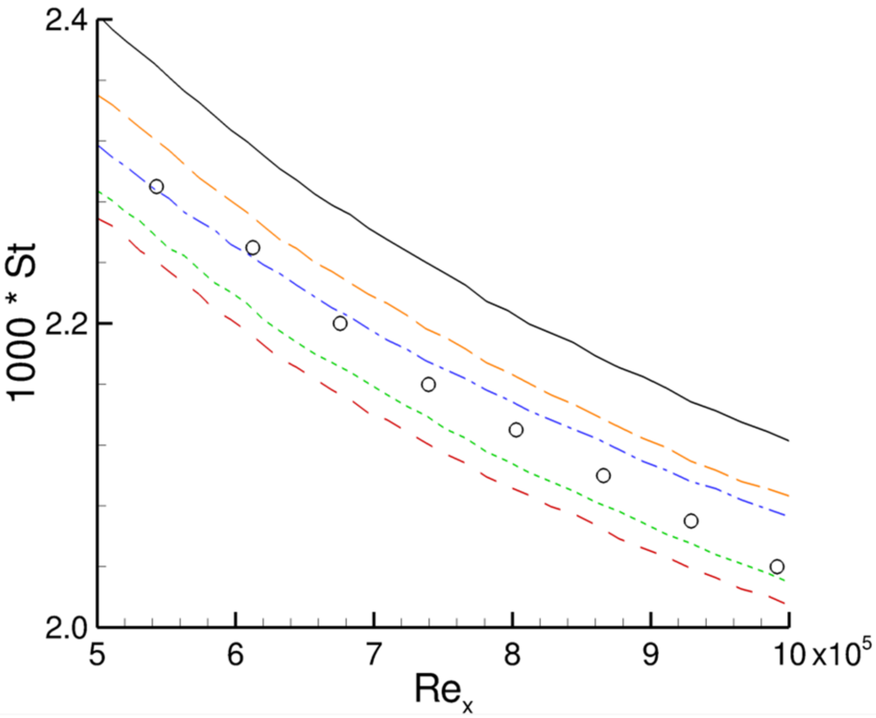


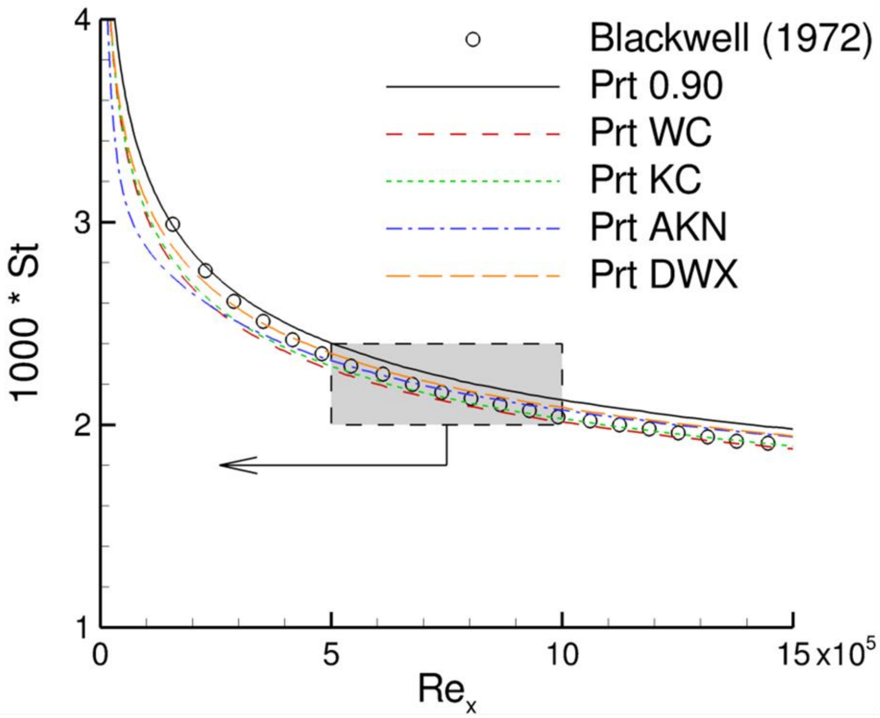
Heat Transfer



Heated Boundary Layer

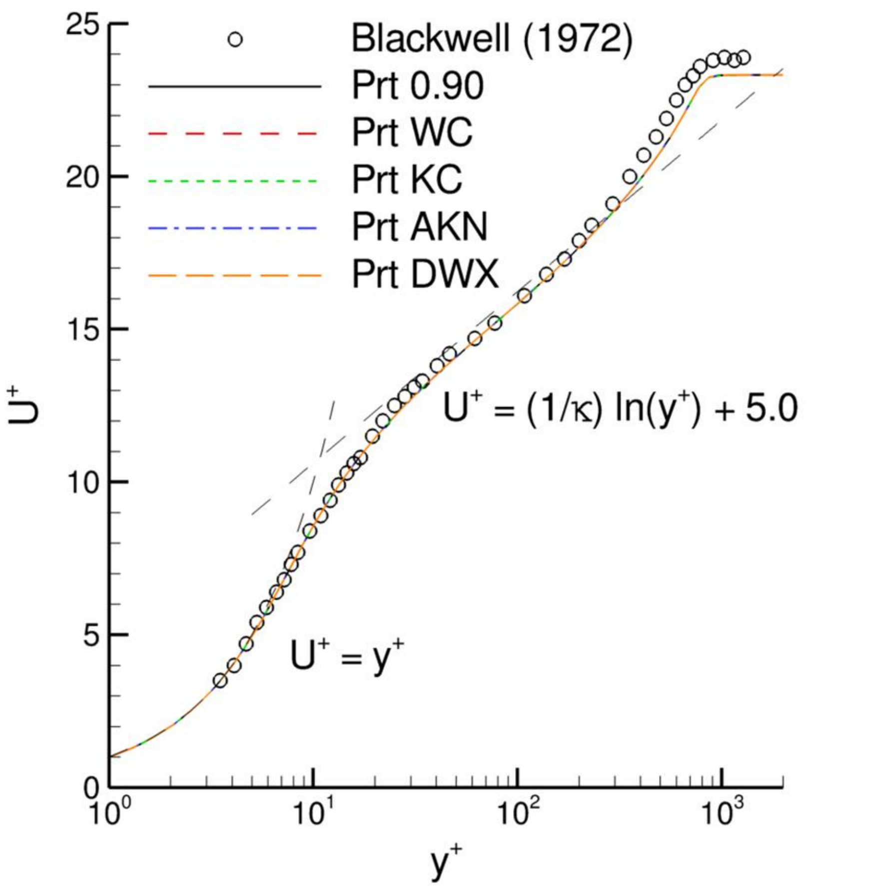
Heat Transfer



