Initial Satellite Formation Flight Results from the Magnetospheric Multiscale Mission

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This paper describes the underlying dynamics of formation flying in a high-eccentricity orbit such as that of the Magnetospheric Multiscale mission. The GPS-based results used for MMS navigation is summarized, as well as the procedures that are used to design the maneuvers used to place the spacecraft into a tetrahedron formation and then maintain it. The details of how to carry out these maneuvers are then discussed. Finally, the numerical results that have been obtained concerning formation flying for the MMS mission to date (e.g. tetrahedron sizes flown, maneuver execution error, fuel usage, etc.) are presented in detail.

I. Introduction

The NASA Magnetospheric Multiscale (MMS) mission is flying four spinning spacecraft in highly elliptical orbits to study the magnetosphere of the Earth [1]. Launch occurred on Mar. 12, 2015, with insertion into a high-eccentricity orbit that was designed to satisfy a complex set of science and engineering constraints [2]. After roughly 5 months of commissioning, the spacecraft have been flown in tetrahedron formations of varying dimensions in order to perform magnetospheric science measurements. In the first phase of the mission, these measurements are being taken on the dayside of the Earth, in a Region of Interest surrounding the apogee of the MMS orbit (radius 12 \(R_E\)). The goal during Phase 1 is to observe the magnetospheric reconnection events that are expected to occur near the bow shock where the solar wind impinges upon the magnetosphere. Measurements during the Phase 2b, after apogee radius has been increased to 25 \(R_E\), will be taken in the magnetotail [3], to similarly observe nightside magnetic reconnection events. Taking simultaneous measurements from four spacecraft allows spatial derivatives of the electric and magnetic fields to be determined, allowing variations that are functions of distance to be distinguished from those that are functions of time.

This paper will describe the results that have been obtained to date concerning MMS formation flying. The MMS spacecraft spin at a rate of 3.1 RPM, with spin axis roughly aligned with Ecliptic North. Several booms are used to deploy instruments: two 5 m magnetometer booms in the spin plane, two rigid booms of length 12.5 m along the positive and negative spin axes, and four flexible wire booms of length 60 m in the spin plane. Minimizing flexible motion of the wire booms requires that reorientation of the spacecraft spin axis be kept to a minimum: this is limited to attitude maneuvers to counteract the effects of gravity-gradient and apparent solar motion. Orbital maneuvers must therefore be carried out in essentially the nominal science attitude. These burns make use of a set of monopropellant hydrazine thrusters: two (of thrust 4.5 N) along the spin axis in each direction, and eight (of thrust 18 N) in the spin plane; the latter are pulsed at the spin rate to produce a net delta-v. An on-board accelerometer-based controller [4] is used to accurately achieve a commanded delta-v. Navigation makes use of a weak-signal GPS-based system [5]: this allows signals to be detected even when MMS is flying above the GPS constellation, producing a highly accurate determination of the four MMS orbits. This data is downlinked to the MMS Mission Operations Center (MOC) and used by the MOC Flight Dynamics Operations Area (FDOA) for maneuver design and evaluation. These commands are then uplinked to the spacecraft and executed autonomously using the controller, with the ground monitoring the burns in real time.

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MMS formation flying is driven by the Quality Factor: this function, evaluated to a number between 0 and 1, describes whether the formation is in the desired size range, and how close it is to an ideal tetrahedron geometry. A set of maneuvers to achieve this are produced using the Formation Design Algorithm, which will be described below. The mechanics of the design and execution of these maneuvers will be discussed, including how spacecraft attitude corrections are dealt with in conjunction with delta-v. Further, the sequence of maneuvers that have been carried out to date during the MMS mission will be described: these go from an initial series of perigee-raise maneuvers to phasing clean-up trims, followed by insertion into an initial 160 km tetrahedron formation. A series of formations of gradually decreasing sizes (60 km, 25 km, and 10 km) was then entered into: this resize campaign, with two weeks spent at each size, was designed to allow the scientists to determine what size(s) produced the best data. At the completion of this campaign, the formation was held at 10 km, as selected by the science team. Formation maintenance maneuvers to correct the slow drifts that are produced by maneuver execution errors are typically required every 4 or 5 weeks. This is considerably better than the pre-flight predictions of a 2-week cadence, reflecting the better than expected performance of navigation solutions and the delta-v controller.

