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Martian Environment Electrostatic Precipitator

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As part of the planned manned mission to Mars, NASA has noticed that shipping oxygen as a part of life support to keep the astronauts alive continuously is overly expensive, and impractical. As such, noting that the Martian atmosphere is 95.37% $\mathrm{CO_2}$, NASA chemists noted that one could obtain oxygen from the Martian atmosphere. The plan, as part of a larger ISRU (in-situ resource utilization) initiative, would extract water from the regolith, or the Martian soil which can be electrolyzed by solar panel produced voltage into hydrogen and oxygen. The hydrogen can then be used in the Sabatier reaction with carbon dioxide to produce methane and water producing a net reaction that does not lose water and outputs methane and oxygen for use as rocket fuel and breathing

Electrolyze water from regolith: $\mathrm{2H_2O \rightarrow 2H_2 + O_2}$

Sabatier reaction: $\mathrm{CO_2 + 4H_2 \rightarrow CH_4 + 2H_2O}$

Net reaction: $\mathbf{CO_2 + 2H_2O \rightarrow CH_4 + 2O_2}$

Additionally, oxygen could be produced by Solid-Oxide Electrolysis,

Reduce Carbon Dioxide: $\mathrm{CO_2} + 2e^- \rightarrow \mathrm{O}^{2-} + \mathrm{CO}$

Recombine monatomic oxygen: $2\mathrm{O}^{2-} \rightarrow 4e^- + \mathrm{O_2}$

Net reaction: $\mathbf{2CO_2 \rightarrow O_2 + 2CO}$

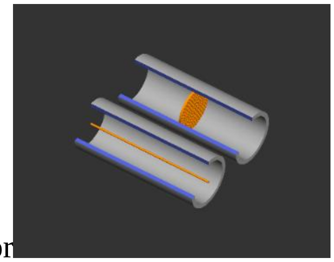
However this produces waste products of Carbon Monoxide.

For six astronauts, 2.2 kg per hour production of breathable oxygen is required. Given the conversion efficiency of carbon dioxide to oxygen and the composition of the Martian atmosphere, as well as the required oxygen production, we can calculate the mass intake required in the Martian atmosphere

$$\dot{m}_{\mathrm{mars}} = \dot{m}_{\mathrm{CO_2}} \frac{1}{\eta} \frac{1}{w_{\mathrm{CO_2}}} = \dot{m}_{\mathrm{O_2}} \frac{M_{\mathrm{CO_2}}}{M_{\mathrm{O_2}}} \frac{n_{\mathrm{CO_2}}}{n_{\mathrm{O_2}}} \frac{1}{\eta} \frac{1}{w_{\mathrm{CO_2}}}$$

And the volume of gas this represents

$$\dot{V}_{\mathrm{mars}} = \frac{\dot{m}_{\mathrm{mars}}}{M_{\mathrm{CO_2}}} R \frac{T}{P} = \frac{\dot{m}_{\mathrm{O_2}}}{M_{\mathrm{O_2}}} \frac{n_{\mathrm{CO_2}}}{n_{\mathrm{O_2}}} \frac{1}{\eta} \frac{1}{w_{\mathrm{CO_2}}} R \frac{T}{P} \text{[1]}.$$

The problem is that this requires such an intake of carbon dioxide that fine dust in the Martian atmosphere kicked up by frequent dust storms on the planet can interfere with the production of oxygen by adversely affecting the ISRU systems. Thus, it must be filtered out. The low pressure of the Martain Atmosphere would limit the flow rate on a conventonal filter thanks to the pressure gradient across the filter so what is required is a plasma device called an electrostatic precipitator  [2].

In an electrostatic precipitator, electric fields push dust out of a gas it is suspended in so the particles need to overcome the flow velocity $\dot{x} = \frac{Q}{\pi R^2}$ with a radial velocity $\dot{r} = \frac{Q}{\pi R}\frac{1}{L}$ thus dust particles will have a 100% efficiency rate with a velocity $\dot{r} > \frac{R\dot{x}}{L}$. Thus, given that the fraction of the dust particles taken out of the gas is

$\frac{dN}{N} = -\frac{2\pi R\dot{r}dt}{\pi R^2} = -\frac{2\dot{r}dt}{R}$, we can integrate this to get

$$\int_{N_0}^{N(t)} \frac{dN}{N} = \int_0^t -\frac{2\dot{r}}{R}dt$$

Revealing the fractional change in the number of particles over time as

$$\frac{N(t)}{N_0} = \exp\left(-\frac{2\dot{r}t}{R}\right)$$

And given the residence time in the tube $t = \frac{\pi R^2 L}{Q}$, this gives a penetration of

$$P = \frac{N_{out}}{N_0} = \exp\left(-\frac{2\pi\dot{r}RL}{Q}\right)$$

And an efficiency of

$$\eta = 1 - \exp\left(-\frac{\dot{r}A_C}{Q}\right).$$

From a conservation of energy perspective,

$$qV = \frac{1}{2}m\dot{r}^2 \rightarrow \dot{r} = \sqrt{\frac{2q}{m}V}$$

So given that $\dot{x} = \frac{Q}{\pi R^2}$, combining these equations yields

$$\sqrt{\frac{2q}{m}V} > \frac{Q}{\pi RL} \text{ [3].}$$

To calculate the breakthrough voltage a homogeneous electrical field is assumed. This is the case in a parallel plate capacitor setup. The electrodes may have the distance d. The cathode is located at the point $x = 0$. To get impact ionization, the electron energy E_e must become greater than the ionization energy E_I of the gas atoms between the plates. Per length of path x a number of α ionizations will occur. α is known as the first Townsend coefficient as it was introduced by Townsend. The increase of the electron current Γ_e can be described for the assumed setup as

$$\Gamma_e(x = d) = \Gamma_e(x = 0)e^{\alpha d}$$

(so the number of free electrons at the anode is equal to the number of free electrons at the cathode that were multiplied by impact ionization. The larger d and/or α the more free electrons are created.) The number of created free electrons is

$$\Gamma_e(d) - \Gamma_e(0) = \Gamma_e(0)(e^{\alpha d} - 1).$$

Neglecting possible multiple ionizations of the same atom, the number of created ions is the same as the number of created electrons:

$$\Gamma_i(0) - \Gamma_i(d) = \Gamma_e(0)(e^{\alpha d} - 1)$$

is the ion current. To keep the discharge going on, free electrons must be created at the cathode surface. This is possible because the ions hitting the cathode release secondary electrons at the impact. (For very large applied voltages also field electron emission can occur) Without field emission, we can write

$$\Gamma_e(0) = \gamma \Gamma_i(0)$$

Where γ is the mean number of generated secondary electrons per ion. This is also known as the second Townsend coefficient. Assuming that $\Gamma_i(d) = 0$ one gets the relation between the Townsend coefficients by substitution and transforming:

$$ad = \ln\left(1 + \frac{1}{\gamma}\right).$$

What is the amount of α? The number of ionization depends upon the probability that an electron hits an ion. This probability P is the relation of the cross-sectional area of a collision between electron and ion σ in relation to the overall area A that is available for the electron to fly through:

