



# **Multivariable Parametric Cost Models for Space and Ground Telescopes**

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**All Cost Models are Wrong!**

**But Some are Useful.**

**The Rest will get you into Trouble.**



## Parametric Cost Models

Parametric cost models have uses:

- high level mission concept design studies,
- identify major architectural cost drivers,
- allow high-level design trades,
- enable cost-benefit analysis for technology development investment, and
- provide a basis for estimating cost.



## HPS Intuitive Supposition

While space telescopes cost more than ground telescopes, the underlying physics & engineering principles of making telescopes are common.

Scaling laws related to engineering are common

For example:

- Cost versus Diameter depends on substrate manufacture, grind and polish methods; e.g. large tool versus small tool polishing.
- Cost difference between ground and space relates to mirror stiffness from lightweighting – but processing steps are similar for both.

This is important because ground dataset has better wavelength diversity (optical to Radio) and space dataset has better temperature diversity (to 5K)

Program Management practice is different and impacts cost.



## Telescope Cost Model

Potential 'generic' model (combination of Ground and Space):

$$\text{OTA Cost} \sim (A) \text{ SF}^{0.7} D^{(1.65 \pm 0.05)} \lambda^{(-0.5 \pm 0.2)} T^{-0.25} e^{(-0.035 \pm 0.05) Y}$$

**OTA Cost in Millions of FY2000\$**

**A =**      **\$1M**      **Ground**  
              **\$100M**    **Space**

**D =**      **Primary Mirror Diameter (meters)**

**$\lambda$  =**      **Wavelength Diffraction Limited (microns)**

**Y =**      **Year of Development – 2000**

**SF =**      **(#of Segments)<sup>0.7</sup> (Ds/D)<sup>1.7</sup>**

Note: SF fits the data but is not very predictive. Is missing something, probably difficulty of making the backplane.



## DISCLAIMERS

- Cost Models CANNOT predict the cost of a specific mission.
- Cost Models are a RELATIVE tool. They estimate a potential mission's cost relative to known missions in the Data Base.
- Cost Model interpretation must be consistent with laws of physics, engineering practice and program management.
- Blindly using an incorrect and unjustified cost estimating relationship without understanding its assumptions & limitations will lead to wrong conclusions and potentially very expensive decisions.



## DISCLAIMER

### **Cost Models are only as good as their databases**

#### Ground Database

- 10 monolithic and 5 segmented telescopes since 1979
- Data on 20 Programmatic and Engineering parameters
- Data sources:
  - Interviews
  - REDSTAR Library (Research Data Storage and Retrieval System)
  - RSIC (Redstone Scientific Information Center)

#### Space Database

- 33 UVOIR & IR, 5 X-Ray, 7-Radio;
- Completeness only for 15 'free-flying', 4 'attached', 1 'planet'
  - 8 are spectroscopic
- 59 Programmatic & Engineering parameters
- Detailed WBS data on 7 Mission.
- Data sources:
  - NAFCOM (NASA/ Air Force Cost Model) database
  - NICM (NASA Instrument Cost Model)
  - NSCKN (NASA Safety Center Knowledge Now)
  - RSIC (Redstone Scientific Information Center)
  - REDSTAR (Resource Data Storage & Retrieval System)
  - SICM (Scientific Instrument Cost Model)
  - project websites, and interviews

Normal Incidence Database (8.6.11)	
Free Flying Telescopes	Attached Telescopes
Cloud SAT	HUT
Commercial #1	SOFIA
Commercial #2	UIT
Copernicus (OAO-3/PEP)	WUPPE
GALEX	
Herschel	
HST	
IRAS	
JWST	
Kepler	
OAO-B/GEP	
Planck	
Spitzer (SIRTF)	
WIRE	
WISE	
	Planetary Telescopes
	MRO/HIRISE



## Please Help

Please contribute Cost and Technical Data for the Database.

To gain wavelength diversity, seeking data on ground or space:

- Far-IR, Radio and Microwave missions
- Particularly segmented Radio and Microwave dishes
- Also, UV and EUV missions



## Definitions

### **Total Mission:**

- . Spacecraft
- . Science Instruments
- . Telescope

### **Optical Telescope Assembly (OTA):**

- . Primary mirror
- . Secondary (and tertiary if appropriate) mirror(s)
- . Support structure
- . Mechanisms (actuators, etc.), Electronics, Software, etc.
- . Assembly, Integration & Test



## Definitions (2)

### **Cost includes:**

- . Phase A-D (design, development, integration and test)

### **Cost excludes:**

- . Pre-phase A (formulation)
- . Phase E (launch/post-launch)
- . Government labor costs (NASA employees: CS or support contractors)
- . Government Furnished Equipment (GFE)
- . Existing Contractor infrastructure which is not 'billed' to contract.
- . These are 'First Unit' Costs only – no HST Servicing & there are no 2<sup>nd</sup> Systems.

### **Mass includes:**

- . Dry mass only (no propellant)



## FINDING

### **OTA is not Largest Mission Cost Element**

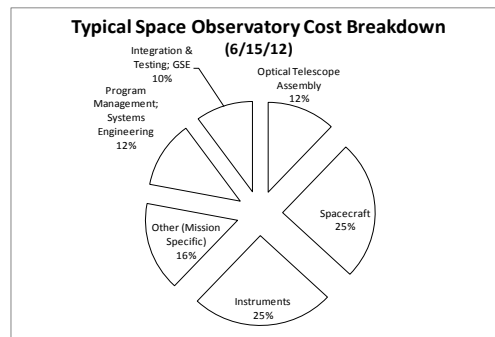
OTA ~12%

Spacecraft and Instruments ~ 50% (**Invest here to reduce \$**)

Program Management & Systems Engineering equals OTA (\$\$\$)

I&T ~ 10% (maybe another 10 to 15% of Subsystems)

Example of Mission Specific is Sun Shade for JWST



Composite WBS for 7 of 14 free flying missions.

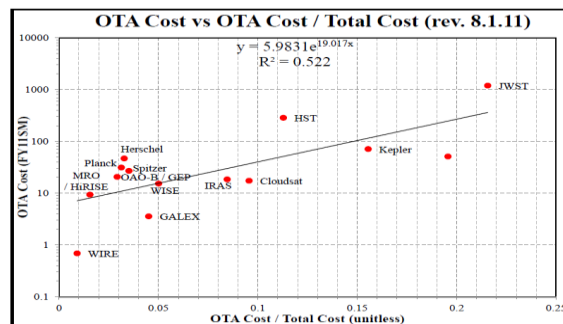


## FINDING

### **Mission Cost is not Proportional to OTA Cost**

OTA Cost varies from ~ 1% to ~ 25% of the Total.

OTA's cost as % of Total may depend on infrastructure cost.



Notes:

WIRE is clearly questionable.

GALEX CADRe cost may be missing Structure cost.



## Want to Build a Cost Model?



### Model Creation

Start with Correlation Matrix.