This paper will describe the underlying dynamics of formation flying in a high-eccentricity orbit. The GPS-based results used for MMS navigation is then summarized, as well as the procedures that are used to design the maneuvers used to place the spacecraft into a tetrahedron formation and then maintain it. The details of how these maneuvers are designed are then discussed. Finally, the numerical results that have been obtained concerning formation flying for the MMS mission to date (e.g. tetrahedron sizes flown, maneuver execution error, fuel usage, etc.) are presented in detail.

II. MMS Formation Flying Dynamics

In order to collect the science data that is required in order to study magnetic reconnection, the MMS spacecraft must fly in a tetrahedron formation (so spanning all three axes) in the Region of Interest (RoI) of the orbit. This region is an extended arc that contains apogee, where the radius of the orbit is in the range where reconnection is expected to occur: for the MMS Phase 1 orbit, this is all radii of 9 Earth radii \( R_E \) or greater. The science team can select to fly the various sub-phases of Phase 1 at any formation size in the range 10 to 160 km (a study is currently under way to see if this lower limit can be reduced to a range between 5 and 10 km). It should be noted that the orientation of the tetrahedron is not specified: any orientation will give the spread across each of the three axes that is required for science.

In order for the formation to persist over multiple orbits (referred below as revs), it is necessary for the spacecraft to take up essentially the same relative positions from one rev to the next. If this is to occur, the four MMS orbits must have essentially equal periods. Since period is directly related to semi-major axis (SMA), the key is to have the SMAs of the four orbits be closely matched. Any differences will lead to drift rates (which can be either expanding or closing) between the spacecraft. Once the accumulated drift is sufficiently large, the tetrahedron will be distorted enough that it is no longer suitable for science data generation: maneuvers will then be required in order to set up a new formation.

Since the MMS orbit is highly eccentric (eccentricity 0.8182 for Phase 1, 0.9084 for Phase 2b), the behavior of the formation when traveling from apogee to perigee and back is quite complicated. Consider the along-track separation between any pair of spacecraft: this is created by setting up the appropriate difference in phasing between the satellites. For instance, suppose that an along-track spacing of 18 km is required at apogee: since orbital speed at apogee is approximately 0.9 km/s, this will be achieved by having one spacecraft fly 20 s ahead of the other. But the orbital speed at perigee is around 9 km/s: this same lead time will therefore result in an along-track spacing of 180 km at perigee, a tenfold increase. This “breathing mode” around the orbit implies that along-track separation in the RoI will be at its minimum at apogee, increasing somewhat in the vicinity of RoI entry and exit.

The out-of-plane (OOP) relative motion is quite different. To have a specified (and maximal) OOP spacing at apogee, the common (or node) points of the corresponding pair of MMS orbits should occur at true anomalies in the vicinity of 90 deg and 270 deg. As a result of this geometry, and since apogee radius is 10 times as large as perigee radius, the OOP spacing will be roughly 10 times greater at apogee than at perigee. Furthermore, the sign of the relative displacement changes going from apogee to perigee: the spacecraft swap from side to side. Consequently, the most likely places for close approaches to occur between a pair of MMSs are at true anomalies around 90 deg or

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270 deg. Care is taken in the formation maneuver design process to ensure that this does not occur (see Section IV). Furthermore, if a close approach (CA) were predicted to occur, a “Dodge” maneuver has been specifically designed to push one of the CA MMS pair away from the other along-track, thus increasing the miss distance significantly. This same type of maneuver can also be employed in the event of a CA being detected between an MMS and another satellite: since these other spacecraft are typically well below the MMS orbit except around perigee, any such CA will be expected to occur for true anomalies roughly in the range of 270 deg to 90 deg. This makes the Dodge maneuver design suitable for dealing with these as well, since it is designed specifically for CAs in this true anomaly range.

Finally, since the orbits must have the same SMAs, any radial separation between these spacecraft at apogee must be set up by introducing a small difference in eccentricity, \( \Delta e \): one orbit will therefore have apogee radius \( a + \Delta e \) above the other, and perigee radius \( a - \Delta e \) below it. The radial separation therefore changes sign when going from apogee to perigee, as for the OOP case, but the magnitude stays the same at perigee and apogee.

