Technical Note

Re: Penetration behavior of opposed rows of staggered secondary air jets depending on jet penetration coefficient and momentum flux ratio

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ARTICLE INFO

ABSTRACT

The purpose of this article is to explain why the extension of the previously published $C = (S/H_o)\mathrm{sqrt}(J)$ scaling for opposed rows of staggered jets wasn't directly successful in the study by Choi et al. (2016).

It is not surprising that staggered jets from opposite sides do not pass each other at the expected C value, because H_o/D and $\mathrm{sqrt}(J)$ are much larger than the maximum in previous studies. These, and large x/D's, tend to suggest development of 2-dimensional flow.

Although there are distinct optima for opposed rows of in-line jets, single-side injection, and opposed rows of staggered jets based on C, opposed rows of staggered jets provide as good or better mixing performance, at any C value, than opposed rows of in-line jets or jets from single-side injection.
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1. Introduction

The extensive studies of rows of non-reacting jets in crossflow (JIC) that are summarized in Ref. [1] were motivated by mixing of dilution jets in conventional gas turbine combustors.

The orifice spacing, S/H_o and S/D, the density ratio, DR, and the jet-to-mainstream mass-flow ratio, MR, in Ref. [2] are within the range of the experiments in Ref. [1]. However, the jet-to-mainstream momentum-flux ratio, J ($=(\rho_j V_j^2)/(\rho_m U_m^2)$), and the reciprocal of the orifice size, H_o/D, are substantial extrapolations. The minimum J in Ref. [2] is approximately an order of magnitude greater than the maximum in Ref. [1] ($J \simeq 100$), and the holes in Ref. [2] are much smaller than the minimum size in Ref. [1] (i.e. $(1/(H_o/D)) = 0.125$).

Since both J and H_o/D are significantly larger in the application that motivated Ref. [2] than they are in gas turbine combustors, it is not surprising that the jets from opposite sides cannot pass each other without interacting as is required for optimum mixing of opposed rows of staggered jets.

The purpose of this article is to investigate why the extension of the $C = (S/H_o)\mathrm{sqrt}(J)$ scaling in Ref. [1] wasn't quantitatively successful for the application that motivated Ref. [2].

From the experimental results in Ref. [1], it is apparent that midplane values approach centerplane values (the mean flow becomes more 2-dimensional) as the momentum-flux ratio, J, and the dimensionless downstream distance, x/D ($=(x/H_o)(H_o/D)$), increase.

2. History and previous usage of the NASA empirical model for confined JIC's

The effect of high momentum-flux ratio and small orifice size can be shown for conditions similar to those in Ref. [2] using the NASA JIC empirical model. This model was used in Refs. [3,4] to represent data obtained in the 1970s and 1980s. The original Applesoft® BASIC code (in Appendixes A and B of Ref. [5]) was subsequently rendered in an Excel® spreadsheet (unchanged – except for correction of known errors) in Refs. [5–8] using the correlation equations in Appendix C of Ref. [5]. (These correlations are the same as those in Ref. [1], but with the curvature effect equations in Ref. [1] deleted). The NASA JIC spreadsheet was also used in

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Nomenclature

C	$(S/H_o)(\sqrt{J})$
C_d	orifice discharge coefficient = (effective area)/(physical area)
D	actual orifice diameter
DR	jet-to-mainstream density ratio, ρ_j/ρ_m (for an incompressible flow, and for gases of the same chemical composition, this ratio is approximately equal to the mainstream-to-jet temperature ratio, T_m/T_j)
H_o	duct height at center of row of holes (called H in several previous publications)
J	jet-to-mainstream momentum-flux ratio
MR	jet-to-mainstream mass-flow ratio
S	lateral spacing between equivalent locations of adjacent orifices, e.g. between orifice centerplanes
S/D	ratio of orifice spacing to orifice diameter

T	local scalar variable
T_j	jet exit scalar variable
T_m	unmixed mainstream scalar variable
U_m	velocity of unmixed mainstream
V_j	jet exit velocity
x	downstream coordinate; $x = 0$ at center of the row of orifices
y	cross-stream coordinate
y_c/H_o	scalar trajectory = location of maximum scalar difference ratio in centerplane, θ_c
z	lateral coordinate; $z = 0$ at centerplane
θ	$(T_M - T)/(T_M - T_J)$
θ_c	maximum scalar difference ratio, defines location of scalar trajectory, y_c/H_o

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Ref. [9] for opposed rows of both aligned and staggered jets. (actually Refs. [9,10] are both subsets of Ref. [7].)

The several versions of the spreadsheet published to date are all capable of performing previous calculations, so the one included with Ref. [8] can duplicate all the results shown in Refs. [5–10] and is the version used in this paper.

Note that the NASA JIC spreadsheet assumes Gaussian profiles on both sides of the jet trajectory. Thus it should not be used if the jet trajectory exceeds H_o, i.e. if $y_c/H_o > 1$.

The NASA JIC spreadsheet doesn't have $C = (S/H_o)\sqrt{J}$ scaling "hardwired" so it should give a fair and adequate representation of the results in Ref. [2]. Although it returns results, the spreadsheet should not be used upstream of the trailing edge of the orifices ($x/D = 0.5$ ($x/H_o = 0.5/(H_o/D)$).