Look for Variables which are Highly Correlated with Cost.

The higher the correlation the greater the Cost Variation which is explained by a given Variable.

Sign of correlation is important and must be consistent with Engineering Judgment.

Important for Multi-Variable Models:

We want Variables which Independently effect Cost.

When Variables 'cross-talk' with each other it is called Multi-Collinearity.

Thus, avoid Variables which are highly correlated with each other.



## Goodness of Correlation, Fits and Regressions

‘Correlation’ between variables and ‘Goodness’ of single variable models is evaluated via Pearson’s  $r^2$  standard percent error (SPE), and Student’s T-Test p-value.

‘Goodness’ of multivariable fits are evaluated via Pearson’s Adjusted  $r^2$  which accounts for number of data points and number of variables.

Pearson’s  $r^2$  coefficient describes the percentage of agreement between the fitted values and the actual data.

The closer  $r^2$  is to 1, the better the fit.

SPE is a normalized standard deviation of the fit residual (difference between data and fit) to the fit.

The closer SPE is to 0, the better the fit



## Significance

The final issue is whether or not a correlation or fit is significant.

p-value is the probability that the fit or correlation would occur if the variables are independent of each other.

The closer p-value is to 0, the more significant the fit or correlation.

The closer p-value is to 1, the less significant.

If the p-value for a given variable is small, then removing it from the model would cause a large change to the model.

If p-value is large, then removing the variable will have a negligible effect

It is only possible to ‘test’ if the correlation between two variables is significant.

It is not possible to ‘test’ if two variables are independent.





## Cross-Correlation Matrix

rev. 8.1.11	Total Cost	OTA Cost	Aperture Diameter	PMF Len.	System F Len.	FOV	Pointing Stability	Total Mass	OTA Mass	Spectral Range minimum	Diff Lim. $\lambda$	Operating Temp.	Total Avg. Input Power	Data Rate	Design Life	TRL	Year of Dev.	Dev. Period	Date of Launch	Orbit
units	(FY11\$M)	(FY11\$M)	(m)	(m)	(m)	(°)	(Arc-Sec / Sec)	(kg)	(kg)	( $\mu$ )	( $\mu$ )	(K)	(Watts)	(Kbps)	(months)		(year)	(months)	(year)	(km)
Total Cost	<b>1.00</b>	<b>0.85</b>	<b>0.69</b>	0.21	0.52	0.13	<b>-0.72</b>	<b>0.68</b>	<b>0.85</b>	0.21	-0.05	-0.11	0.57	0.05	0.30	-0.45	-0.15	<b>0.70</b>	0.03	0.46
OTA Cost		<b>1.00</b>	<b>0.78</b>	<b>0.88</b>	<b>0.72</b>	-0.13	<b>-0.80</b>	<b>0.84</b>	<b>0.95</b>	-0.16	-0.20	0.04	0.37	0.22	<b>0.64</b>	<b>-0.61</b>	-0.03	<b>0.62</b>	0.16	0.07
Aperture Diameter			<b>1.00</b>	0.36	<b>0.72</b>	-0.04	<b>-0.71</b>	0.59	<b>0.84</b>	0.51	0.46	-0.05	0.44	-0.10	0.43	-0.28	-0.08	0.45	0.05	0.05
PMF Len.				<b>1.00</b>	0.70	0.13	-0.77	<b>0.69</b>	<b>0.89</b>	-0.48	-0.38	0.18	0.21	0.04	0.56	-0.35	-0.11	0.41	0.11	0.00
System F Len.					<b>1.00</b>	-0.29	-0.47	<b>0.61</b>	<b>0.70</b>	0.02	-0.13	-0.04	0.13	-0.16	0.60	-0.38	-0.22	0.43	-0.03	0.18
FOV						<b>1.00</b>	-0.33	0.03	0.27	0.24	0.26	-0.12	0.10	0.19	-0.15	-0.31	0.19	-0.01	0.18	0.06
Pointing Stability							<b>1.00</b>	<b>-0.62</b>	<b>-0.87</b>	0.16	0.18	-0.13	-0.46	-0.03	-0.54	0.26	-0.04	<b>-0.63</b>	-0.24	-0.03
Total Mass								<b>1.00</b>	<b>0.92</b>	-0.10	0.10	0.01	0.39	-0.19	0.46	-0.54	-0.31	0.51	-0.16	0.24
OTA Mass									<b>1.00</b>	-0.22	-0.22	0.00	0.27	-0.17	0.41	<b>-0.62</b>	-0.04	<b>0.69</b>	0.18	-0.35
Spectral Range minimum										<b>1.00</b>	<b>0.96</b>	-0.24	0.16	0.10	0.06	-0.09	0.19	0.17	0.19	0.12
Diff. Lim. $\lambda$											<b>1.00</b>	-0.28	0.10	0.15	-0.20	0.12	0.35	0.23	0.32	0.26
Operating Temp.												<b>1.00</b>	0.12	-0.05	0.27	0.11	-0.06	-0.39	-0.09	-0.06
Total Avg. Input Power													<b>1.00</b>	0.50	0.57	0.13	0.59	0.06	0.57	0.25
Data Rate														<b>1.00</b>	0.14	<b>0.63</b>	<b>0.72</b>	-0.09	0.70	0.28
Design Life															<b>1.00</b>	-0.15	0.12	0.15	0.25	0.33
TRL																<b>1.00</b>	<b>0.67</b>	-0.17	<b>0.64</b>	0.32
Year of Dev.																	<b>1.00</b>	-0.13	<b>0.97</b>	0.22
Dev. Period																		<b>1.00</b>	0.09	0.36
Date of Launch																			<b>1.00</b>	0.28
Orbit																				<b>1.00</b>