As a result of these effects, a tetrahedron that is suitable for science will approximate a regular tetrahedron throughout the RoI, although being slightly “squashed” at apogee. At perigee, it will be extremely elongated, no longer even remotely resembling a regular tetrahedron, and will have “flipped” in the OOP and radial directions (see Fig. 1: the blue ribbon marks the orbit plane of one spacecraft). This complicated motion to some extent approximates a tumble, with superimposed elongation and then recompression as the formation passes from apogee to perigee and back, while the same behavior is repeated from one rev to the next.

![Figure 1. Formation at apogee (left) and perigee (right; not to scale).](image)

**III. MMS Onboard Navigation System**

The Goddard Enhanced Onboard Navigation System (GEONS) is employed on each of the four MMS spacecraft to provide the navigation solutions for orbit determination, contact acquisition, and maneuver planning. GEONS estimates the spacecraft’s position, velocity, clock bias, clock bias rate, and clock bias acceleration using an Extended Kalman Filter (EKF) coupled with a high-fidelity dynamics model to process GPS L1 pseudorange (PR) measurements referenced to the Ultra-Stable Oscillator (USO) clock. The Navigator’s weak signal acquisition capability allows the receiver to acquire and track GPS signals well above the GPS constellation and deliver highly accurate navigation solutions [5].

The GEONS Ground Support System (GGSS) provides the tools needed to support MMS navigation operations at MMS Flight Dynamics Operations Area (FDOA). During the Navigator/GEONS commissioning period (the first
nine weeks of the mission), the GGSS was used to compare GEONS solutions against independent navigation solutions provided by the Flight Dynamics Facility (FDF) at NASA Goddard Space Flight Center (GSFC). As discussed in detail in Ref. 5, Navigator/GEONS performance during the MMS commissioning phase exceeded expectations. The high accuracy navigation solutions were used extensively to generate definitive, predictive, and maneuver planning products even prior to the completion of GEONS calibration.

The key MMS on-board orbit determination (OD) requirements were designed to ensure that the FDOA team would be able to safely and accurately maintain the range of nominal formation sizes throughout the mission. The most critical requirement is to determine the spacecraft SMA, since error in SMA knowledge largely determines the growth rate of propagation error [8]. In Phase 1, the SMA must be known to within 50 m (above 3 $\text{R}_E$ and outside maneuver recovery periods) for 99% of the time.

Figs. 2 and 3 below show the typical results of the Navigator/GEONS performance during Phase 1a of the mission. The plots were generated as part of GEONS weekly trending analysis using the GGSS for MMS1 over the time span of 2016-32-16:10 to 2016-40-16:00. Fig. 1 shows the differences between GEONS and predicted solutions in the position components as a function of time and mean anomaly. The predicted solutions are based on GEONS solutions downloaded from spacecraft at a post perigee epoch and generated by propagating in the Planning Products segment of the MMS FDOA ground system using FreeFlyer 6.9.1. The typical differences are within +/-100 m along Velocity (V), Normal (N), and Binormal (B) directions. The differences in SMA computed based on GEONS and predicted solutions is shown in the last subplot of Fig. 2. The SMA differences are usually within +/-5 m, well below the requirement. The thick black line in Fig. 1 indicates the mean value and the grey lines indicate the upper and lower 99% probability confidence interval bounds.

![Fig. 2. GEONS vs predicted solutions differences.](image)

Fig. 3 displays the number of GPS Space Vehicles (SV) tracked as a function of time and mean anomaly for MMS1. The number of GPS SV decreases near apogee but the average number over consecutive orbits remains greater than four. Simulations performed prior to launch were based on the expectation that four or more GPS measurements would be tracked for only $\pm$ 3 hours around perigee.
Based on the exceptional performance of the Navigator in Phase 1, MMS Navigator team has recently run the simulations of Phase 2b orbit to update the performance predictions. The performance level is proved to be 2-3 times better than the preflight simulations and suggest Phase 2b requirements will be met with significant margin, Ref. [8].