$$P = \frac{N\sigma}{A} = \frac{x}{\lambda}.$$

As expressed by the second part of the equation, it is also possible to express the probability as The relation of the path traveled by the electron x to the mean free path length λ (the distance at which another collision occurs). N is the number of molecules which electrons can hit. It can be calculated using the equation of state of the ideal gas

$$pV = Nk_BT.$$

The collision cross section can be written as $\sigma = \pi(r_a^2 + r_b^2)$. As the radius of an electron can be neglected compared to the radius of an ion r_I it simplifies to $\sigma = \pi r_I^2$. Using this relation, substitution for the mean free path length yields

$$\lambda = \frac{k_B T}{p\pi r_I^2} = \frac{1}{Lp}$$

Where the factor L was only introduced for a better overview. The alteration of the current of not yet collided electrons at every point in the path x can be expressed as

$$d\Gamma_e(x) = -\Gamma_e(x)\frac{dx}{\lambda_e}.$$

This differential equation can easily be solved:

$$\Gamma_e(x) = \Gamma_e(0)\exp\left(-\frac{x}{\lambda_e}\right).$$

The probability that $\lambda > x$ (that there was not yet a collision at the point x) is

$$P(\lambda > x) = \frac{\Gamma_e(x)}{\Gamma_e(0)} = \exp\left(-\frac{x}{\lambda_e}\right).$$

According to its definition α is the number of ionizations per length of path and thus the relation of the probability that there was no collision in the mean free path of the ions, and the mean free path of the electrons:

$$\alpha = \frac{P(\lambda > \lambda_I)}{\lambda_e} = \frac{1}{\lambda_e}\exp\left(-\frac{\lambda_I}{\lambda_e}\right) = \frac{1}{\lambda_e}\exp\left(-\frac{E_I}{E_e}\right).$$

It was hereby considered that the energy E that a charged particle can between a collision depends on the electric field strength ε and the charge Q:

$$E = \lambda Q \varepsilon.$$

For the parallel –plate capacitor we have $\varepsilon = \frac{U}{d}$ where U is the applied voltage. As a single ionization was assumed Q is the elementary charge e. We can now substitute and obtain

$$\alpha = L \cdot p \exp\left(-\frac{L \cdot p \cdot d \cdot E_I}{eU}\right).$$

Substitution and transforming to V we get the Paschen law for the breakdown voltage

$$U_B(pd) = \frac{LpdE_I}{e\ln(Lpd) - \ln\left[\ln\left(1 + \frac{1}{\gamma}\right)\right]}$$

However, $L = \frac{\pi r_I^2}{k_B T}$ so we can rewrite this in terms of the ionization saturation and ionization energy constants as

$$V_B(pd) = \frac{Bpd}{\ln(Apd) - \ln\left[\ln\left(1 + \frac{1}{\gamma}\right)\right]}$$

And solving for the minimum pd we obtain the minimum breakdown voltage of

$$V_{\mathrm{B,\ min}} = \frac{B}{A} e \ln\left(1 + \frac{1}{\gamma}\right) \text{ [4].}$$

Assuming the upper region of Paschen's law is roughly linear gives the following relation $V_B = \alpha PR = E_B R$ where E_B is the electric field required for breakdown.

Given that some data for California Polytechnic [5] suggests the slope of the linear region in carbon dioxide is approximately $\alpha = 125\ V\ \mathrm{Torr}^{-1}\mathrm{cm}^{-1}$ and since the Martian atmosphere has a pressure of $P = 4.75$ Torr, the slope of the Paschen curve for Martian pressure is $E_B = 60\ kV\ \mathrm{m}^{-1}$ and the breakdown voltage for a cylindrical precipitator as a function of it's radius is roughly:

$$V_B = \alpha PR = E_B R = (125\ V\ \mathrm{Torr}^{-1}\mathrm{cm}^{-1})(4.75\ \mathrm{Torr})R = (60 kV\ \mathrm{m}^{-1})R.$$

For Corona operation, a voltage of approximately half the breakdown voltage will be assumed

$$V_c = \frac{1}{2}V_b = \frac{1}{2}E_b R = E_c R = (30\ kV \mathrm{m}^{-1})R$$

Where E_C is the electric field of the corona. Plugging this into the earlier constraint yields

$$\sqrt{2\frac{q}{m}E_c R} > \frac{Q}{\pi RL}$$

And thus

$$R^3 > \frac{1}{2}\frac{1}{E_c}\frac{m}{q}\left(\frac{Q}{\pi L}\right)^2$$

Indicating that the radius should increase at a rate greater than the length to maintain the same efficiency and thus we can write that the overall efficiency as

$$\eta = 1 - e^{-\left(\frac{2\pi\sqrt{\frac{2q}{m}}VRL}{Q}\right)}.$$

If the charge density of the gas in the tube is given, we can write in cylindrical coordinates

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \Phi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 \Phi}{\partial \phi^2} + \frac{\partial^2 \Phi}{\partial z^2} = -\frac{\rho}{\varepsilon_0}$$

But the electric field does not vary much along the length and along the circumference so the equation reduces to

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{d\Phi}{dr}\right) = -\frac{\rho}{\varepsilon_0}.$$

The volume charge density in the radial direction can be related to the steady state current I between the inner and outer conductors. The current is assumed entirely due to conduction. The volume current density J is related to the electric field and charged particles flowing through the two conductors via the expression

$$J(r) = b|\rho(r)|E(r)$$

Where b is the mobility of the charged particles, assumed to be constant. Notice that both the steady-state volume charge density and electric field can vary with the radial distance from the center of the wire. The volume current density is simply related to the steady-state current, by multiplying the current density by the area that the current density is passing through:

$$I = 2\pi rLJ = 2\pi rLb|\rho(r)|E(r)$$

Where L is the length of the outer conducting tube. Solving for the volume charge density, assuming that the volume charge density is positive, and substituting into Poisson's equation,

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{d\Phi}{dr}\right) = -\frac{1}{\varepsilon_0}\left(\frac{I}{2\pi rLbE(r)}\right).$$

To eliminate the potential function, substitute $E(r) = -\frac{d\Phi}{dr}$ which gives

$$\frac{1}{r}\frac{d}{dr}\left(-rE(r)\right) = -\frac{1}{\varepsilon_0}\left(\frac{I}{2\pi rLbE(r)}\right)$$

Or

$$\frac{d}{dr}\,(rE(r)) = \frac{I}{2\pi\varepsilon_0 LbE(r)}.$$

One way of determining the solution of this differential equation is to let $m = rE(r)$:

$$\frac{dm}{dr} = \frac{Ir}{2\pi\varepsilon_0 Lbm}$$

Then integrating in the standard way,

$$\int_{m(a)}^{m(r)} mdm = \int_a^r \frac{Irdr}{2\pi\varepsilon_0 Lb} \rightarrow \frac{1}{2}[m^2(r) - m^2(a)] = \frac{I}{4\pi\varepsilon_0 Lb}(r^2 - a^2)$$