## Cross-Correlation Matrix

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Total Cost	<b>1.00</b>	<b>0.85</b>	<b>0.69</b>	0.21	0.52	0.13	<b>-0.72</b>	<b>0.68</b>	<b>0.85</b>	0.21	-0.05	-0.11	0.57	0.05	0.30	-0.45	-0.15	<b>0.70</b>	0.03	0.46
OTA Cost		<b>1.00</b>	<b>0.78</b>	<b>0.88</b>	<b>0.72</b>	-0.13	<b>-0.80</b>	<b>0.84</b>	<b>0.95</b>	-0.16	-0.20	0.04	0.37	0.22	<b>0.64</b>	<b>-0.61</b>	-0.03	<b>0.62</b>	0.16	0.07
Aperture Diameter			<b>1.00</b>	0.36	<b>0.72</b>	-0.04	<b>-0.71</b>	0.59	<b>0.84</b>	0.51	0.46	-0.05	0.44	-0.10	0.43	-0.28	-0.08	0.45	0.05	0.05
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Total Mass								<b>1.00</b>	<b>0.92</b>	-0.10	0.10	0.01	0.39	-0.19	0.46	-0.54	-0.31	0.51	-0.16	0.24
OTA Mass									<b>1.00</b>	-0.22	-0.22	0.00	0.27	-0.17	0.41	<b>-0.62</b>	-0.04	<b>0.69</b>	0.18	-0.35
Spectral Range minimum										<b>1.00</b>	<b>0.96</b>	-0.24	0.16	0.10	0.06	-0.09	0.19	0.17	0.19	0.12
Diff. Lim. $\lambda$											<b>1.00</b>	-0.28	0.10	0.15	-0.20	0.12	0.35	0.23	0.32	0.26
Operating Temp.												<b>1.00</b>	0.12	-0.05	0.27	0.11	-0.06	-0.39	-0.09	-0.06
Total Avg. Input Power													<b>1.00</b>	0.50	0.57	0.13	0.59	0.06	0.57	0.25
Data Rate														<b>1.00</b>	0.14	<b>0.63</b>	<b>0.72</b>	-0.09	0.70	0.28
Design Life															<b>1.00</b>	-0.15	0.12	0.15	0.25	0.33
TRL																<b>1.00</b>	<b>0.67</b>	-0.17	<b>0.64</b>	0.32
Year of Dev.																	<b>1.00</b>	-0.13	<b>0.97</b>	0.22
Dev. Period																		<b>1.00</b>	0.09	0.36
Date of Launch																			<b>1.00</b>	0.28
Orbit																				<b>1.00</b>

Correlations which are at least 95% significant are **Bolded**, e.g. for 12 data points a correlation of greater than 60% is significant to better than 95%.



## Cross-Correlation Matrix

rev. 8.1.11	Total Cost	OTA Cost	Aperture Diameter	PMF Len.	System F Len.	FOV	Pointing Stability	Total Mass	OTA Mass	Spectral Range minimum	Diff Lim. $\lambda$	Operating Temp.	Total Avg. Input Power	Data Rate	Design Life	TRL	Year of Dev.	Dev. Period	Date of Launch	Orbit
units	(FY11\$M)	(FY11\$M)	(m)	(m)	(m)	(°)	(Arc-Sec / Sec)	(kg)	(kg)	(μ)	(μ)	(K)	(Watts)	(Kbps)	(months)		(year)	(months)	(year)	(km)
Total Cost	1.00	0.85	0.69	0.21	0.52	0.13	-0.72	0.68	0.85	0.21	-0.05	-0.11	0.57	0.05	0.30	-0.45	-0.15	0.70	0.03	0.46
OTA Cost		1.00	0.78	0.88	0.72	-0.13	-0.80	0.84	0.95	-0.16	-0.20	0.04	0.37	0.22	0.64	-0.61	-0.03	0.62	0.16	0.07
Aperture Diameter			1.00	0.36	0.72	-0.04	-0.71	0.59	0.84	0.51	0.46	-0.05	0.44	-0.10	0.43	-0.28	-0.08	0.45	0.05	0.05
PMF Len.				1.00	0.70	0.13	-0.77	0.69	0.89	-0.48	-0.38	0.18	0.21	0.04	0.56	-0.35	-0.11	0.41	0.11	0.00
System F Len.					1.00	-0.29	-0.47	0.61	0.70	0.02	-0.13	-0.04	0.13	-0.16	0.60	-0.38	-0.22	0.43	-0.03	0.18
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Pointing Stability							1.00	-0.62	-0.87	0.16	0.18	-0.13	-0.46	-0.03	-0.54	0.26	-0.04	-0.63	-0.24	-0.03
Total Mass								1.00	0.92	-0.10	0.10	0.01	0.39	-0.19	0.46	-0.54	-0.31	0.51	-0.16	0.24
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Spectral Range minimum										1.00	0.96	-0.24	0.16	0.10	0.06	-0.09	0.19	0.17	0.19	0.12
Diff Lim. $\lambda$											1.00	-0.28	0.10	0.15	-0.20	0.12	0.35	0.23	0.32	0.26
Operating Temp.												1.00	0.12	-0.05	0.27	0.11	-0.06	-0.39	-0.09	-0.06
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Total Cost has significant correlations with:

Aperture Diameter  
Pointing Stability (inverse correlation)  
OTA & Total Mass  
Average Power (weak)  
Development Period

OTA Cost has significant correlations with:

Aperture Diameter  
Primary Mirror & System Focal Length (Volume)  
Pointing Stability (inverse correlation)  
OTA Mass  
Design Life  
TRL (inverse)  
Development Period

Wavelength & Temperature correlation is weak



## Cross-Correlation Matrix

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Diff Lim. $\lambda$											1.00	-0.28	0.10	0.15	-0.20	0.12	0.35	0.23	0.32	0.26
Operating Temp.												1.00	0.12	-0.05	0.27	0.11	-0.06	-0.39	-0.09	-0.06
Total Avg. Input Power													1.00	0.50	0.57	0.13	0.59	0.06	0.57	0.25
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Date of Launch																			1.00	0.28
Orbit																				1.00

Beyond Cost Modeling,  
Correlation Matrix provides  
insight into Engineering  
connections



## Not all Correlated Variables are Independent

rev. 8.1.11	Total Cost	OTA Cost	Aperture Diameter	PM F Len.	System F Len.	FOV	Pointing Stability	Total Mass	OTA Mass	Spectral Range minimum	Diff Lim. $\lambda$	Operating Temp.	Total Avg. Input Power	Data Rate	Design Life	TRL	Year of Dev.	Dev. Period	Date of Launch	Orbit
units	(FY11\$M)	(FY11\$M)	(m)	(m)	(m)	(°)	(Arc-Sec / Sec)	(kg)	(kg)	( $\mu$ )	( $\mu$ )	(K)	(Watts)	(Kbps)	(months)		(year)	(months)	(year)	(km)
Total Cost	1.00	0.85	0.69	0.21	0.52	0.13	-0.72	0.68	0.85	0.21	-0.05	-0.11	0.57	0.05	0.30	-0.45	-0.15	0.70	0.03	0.46
OTA Cost		1.00	0.78	0.88	0.72	-0.13	-0.80	0.84	0.95	-0.16	-0.20	0.04	0.37	0.22	0.64	-0.61	-0.03	0.62	0.16	0.07
Aperture Diameter			1.00	0.36	0.72	-0.04	-0.71	0.59	0.84	0.51	0.46	-0.05	0.44	-0.10	0.43	-0.28	-0.08	0.45	0.05	0.05
PM F Len.				1.00	0.70	0.13	-0.77	0.69	0.89	-0.48	-0.38	0.18	0.21	0.04	0.56	-0.35	-0.11	0.41	0.11	0.00
System F Len.					1.00	-0.29	-0.47	0.61	0.70	0.02	-0.13	-0.04	0.13	-0.16	0.60	-0.38	-0.22	0.43	-0.03	0.18
FOV						1.00	-0.33	0.03	0.27	0.24	0.26	-0.12	0.10	0.19	-0.15	-0.31	0.19	-0.01	0.18	0.06
Pointing Stability							1.00	-0.62	-0.87	0.16	0.18	-0.13	-0.46	-0.03	-0.54	0.26	-0.04	-0.63	-0.24	-0.03
Total Mass								1.00	0.92	-0.10	0.10	0.01	0.39	-0.19	0.46	-0.54	-0.31	0.51	-0.16	0.24
OTA Mass									1.00	-0.22	-0.22	0.00	0.27	-0.17	0.41	-0.62	-0.04	0.69	0.18	-0.35
Spectral Range minimum										1.00	-0.22	-0.22	0.00	0.27	-0.17	0.41	-0.62	-0.04	0.69	0.18
Diff Lim. $\lambda$											1.00	-0.22	-0.22	0.00	0.27	-0.17	0.41	-0.62	-0.04	0.69
Operating Temp.												1.00	-0.22	-0.22	0.00	0.27	-0.17	0.41	-0.62	-0.04
Total Avg. Input Power													1.00	-0.22	-0.22	0.00	0.27	-0.17	0.41	-0.62
Data Rate														1.00	-0.22	-0.22	0.00	0.27	-0.17	0.41
Design Life															1.00	-0.22	-0.22	0.00	0.27	-0.17
TRL																1.00	-0.22	-0.22	0.00	0.27
Year of Dev.																	1.00	-0.22	-0.22	0.00
Dev. Period																		1.00	-0.22	-0.22
Date of Launch																			1.00	-0.22
Orbit																				1.00