3. Results

Profile and contour plots from the NASA JIC spreadsheet are shown in Figs. 1–7 for the conditions in Table 1 in the present communication. The conditions for Figs. 3–6 are the same as those in Table 1 in Ref. [2].

The parameter shown in the profile and contour plots is $\theta = (T - T)/(T_m - T_j)$, where T_m is the mainstream temperature, T_j is the jet temperature, and T is the local temperature. Note that $\theta = 1$ for pure jet fluid, $\theta = 0$ for pure mainstream fluid, and $\theta = MR/(MR + 1)$ at the fully mixed condition. Although temperature is used here, any conserved scalar is acceptable.

The profile and contour plots in this paper always cover a span of two orifices on each side. Thus, the physical distance shown will vary with orifice spacing.

The conditions for $C = 1.3$ in Fig. 1 (column 1) represent an optimum mixer for opposed rows of aligned jets. The results shown in

Fig. 1 suggest that the flow becomes 2-dimensional very quickly, which is not surprising given the high J. high x/D, small orifice size, and small orifice spacing.

Nonetheless, the results in Fig. 1 agree with the $C = (S/H_o)\sqrt{J}$ scaling in Ref. [1]. Although the jets on top and bottom are aligned, results for in-line and staggered jets are indistinguishable for high C values at the C value that is optimum for opposed rows of in-line jets.

The conditions in column 2 for $C = 2.6$ are nearly optimum for one-side injection, and have been changed from the staggered case in Ref. [2] to one-side injection at the jet-to-mainstream mass-flow ratio used in Ref. [2]. Note that the diameter of the orifices is double that for each row of the single-side jets in column 2 (and Fig. 2) because there are only half as many orifices per row and only half as many rows in the single-side configuration in column 2.

The one-side injection in Fig. 2 is also in approximate agreement with the C scaling in Ref. [1], but note that the jet penetration also depends significantly on downstream distance.
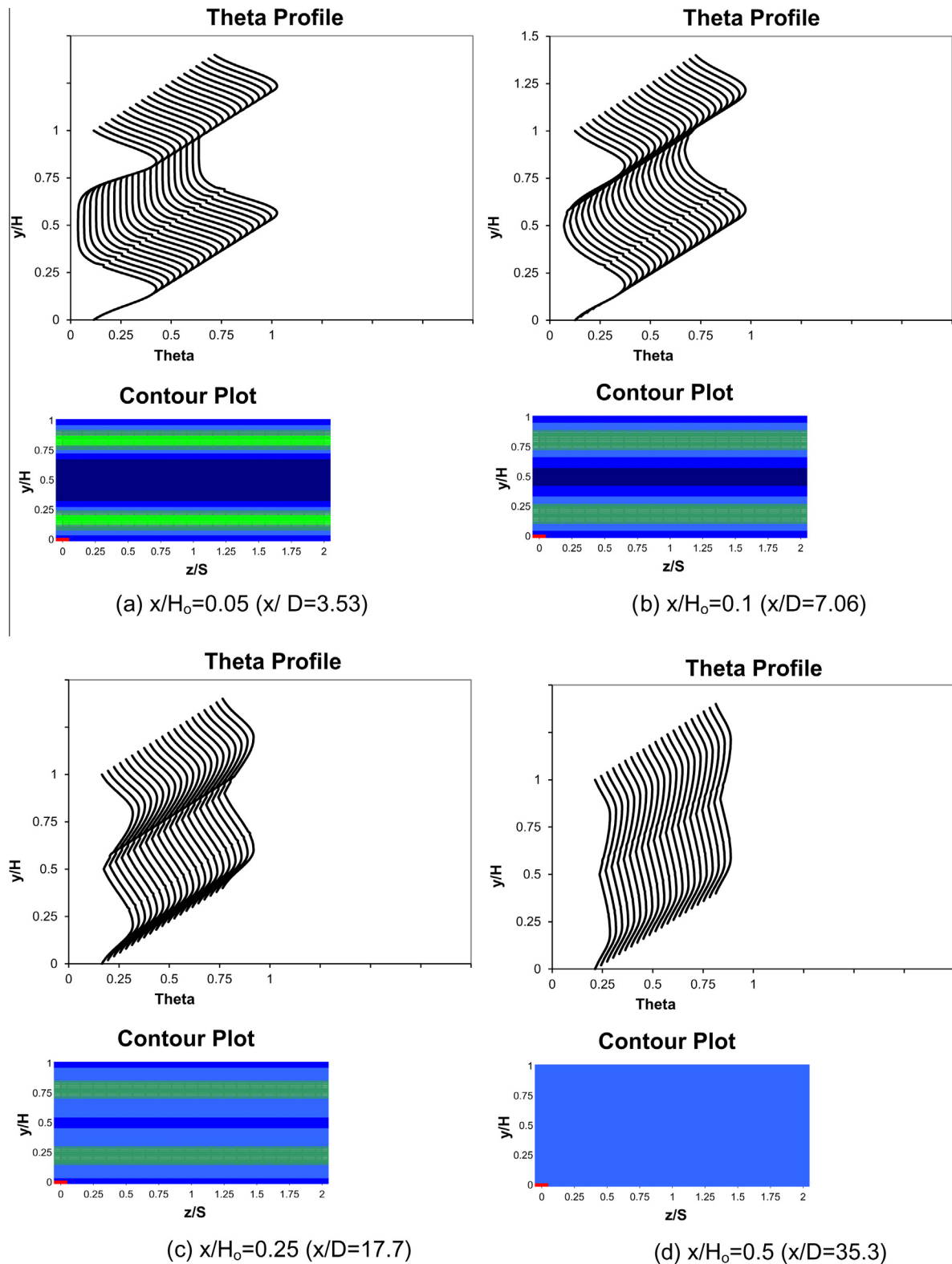
The results in Fig. 2 are very similar to the results for the upper jets in Fig. 1 when the x/H_o for single-side injection (Fig. 2) is twice that in opposed rows of in-line jets (Fig. 1) since the "effective" mixing height for opposed rows of in-line jets is $H_o/2$. Thus, better mixing is obtained at a specified downstream distance (x/H_o) with opposed rows of jets.