And solving for m,

$$m(r) = \sqrt{\frac{I}{2\pi\varepsilon_0 Lb}(r^2 - a^2) + m^2(a)}.$$

In terms of the electric field,

$$E(r) = \sqrt{\frac{I}{2\pi\varepsilon_0 Lb} + \left[1 - \left(\frac{a}{r}\right)^2\right]\frac{m^2(a)}{r^2}}.$$

Since $m(a) = aE(a) = aE_c$, then

$$E(r) = \sqrt{\frac{I}{2\pi\varepsilon_0 Lb} + \left(\frac{a}{r}\right)^2\left(E_c - \frac{I}{2\pi\varepsilon_0 Lb}\right)}.$$

The fields generated by the electrodes and space charge are included in this result. Not too close to the wire and for larger currents, the field is approximately uniform and independent of position:

$E(r) = \sqrt{\frac{I}{2\pi\varepsilon_0 Lb} - \left(\frac{a}{r}\right)^2\frac{I}{2\pi\varepsilon_0 Lb} + \left(\frac{a}{r}\right)^2 E_c^2} \approx \sqrt{\frac{I}{2\pi\varepsilon_0 Lb}}$ if $\frac{I}{2\pi\varepsilon_0 Lb} \gg \left(\frac{a}{r}\right)^2\frac{I}{2\pi\varepsilon_0 Lb}$ and $\frac{I}{2\pi\varepsilon_0 Lb} \gg \left(\frac{a}{r}\right)^2 E_c^2$. Also $E(r) \approx \sqrt{\frac{I}{2\pi\varepsilon_0 Lb}}$ if $r \gg a$ and $I \gg \left(\frac{a}{r}\right)^2 E_c^2 2\pi\varepsilon_0 Lb$.

The electric field of the coronal discharge produces a voltage that is roughly constant while the voltage input required from the power supply increases exponentially with the ratio of the radius of the tube and the radius of the wire. As such we can write that $E(r) = \frac{V}{r\ln\frac{R}{a}}$, and from this we obtain the relation $E_c = \frac{V_c}{a\ln\left(\frac{R}{a}\right)}$ for the coronal electric field. This yields the general expression for the electric field in total as

$$E(r) = \sqrt{\frac{I}{2\pi\varepsilon_0 Lb} + \left(\frac{a}{r}\right)^2 \left[\left(\frac{V}{a\ln\frac{R}{a}}\right)^2 - \frac{I}{2\pi\varepsilon_0 Lb}\right]}.$$

To determine the voltage of the wire relative to the outer conductor, one must integrate and obtain

$$V_0 \approx \int_a^R \sqrt{\frac{I}{2\pi\varepsilon_0 Lb} + \left(\frac{a}{r}\right)^2 E_c^2}\,dr = \int_a^R \frac{a}{r} E_c \sqrt{\frac{I}{2\pi\varepsilon_0 Lb}\left(\frac{r}{a}\right)^2 \frac{1}{E_c^2} + 1}\,dr$$

If $E_c^2 \gg \frac{I}{2\pi\varepsilon_0 Lb}$. Likewise,

$$V_0 \approx \int_a^R \frac{a}{r} E_c \left[1 + \frac{1}{2}\frac{I}{2\pi\varepsilon_0 Lb}\left(\frac{r}{a}\right)^2 \frac{1}{E_c^2}\right] dr$$

If $\frac{I}{2\pi\varepsilon_0 Lb}\left(\frac{r}{a}\right)^2 \frac{1}{E_c^2} \ll 1$ using the binomial expansion $(1+x)^{1/2} \approx 1 + \frac{x}{2}$ if $|x| \ll 1$.

Integrating, we get

$$V_0 = \int_a^R \left(\frac{aE_c}{r} + \frac{Ir}{4\pi\varepsilon_0 LbE_c a}\right) dr$$

If $\left(\frac{a}{r}\right)^2 E_c^2 \gg \frac{I}{2\pi\varepsilon_0 Lb}, R > a$, which solves to

$$V_0 = aE_c \ln\left(\frac{R}{a}\right) + \frac{I(R^2 - a^2)}{8\pi\varepsilon_0 LbE_c a}$$

If $\left(\frac{a}{r}\right)^2 E_c^2 \gg \frac{I}{2\pi\varepsilon_0 Lb}$ since $E_c = \frac{V_c}{a\ln\left(\frac{R}{a}\right)}$ [6].

Additionally, the drop in pressure can be obtained through dimensional analysis or noting that the change in pressure is really proportional to the kinetic energy per unit volume and the ratio to the diameter over the length of the pipe $\Delta p \propto \frac{L}{D}\frac{1}{2}\rho v^2$ and incorporating a proportionality factor yields the Darcy-Weisbach equation $\Delta p = f_D \frac{L}{D}\frac{1}{2}\rho v^2$. In reality, the Darcy factor is not a constant and is defined as $f_D = \frac{64}{Re}$ where Re is the Reynolds number for predicting the flow of the fluid $Re = \frac{\rho}{\mu} vD = \frac{vD}{\eta}$ where η is the kinematic viscosity[7]. Given this, and the fact that for carbon dioxide the viscosity is $\mu = 1.5 * 10^{-5} N * \frac{s}{m^2}$ and $\rho \sim 0.02 \frac{kg}{m^3}$ while our target flow velocity is $Q = 0.01333 \frac{m^3}{s} \rightarrow v = 3.36 \frac{m}{s}$ then $Re = 318.808$ indicating Laminar flow. The entrance length to ensure fully developed flow however, is $L_e = .05 ReD = 1.13m$.

Laminar flow, or fluid flow of parallel vectors through a pipe of uniform circular cross-section means that the flow is steady so that $\frac{\partial v_x}{\partial t} = \frac{\partial v_y}{\partial t} = \frac{\partial v_z}{\partial t} = 0$, the radial and swirl components of the fluid velocity are zero, $v_r = v_\theta = 0$, and the flow is axisymmetric $\frac{\partial v_x}{\partial \theta} = \frac{\partial v_y}{\partial \theta} = \frac{\partial v_z}{\partial \theta} = 0$ and fully developed $\frac{\partial v_z}{\partial z} = 0$. Thus, using cylindrical coordinates the pressure becomes a function of the z coordinate where $\frac{\partial p}{\partial r} = 0$ giving $\frac{1}{r} \frac{\partial}{\partial r}\left(r \frac{\partial v_z}{\partial r} \right) = \frac{1}{\mu} \frac{\partial p}{\partial z}$ where μ is the dynamic viscosity of the fluid. This solves to $v_z = \frac{1}{4\mu} \frac{\partial p}{\partial z} r^2 + c_1 \ln r + c_2$. Since u_z must be finite at $r = 0, c_1 = 0$, then the boundary condition at the edge of the pipe means that $u_z = 0$ at $r = R$ so $c_2 = -\frac{1}{4\mu} \frac{\partial p}{\partial z} R^2$ or generally $v_z = -\frac{1}{4\mu} \frac{\partial p}{\partial z} (R^2 - r^2)$ with a maximum velocity at the center of $v_{zmax} = \frac{R^2}{4\mu}\left(-\frac{\partial p}{\partial z}\right)$. Integrating over the pipe cross section gives the average velocity of