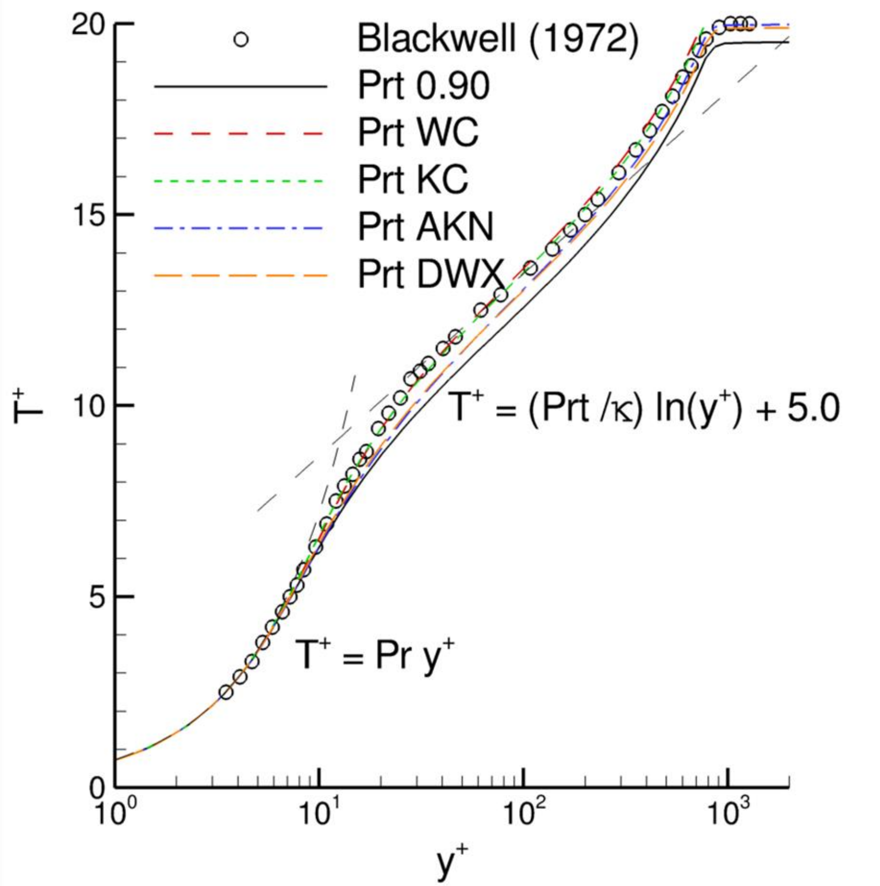


Heated Boundary Layer Profiles

Velocity

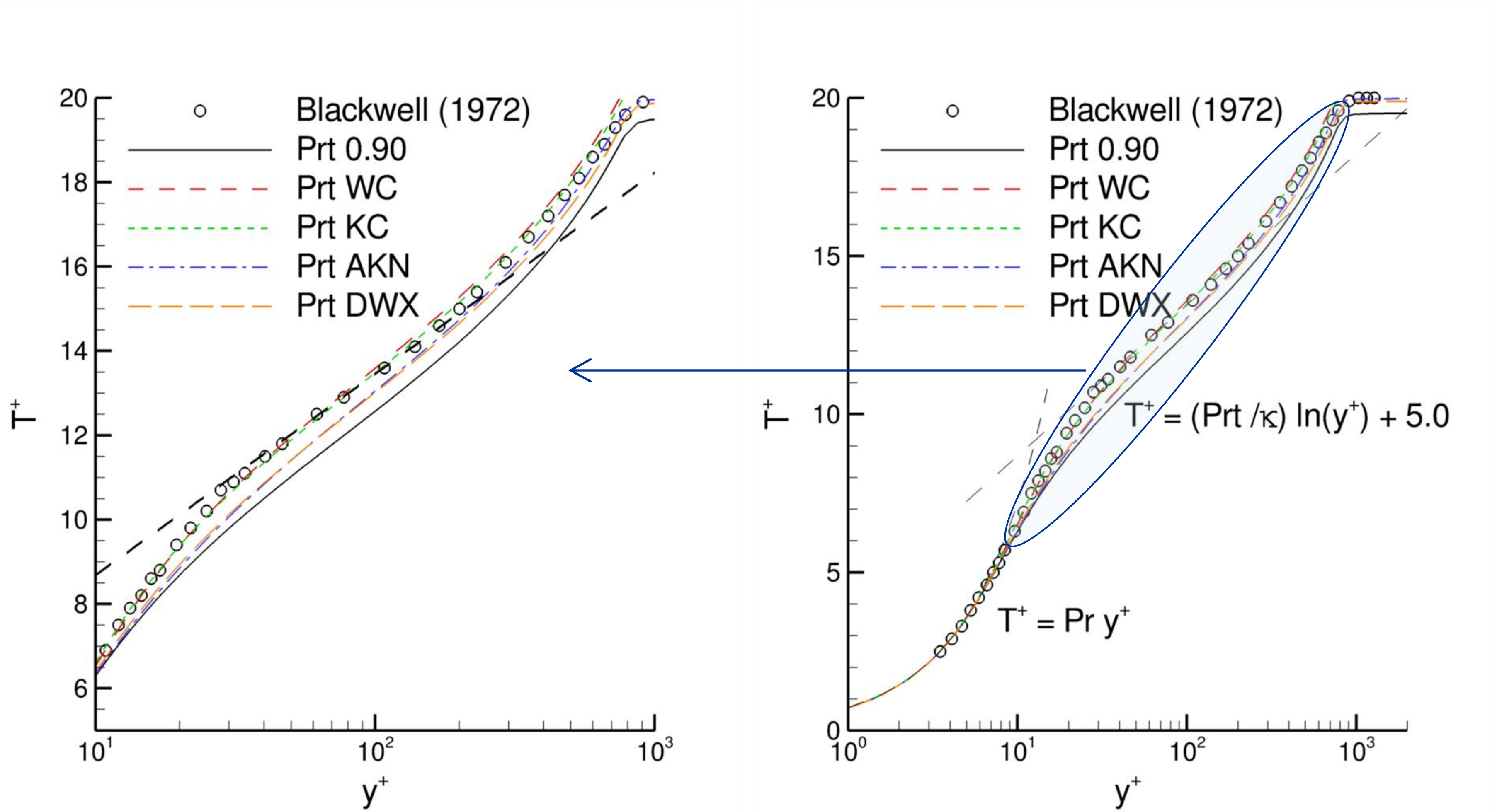


Temperature



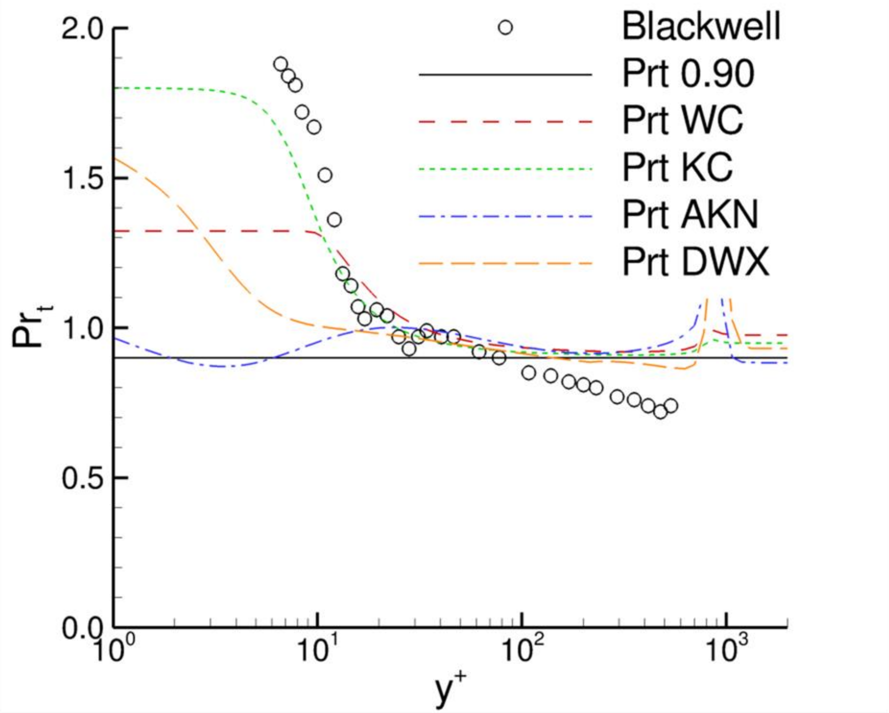
Heated Boundary Layer Profiles