IV. Formation Maneuver Design Process
The MMS formation maneuver sequence is based shifting the spacecraft into a fresh formation arrangement. To do so, one of the spacecraft is selected as a reference and does not performing any DV maneuvers. The other three spacecraft perform DV maneuvers so as to position themselfs into desired orbits relative to the reference spacecraft, such that all four create a tetrahedron while within the RoI. Each of the non-reference spacecraft carries out two burns: the first (FM1) on the orbit flank after apogee, and the second (FM2) in the vicinity of the subsequent apogee. This can be considered as a rendezvous pair: the first burn transfers the spacecraft from its position in the existing formation to its desired location in the new formation; the second burn then modifies its velocity so as to ensure that it continues to track the new formation geometry.

The MMS Formation Design Algorithm (FDA) (see Ref. [6] for further details) designs these maneuvers by optimizing the Quality Factor (QF), a normalized measure of the “closeness” of a formation to a uniform tetrahedron of the specified size [7] that is evaluated at all points in the RoI. The QF has two multiplicative components: one based on the size of the instantaneous formation, the other based on the shape. The size component makes use of the mean of the 6 sidelengths between the 4 spacecraft: if this distance lies in a specified range, this component of the QF is one; if not, it tapers to zero. Given the previous discussion on the evolution of the formation geometry throughout the RoI, it is necessary to allow a range of sizes throughout the RoI: for instance, for a 10 km formation the instantaneous mean sidelenath is deemed acceptable if lying in the range of 6 to 18 km. The shape component of the QF consists of the ratio of the volume of the instantaneous formation to that of a uniform tetrahedron: the closer the shape of the formation is to the desired tetrahedron, the closer this QF term is to unity. The FDA performs a numerical optimization of the QF, subject to two key constraints: firstly, as previously noted, in order for the formation to persist, the SMAs of the four orbits must be matched; secondly, in order to ensure safety of the spacecraft, no inter-satellite range (ISRs) is allowed to go below a specified lower limit at any point on the orbit.
The reference spacecraft is normally chosen to be the one that has the least amount of fuel remaining, to attempt to keep fuel balanced among the fleet. However, in certain circumstances other considerations can come into play that outweigh the fuel balancing effort. Notably, for large formations, if the reference has a low perigee altitude, this can cause the perigees of the other three spacecraft to be lowered down to match. The reference spacecraft for larger formations is therefore selected to be the one with the highest perigee. Selecting a different reference can also be driven by the advantage of reducing the total fuel consumption across the fleet for a given maneuver sequence. This is a consequence of the fact that the QF optimization problem has a great many local minima, as a result of the many variables involved and the non-linear dynamics of the problem. This implies that minor changes in inputs can, in some circumstances, lead to quite different FDA solutions. The choice of reference is a notable example of such an input.

The MMS spacecraft must be in communications with the Flight Operations Team (FOT) when carrying out any formation maneuvers, so that the progress of the burns can be monitored in real time. Furthermore, it is only possible to communicate with one spacecraft at a time, as they share a single frequency. Maneuvers must therefore be “staggered”, taking place during a series of consecutive Deep Space Network (DSN) or Near Earth Network (NEN) passes. Including the reference spacecraft (which performs an attitude maneuver in its FM1 slot, and no maneuver in the FM2 slot), the possible number of permutations of spacecraft burn order amounts to $4!^2$, or a total of 576. The MMS maneuver design code takes advantage of this degree of freedom by searching for a staggering order that keeps fuel use as low as possible while preserving spacecraft safety by keeping all ISRs on the orbit arc between the FM1 and FM2 burns above a given threshold. The precise sequence is: find the top 12 candidate staggering sequences in terms of minimizing fuel usage; rank these for safety, in terms of maximizing ISRs between the burns; re-rank these in terms of fuel usage and select the top candidate as the preferred staggering sequence.

V. Formation Maneuver Execution Process

MMS Delta-V (DV) maneuvers, of which the formation maneuvers are the most prevalent type, are executed by an on-board DV controller [4] that autonomously pulses the radial thrusters on the rotating spacecraft, and fires the axial thrusters, as required to produce the net delta-v that has been computed by the ground system maneuver planning algorithm. This data is uploaded to the spacecraft before the time of the burn in the form of a table (known as the DV table) of 10-second time steps of dictating the cumulative delta-v target during the maneuver. The maneuver is executed by having the DV controller match the delta-v table. The role of the ground operations team prior to a maneuver is to first go through a GO/NO GO decision process and then monitor the maneuver performance in real time, and finally analyze the results based on the data downlinked from the spacecraft. Burns cannot be aborted from the ground: this can only be done by the on-board fault detection and correction system.