$$v_{zavg} = \frac{1}{\pi R^2} \int_0^R v_z \cdot 2\pi r dr = 0.5 v_{zmax}.$$

Assuming that along the length of the pipe L the pressure drops linearly or $-\frac{\partial p}{\partial z} = \frac{\Delta p}{L}$, then we can substitute this and the maximum velocity into the equation for the average velocity to give $v_{zavg} = \frac{D^2}{32\mu} \frac{\Delta p}{L}$ where D is the diameter of 2R. Rearranging this gives $\Delta p = \frac{32 \mu L v}{D^2}$ or $\Delta p = \frac{8 \mu L Q}{\pi R^4}$ where Q is the volumetric flow rate [8]. This can be rewritten as $Q = \frac{\pi R^4 \Delta p}{8 \mu L}$ or $Q = \pi R^2 v$ which given that we have already derived $R^3 > \frac{1}{2} \frac{1}{E_c} \frac{m}{q} \left(\frac{Q}{\pi L} \right)^2$ then by substitution we obtain $R^3 > \frac{1}{2} \frac{1}{E_c} \frac{m}{q} \left(\frac{R^2 v}{L} \right)^2$ or else simplified as $R > \frac{1}{2E_c} \frac{m}{q} \frac{v}{L}$. From this we can determine that the radius should increase at a rate proportional to the rate of flow along the tube.
Then, the efficiency

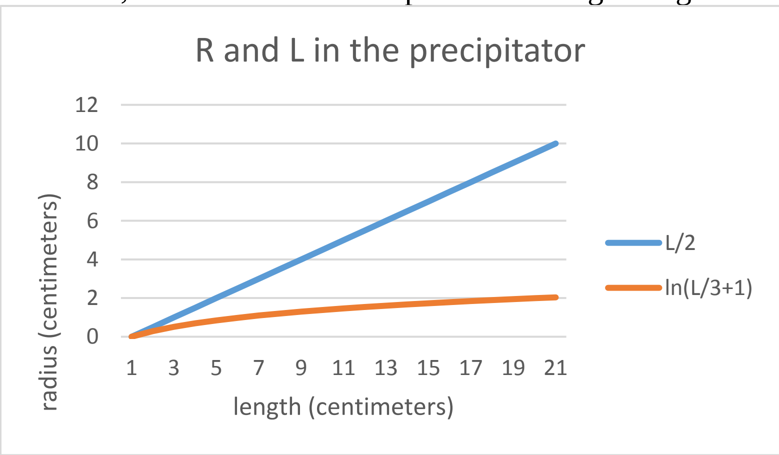
$$\eta = 1 - e^{-\left(\frac{2\pi \sqrt{\frac{2q}{m}} VRL}{Q} \right)}$$

Becomes

$$\eta = 1 - e^{-\left(\frac{2 \sqrt{\frac{2q}{m}} VL}{Rv} \right)}$$

Where we see that the efficiency will decrease as the flow velocity increases because of the increasing rate of fluid flow that prevents dust charging by pushing it away from the electric field. However, rearranging our inequality to yield $\frac{1}{v} > \frac{1}{2E_c}\frac{m}{q}\frac{1}{RL}$ or $v > \frac{2E_C q}{m}RL$. The voltage of the corona wire relative to the side of the tube is largely unaffected by the geometry of the tube, provided the tube is decently large given that the voltage function plateaus as a function of R and L beyond a certain point, leaving the only real variable for the voltage being the input voltage from the power supply and thus the current through the gas. Likewise, the ratio of $\frac{q}{m}$ is by Paschen's law also linearly proportional to the voltage, which indicates that a maximum efficiency can be reached for large precipitators that are much longer than wide but have a thin corona wire and a low flow velocity.

However, given that the efficiency is proportional to $\frac{L}{R}$ and the pressure drop is proportional to $\frac{L}{R^4}$, the pressure drop is lowest when $R > \ln\left(\frac{L}{3}+1\right)$ and the efficiency is greatest when $R \ll \frac{L}{2}$. However, the pressure drop remains relatively constant above the curve and the efficiency spikes up exponentially as the length increases. As such, the areas between these two curves is the prime area of concern but we want the area that is far closer to the logarithmic function than to the linear, which is much more practical at larger length scales.



The flow velocity appears to be proportional to the cross sectional area of the pipes, so to keep the efficiency high, it makes sense to have again, a relatively small radius above but closer to the orange logarithmic curve. When combined with a high voltage, this allows for the highest combination of efficiency and flow rate. Additionally, to obtain an even higher flow rate, it makes sense to not only make a single precipitator and instead use multiple precipitators together in a tube to obtain a maximum flow rate overall and a maximum efficiency combination. This is particularly important when one factors in the necessity to obtain a low entry length to ensure fully developed flow in the precipitator tube.

The fact that the Martian atmosphere saturated with dust is being charged qualifies it as a dusty plasma. Dusty plasmas are defined as having nanometer or micrometer size particles embedded in them, and being extremely low temperature overall. The forces acting on the dust in a dusty plasma is ultimately determined by electromagnetism, viscosity, gravity, and body forces or $m\frac{dv}{dt} = mg + q(E + v \times B) - m v_c v + f$. Which force dominates the behavior of the plasma however, depends on the size of the dust particles. The ratio of $\frac{q}{m}$ ultimately determines the role the viscous forces, the gravitational forces, the electromagnetic, and body forces play on the system [9]. In order to determine the Martian atmospheric plasma in the Electrostatic precipitator's behavior, we need to find $\frac{q}{m}$ for Martian dust, which can range in size from around 0.1 μm to about 5 μm in diameter, but typically exist at around 3 μm in diameter. However, it gets more complicated. Charging of this dust occurs over a real finite experimentally derived charging time of $\tau = \frac{4\varepsilon_0}{N_i e b}$ where N_i is the number density of ions and the ion mobility constant is b. However, this cannot merely be due to pure collision, because we immediately notice that because the mean free path length is defined as $\mu = \frac{1}{n\pi d^2}$ or $\mu = \frac{RT}{\sqrt{2} N_A P \pi d^2}$ [10]. Since the Martian atmosphere has a pressure of $P = 4.75$ Torr and an average temperature of $218.15\ K$, the number density n could be calculated to give a mean free path length of about $9.8 \mu m$. As such, this length is essentially negligible.