The results for staggered jets at $C = 2.6, 3.5, 5.7$ and 9.4 shown in Figs. 3–6 essentially agree with the results in Ref. [2].

Fig. 3 has the same C value as Fig. 2 ($C = 2.6$), but the configuration in Fig. 3 is opposed rows of staggered jets. Note that the orifices are smaller than for single-side injection since there are twice as many rows, but the orifice spacing (S/H_o) and MR are the same.

Table 1
Parameters for cases calculated with the NASA JIC spreadsheet.

| Figure	1	2	3	4	8	6	7
C	1.3	2.6	2.6	3.5	5.7	9.4	18.8
DR	1	1	1	1	1	1	1
J	800	800	800	800	800	800	800
H_o/D	70.6	35.3	49.9	42.4	32.8	24.4	17.3
S/H_o	045	.091	.091	.125	.200	.333	.667
C_d	1	1	1	1	1	1	1
S/D	3.2	3.2	4.5	5.4	6.6	8.2	11.5
D/H_o	.014	.028	.02	.024	.031	.041	.058

(a) $x/H_o = 0.05$ $(x/D = 3.53)$

(b) $x/H_o = 0.1$ $(x/D = 7.06)$

(c) $x/H_o = 0.25$ $(x/D = 17.7)$

(d) $x/H_o = 0.5$ $(x/D = 35.3)$

Fig. 1. Profile and contour plots from NASA JIC Excel® spreadsheet for opposed rows of in-line jets at $C = 1.3$: $DR = 1$, $J = 800$, $H_o/D = 70.6$, $S/H_o = 0.045$ ($S/D = 3$ 2), and $C_d = 1$.

Fig. 3 shows equivalent mixing to that in Fig. 2 at the C value that is optimum for single-side injection. However, opposed rows of staggered jets fill in the gaps, and are less sensitive to variations in downstream distance.

The profiles and contours in Figs. 4 and 5 show that the jets from opposite sides do not pass each other. Although optimum penetration for opposed rows of staggered jets was expected for $C = 5.7$ (Fig. 5), this probably does not occur because the high J and H_o/D values promote jet interaction and 2-dimensional flow.