The performance of each maneuver is evaluated shortly after it is completed by making use of data that is downlinked shortly before the end of the maneuver pass. A critical measurement that comes out of the maneuver evaluation and reconstruction process is an estimate of the fuel consumed, which is compared with amount predicted from the pre-maneuver simulations. In addition, since SMA is such an important factor for MMS, an estimate is computed of the actual change in SMA that was produced by the burn, compared with the pre-maneuver prediction. The navigation data that is downlinked during the post-perigee pass between FM1 and FM2 provides an more accurate estimate for the SMA change. There is a provision in the MMS maneuver design code to use this data to “tweak” the FM2 burn to compensate for the SMA errors stemming from FM1. However, to date it has been found that these errors are negligible and tweaking makes little to no improvement in formation quality or lifetime: tweaking was only carried out on the first, large, Formation Initialization maneuvers.

Maneuvers during MMS Phase 1 are nominally carried out on pre-scheduled DSN or NEN contacts on Wednesdays (FM1) and Thursdays (FM2), with contingency contacts scheduled on the following Saturday/Sunday. (Note that DSN contacts are preliminarily scheduled up to 26 weeks in advance, and confirmed 10 weeks in advance.) The MMS Science Operations Center (SOC) requires sufficient time to plan their instrument command uploads, or a minimum of 10 days of advance notice of a maneuver design. Taking this and other factors into account, it requires two weeks to evaluate the results of a previous set of maneuvers and plan the next, meaning that a maneuver cadence of two weeks is the fastest that can be accommodated. The activities that must be carried out during this period can be summarized as follows:

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• Perform preliminary design of maneuvers (this is actually done daily for weeks in advance)
• Use these results to decide which spacecraft should be the reference and the staggering sequence
• Check the results based on daily OD data and the FDA reruns
• Finalize the staggering sequence; submit this to scheduling/operations teams
• Deliver preliminary DV tables to GEONS team
• Perform preliminary and then final detailed simulation (using the tool CHiFi [a high fidelity simulator of the onboard controller]) to verify expected fuel use and check that the maneuvers do not violate any spacecraft safety constraints (boom bending moments, etc.)
• Evaluate the effects of the maneuvers on SMA values, orbit planes, QF evolution
• Perform Monte Carlo runs to determine safety in the face of maneuver execution errors, as well as QF lifetime, thus an estimate to the time until the next maneuvers
• Present these results at Command Authorization Meeting (CAM) that is held between the FOT and FDOA teams (with SOC members also usually remotely in attendance)
• Monitor FM1 burns; reconstruct fuel use and SMA changes from spacecraft data downlinked at end of maneuver pass
• Use the navigation data downlinked at the post-perigee passes to evaluate maneuver errors and determine if tweaking of FM2 is necessary: if so, perform Delta-CAM and upload new maneuver commands
• Monitor second maneuvers, reconstruct fuel use and SMA changes from maneuver pass data
• Use navigation data downlinked at the post-perigee passes to evaluate the final formation orbits, which results in the initial data for the generation of the next set of maneuvers occurring in 2 or more weeks
• Repeat the steps above.

A theoretical upper limit on the possible interval between formation maneuver sequences is approximately two months: this is driven by differential Earth oblateness effects that gradually pull the spacecraft out of formation. However, in reality this is dominated by the effects of maneuver execution errors. Pre-launch simulations, based on DV controller performance requirements, indicated that formation maneuvers would be required approximately every two weeks: from the list of activities above, it can be seen that this would require essentially continuous maneuver planning and execution. Fortunately, the DV controller, as well as the GEONS navigation system, have performed significantly better than their specifications (see Section VI for these results): as a result, maneuver cadence has typically been every 4 weeks. In fact, during the season of long eclipses during June-July 2016, when there was insufficient power to operate all the systems (heaters, communications, thrusters) that are required for maneuvers, an interval of 6 weeks between maneuvers was scheduled.