Therefore, we need to look deeper into plasma physics for an explanation. The Debye length arises naturally in thermodynamic description of large systems of mobile charges. In a system N different species of charges, the j-th species carries charge q_j and has concentration $n_j(r)$ at position r. According to the so-called "primitive model", these charges are distributed in a continuous medium that is characterized only by its relative static permittivity, ε_r. This distribution of charges within this medium gives rise to an electric potential $\Phi(r)$ that satisfies Poisson's equation:

$$\varepsilon \nabla^2 \Phi(r) = -\sum_{j=1}^{N} q_j n_j(r) - \rho_E(r)$$

Where $\varepsilon \equiv \varepsilon_r \varepsilon_0$, and ρ_E is a charge density external (not spatially) to the medium. The mobile charges not only establish $\Phi(r)$ but also move in response to the associated Coulomb force $-q_j \nabla \Phi(r)$. If we further assume the system to be in thermodynamic equilibrium with a heat bath at absolute temperature T, then the concentrations of discrete charges $n_j(r)$, may be considered to be thermodynamic (ensemble) averages and the associated electric potential to be a thermodynamic mean field. With these assumptions, the concentration of the j-th charge species is described by the Boltzmann distribution

$$n_j(r) = n_j^0 \exp\left(-\frac{q_j \Phi(r)}{k_B T}\right),$$

Where n_j^0 is the mean concentration of charges of species j. Identifying the instantaneous concentrations and potential in the Poisson equation with their mean-field counterparts in Boltzmann's distribution yields the Poisson-Boltzmann equation:

$$\varepsilon\nabla^2\Phi(r) = -\sum_{j=1}^{N} q_j n_j^0 \exp\left(-\frac{q_j\Phi(r)}{k_BT}\right) - \rho_E(r).$$

Solutions to this nonlinear equation are known for some simple systems. Solutions for more general systems may be obtained in the high temperature (weak coupling) limit $q_j\Phi(r) \ll k_BT$ by Taylor expanding the exponential:

$$\exp\left(-\frac{q_j\Phi(r)}{k_BT}\right) \approx 1 - \frac{q_j\Phi(r)}{k_BT}.$$

This approximation yields the linearized Poisson-Boltzmann equation

$$\varepsilon\nabla^2\Phi(r) = \left(\sum_{j=1}^{N}\frac{n_j^0 q_j^2}{k_BT}\right)\Phi(r) - \sum_{j=1}^{N} q_j n_j^0(r) - \rho_E(r)$$

Which is also known as the Debye-Hückel equation: the second term on the right hand side vanishes for systems that are electrically neutral. The term in parentheses divided by ε, has the units of an inverse length squared and by dimensional analysis leads to the definition of the characteristic length scale

$$\lambda_D = \left(\frac{\varepsilon k_BT}{\sum_{j=1}^{N} n_j^0 q_j^2}\right)^{\frac{1}{2}}$$

That commonly is referred to as the Debye-Hückel length. As the only characteristic length scale in the equation, it sets the scale for variations in the potential and in concentrations of charged species. All charged species contribute to the Debye length in the same way, regardless of the sign of their charges. For an electrically neutral system, the Poisson equation becomes

$$\nabla^2\Phi(r) = \lambda_D^{-2}\Phi(r) - \frac{\rho_E(r)}{\varepsilon}.$$

To illustrate Debye screening, the potential produced by an external point charge $\rho_E = Q\delta(r)$ is

$$\Phi(r) = \frac{Q}{4\pi\varepsilon r}e^{-\frac{r}{\lambda_D}}.$$

The bare Coulomb potential is exponentially screened by the medium over a distance of the Debye length. In a plasma, the background medium may be treated as the vacuum ($\varepsilon_r = 1$), and the Debye length is

$$\lambda_D = \sqrt{\frac{\frac{\varepsilon_0 k_B}{q_e^2}}{\frac{n_e}{T_e} + \sum_j \frac{z_j^2 n_j}{T_j}}}$$

And the ion term is often dropped due to the fact electrons are much more mobile than ions giving

$$\lambda_D = \sqrt{\frac{\varepsilon_0 k_B T_e}{n_e q_e^2}} \text{ [11].}$$

When one inserts a Langmuir probe into such a plasma, this can cause a Debye sheath to form. The quantitative physics of the Debye sheath is determined by four phenomena:

Energy conservation of the ions: If we assume for simplicity cold ions of mass m_i entering the sheath with a velocity u_0, having charge opposite to the electron, conservation of energy in the sheath potential requires

$$\frac{1}{2} m_i u(x)^2 = \frac{1}{2} m_i u_0^2 - e\varphi(x)$$

Where e is the positive elementary charge.

Ion continuity: In the steady state, the ions do not build up anywhere so the flux is everywhere the same:

$$n_0 u_0 = n_i(x) u(x).$$

The Boltzmann relation for the electrons: Since most of the electrons are reflected, their density is given by

$$n_e(x) = n_0 \exp\left(\frac{e\varphi(x)}{k_B T_e}\right).$$

Poisson’s equation: The curvature if the electrostatic potential is related to the net charge density as follows:

$$\frac{d^2\varphi(x)}{dx^2} = \frac{e(n_e(x) - n_i(x))}{\varepsilon_0}.$$

Combining these equations and writing them in terms of the dimensionless potential, position, and ion speed,

$$\chi(\xi) = -\frac{e\varphi(\xi)}{k_B T}$$

$$\xi = \frac{x}{\lambda_D}$$

$$\mathcal{M} = \frac{u_0}{\left(\frac{k_B T_e}{m_i}\right)^{\frac{1}{2}}}$$

We arrive at the sheath equation:

$$\chi'' = \left(1 + \frac{2\chi}{\mathcal{M}^2}\right)^{-\frac{1}{2}} - e^{-\chi}.$$

The sheath equation can be integrated once by multiplying by χ':

$$\int_0^\xi \chi'\chi'' d\xi_1 = \int_0^\xi \left(1 + \frac{2\chi}{\mathcal{M}^2}\right)^{-\frac{1}{2}} \chi' d\xi_1 - \int_0^\xi e^{-\chi}\chi'\, d\xi_1.$$

At the sheath edge ($\xi = 0$), we can define the potential to be zero ($\chi = 0$) and assume that the electric field is also zero ($\chi' = 0$). With these boundary conditions, the integrations yield

$$\frac{1}{2}\chi'^2 = \mathcal{M}^2 \left[\left(1 + \frac{2\chi}{\mathcal{M}^2}\right)^{\frac{1}{2}} - 1\right] + e^{-\chi} - 1.$$

This is easily rewritten as an integral in closed form, although one that can only be solved numerically. Nevertheless, an important piece of information can be derived analytically. Since the left-hand-side is a square, the right-hand-side must also be non-negative for every value of χ, in particular for small values. Looking at the Taylor expansion around $\chi = 0$, we see that the first term that does not vanish is the quadratic one, so that we can require

$$\frac{1}{2}\chi^2(-\frac{1}{\mathcal{M}^2} + 1) \geq 0$$

Or

$$\mathcal{M}^2 \geq 1$$

Or

$$u_0 \geq \left(\frac{k_B T_e}{m_i}\right)^{\frac{1}{2}}.$$

This inequality is known as the Bohm sheath criterion. If the ions are entering the sheath too slowly, the sheath potential will "eat" its way into the plasma to accelerate them. Ultimately a so-called pre-sheath will develop with a potential drop on the order of $\left(\frac{k_B T_e}{2e}\right)$ and a scale determined by the physics of the ion source (often the same as the dimensions of the plasma). Normally the Bohm criterion will hold with equality, but there are some situations where the ions enter the sheath with supersonic speed.