The profiles and contours for $C = 9.4$ in Fig. 6 suggest that the jets from opposite sides are trying to pass, as the initial profiles are similar to the initial ones in sequence 18 in the slideshow in the
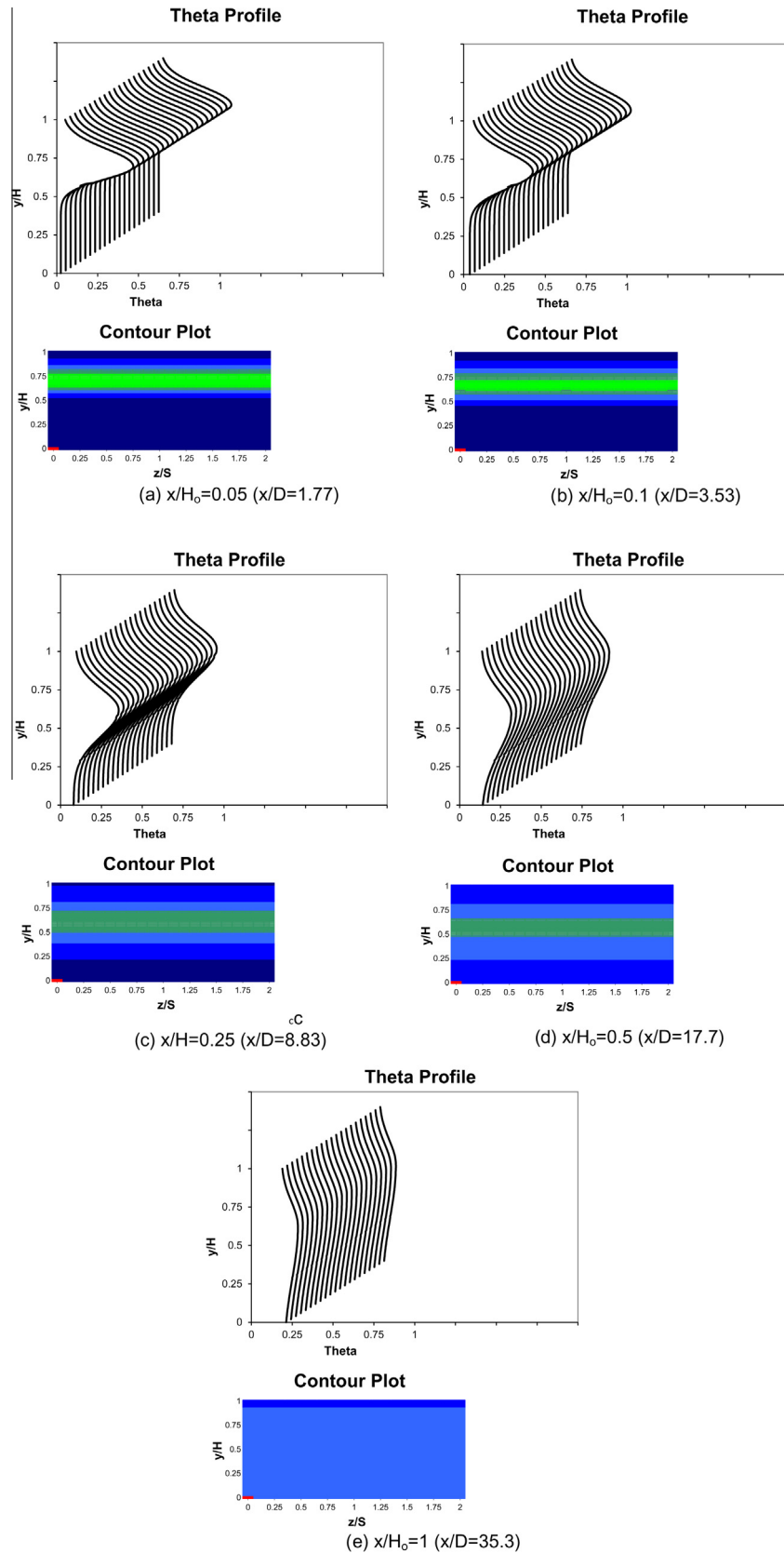
Fig. 2. Profile and contour plots from NASA JIC Excel® spreadsheet for single-side injection at $C = 2.6$: $DR = 1$, $J = 800$, $H_o/D = 35.3$, $S/H_o = 0.091$ ($S/D = 3.2$), and $C_d = 1$.