Temperature

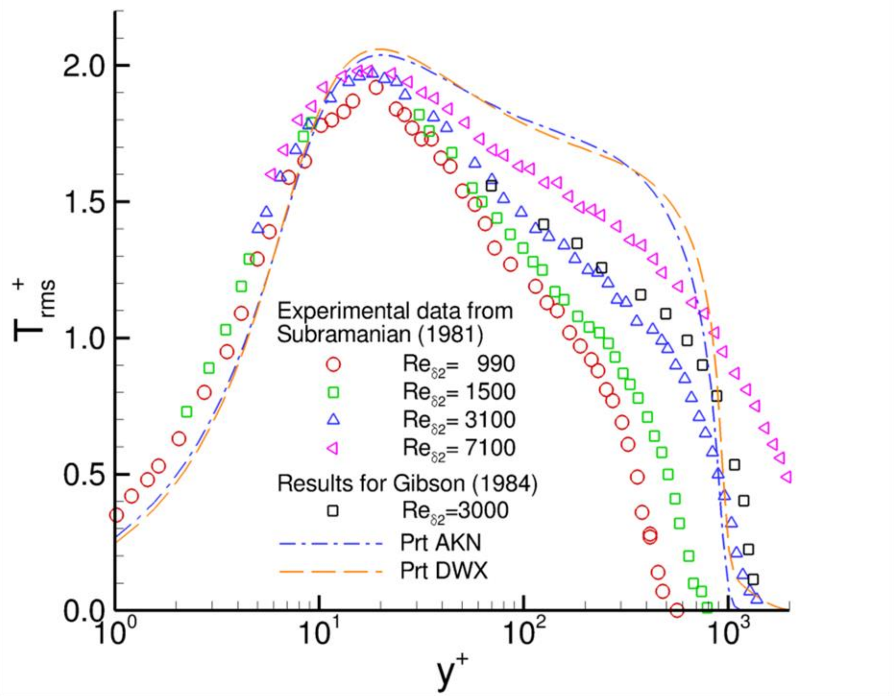


Heated Boundary Layer Profiles

Turbulent Prandtl Number

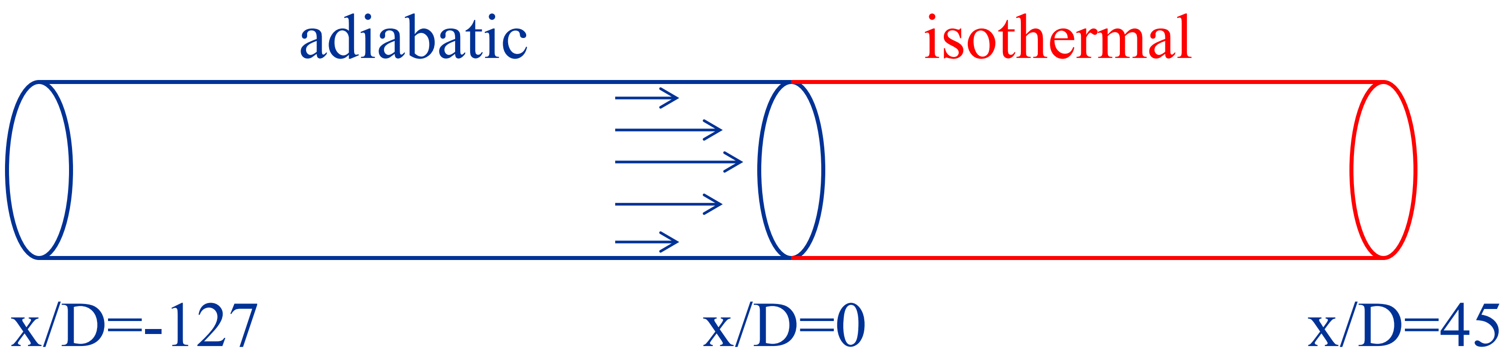


Temperature Variance