Although the sheath equation must generally be integrated numerically, we can find an approximate solution analytically by neglecting the $e^{-\chi}$ term. This amounts to neglecting the electron density in the sheath, or only analyzing that part of the sheath where there are no electrons. For a "floating' surface, i.e. one that draws no net current from the plasma, this is a useful if rough approximation. For a surface biased strongly negative so that it draws the ion saturation current, the approximation is very good. It is customary, although not strictly necessary, to further simplify the equation by assuming that $\frac{2\chi}{\mathcal{M}^2}$ is much larger than unity. Then the sheath equation takes on the simple form

$$\chi'' = \frac{\mathcal{M}}{(2\chi)^{\frac{1}{2}}}.$$

As before, we multiply by χ' and integrate to obtain

$$\frac{1}{2}\chi'^2 = \mathcal{M}(2\chi)^{\frac{1}{2}}$$

Or

$$\chi^{-\frac{1}{4}}\chi' = 2^{\frac{3}{4}}\mathcal{M}^{\frac{1}{2}}.$$

This is easily integrated over ξ to yield

$$\frac{4}{3}\chi_w^{\frac{3}{4}} = 2^{\frac{3}{4}}\mathcal{M}^{\frac{1}{2}}d,$$

Where χ_W is the normalized potential at the wall (relative to the sheath edge), and d is the thickness of the sheath. Changing back the variables u_0 and φ and noting that the ion current into the wall is $J = e\, n_0 u_0$, we have

$$J = \frac{4}{9}\left(\frac{2e}{m_i}\right)^{\frac{1}{2}}\frac{|\varphi_w|^{\frac{3}{2}}}{4\pi d^2}.$$

This equation is known as Child's Law. It was first used to give the space-charge-limited current in a vacuum diode with electrode spacing d. It can also be inverted to give the thickness of the Debye sheath as a function of the voltage drop by setting $J = j_{ion}^{sat}$:

$$J = \frac{2}{3}\left(\frac{2e}{m_i}\right)^{\frac{1}{4}} \frac{|\varphi_w|^{\frac{3}{4}}}{2\sqrt{\pi j_{ion}^{sat}}}.$$

As such, we see here clearly that as the dust particles act as low potentials relative to the ions and electrons and the electrons are far more mobile and energetic, the potential that arises from their separation causes the dust particles to act as a negative potential relative to the positive ions and attract a sheath of ions of the thickness of the Debye length [12]. This is the mechanism by which the dust becomes charged.

This can be rewritten in terms of our plasma as $d = \frac{2}{3}\left(\frac{2e}{m_i}\right)^{\frac{1}{4}}\left(\frac{\varepsilon_0}{J}\right)^{\frac{1}{2}} V^{\frac{3}{4}}$ and given the Bohm criterion $J \geq \frac{1}{2} n_0 e \left(\frac{k_B T_e}{m_i}\right)^{\frac{1}{2}}$, thus $d = \frac{2}{3}(2e)^{\frac{1}{4}} \frac{\sqrt{\varepsilon_0}}{\sqrt{n_0} e (k_B T_e)^{\frac{1}{4}}} V^{\frac{3}{4}} = \frac{(2)^{\frac{3}{4}}}{3}\sqrt{\frac{\varepsilon_0 k_b T_e}{n_e q_e^2}} = \frac{(2)^{\frac{3}{4}}}{3}\lambda_D$.

In order to calculate the ratio of $\frac{q}{m}$, we approximate by using a spherical shell of carbon dioxide molecules outside a spherical dust particle to calculate the number of carbon dioxide ions. Because the ionization reaction is $\mathrm{CO_2} \rightarrow \mathrm{CO_2^+} + e^-$, we then multiply it by the elementary charge.

$$q = \frac{e4\pi\left(\frac{(2)^{\frac{3}{4}}}{3}\lambda_D\right)^3}{3(4\pi r_{CO_2}^3)} = \frac{(2)^{\frac{3}{4}}}{3} \frac{e\left(\frac{\varepsilon_0 k_B T_e}{n_e e^2}\right)^{\frac{3}{2}}}{3r_{CO_2}^3} = \left(\frac{\varepsilon_0 k_B T_e}{n_e}\right)^{\frac{3}{2}} \frac{(2)^{\frac{3}{4}}}{9e^2 r_{CO_2}^3}.$$

If we approach thermodynamics from a statistical standpoint, we can write the Boltzmann constant $k_B = \frac{R}{N_A}$ and thus the number of particles $N = nN_A$ where n is the number of moles. Therefore $Nk_B = nR$ and $PV = nk_B T$. For impulses of particle collisions since

$$\int_t^{t+\Delta t} F_x(t')dt' = \int_t^{t+\Delta t} ma(t')dt' = mv_x(t+\Delta t) - mv_x(t)$$

Then the average force is

$$F_x^{av} = \frac{1}{\Delta t}\int_t^{t+\Delta t} F_x(t')dt' .$$

Assuming flat, rigid walls, a collision means that the x component will be reversed so

$mv_x(t+\Delta t) - mv_x(t) = m|v_x| - m(-|v_x|) = 2m|v_x|$. Thus, $F_x^{av}\Delta t = 2m|v_x|$. If L is the length of the gas container, then to go back and forth with $2L$ it takes a time of $\Delta t = \frac{2L}{|v_x|}$ so $F_x^{av} = \frac{mv_x^2}{L}$. Statistically the total would be

$$F_x^{tot} = \sum_{i=1}^{N} \frac{m_i v_{x,i}^2}{L}$$

This of course leads to the idea that

$$P = \frac{1}{V} \sum_{i=1}^{N} m_i v_{x,i}^2$$

Where $V = L^3$. Averaging the three for x, y, and z gives

$$P = \frac{1}{3V} \sum_{i=1}^{N} m_i (v_{x,i}^2 + v_{y,i}^2 + v_{z,i}^2)$$

Or

$$P = \frac{N}{3V} \left(\frac{1}{N} \sum_{i=1}^{N} m_i v_i^2 \right).$$

More generally, the average of a value is

$$\bar{X} = \frac{1}{N} \sum_{i=1}^{N} X_i$$

Thus, $PV = \frac{2N}{3}\left(\frac{1}{2}\overline{mv^2}\right)$ or $\frac{PV}{N} = \frac{2}{3}\left(\frac{1}{2}\overline{mv^2}\right) = k_B T$. Thus the average kinetic energy of a gas is $\frac{3}{2} k_B T$. This is equivalent to three degrees of freedom with each one at $\frac{1}{2} k_B T$ [13]. Thus, the temperature of the electrons can be calculated as $eV_0 = \frac{3}{2} k_B T_e$ or rearranged as $T_e = \frac{2eV_C}{3k_B}$. The number density can be calculated from the ideal gas law $P = \frac{n_e R T_e}{N_A}$ to obtain $n_e = \frac{N_A P}{R T_e}$ or $n_e = \frac{P}{k_B T_e}$ Giving

$$q = \left(\frac{\varepsilon_0}{P}\right)^{\frac{3}{2}} \frac{8eV_C}{81 r_{CO_2}^3}$$

Where $P = 4.75$ Torr and $r_{CO_2} = 116.3\ pm$. Given an approximation of a corona voltage of $(30\ kV\mathrm{m}^{-1})R$ for a radius of $3.55\ cm$, we obtain an operating voltage of approximately $1\ kV$.This allows us to obtain a charge of $3.07829277 \times 10^{-8}$ coulombs. This calculation, it should be noted, is the charge of a single dust particle, but also this is the charge for a volume of carbon dioxide gas.