Appendix of Ref. [6] and in Figs. 12 and 13 of Ref. [7] and Figs. 7 and 8 of Ref. [9]. However, further penetration is masked by 2-dimensional flow. However, the mixing is quite uniform at $x/H_o = 0.5$.
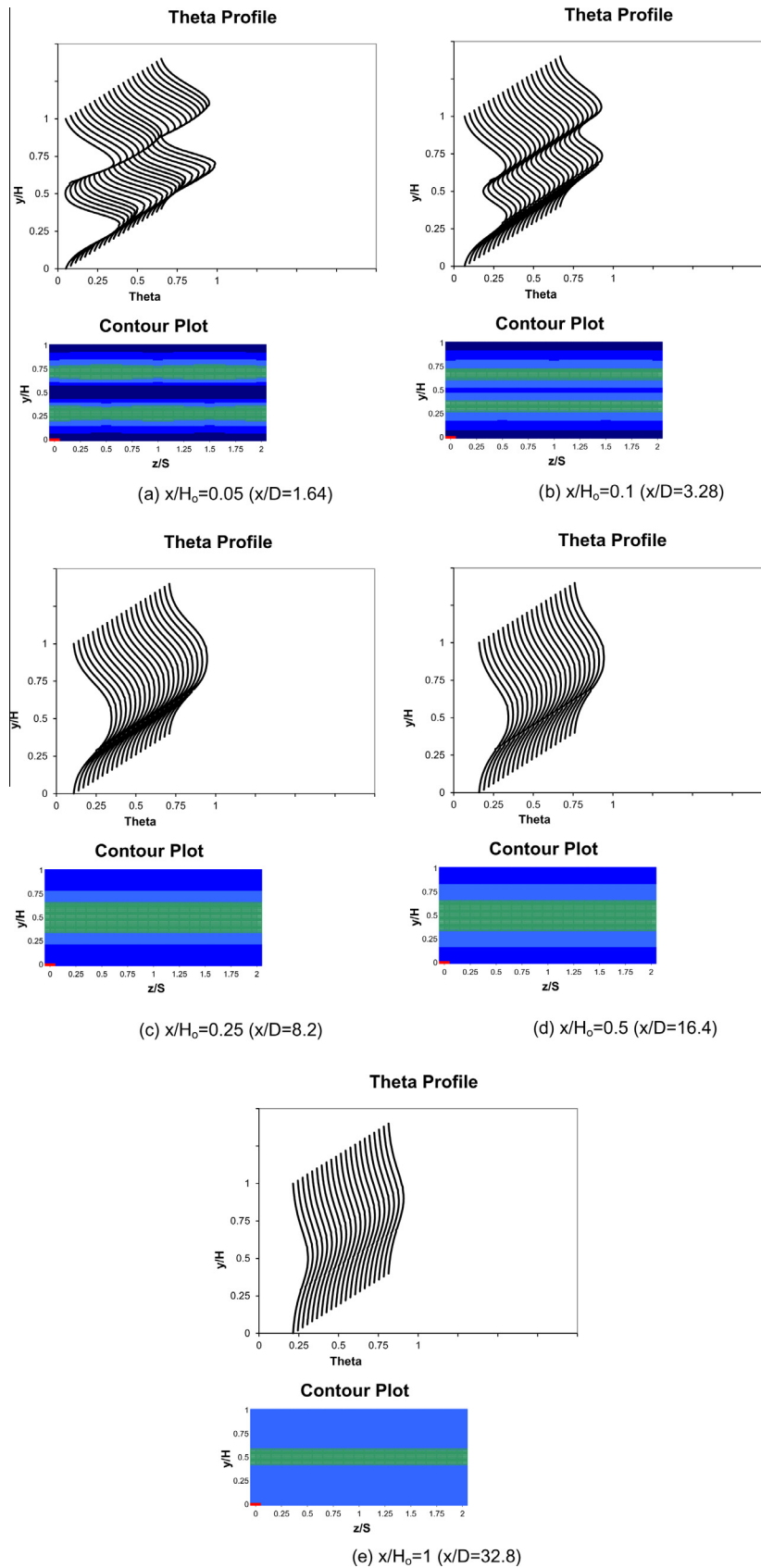
The conditions for $C = 18.8$ in column (and Fig. 7) are not possible in the facility used in Ref. [2], but are included here to show what might be expected for a further increase in C. Here
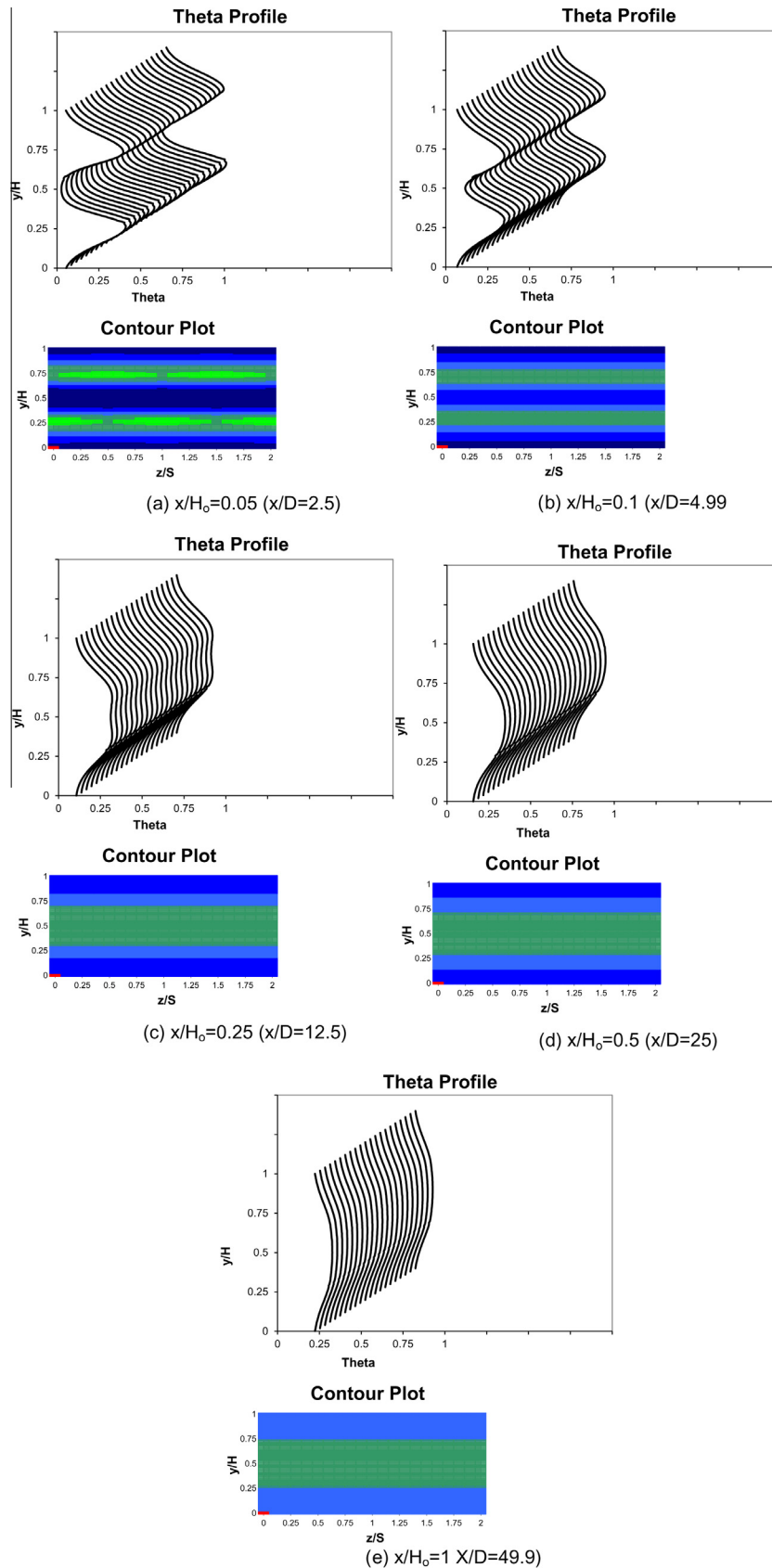
Fig. 3. Profile and contour plots from NASA JIC Excel® spreadsheet for opposed rows of staggered jets at $C = 2.6$: $DR = 1$, $J = 800$, $H_o/D = 49.9$, $S/H_o = 0.091$ ($S/D = 4.5$), and $C_d = 1$.