Heated Pipe Flow

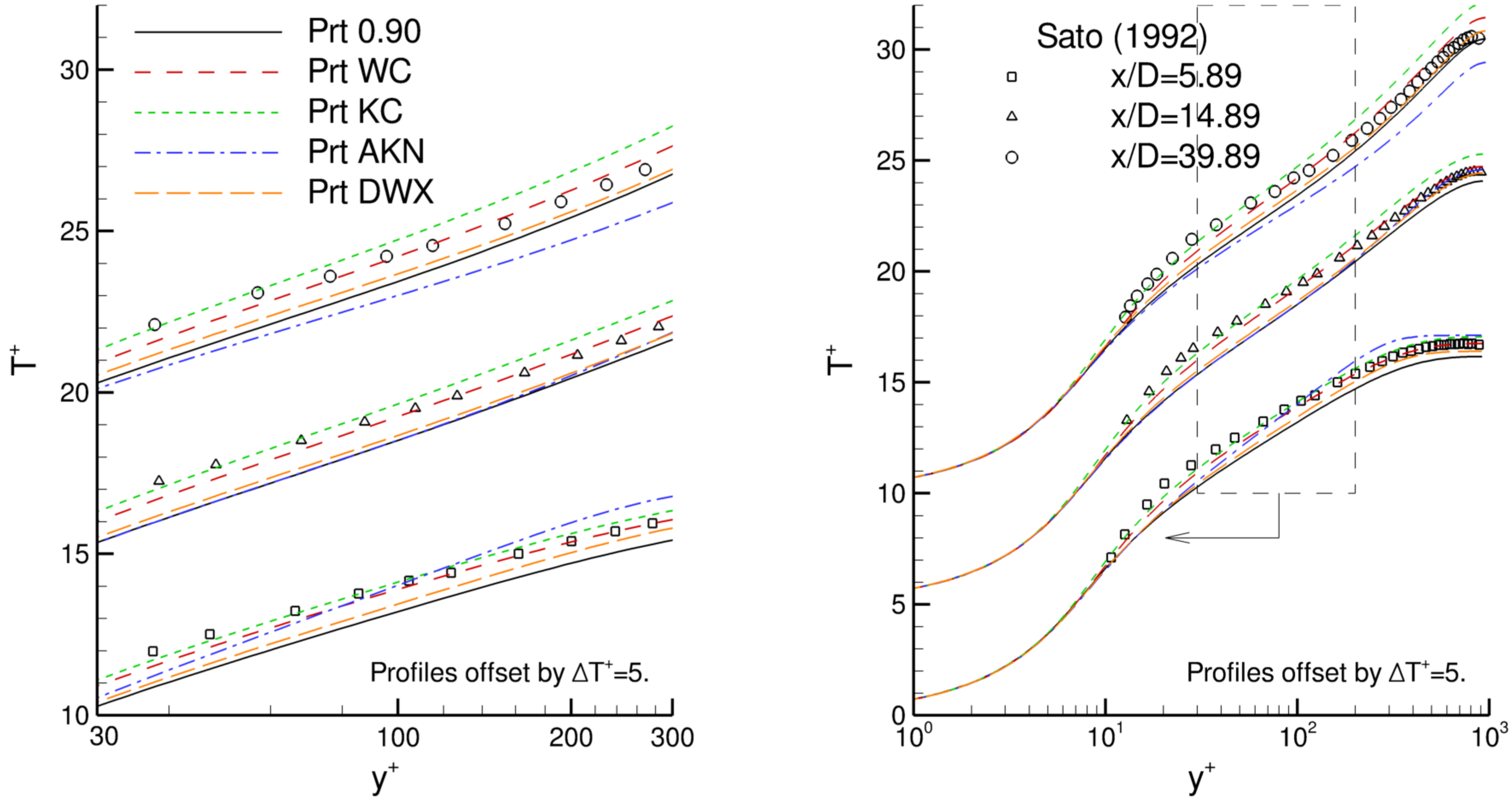
- Configuration of Hishida & Nagano (1978)
  - Upstream section is adiabatic, velocity fully developed.
  - Downstream section is isothermal.