Correlation Matrix implies that Larger Diameter OTAs:

have longer Focal Lengths

have smaller Pointing Stability Requirements

are more Massive

require bigger spacecraft which are more Massive & require Power

have larger instruments that are more Massive & require Power

need a long Design Life

take longer to Develop

Aperture Diameter is co-linear with System F/#, Pointing, OTA Mass.



## Variable Linkages

rev. 8.1.11	Total Cost	OTA Cost	Aperture Diameter	PM F Len.	System F Len.	FOV	Pointing Stability	Total Mass	OTA Mass	Spectral Range minimum	Diff Lim. $\lambda$	Operating Temp.	Total Avg. Input Power	Data Rate	Design Life	TRL	Year of Dev.	Dev. Period	Date of Launch	Orbit
units	(FY11\$M)	(FY11\$M)	(m)	(m)	(m)	(°)	(Arc-Sec / Sec)	(kg)	(kg)	( $\mu$ )	( $\mu$ )	(K)	(Watts)	(Kbps)	(months)		(year)	(months)	(year)	(km)
Total Cost	1.00	0.85	0.69	0.21	0.52	0.13	-0.72	0.68	0.85	0.21	-0.05	-0.11	0.57	0.05	0.30	-0.45	-0.15	0.70	0.03	0.46
OTA Cost		1.00	0.78	0.88	0.72	-0.13	-0.80	0.84	0.95	-0.16	-0.20	0.04	0.37	0.22	0.64	-0.61	-0.03	0.62	0.16	0.07
Aperture Diameter			1.00	0.36	0.72	-0.04	-0.71	0.59	0.84	0.51	0.46	-0.05	0.44	-0.10	0.43	-0.28	-0.08	0.45	0.05	0.05
PM F Len.				1.00	0.70	0.13	-0.77	0.69	0.89	-0.48	-0.38	0.18	0.21	0.04	0.56	-0.35	-0.11	0.41	0.11	0.00
System F Len.					1.00	-0.29	-0.47	0.61	0.70	0.02	-0.13	-0.04	0.13	-0.16	0.60	-0.38	-0.22	0.43	-0.03	0.18
FOV						1.00	-0.33	0.03	0.27	0.24	0.26	-0.12	0.10	0.19	-0.15	-0.31	0.19	-0.01	0.18	0.06
Pointing Stability							1.00	-0.62	-0.87	0.16	0.18	-0.13	-0.46	-0.03	-0.54	0.26	-0.04	-0.63	-0.24	-0.03
Total Mass								1.00	0.92	-0.10	0.10	0.01	0.39	-0.19	0.46	-0.54	-0.31	0.51	-0.16	0.24
OTA Mass									1.00	-0.22	-0.22	0.00	0.27	-0.17	0.41	-0.62	-0.04	0.69	0.18	-0.35
Spectral Range minimum										1.00	-0.22	-0.22	0.00	0.27	-0.17	0.41	-0.62	-0.04	0.69	0.18
Diff Lim. $\lambda$											1.00	-0.22	-0.22	0.00	0.27	-0.17	0.41	-0.62	-0.04	0.69
Operating Temp.												1.00	-0.22	-0.22	0.00	0.27	-0.17	0.41	-0.62	-0.04
Total Avg. Input Power													1.00	-0.22	-0.22	0.00	0.27	-0.17	0.41	-0.62
Data Rate														1.00	-0.22	-0.22	0.00	0.27	-0.17	0.41
Design Life															1.00	-0.22	-0.22	0.00	0.27	-0.17
TRL																1.00	-0.22	-0.22	0.00	0.27
Year of Dev.																	1.00	-0.22	-0.22	0.00
Dev. Period																		1.00	-0.22	-0.22
Date of Launch																			1.00	-0.22
Orbit																				1.00

Correlation Matrix can be used to identify variable cross-linkages which should be reconciled with Engineering Judgment.

Aperture Diameter and Pointing Stability have a large negative correlation: Larger Diameter OTAs required smaller Pointing Stability.

Pointing Stability and OTA Mass have a large negative correlation: Small Pointing Stability requires a very stiff, i.e. Massive, OTA.