The mass of the Martian dust can be approximated from the fact that the Martian soil appears to be a combination of hematite, Fe_2O_3, silica SiO_2 and titanium enriched magnetite $Fe^{2+}(Fe^{3+},Ti)_2O_4$. Essentially, volcanic basalt on the planet was consistently weathered to the point where magnetide powder and quartz powder could convert to fine hematite, which accounts for the red color of Mars, and the volumetric majority of the dust in the Martian regolith[14][15]. This would mean that the most fine hematite particles of only a few microns would largely be the ones to be in the Martian atmosphere accounting for the redness of the Martian sky. Given that hematite has a density of $5.26\frac{g}{cm^3}$,

$$m = 5.26\frac{g}{cm^3} * \frac{4}{3}\pi r_d^3 = 7.436 \times 10^{-11}\ g.$$

This gives a $\frac{q}{m}$ ratio of $413.97\frac{C}{kg}$ showing a dominance of the electric force in the dynamics of the dusty plasma.

Given that for an atmospheric target flow rate of $Q_a = 5\ slpm$ then $Q_r = \frac{760}{4.75*.7} *\ 5\ slpm = 1142.86\ lpm$ and thus $v = \frac{Q}{\pi R^2}$. Thus for our existing system, $v = 4.811\frac{m}{s}$ and considering that $V_c = (30\ kV\mathrm{m}^{-1})R,$ then we can write for our current radius of $3.55\ cm$ and our length of ¾ a meter, that our efficiency is essentially 1, and increasing the diameter of the radius of the electrode wire to 12 times the thickness for stability only decreases the electric field and by extension the efficiency to .984 times the original electric field. The natural logarithm relation in units of centimeters of a one meter length in our system reveals as well that the radius should be above $3.258\ cm$ to minimize the pressure. For our system, we utilize a $3.55cm$ radius to optimize performance and minimize pressure loss. Given this same radius, we should be able to go up to 1 meter in length to obtain an optimal collection efficiency and minimal pressure loss given that the logarithmic curve at that point would be at a radius of $3.53\ cm$.

For a precipitator of a smaller length of $\frac{1}{4}$ of a meter, the radius would be have to be larger than $2.23\ cm$ and at a length of 1.25 meters, the radius would have to be $3.75\ cm$. At a radius of a value like $30\ cm$, the length would have to be around 2 light years long to have the exact same charge and pressure drop ratio. Even at a 10 centimeter radius the tube would have to be 800 meters long. Therefore, the system works the best at small radii.

In an EHD system, a small positively charged electrode can concentrate the charge density and by extension the electric field which will then be spread out over a larger negative electrode to minimize the chance of arcing.

Our Electrostatic precipitator fits this description given the radius of the tube relative to the wire. Thus, given this we can determine that the force of voltage driven ions is written as $F = \frac{Id}{k}$ where k is the ion mobility coefficient and d is the distance between the electrodes [16]. The potential energy is the integral of the force over space or $U = \frac{Id^2}{2k} = qV$ and thus we can write the current as $I = \frac{2kqV}{d^2}$ Because q is proportional to the voltage, we see that the IV curve is going to be of the form $y = \frac{x^2}{d^2}$ or $I = k_E V^2$which would indeed indicate a parabolic graph, which is what we appear to see in the measurements. Indeed, we can use the EHD calculation to obtain

$$k_E = \frac{\left(\frac{\varepsilon_0}{P}\right)^{\frac{3}{2}} \frac{8e}{81 r_{CO_2}^3}}{d^2}$$

Which given an ion mobility constant for Martian Atmospheric conditions of $k = 0.008 \frac{m^2}{V*s}$, can calculate IV curves given a 100 μm diameter wire and a 7.1 cm diameter tube we obtain a constant of

$$k_E = 2.13 * 10^{-8} \frac{A}{V^2}$$

The Saha equation describes the degree of ionization of this plasma as a function of the temperature, density, and ionization energies of the atoms. The Saha equation only holds for weakly ionized plasmas for which the Debye length is large. This means that the "screening" of the Coulomb charge of ions and electrons by other ions and electrons is negligible. The subsequent lowering of the ionization potentials and the "cutoff" of the partition function is therefore also negligible. For a gas composed of a single atomic species, the Saha equation is written:

$$\frac{n_{i+1} n_e}{n_i} = \frac{2}{\Lambda^3} \frac{g_{i+1}}{g_i} \exp\left[-\frac{(\epsilon_{i+1} - \epsilon_i)}{k_B T}\right]$$

Where:

- n_i is the density of atoms in the i-th state of ionization, that is with i electrons removed.
- g_i is the degeneracy of states for the i-ions
- ϵ_i is the energy required to remove i electrons from a neutral atom, creating an i-level ion
- n_e is the electron density
- Λ is the thermal de Broglie wavelength of an electron $\Lambda = \sqrt{\frac{h^2}{2\pi m_e k_B T}}$.

The expression $(\epsilon_{i+1} - \epsilon_i)$ is the energy required to remove the $(i+1)^{th}$ electron. In the case where only one level of ionization is important, we have $n_1 = n_e$ and defining the total density n as $n = n_0 + n_1$, the Saha equation simplifies to:

$$\frac{n_e^2}{n - n_e} = \frac{2}{\Lambda^3} \frac{g_1}{g_0} \exp\left[-\frac{\epsilon}{k_B T}\right]$$

Where ϵ is the energy of ionization. The Saha equation is useful for determining the ratio of particle densities for two different ionization levels. The most useful form of the Saha equation for this purpose is $\frac{Z_i}{N_i} = \frac{Z_{i+1} Z_e}{N_{i+1} N_e}$, where Z denotes the partition function. The Saha equation can be seen as a restatement of the equilibrium condition for the chemical potentials: $\mu_i = \mu_{i+1} + \mu_e$. This equation simply states that the potential for an atom of ionization state i to ionize is the same as the potential for an electron and an atom of ionization state $i + 1$; the potentials are equal, therefore the system is in equilibrium and no net change of ionization will occur.

Thus we can write the Saha equation for our system as

$$\frac{n_i n_e}{n_a} = 2\frac{g_i}{g_a}\left(\frac{m_e k_B T_e}{2\pi\hbar^2}\right)^{\frac{3}{2}} \exp\left[-\frac{\epsilon_i}{k_B T}\right] \text{ [17].}$$

In a plasma, a Coulomb collision rarely results in a large deflection. The cumulative effect of the many small angle collisions, however, is often larger than the effect of the few larger angle collisions that occur, so it is instructive to consider the collision dynamics in the limit of small deflections.