- Data from Sato, Nagano, & Tagawa (1992)
  - Re=40,000
  - U=17 m/s
  - ΔT=74 K

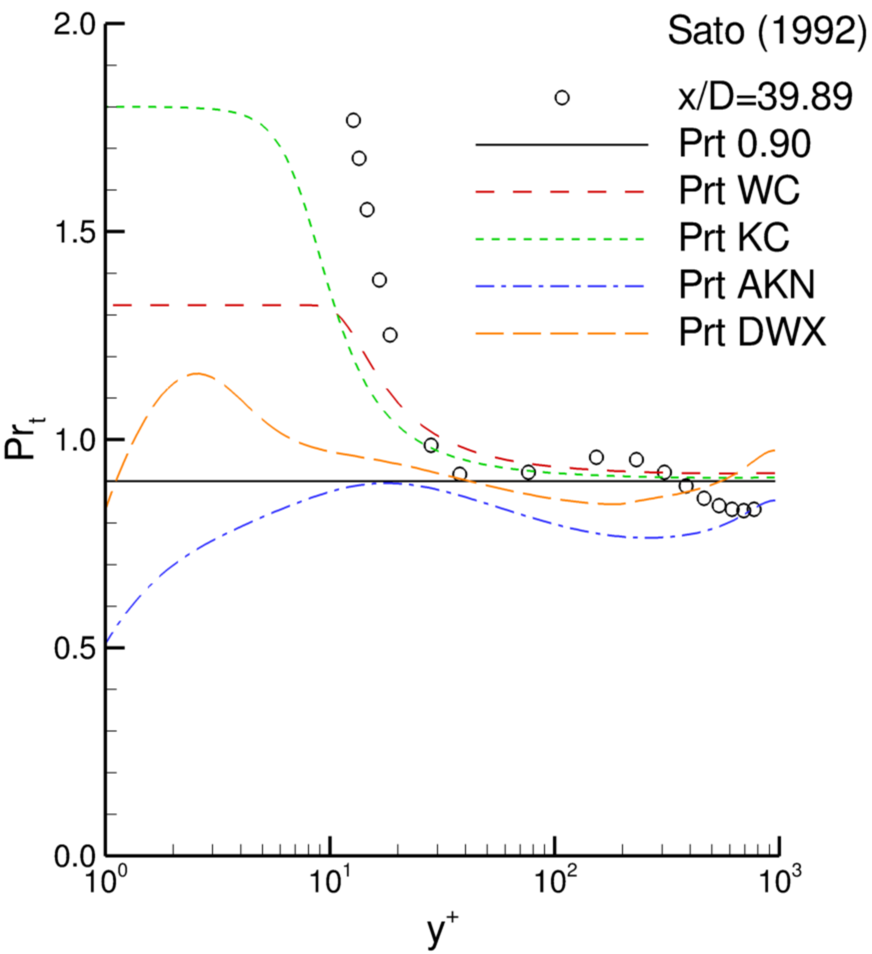
Heated Pipe Flow Profiles

Temperature



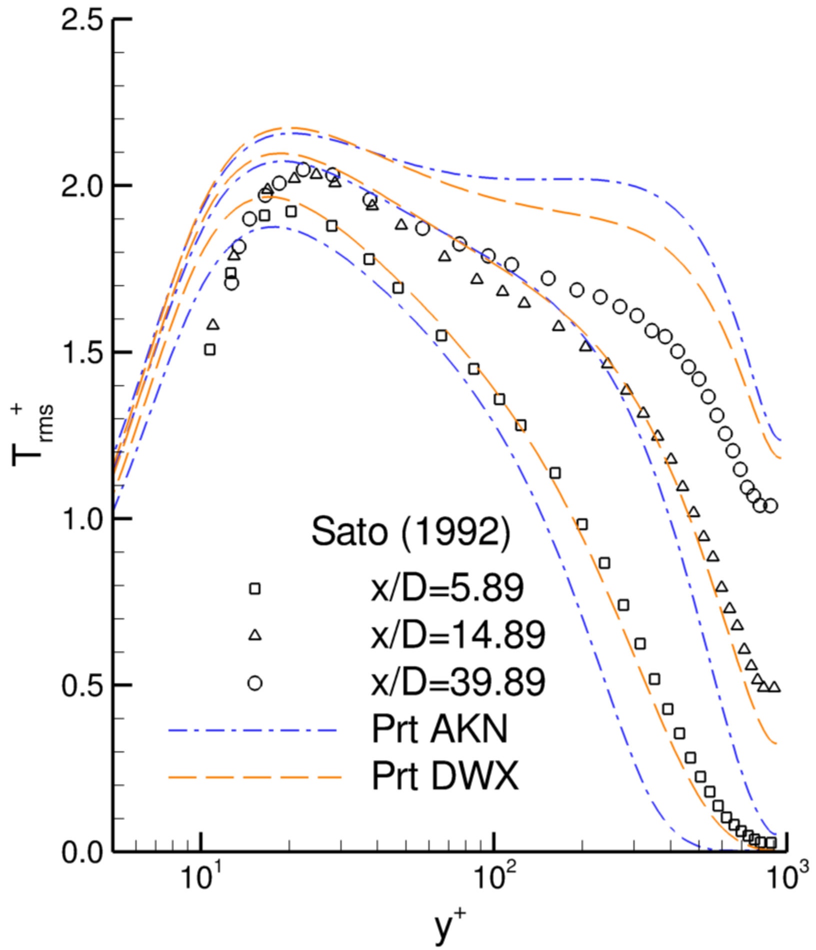
Heated Pipe Flow Profile

Turbulent Prandtl Number

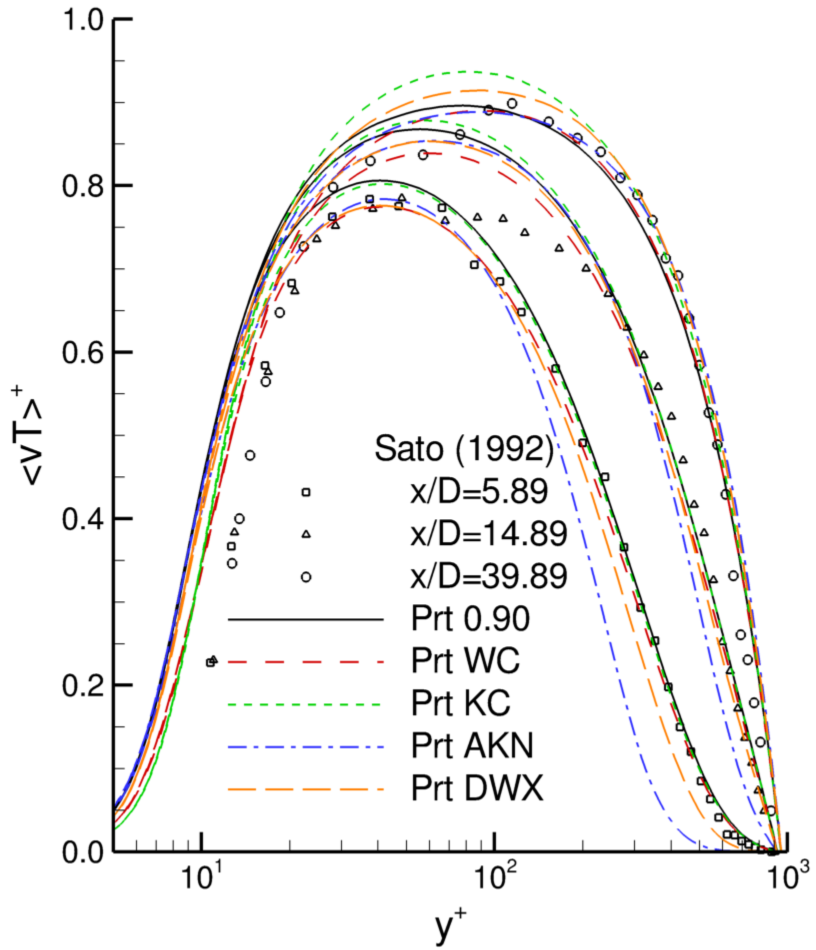


Heated Pipe Flow Profiles

Temperature Variance



Turbulent Heat Flux