## Wavelength and Temperature

rev. 8.1.11	Total Cost	OTA Cost	Aperture Diameter	PM F Len.	System F Len.	FOV	Pointing Stability	Total Mass	OTA Mass	Spectral Range minimum	Diff Lim $\lambda$	Operating Temp.	Total Avg. Input Power	Data Rate	Design Life	TRL	Year of Dev.	Dev. Period	Date of Launch	Orbit
units	(FY11\$M)	(FY11\$M)	(m)	(m)	(m)	(°)	(Arc-Sec / Sec)	(kg)	(kg)	(μ)	(μ)	(K)	(Watts)	(Kbps)	(months)		(year)	(months)	(year)	(km)
Total Cost	1.00	0.85	0.69	0.21	0.52	0.13	-0.72	0.68	0.85	0.21	-0.05	-0.11	0.57	0.05	0.30	-0.45	-0.15	0.70	0.03	0.46
OTA Cost		1.00	0.78	0.88	0.72	-0.13	-0.80	0.84	0.95	-0.16	-0.20	0.04	0.37	0.22	0.64	-0.61	-0.03	0.62	0.16	0.07
Aperture Diameter			1.00	0.36	0.72	-0.04	-0.71	0.59	0.84	0.51	0.46	-0.05	0.44	-0.10	0.43	-0.28	-0.08	0.45	0.05	0.05
PM F Len.				1.00	0.70	0.13	-0.77	0.69	0.89	-0.48	-0.38	0.18	0.21	0.04	0.56	-0.35	-0.11	0.41	0.11	0.00
System F Len.					1.00	-0.29	-0.47	0.61	0.70	0.02	-0.13	-0.04	0.13	-0.16	0.60	-0.38	-0.22	0.43	-0.03	0.18
FOV						1.00	-0.33	0.03	0.27	0.24	0.26	-0.12	0.10	0.19	-0.15	-0.31	0.19	-0.01	0.18	0.06
Pointing Stability							1.00	-0.62	-0.87	0.16	0.18	-0.13	-0.46	-0.03	-0.54	0.26	-0.04	-0.63	-0.24	-0.03
Total Mass								1.00	0.92	-0.10	0.10	0.01	0.39	-0.19	0.46	-0.54	-0.31	0.51	-0.16	0.24
OTA Mass									1.00	-0.22	-0.22	0.00	0.27	-0.17	0.41	-0.62	-0.04	0.69	0.18	-0.35
Spectral Range minimum										1.00	0.96	-0.24	0.16	0.10	0.06	-0.09	0.19	0.17	0.19	0.12
Diff Lim $\lambda$											1.00	-0.28	0.10	0.15	-0.20	0.12	0.35	0.23	0.32	0.26
Operating Temp.												1.00	0.12	-0.05	0.27	0.11	-0.06	-0.39	-0.09	-0.06
Total Avg. Input Power													1.00	0.57	0.25					
Data Rate														1.00	0.70	0.28				
Design Life															1.00	0.25	0.33			
TRL																1.00	0.64	0.32		
Year of Dev.																	1.00	0.97	0.22	
Dev. Period																		1.00	0.09	0.36
Date of Launch																			1.00	0.28
Orbit																				1.00

As expected Spectral Range and Diffraction Limit are highly correlated. Operating Temperature are inversely correlated.

But neither are significantly correlated with Cost – because they cancel either other out.



## Year and TRL

rev. 8.1.11	Total Cost	OTA Cost	Aperture Diameter	PM F Len.	System F Len.	FOV	Pointing Stability	Total Mass	OTA Mass	Spectral Range minimum	Diff Lim $\lambda$	Operating Temp.	Total Avg. Input Power	Data Rate	Design Life	TRL	Year of Dev.	Dev. Period	Date of Launch	Orbit
units	(FY11\$M)	(FY11\$M)	(m)	(m)	(m)	(°)	(Arc-Sec / Sec)	(kg)	(kg)	(μ)	(μ)	(K)	(Watts)	(Kbps)	(months)		(year)	(months)	(year)	(km)
Total Cost	1.00	0.85	0.69	0.21	0.52	0.13	-0.72	0.68	0.85	0.21	-0.05	-0.11	0.57	0.05	0.30	-0.45	-0.15	0.70	0.03	0.46
OTA Cost		1.00	0.78	0.88	0.72	-0.13	-0.80	0.84	0.95	-0.16	-0.20	0.04	0.37	0.22	0.64	-0.61	-0.03	0.62	0.16	0.07
Aperture Diameter			1.00	0.36	0.72	-0.04	-0.71	0.59	0.84	0.51	0.46	-0.05	0.44	-0.10	0.43	-0.28	-0.08	0.45	0.05	0.05
PM F Len.				1.00	0.70	0.13	-0.77	0.69	0.89	-0.48	-0.38	0.18	0.21	0.04	0.56	-0.35	-0.11	0.41	0.11	0.00
System F Len.					1.00	-0.29	-0.47	0.61	0.70	0.02	-0.13	-0.04	0.13	-0.16	0.60	-0.38	-0.22	0.43	-0.03	0.18
FOV						1.00	-0.33	0.03	0.27	0.24	0.26	-0.12	0.10	0.19	-0.15	-0.31	0.19	-0.01	0.18	0.06
Pointing Stability							1.00	-0.62	-0.87	0.16	0.18	-0.13	-0.46	-0.03	-0.54	0.26	-0.04	-0.63	-0.24	-0.03
Total Mass								1.00	0.92	-0.10	0.10	0.01	0.39	-0.19	0.46	-0.54	-0.31	0.51	-0.16	0.24
OTA Mass									1.00	-0.22	-0.22	0.00	0.27	-0.17	0.41	-0.62	-0.04	0.69	0.18	-0.35
Spectral Range minimum										1.00	0.96	-0.24	0.16	0.10	0.06	-0.09	0.19	0.17	0.19	0.12
Diff Lim $\lambda$											1.00	-0.28	0.10	0.15	-0.20	0.12	0.35	0.23	0.32	0.26
Operating Temp.												1.00	0.12	-0.05	0.27	0.11	-0.06	-0.39	-0.09	-0.06
Total Avg. Input Power													1.00	0.57	0.25					
Data Rate														1.00	0.14	0.63	0.72	-0.05	0.70	0.28
Design Life															1.00	-0.15	0.12	0.15	0.25	0.33
TRL																1.00	0.67	0.17	0.64	0.32
Year of Dev.																	1.00	-0.13	0.97	0.22
Dev. Period																		1.00	0.09	0.36
Date of Launch																			1.00	0.28
Orbit																				1.00

As expected, Year of Development and Launch year are highly correlated.

TRL is correlated with Year of Development – more recent missions require higher TRL

Data Rate is correlated with Date of Launch – more recent mission require higher Data Rate.