$S/H_o = 0.667$ which seems acceptable, but it is unlikely that a jet would spread laterally more than in the direction of jet penetration so care must be taken to insure that $S/H_o \leqslant 1$.

Fig. 7 shows jets from opposite sides that pass each other, as shown in Fig. 12 of Ref. [7] and Fig. 7 of Ref. [9]. Apparently the high momentum-flux ratio jets are finally spaced far enough apart

(a) x/H_o=0.05 (x/D=1.64)

(b) x/H_o=0.1 (x/D=3.28)

(c) x/H_o=0.25 (x/D=8.2)

(d) x/H_o=0.5 (x/D=16.4)

(e) x/H_o=1 (x/D=32.8)
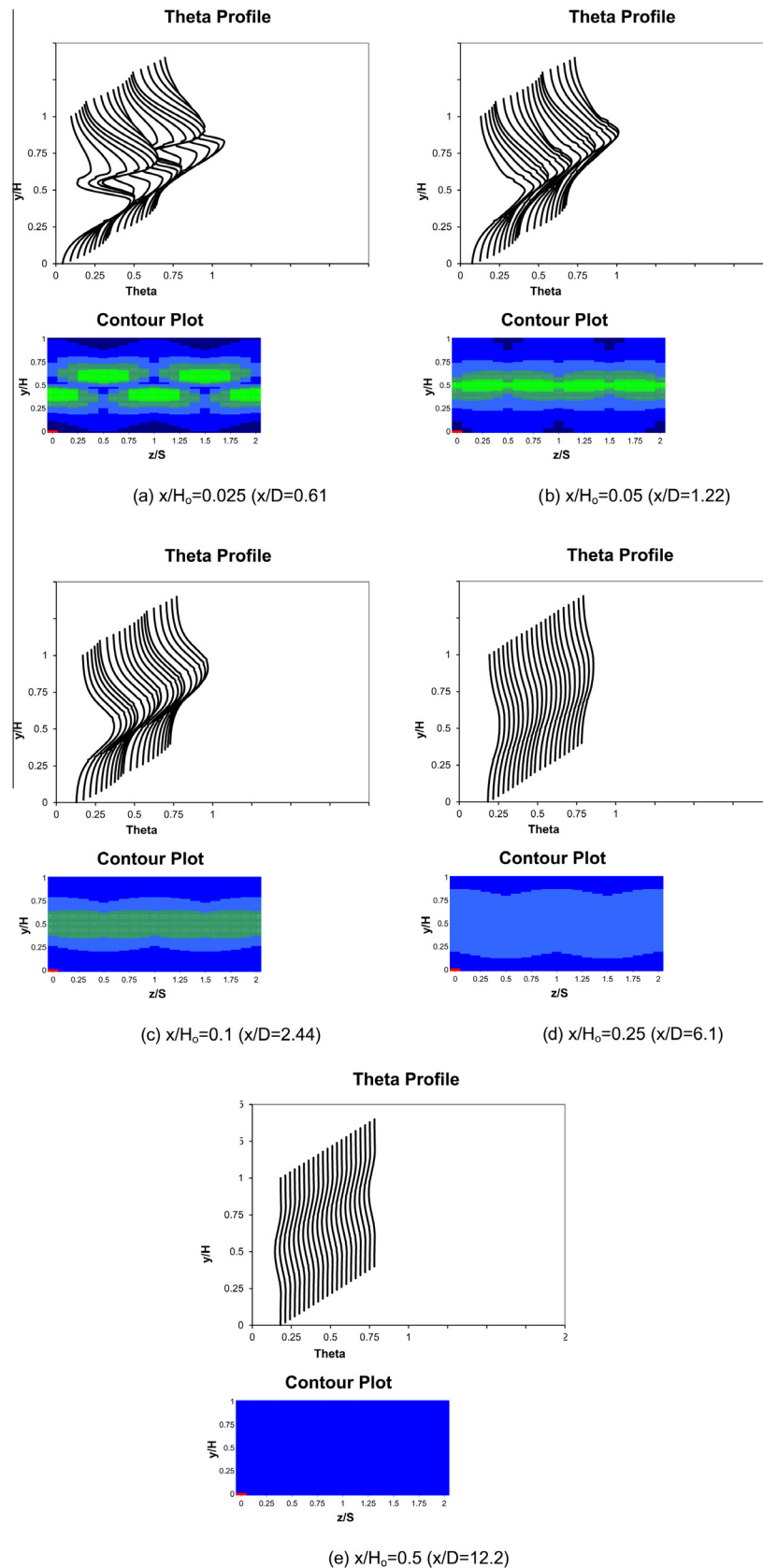
Fig. 4. Profile and contour plots from NASA JIC Excel® spreadsheet for opposed rows of staggered jets at C = 3.5: DR = 1, J = 800, H_o/D = 42.8, S/H_o = 0.125 (S/D = 5.4), and C_d = 1.