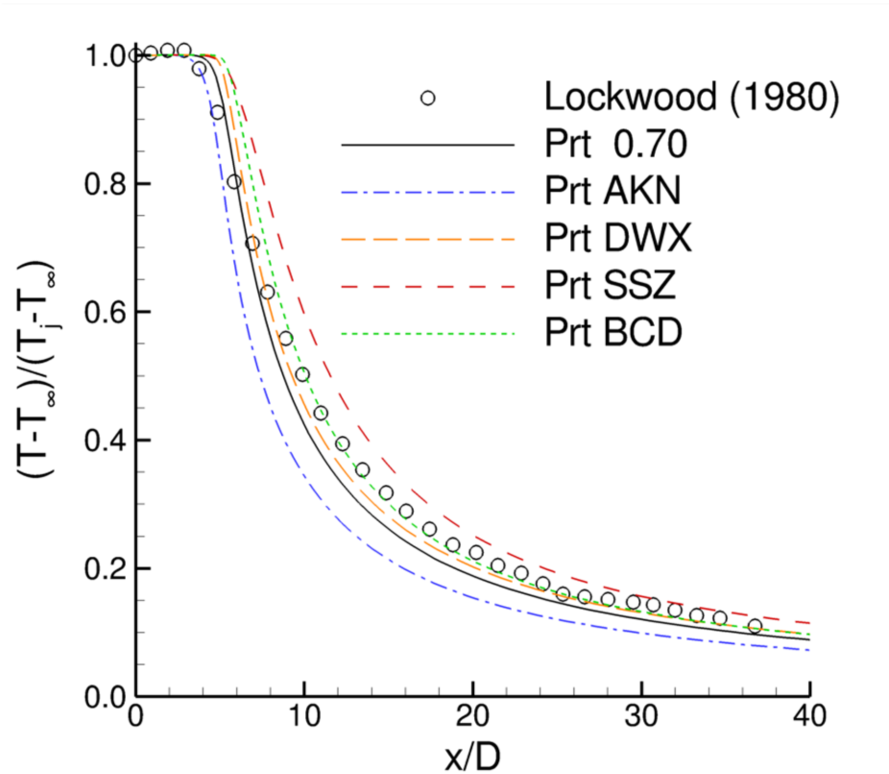


Heated Jets

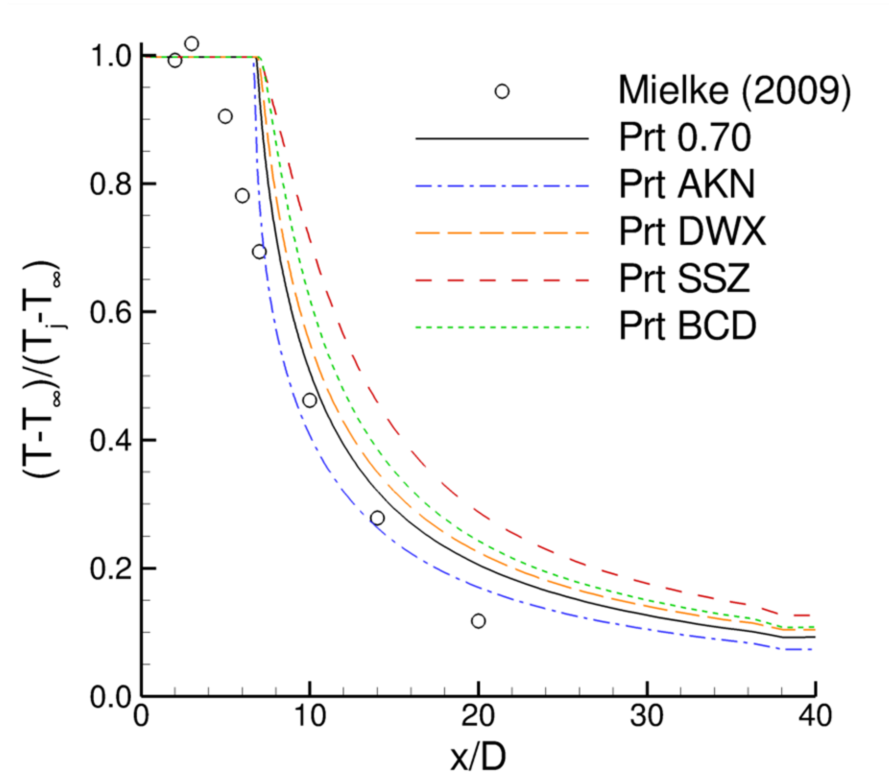
- Lockwood & Moneib (1980) Pipe Exhaust
  - Velocity profile fully developed, flat temperature.
  - Re=50,000
  - M_j=0.25
  - U_j=117 m/s
  - ΔT=255 K
  - T_j/T_∞=1.86
- Mielke, et al. (2008) Convergent Nozzle Exhaust
  - Re=200,000
  - M_j=0.37
  - U_j=167 m/s
  - ΔT=215 K
  - T_j/T_∞=1.76