	Total Cost	OTA Cost	Total Cost - OTA Cost	Areal Total Cost	Areal OTA Cost	Total Cost / Areal	OTA Cost / Areal	System Element	PM F/Len	PM F#	System F#	System F#	OTA Volume	FOV	Pointing Stability	Total Mass	OTA Mass	Total Area Density	Areal Density	Special Features	Diffraction Limited Wavelength	Operating Temperature	Ave. Input Power	OTAs per Power	Design Life	TRL	Year Offsets	Time Period	Rate of Launch	Cost
per: 11.6 10	(\$/mm^2)	(\$/mm^2)	(\$/mm^2)	(\$/mm^2)	(\$/mm^2)	(\$/mm^2)	(\$/mm^2)	(\$/mm^2)	(\$/mm^2)	(\$/mm^2)	(\$/mm^2)	(\$/mm^2)	(\$/mm^2)	(\$/mm^2)	(\$/mm^2)	(\$/mm^2)	(\$/mm^2)	(\$/mm^2)	(\$/mm^2)	(\$/mm^2)	(\$/mm^2)	(\$/mm^2)	(\$/mm^2)	(\$/mm^2)	(\$/mm^2)	(\$/mm^2)	(\$/mm^2)	(\$/mm^2)	(\$/mm^2)	
Total Cost	1.00	0.88	1.00	-0.16	0.01	0.52	0.37	0.64	0.80	0.11	0.68	0.31	0.81	0.07	-0.62	0.93	0.76	-0.29	0.11	0.08	0.12	-0.05	0.68	0.19	0.69	-0.46	-0.07	0.58	0.17	0.36
OTA Cost		1.00	0.85	-0.45	0.18	0.12	0.23	0.82	0.82	0.03	0.72	0.33	0.85	-0.25	-0.82	0.90	0.91	-0.49	-0.05	-0.05	-0.07	-0.01	0.51	-0.06	0.68	-0.38	0.04	0.61	0.26	-0.02
Total Cost - OTA			1.00	-0.14	-0.01	0.54	0.38	0.68	0.81	0.19	0.65	0.31	0.87	-0.01	-0.61	0.95	0.73	-0.28	0.08	0.09	0.16	0.02	0.71	0.29	0.68	-0.46	-0.02	0.59	0.19	-0.08
Areal Total Cost			1.00	0.62	0.08	0.20	-0.86	-0.42	0.52	-0.53	0.18	-0.71	0.35	0.34	-0.21	0.70	0.97	-0.17	-0.13	-0.28	0.14	0.14	0.24	-0.29	-0.39	-0.09	-0.15	0.21	0.22	
Areal OTA Cost			1.00	0.10	0.37	-0.42	-0.11	0.44	-0.28	-0.05	-0.28	0.26	-0.53	-0.04	0.05	0.64	0.47	-0.32	-0.58	-0.31	0.04	0.04	0.14	-0.36	-0.33	0.09	-0.28	0.14	0.1	
Total Cost / Areal				1.00	0.62	0.22	0.24	0.15	0.43	0.33	0.18	0.11	-0.15	0.18	0.14	-0.18	-0.20	0.23	0.01	-0.19	0.25	0.31	0.39	0.08	-0.01	0.33	0.09	0.55		
OTA Cost / Areal				1.00	0.10	-0.08	-0.46	0.29	0.35	0.03	-0.35	-0.03	0.08	-0.19	-0.11	-0.65	0.13	0.20	-0.33	0.37	0.50	0.17	0.89	0.33	0.15	0.28	0.68			
Apertures Diameter				1.00	0.76	-0.29	0.80	0.11	0.97	-0.30	-0.79	0.64	0.87	-0.88	0.15	0.11	0.28	-0.08	-0.14	-0.16	0.60	-0.29	0.01	0.40	0.19	-0.08				
PM F/Len					1.00	0.40	0.78	0.39	0.89	0.19	-0.84	0.80	0.90	-0.45	-0.27	-0.41	-0.21	0.18	0.15	0.09	0.59	-0.39	0.07	-0.52	-0.12	-0.13				
PM F#					1.00	-0.05	0.28	-0.06	0.49	-0.53	0.11	0.65	0.49	0.50	-0.16	-0.68	0.22	-0.16	-0.05	0.09	-0.37	0.23	0.26	-0.21	-0.20					
System F/Len						1.00	0.69	0.86	-0.33	-0.40	0.65	0.71	-0.57	0.09	-0.07	-0.16	-0.04	-0.04	-0.26	0.65	-0.39	-0.23	0.35	-0.06	0.07					
System F#						1.00	0.27	-0.18	-0.01	0.28	0.28	0.13	-0.01	-0.26	-0.57	0.04	-0.19	-0.19	0.36	-0.32	-0.38	0.06	-0.32	0.20						
OTA Volume						1.00	-0.08	-0.90	0.83	0.89	-0.76	0.17	-0.27	0.05	0.09	0.28	0.14	0.60	-0.31	0.05	0.53	0.24	-0.04	-0.07						
FOV						1.00	-0.28	0.02	0.46	0.35	0.64	0.04	-0.11	0.08	0.04	0.26	-0.16	-0.31	0.10	0.04	0.10	0.03	0.05							
Pointing Stability						1.00																								



1. Perform a single-variable regression to identify key variable.
2. Fix 1<sup>st</sup> Variable and perform a 2-variable regression to identify next key variable.
3. Select 2nd variable based on:
  - Change in 1<sup>st</sup> Variable's Significance
  - Significance of Variable #2
  - Increase in  $r^2_{\text{adj}}$
  - Decrease in SPE
  - Multi-Collinearity
4. Repeat for 3<sup>rd</sup> Variable.

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## Single Variable Space OTA

Regressing on 15 normal incidence, 'free-flying' UVOIR OTAs

Significant Variables: Diameter, Focal Length, Volume, Pointing & Mass

Diameter is co-linear with Volume, Pointing & Mass.

Focal Length has the highest  $R^2_{adj}$  and Mass has the lowest SPE

Diameter is most relevant for Science and Engineering.

rev. 8.1.11		OTA Cost vs V1						
Variable Name		Aperture Diameter	PM F Len.	PM f/#	OTA Volume	FOV	Pointing Stability	OTA Mass
Var.	p-value	1.42 0.00	1.55 0.00	0.58 0.57	0.58 0.00	-0.12 0.69	-0.76 0.02	1.08 0.00
Adjusted $r^2$		81%	94%	-3%	92%	4%	6%	86%
SPE		123%	92%	707%	80%	400%	242%	58%
n		15	11	11	11	12	8	13

Variable Name		OTA Areal Density	Spectral Range minimum	Diff. Lim. $\lambda$	Operating Temp.	Year of Dev. (exp)	Date of Launch (exp)
Var.	p-value	0.06 0.90	-0.07 0.56	-0.11 0.54	0.04 0.88	0.00 0.91	0.02 0.56
Adjusted $r^2$		-8%	-4%	-7%	-8%	-7%	3%
SPE		810%	830%	787%	979%	1007%	747%
n		12	15	12	14	14	15



## Single Variable Cost Model

Diameter yields similar CER for Space & Ground OTA Cost.

**Ground OTA Cost ~ \$2M  $D^{1.4}$**

**Space OTA Cost ~ \$30M  $D^{1.4}$**

( $N = 15$ ;  $r^2 = 81\%$ ;  $SPE = 123$ ) (2012)

While single variable model is informative, it is of limited value:

- Diameter exponent is artificial because this model does not include year of development. More recent telescopes use advances in technology to produce larger aperture diameters at a lower cost.
- Diameter model only explains 81% of Cost Variation. Need additional variables to explain cost variation.