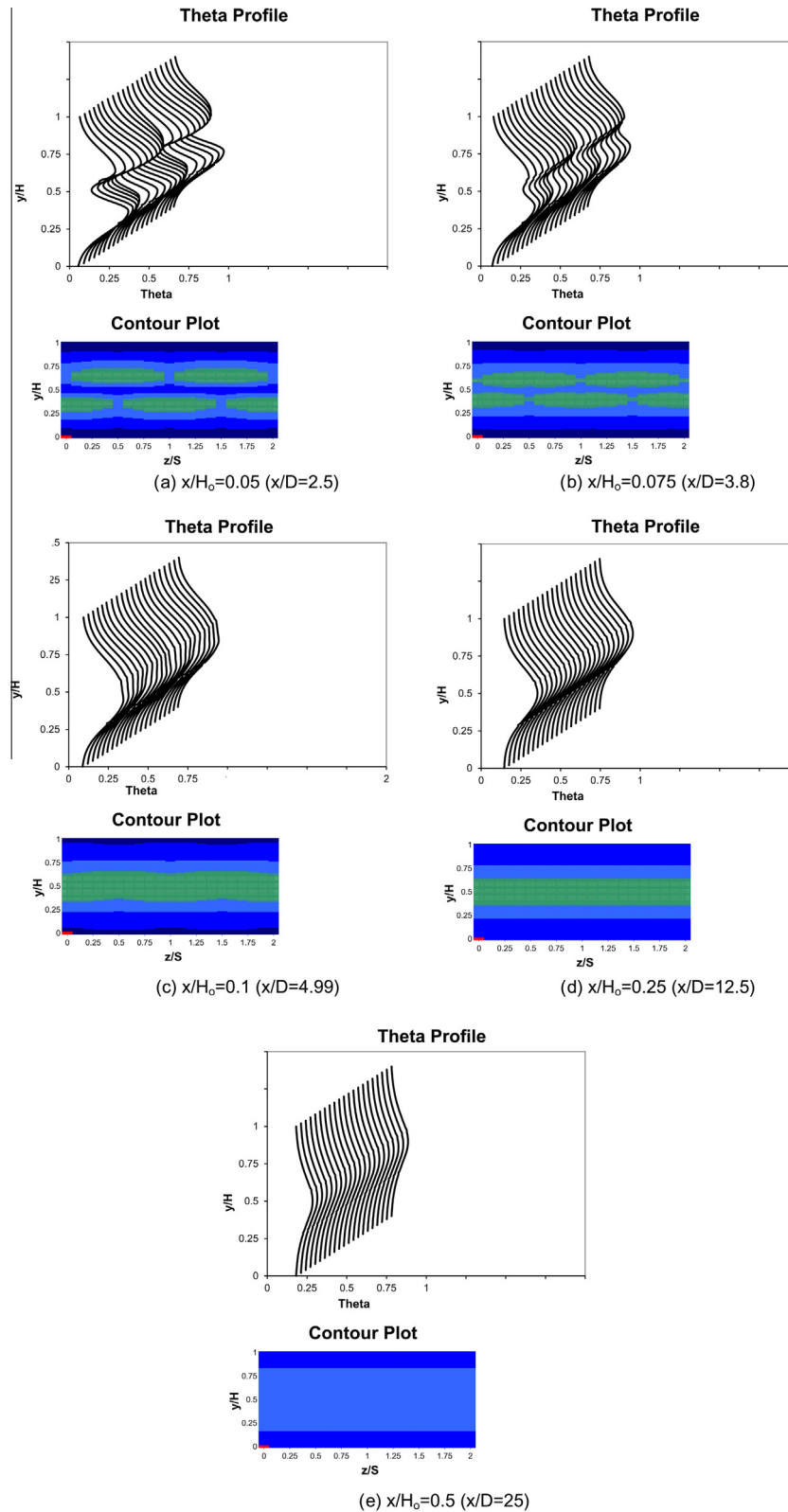
Fig. 5. Profile and contour plots from NASA JIC Excel® spreadsheet for opposed rows of staggered jets at $C = 5.7$: $DR = 1$, $J = 800$, $H_o/D = 32.8$, $S/H_o = 0.2$ ($S/D = 6.6$), and $C_d = 1$.

that jets from opposite sides can pass each other without interacting. Distributions from the JIC spreadsheet are not shown farther downstream than $x/H_o = 0.1$ as the jet trajectory exceeds H_o, and significant mass is "lost" (i.e. the profiles are truncated).

Although calculating S/D ($=(S/H_o)(H_o/D)$) is straight-forward, S/D values are included in Table 1 and the titles of Figs. 1–7 as they highlight the importance of S/D spacing.