We can consider an electron of charge $-e$ and mass m_e passing a stationary ion of charge $+Ze$ and much larger mass at a distance b with a speed v. The perpendicular force is $\left(\frac{1}{4\pi\varepsilon_0}\right)\frac{Ze^2}{b^2}$ at the closest approach and the duration of the encounter is about $\frac{b}{v}$. The product of these expressions divided by the mass is the change in perpendicular velocity:

$$\Delta m_e v_\perp \approx \frac{Ze^2}{4\pi\varepsilon_0}\left(\frac{1}{vb}\right)$$

Note that the deflection angle is proportional to $\frac{1}{v^2}$. Fast particles are "slippery" and thus dominate many transport processes. The efficiency of velocity-matched interactions is also the reason that fusion products tend to heat the electrons rather than (as would be desirable) the ions. If an electric field is present, the faster electrons feel less drag and become even faster in a "run-away" process. In passing through a field of ions with density n, an electron will have many such encounters simultaneously, with various impact parameters (distance to the ion) and directions. The cumulative effect can be described as a diffusion of the perpendicular momentum. The corresponding diffusion constant is found by integrating the squares of the individual changes in momentum. The rate of collisions with impact parameter between b and $(b + db)$ is $nv(2\pi b\, db)$, so the diffusion constant is given by

$$D_{v_\perp} = \int \left(\frac{Ze^2}{4\pi\varepsilon_0}\right)^2 \frac{1}{v^2 b^2}\, nv(2\pi b db) = \left(\frac{Ze^2}{4\pi\varepsilon_0}\right)^2 \frac{2\pi n}{v}\int \frac{db}{b}.$$

Obviously the integral diverges toward both small and large impact parameters. At small impact parameters, the momentum transfer also diverges. This is clearly unphysical since under the assumptions used here, the final perpendicular momentum cannot take on a value higher than the initial momentum. Setting the above estimate for $\Delta m_e v_\perp$ equal to mv, we find the lower cut-off to the impact parameter to be about

$$b_0 = \frac{Ze^2}{4\pi\varepsilon_0}\frac{1}{m_e v^2}.$$

We can also use πb_0^2 as an estimate of the cross section for large-angle collisions. Under some conditions there is a more stringent lower limit due to quantum mechanics, namely the de Broglie wavelength of the electron.

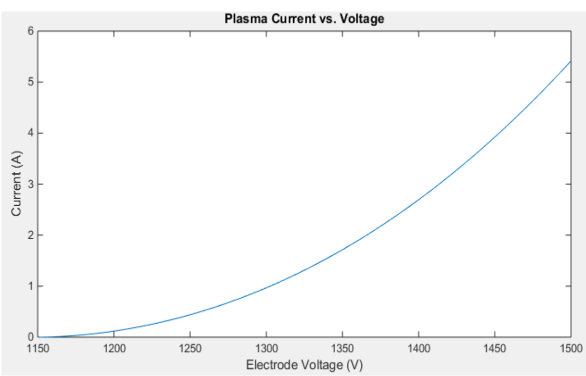
At large impact parameters, the charge of the ion is shielded by the tendency of electrons to cluster in the neighborhood of the ion and other ions to avoid it. The upper cut-off to the impact parameter should thus be approximately equal to the Debye length.

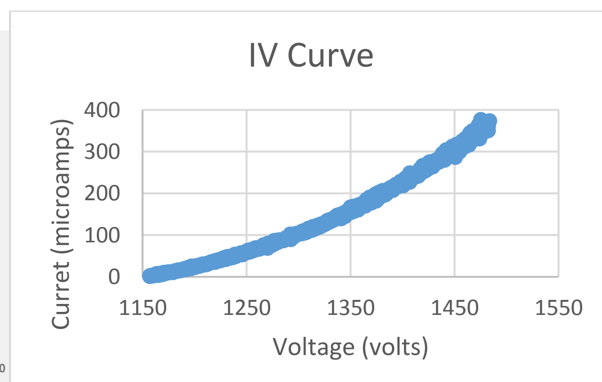
The integral of $\frac{1}{b}$ thus yields the logarithm of the ratio of the upper and lower cut-offs. This number is known as the Coulomb logarithm and is designated by either $\ln \Lambda$ or λ. It is the factor by which small-angle collisions are more effective than large-angle collisions. For many plasmas of interest it takes on values between 5 and 15. The limits of the impact parameter integral are not sharp, but are uncertain by factors on the order of unity, leading to theoretical uncertainties on the order of $\frac{1}{\lambda}$. For this reason it is often justified to simply take the convenient choice

$\lambda = 10$[18]. From these equations we can calculate the resistivity of such a plasma as

$$\eta_S = \frac{Ze^2\sqrt{m_e}2\pi \ln \Lambda_e}{(4\pi\varepsilon_0)^2 3\sqrt{2\pi}(k_B T)^{\frac{3}{2}}}$$

Where $\Lambda_e = \left(\frac{k_B\varepsilon_0 T_e}{ne^2}\right)^{\frac{1}{2}}\left(\frac{q_1 q_2}{4\pi\varepsilon_0 m_{12} v_1^2}\right)^{-1}$ [19]. Given that the resistivity is related to the resistance by the cross sectional area and distance between the electrodes or $\eta_S = R\,\frac{A}{d}$, assuming that $Z = 1$ and $m_{12} = \frac{m_e m_i}{m_e + m_i}$, then we can predict the IV curve behavior for the aforementioned system While we do not know the exact degeneracy of the carbon dioxide molecules, and even using a dummy value for the cross sectional distance as $20\ cm$ we see the same behavior for a change in current with respect to voltage even though the exact values are off.





This also however assumes that the energy of the system is written $E = \frac{3}{2}k_B T_e + E_{other}$ where $E_{other} = \{\begin{matrix} c_0 V\ (before\ the\ coronal\ onset) \\ eV_C(after\ the\ coronal\ onset) \end{matrix}$. This would imply that the energy of the ions remains relatively unchanged due to the majority of the energy in the plasma being pumped into the electrons and the electrons will collide and recombine with atoms and re-ionize them causing the energy of the ions to remain relatively unchanged. The only glaring issue is the difference between the curves of 4 orders of magnitude, however, this would be because this model predicts too low a resistance and factors affecting the resistance should largely be affected by phenomena proportional to the temperature of the electrons, which would largely change the magnitude of the predicted curve.

Even then, for the experimental curve the equation of best fit is approximated as the curve $k_E V^2 - 4.9238V + 2646.6$ where we can see the influence of the EHD formulation at play. As such, we can regard the EHD parabolic formulation as an approximation for a much more complicated plasma phenomenon.

Because of this however, the Debye Sheath around the dust particles would determine the charge on the dust similar to the charge in an arbitrary volume of gas, this means that we can approximate the IV curve for a dusty plasma by multiplying it by the collection efficiency.

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