Heated Jet Centerline Temperature

Pipe Exhaust

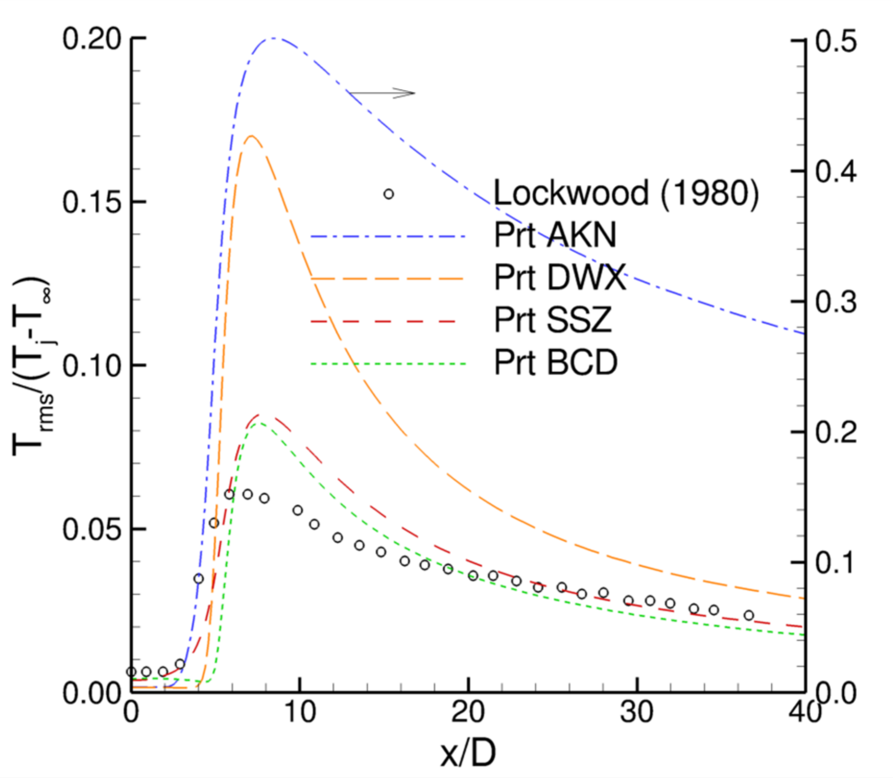


Nozzle Exhaust

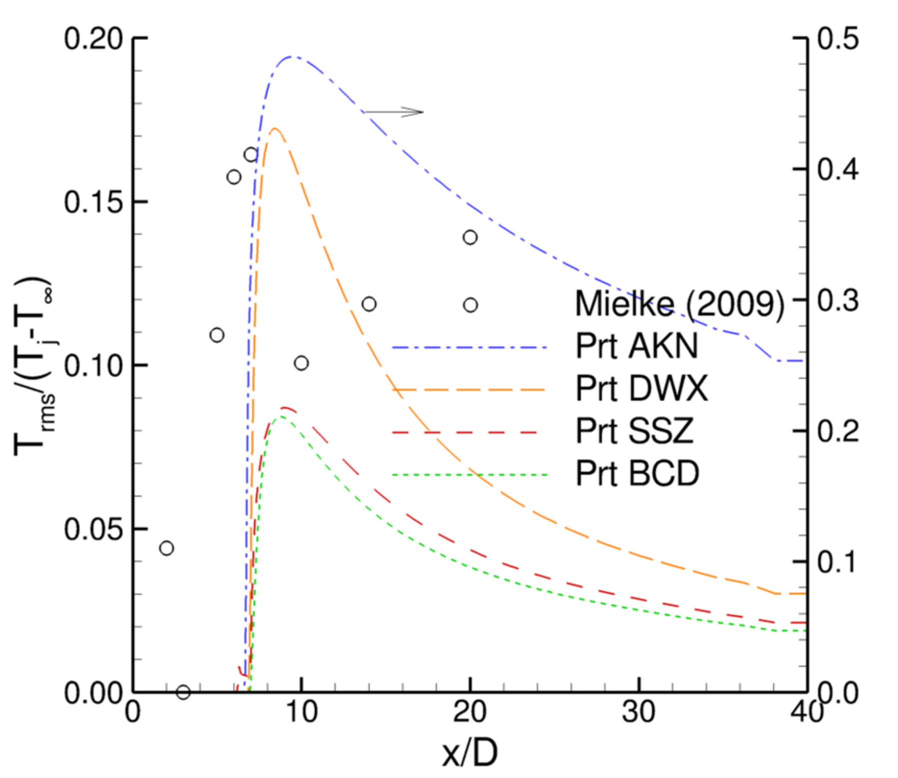


Heated Jet Centerline Temperature Variance

Pipe Exhaust

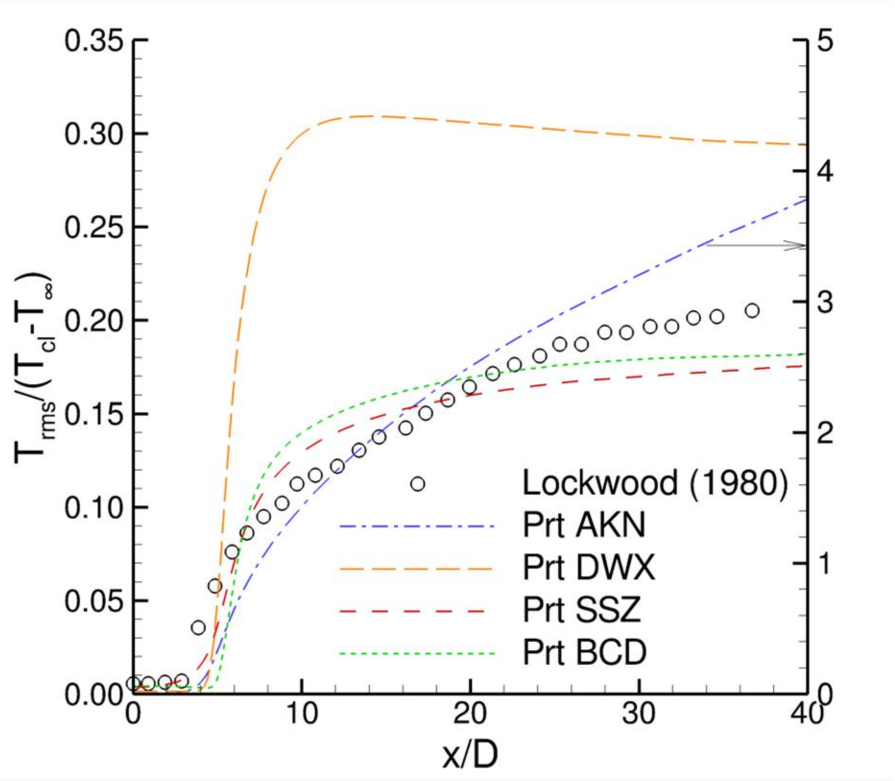


Nozzle Exhaust

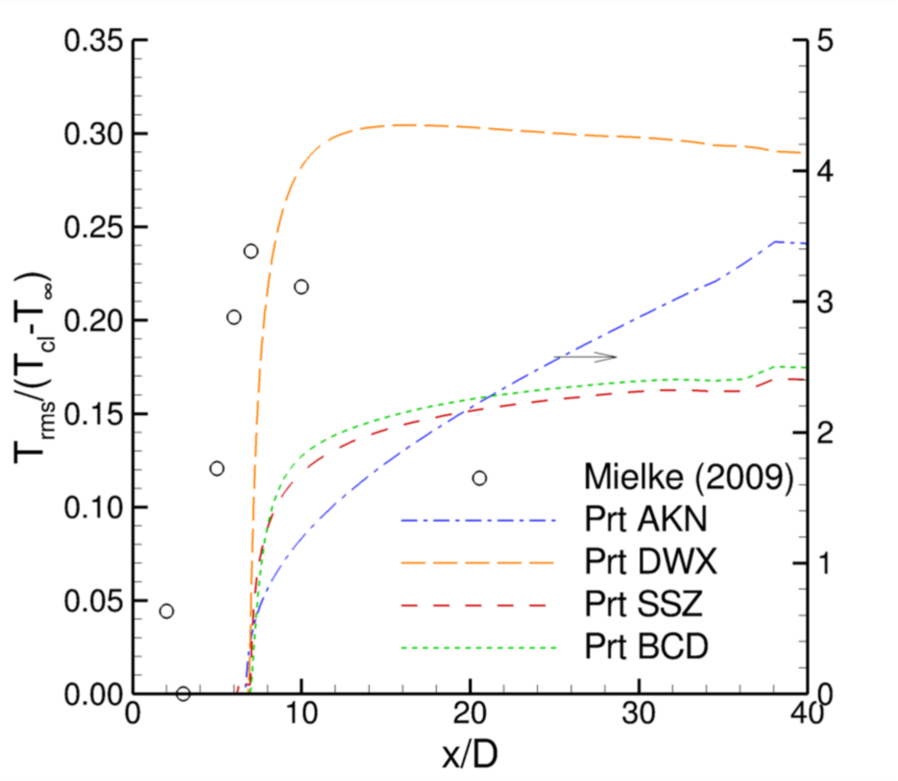


Heated Jet Centerline Temperature Variance

Pipe Exhaust

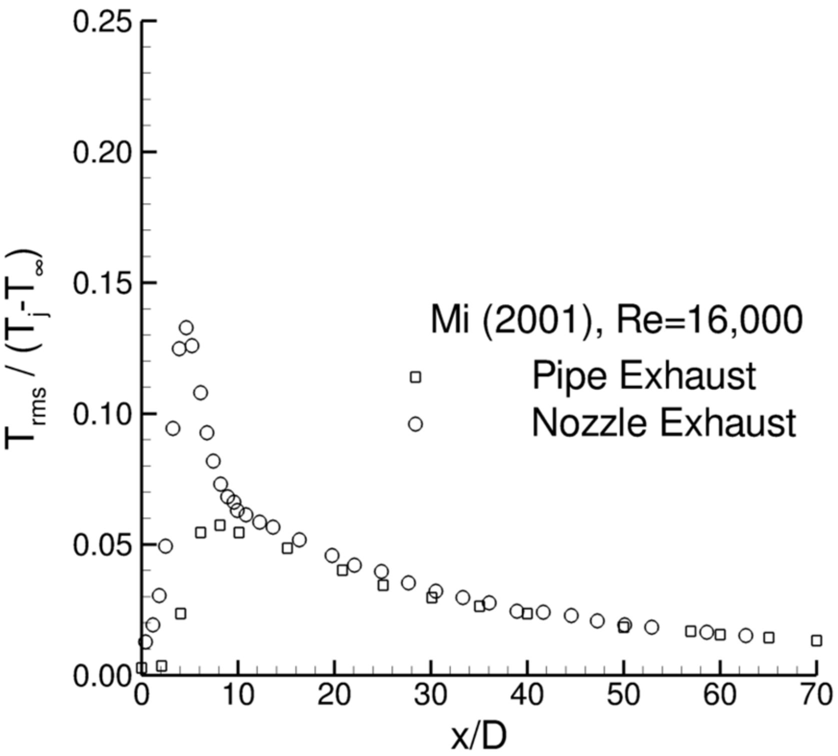


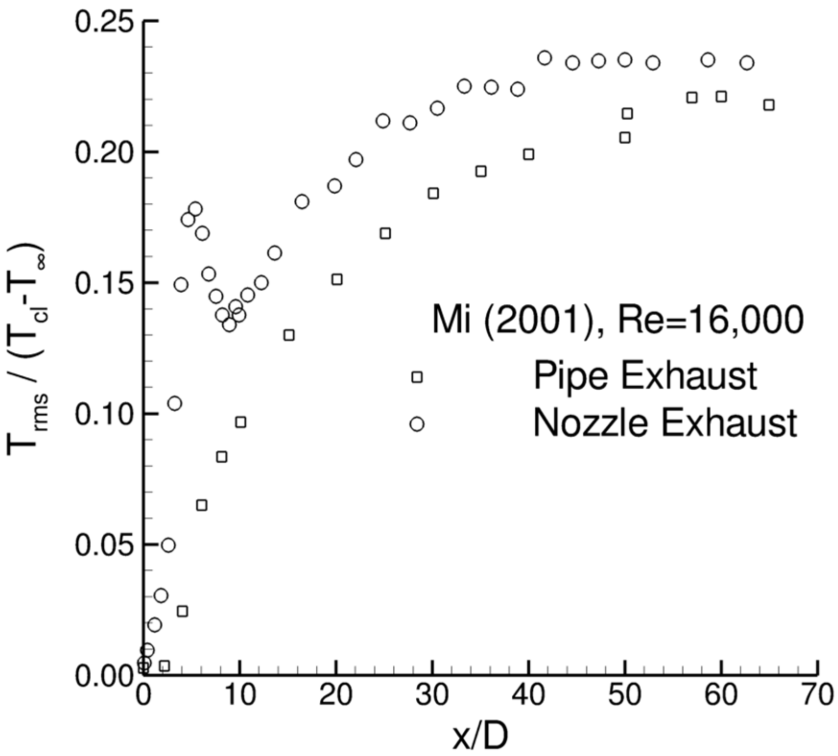
Nozzle Exhaust



Which data is right?

- George (1989) theorizes that differences in turbulent structure affect the scalar field.
- Mi (2001) experiment demonstrates the difference at like conditions.





Conclusions

- Constant Pr_t
  - Crude approximation for boundary layer.
  - Can match the slope of the log-layer temperature profile, but not the offset.
- 0-Eq models
  - Best predict the increase in the near-wall Pr_t and log-law temperature profiles, but formulations are not very general.
  - Do not provide Trms.
  - Are effectively the same as constant Pr_t in free shear flows.
- 2-Eq AKN & DWX models
  - Under predict near-wall Pr_t and log-layer mean temperature.
  - Provide good agreement with near-wall Trms data.
  - Predict higher values of Trms in jets.
    - AKN is >10x larger, perhaps due to choice of mixed timescale.

Conclusions

- 2-Eq SSZ & BCD models
  - Provided unreliable results for wall-bounded flow, perhaps due to near-wall source term implementation.
  - Provide good Trms values for Lockwood jet case.
  - BCD model better predicts mean temperature.
- Outstanding issues with jet predictions
  - For low-Mach jets, Pr_t has little effect on velocity.
    - Cannot explain differences in potential core length for heated/unheated jets.
  - 2-Eq results for pipe and nozzle exhausts are very similar, but data suggests significant differences in Trms.
    - If this is due to differences in turbulent structures at the jet exit, then RANS models may be hopeless.