No matter what the application is, the absolute minimum spacing that allows staggered jets from opposite sides to pass each other
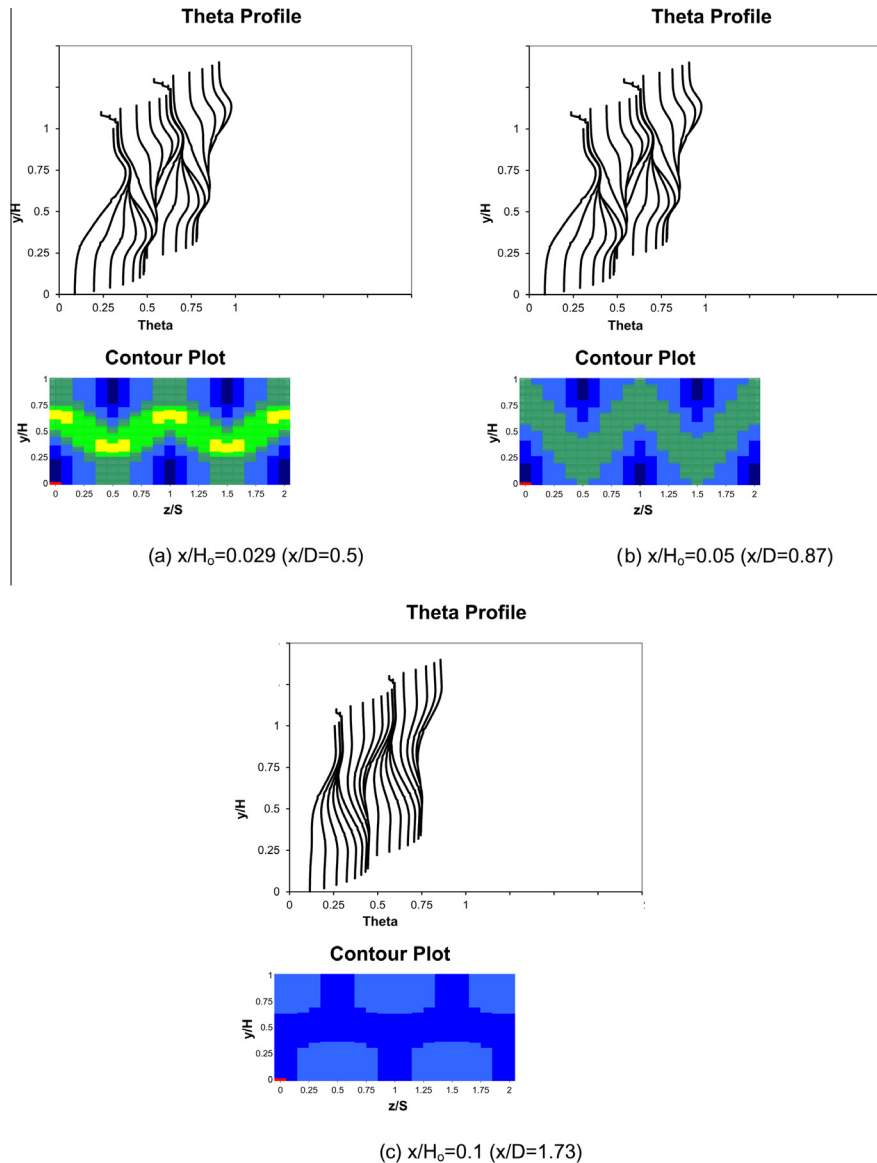
(a) x/H_o=0.025 (x/D=0.61)

(b) x/H_o=0.05 (x/D=1.22)

(c) x/H_o=0.1 (x/D=2.44)

(d) x/H_o=0.25 (x/D=6.1)

(e) x/H_o=0.5 (x/D=12.2)

Fig. 6. Profile and contour plots from NASA JIC Excel® spreadsheet for opposed rows of staggered jets at C = 9.4: DR = 1, J = 800, H_o/D = 24.4, S/H_o = 0.333 (S/D = 8.2), and C_d = 1.

is S/D = 2, provided that the opposed rows are perfectly staggered and the jets do not diffuse as they proceed downstream. Of course, this won't happen, and the minimum S/D will increase as J increases.

Theta Profile

Contour Plot

(a) x/H_o=0.029 (x/D=0.5)

Theta Profile

Contour Plot

(b) x/H_o=0.05 (x/D=0.87)

Theta Profile

Contour Plot

(c) x/H_o=0.1 (x/D=1.73)

Fig. 7. Profile and contour plots from NASA JIC Excel® spreadsheet for opposed rows of staggered jets at C = 18.8: DR = 1, J = 800, H_o/D = 17.3, S/H_o = 0.667 (S/D = 11.5,) and C_d = 1.

In the application that motivated Ref. [2], the optimum S/D for opposed rows of staggered jets appears to be approximately S/D = 10. Thus, $C = ((S/D)/(H_o/D))\mathrm{sqrt}(J) = 283/(H_o/D)$ for J = 800; if $H_o/D \simeq 20$, then $C \simeq 15$. Note that in Fig. 7 of Ref. [9], S/D = 8, H_o/D = 8, and J = 26.4, so C = 5.14, but if J were 800, C would be 28.3 for S/H_o = 1.

4. In Conclusion

Since H_o/D and sqrt(J) are approximately both 3 times larger than the maximum in previous studies, and these, and large x/D's show 2-dimensional flow, it is not surprising that staggered jets from opposite sides do not pass each other at the expected C value.

There are distinct optima for opposed rows of in-line jets, single-side injection, and opposed rows of staggered jets based on C. However, opposed rows of staggered jets provide as good or better mixing performance as opposed rows of in-line jets or jets from single-side injection at any C value.

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