## OTA Cost versus Diameter and V2

rev. 8.1.10		OTA Cost vs Aperture Diameter and V2									
Variable 2		Aperture Diameter	PM F Len.	OTA Volume	FOV	Pointing Stability	OTA Mass	OTA Areal Density			
Diam.	p-value	1.42	0.00	0.73	0.19	-1.28	0.38	1.26	0.02	1.64	0.01
Var. 2	p-value	-	-	1.00	0.06	1.00	0.06	0.00	1.00	-0.21	0.32
Adjusted $r^2$		81%	93%	93%	4%	95%	85%	84%			
SPE		123%	84%	84%	142%	66%	58%	54%			
n		15	11	11	12	8	13	12			
Multicollinearity?		N/A	No	Yes	No	Yes	Yes	No			
Variable 2		Spectral Range minimum	Diffraction Limited Wavelength	Operating Temperature	Design Life (exp)	Year of Dev. (exp)	Dev. Period (exp)	Date of Launch (exp)			
Diam.	p-value	1.62	0.00	1.54	0.00	0.83	0.02	1.45	0.00	1.14	0.04
Var. 2	p-value	-0.18	0.02	-0.22	0.02	-0.08	0.64	0.01	0.01	-0.01	0.46
Adjusted $r^2$		96%	98%	81%	99%	84%	91%	82%			
SPE		74%	60%	136%	71%	124%	128%	120%			
n		15	12	14	15	14	13	15			
Multicollinearity?		No	No	No	No	No	No	No			

Considering variables that are not collinear with Diameter

- Focal Length increases  $r^2$  and decreases SPE but invalidates Diameter significance
- Diffraction Limit & Spectral Min are significant, both increase  $R^2$  & decrease SPE
- YOD or DOL are 'weakly' inverse correlated, slight cost reduction with time; but for Space, each new OTA is new – limited reuse.

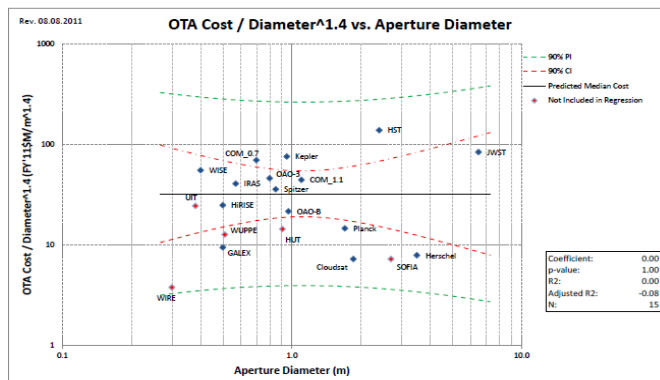


## Residual Error Analysis: Aperture

Divide data by Diameter Model (normalize data) and plot as a function of Variables.

$R^2$  indicates how % of residual error explained by a 2<sup>nd</sup> Variable

For example, as expected diameter explains 'zero' variation

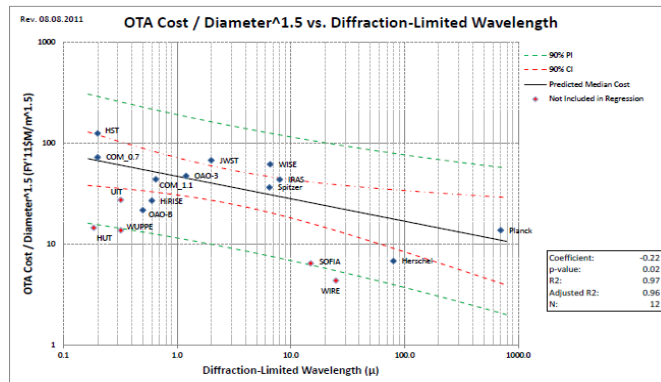




## Residual Error Analysis: Wavelength

Diffraction Limit Wavelength explains 97% of residual variation

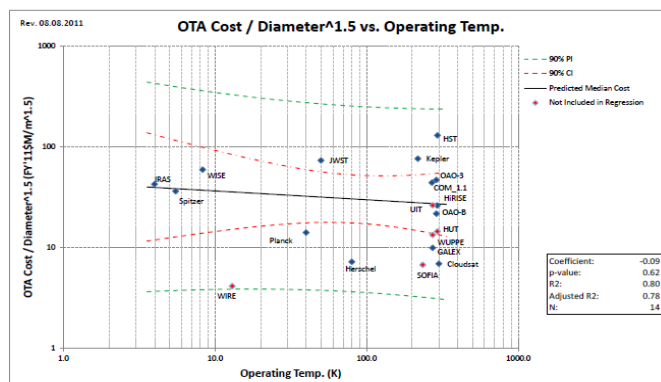
A -0.2 coefficient implies that an OTA with a 10X longer wavelength will cost 40% less.



## Aperture Residual Error Analysis: Temperature

Operating Temperature does not significantly explain residual aperture variation.

But, it might be a good 3<sup>rd</sup> or 4<sup>th</sup> CER parameter







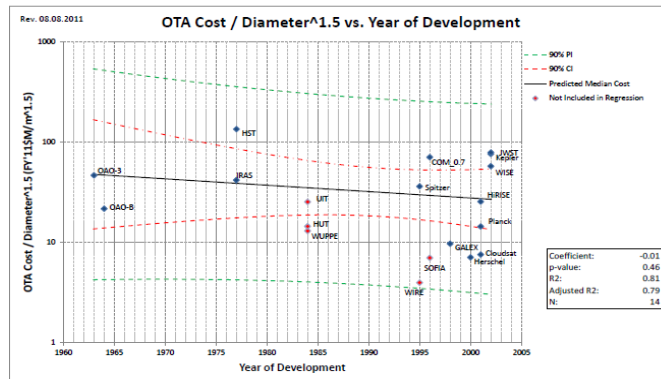
## Aperture Residual Error Analysis: YOD

Year of Development does not significantly explain residual.

But, it might be a good 3<sup>rd</sup> or 4<sup>th</sup> CER parameter

Concern that YOD is correlated with Aperture and Wavelength.

Also, what is role of spectroscopic vs imaging.



## Two Variable Aperture Model

Diffraction Limited Wavelength yields the best model:

$$\text{OTA Cost} \sim \text{Dia}^{1.65} \lambda^{-0.25} \quad (N = 12, r^2 = 99\%; \text{SPE} = 61\%)$$



## OTA Cost versus Diameter, Wavelength and V3

Operating Temperature is the only significant 3<sup>rd</sup> variable

$$\text{OTA Cost} \sim D^{1.7} \lambda^{-0.3} T^{-0.25}$$

( $N = 11$ ,  $r^2 = 96\%$ ;  $SPE = 54\%$ )

rev. 8.1.10		OTA Cost vs Diam., Diff. Lim., and V2									
Variable 2		Diam., Diff. Lim., & V3		FOV		Pointing Stability		OTA Mass		OTA Areal Density	
Diam.	p-value	1.54	0.00	1.37	0.01	1.48	0.10	0.66	0.24	1.97	0.00
Diff. Lim.	p-value	-0.22	0.02	-0.23	0.09	-0.11	0.68	-0.16	0.22	-0.16	0.22
V3	p-value	-	-	0.13	0.66	-0.18	0.56	0.66	0.07	0.66	0.07
Adjusted $r^2$		98%		84%		98%		92%		92%	
SPE		60%		73%		49%		46%		46%	
n		12		10		6		10		10	
Multicollinearity?		N/A		No		Yes		Yes		No	