There are currently studies underway to determine the feasibility of flying the spacecraft in formation sizes smaller than 10 km (the lower limit agreed upon in requirements prior to launch): this desire of the science team is driven by the fact that the electron diffusion region, a key component in understanding magnetic reconnection, has now been found from early MMS data to typically be smaller than 10 km. The key flight dynamics question however is to determine the smallest formation that can be safely flown. If the initial spacing between the spacecraft is too small, the effects of maneuver execution errors can cause them to drift alarmingly close within 2 weeks, i.e. before the next maneuver sequence to reset the formation can be planned, scheduled and executed. The deciding factor on how small a formation can safely be flown is therefore likely to be this interplay between execution error and maneuver cadence.

VI. Formation Maneuver Results

The MMS spacecraft have performed a total of 194 maneuvers as of Jul 2016:

• Five Perigee-Raise (PR) burns shortly after launch, to raise perigee altitude from 585 km to 0.2 $R_e$ (1,276 km)
• Up to eleven burns per spacecraft for system calibration, spin rate adjustment, and boom deployment purposes.
• Two Orbit Stabilization (OS) trims to correct post-PR dispersions and freeze the inter-satellite distances at a range compatible with the initial 160 km formation size
• A set of Formation Initialization (FI) burns to put the spacecraft into an initial 160 km tetrahedron, followed by Formation Maintenance (FM) maneuvers to maintain this formation size

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- A series of Formation Resize (FR) maneuvers, at a two week cadence, to go into tetrahedra with scale sizes 60 km, 25 km and 10 km in turn
- In addition, a series of FM maneuvers were applied to stay at first a 10 km scale size for science data gathering during Phase 1a (with four weeks spent at 40 km, to test the quality of the science data collected at this scale size), and then at 40 km during Phase 1x, when long eclipses preclude maneuvering for a period of six weeks. (Flying in a larger formation size during this mission phase increases safety by reducing the risk that the spacecraft will drift uncomfortably close between maneuvers.)

Fig. 4 shows the achieved average mean side-lengths over each RoI (this yields a single value for each rev, for simplicity) for each of the formations that have been flown to date, together with the bounds on mean side-lengths (green lines) that are allowed from the definition of the formation Quality Factor. It can be seen that each of the flown formations satisfies this requirement for extended periods, typically up to 4 weeks. Fig. 5 then gives more detail on the evolution of the mean side-length throughout the RoI for 10 rev straddling two of the 160 km formations. This not only illustrates how the mean side-length reaches its minimum value at apogee, but also shows the possible difference in size of two formations that have the same official scale size.

![Formation Side Length](image)

**Fig. 4. Evolution of average mean sidelenath over mission to date.**
Fig. 5. Mean instantaneous sidelength in RoI, two 160 km formations.

Fig. 6 shows the evolution of the Quality Factor throughout the RoI for the mission to date and Fig. 7 shows an example of the quality factor over several revs, from one formation to the next: the characteristic “double hump” evident in this plot is a result of the fact, discussed previously, that the formation is somewhat compressed at apogee, leading to a lower QF value at apogee itself than shortly before or after it. It can be seen that one of the two peaks gradually builds up from one rev to the next, making the QF plot increasingly unsymmetrical: this is typical of the effect of inter-satellite drift, and eventually leads to the need to perform a new set of formation maneuvers to initialize a new tetrahedron. In addition to the instantaneous Quality Factor, Figs. 6 and 7 also display the mean quality factor per RoI (Qbar) as well as the percent RoI time with Quality Factor above 0.7 (Tq). Both values are used as single value assessments of the formation quality over an entire RoI. Definitive values for these properties are used as part of science data evaluation and predicted values for these properties are considered when determining when an extra formation maneuver may need to be performed in order to maintain a formation with sufficient quality for science purposes.
Fig. 6. Quality Factor evolution over mission to date.

Fig. 7. Quality Factor evolution, two 40 km formations.
Fig. 8 shows the evolution in inter-satellite ranges between the 6 MMS pairs over the orbit for one of these formations. The large spikes in ISR at perigee that are caused by the increased along-track separation can clearly be seen. This can also be seen by revisiting Figs. 1: these show the tetrahedron geometry at apogee and that at perigee. The extreme elongation of the formation at perigee is clearly evident.

![Inter-Spacecraft Range](image)

**Fig. 8. Six inter-satellite ranges over entire rev.**

As previously noted, a key consideration for MMS formation maneuver design is that the SMAs of the four spacecraft be closely matched after the burns, so as to match the orbital periods and so keep the formation together for extended periods. This is shown in Fig. 9, which covers a complete set of FM maneuvers: it can be seen that the final SMAs are indeed very nearly equal, as indeed were the SMAs before the maneuvers. In addition, it can be observed that the SMA change that is produced by the first burn of each spacecraft is very nearly equal and opposite to that produced by the second: this is a reflection of the fact that the reference satellite does not maneuver. As a result, its SMA does not change across the maneuver, apart from the natural variation from rev to rev produced by orbital perturbations. Since the other spacecraft must match its SMA before and after the burns, the FM1 and FM2 SMA changes must essentially cancel.
As previously noted, the reference spacecraft is generally selected in order to balance fuel consumption across the fleet. This is illustrated in Fig. 10, which shows the evolution in estimated (by book-keeping) fuel remaining in each of the four spacecraft. MMS1 initially had significantly more fuel remaining than the other spacecraft, partly as a result of the details of the PR maneuvers and partly because it was selected as the reference for the large Formation Initialization maneuvers, since it was the spacecraft with highest perigee. However, since that point it has never been selected as the reference, so its fuel quantity is gradually approaching that of the others. The other three spacecraft have been essentially alternating as reference for the long series of FR and FM maneuvers (which typically take something on the order of 1-3 kg per spacecraft to execute), and so are very evenly matched in fuel mass.
The key role of maneuver execution error was discussed previously. Figs. 11 - 13 give the errors as estimated from reconstruction data for the maneuvers that have been performed to date: these figures give the magnitude (Figs. 11-12) and pointing error values (Fig. 13). It can be seen that the errors are typically larger for small maneuver delta-v sizes: this is expected from the design of the Delta-V controller. It should also be noted that the observed errors are considerably smaller than the requirements that are imposed on this controller (see lines on Fig. 11; pointing error requirement is 1.5 deg. 3 sigma): this, in addition to the superior performance of the GEONS navigation system, are the reasons why a interval between maneuvers of 4 weeks has been achievable, rather than the requirement of only 2 weeks. It is also a result of the better than expected performance of the Delta-V controller and GEONS that it will likely prove possible to fly the MMS spacecraft in formation sizes smaller than the originally specified 10 km. (This question is currently under study.)
Fig. 11. Delta-V execution error (magnitude).

Fig. 12. Delta-V execution error (magnitude) Y-axis zoom.
As previously described, the FDA designs the burns of all three non-reference spacecraft together in order to produce a new tetrahedron. One exception to this procedure arose in Dec. 2015, when the MMS4 FM1 contact was lost as a result of a network issue. The other two spacecraft had already maneuvered: they then proceeded with their FM2 burns, putting them into a new, stable formation in relation to the reference. MMS4, however, was not able to maneuver until the contingency Sat./Sun. slots. The original recovery plan was that all three spacecraft would have to maneuver in these slots, setting up a new formation, designed from scratch by the FDA; however, this would use fuel to essentially pull the three-day old formation apart and then form a new one. In addition, it would have required staffing a complete set of maneuver contacts on the weekend before Christmas; for both of these reasons, this approach was undesirable. Fortunately, an approach was found, at short notice, that allowed MMS4 to be maneuvered alone, essentially matching the orbit that it would have followed if it had maneuvered nominally, albeit several days late. This elegant approach saved both fuel and operations costs. MMS4 only required an additional 0.65 kg of fuel compared to its missed maneuver instead of two other spacecraft also needing a potential of 1+ kg of fuel to perform an extra maneuver. In addition to the fuel savings, on the order of 25 hrs of staff time was saved by only having to schedule, monitor, and evaluate one spacecraft maneuvering instead of three.

VII. Conclusions

This paper described the underlying dynamics and findings of formation flying in the Magnetospheric Multiscale mission. The results demonstrate that MMS has been able to carry out formation flying while exceeding requirements for maneuver execution error and maneuver cadence. These results have inspired the science team to request the investigation of evaluating the feasibility of flying formations smaller than 10 km and in return provide significantly enhanced science data.

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