Variable 2		Operating Temperature		Design Life (exp)		Year of Dev. (exp)		Dev. Period (exp)		Date of Launch (exp)	
Diam.	p-value	1.70	0.00	0.87	0.03	1.53	0.00	1.45	0.01	1.49	0.00
Diff. Lim.	p-value	-0.32	0.01	-0.05	0.66	-0.24	0.04	-0.18	0.06	-0.24	0.02
V3	p-value	-0.25	0.10	0.01	0.05	0.01	0.75	0.01	0.42	0.01	0.58
Adjusted $r^2$		96%		99%		97%		96%		97%	
SPE		54%		43%		60%		48%		58%	
n		11		12		11		10		12	
Multicollinearity?		No		Yes		No		No		No	



## Ground Telescopes



## Ground Multivariable Cost Model

Of 20 potential CER parameters, only four have statistically significant impact ( $p < 10\%$ ):

- Primary mirror diameter (D),
- Wavelength Diffraction Limited Performance ( $\lambda$ ),
- Reduction in Technology Cost over Time (where Y = Year of Development),
- Segmentation Factor (SF)



## 2012 Multi-Variable Ground Cost Model

Regressing on ground data set which contains only 5 segmented telescopes and assuming that there are NO cost differences between segment prescriptions (because 'learning' transfers between prescriptions):

$$\text{Ground OTA Cost} \sim (\$1\text{M}) (\text{SF})^{0.7} (\text{D})^{1.7} (\lambda)^{-0.7} e^{-0.04(Y)}$$

( $R^2=91\%$ , adjusted  $R^2=88\%$ , SPE = 37%)

Where:

OTA Cost in Millions of FY2000\$

**D** = Primary Mirror Diameter (meters)

**$\lambda$**  = Wavelength Diffraction Limit (microns)

**Y** = Year of Development - 2000

**SF** =  $(\text{\#of Segments})^{0.7} (\text{Ds/D})^{1.7}$

Luedtke, Alexander and H. Philip Stahl, "Commentary on Multivariable Parametric Cost Model for Ground Optical Telescope Assembly", Optical Engineering, Vol.51, OE-111662C



## Cost as a function of Diameter

An exponent coefficient for Cost vs Diameter of less than 2.0 is consistent with engineering experience.

Cost is a linear combination of diameter & diameter squared.

Some models estimate polishing cost as proportional to area.

But, this assumes a constant tool size. It is possible for tool size to increase with mirror diameter.

Also, ignores perimeter, which is hard to polish & varies with diameter.

Tool and fabrication machine size cost is directly proportional to mirror area.

Substrate cost also is related to Area and Areal Density.



## Wavelength Diffraction Limit (WDL)

Holding variables constant, visible OTA costs more to build than an IR OTA

It takes longer to polish a smooth UV/visible mirror than an infrared mirror.

Stiffer OTA needed to achieve & maintain WDL in UV/visible than infrared/Radio

Ground OTA regression has WDL power of -0.5 to -0.7:

-0.5 exponent predicts that a 2X wavelength change yields a 30% cost reduction

-0.7 exponent predicts that a 2X wavelength change yields a 40% cost reduction

Space OTA regression has WDL power of -0.25 to -0.3:

-0.25 exponent predicts that a 2X wavelength change yields a 15% cost reduction

-0.5 factor is consistent with published data (Meinel – optical to radio):

10X cost decrease for increasing WDL from 1  $\mu\text{m}$  to 0.1 mm

1000X decrease for increasing WDL from 1  $\mu\text{m}$  to 1 meter.

Cost Reduction vs WDL Model				
WDL	-0.3	-0.5	-0.7	Meinel
1 $\mu\text{m}$	na	na	na	na
0.1 mm	4	10	25	10X
1 meter	63	1000	15849	1000X



## Cost as a function of Year of Development

FACT: more recent telescopes tend to cost less than older telescopes because of technology advances.

Our analysis indicates this reduction to be  $\sim e^{-0.04(Y)}$

Horak published the reduction to be  $\sim e^{-0.033(Y)}$

A 4% reduction is cost per year from technology development implies that cost should reduce by 50% every 17 years.

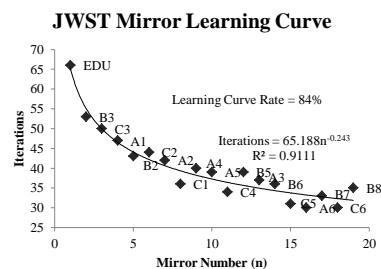
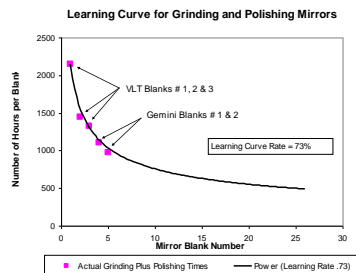
A 3.3% reduction implies a 50% reduction every 21 years.



## Segmentation Factor

Segmentation Factor captures the cost reduction from ‘learning’

- REOSC had ~ 73% learning curve for VLT & Gemini primary mirrors.
- JWST had ~ 84% learning curve.



But, it may only apply to the mirror segments and not the primary mirror assembly or telescope – because it does not include the cost of the support structure.



## Testing the Models

Model without Segmentation Factor better estimates JWST cost.

Cost Model Prediction Hubble versus JWST					
Parameter	HST	JWST	Ratio	$D^{1.8} \lambda^{-0.5} T^{-0.25}$ $e^{-0.033Y}$	$\#S^{0.7} D_s^{1.7} \lambda^{-0.5} T^{-0.25}$ $e^{-0.033Y}$
Diameter	2.4	6.5	2.7X	6X	
Segments	1	18+spare	19X		8X
Seg Dia	2.4	1.5	0.6X		0.4X
Wavelength	0.5	2	4X	0.5X	0.5
Temperature	300	30	0.1X	2X	2X
Year of Dev	1977	2006	29	0.4X	0.4X
Total	~ \$0.5B	~ \$1.2B	2.4X	2.4X	1.2X
Estimate				\$1.2B	\$0.6B

SF is missing something:

- Impact of increased Complexity of Segmented vs Monolithic
- Need to design and make a full size support structure
- Beryllium is 2X harder to fabricate than Glass
- JWST is 10X lower Aeral Density than HST

Horak Model has factors for Material, Off-Axis & Lightweighting.



## Conclusions



## Findings

### Programmatically

Largest Mission Cost drivers are Spacecraft & Instruments

OTA cost is 10% to 15% of Total Mission Cost

I&T cost is 10% to 25% of Total Mission Cost

### Engineering OTA cost drivers are similar for Ground & Space

Larger Diameter OTAs cost more than Smaller.

But Larger Diameter cost less per square meter of Collecting Aperture.

UVO Wavelength OTAs cost more than IR OTAs.

Cryogenic Temperature OTAs cost more than Ambient Temperature OTAs.

Technology Advance reduces cost ~ 50% about every 20 years.

If all parameters are held constant, adding Mass reduces cost.

### Mass is NOT a good Cost Estimating Relationship



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