Trajectory Design Enhancements to Mitigate Risk for the Transiting Exoplanet Survey Satellite (TESS)

Donald Dichmann[1] and Joel Parker[2]
NASA Goddard Space Flight Center, Greenbelt MD

Craig Nickel[3], Stephen Lutz[4]
Applied Defense Solutions, Inc., Columbia MD

The Transiting Exoplanet Survey Satellite (TESS) will employ a highly eccentric Earth orbit, in 2:1 lunar resonance, which will be reached with a lunar flyby preceded by 3.5 phasing loops. The TESS mission has limited propellant and several constraints on the science orbit and on the phasing loops. Based on analysis and simulation, we have designed the phasing loops to reduce delta-V (DV) and to mitigate risk due to maneuver execution errors. We have automated the trajectory design process and use distributed processing to generate and optimal nominal trajectories; to check constraint satisfaction; and finally to model the effects of maneuver errors to identify trajectories that best meet the mission requirements.

Nomenclature

Acronyms

AOP = Argument of Perigee
CR3BP = Circular Restricted 3-Body Problem
DV = delta-V
GEO = Geostationary Orbit
IBEX = Interstellar Boundary Explorer
LRP = Lunar Resonant Phasing
PAM = Period Adjust Maneuver
PLEP = Post Lunar Encounter Perigee
Re = Earth radius
SECO = Secondary Engine Cutoff
SEM = Spacecraft-Earth-Moon
TESS = Transiting Exoplanet Survey Satellite
TLI = Translunar Injection

Variables

a = semimajor axis
e = eccentricity
i = inclination
K = Kozai parameter
T = Tisserand parameter

I.   Introduction

The Transiting Exoplanet Survey Satellite (TESS) will fly in a highly eccentric Earth orbit in 2:1 resonance with the Moon. At mission orbit insertion the apogee will be near 59 Earth radii (Re) and the perigee will be near 17 Re. The lunar resonant orbit exhibits long-term variation due to the Kozai mechanism, a version of the Circular Restricted 3-Body Problem (CR3BP).

The mission design employs the Lunar Resonant Phasing (LRP) condition, where the Moon-Earth-spacecraft angle oscillates around 90 deg at apogee. The LRP condition eliminates the need for orbit maintenance maneuvers, and was used successfully for the Interstellar Boundary Explorer (IBEX) mission[5,7]. The fundamentals of the TESS trajectory design are described in Ref. 1. The TESS launch will be no earlier than 20 December 2017.

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[1] Senior Navigation Engineer, NASA Goddard Space Flight Center, 8800 Greenbelt Road 20771, donald.j.dichmann@nasa.gov
[2] Aerospace Engineer, NASA Goddard Flight Center, joel.j.k.parker@nasa.gov
[3] Astrodynamics Engineer, Applied Defense Solutions Inc., 10440 Little Patuxent Parkway Suite 600, Columbia MD 21044, CNickel@applieddefense.com
[4] Aerospace Engineer, Applied Defense Solutions Inc., Stephen.Lutz@applieddefense.com

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The mission orbit will be reached using a lunar flyby. The arc between the lunar flyby and mission orbit insertion is called the transfer orbit. The lunar flyby serves several purposes: to raise perigee; to raise apogee; to increase ecliptic inclination; and to adjust ecliptic Argument of Perigee (AOP). The perigee radius for the transfer orbit is the initial value needed for the mission orbit. The apogee radius for the transfer orbit is the value needed to achieve the LRP condition.

The flyby will be preceded by 3.5 phasing loops, shown in Figure 1. A sample trajectory for a launch on 02 January 2018 is shown in Figure 2 using the inertial frame and in Figure 3 using the rotating frame. Figure 4 uses a side view to see the plane change caused by the lunar flyby. Figure 5 shows the ground track from Secondary Engine Cutoff-1 (SECO-1) through SECO-2 into the first phasing loop.
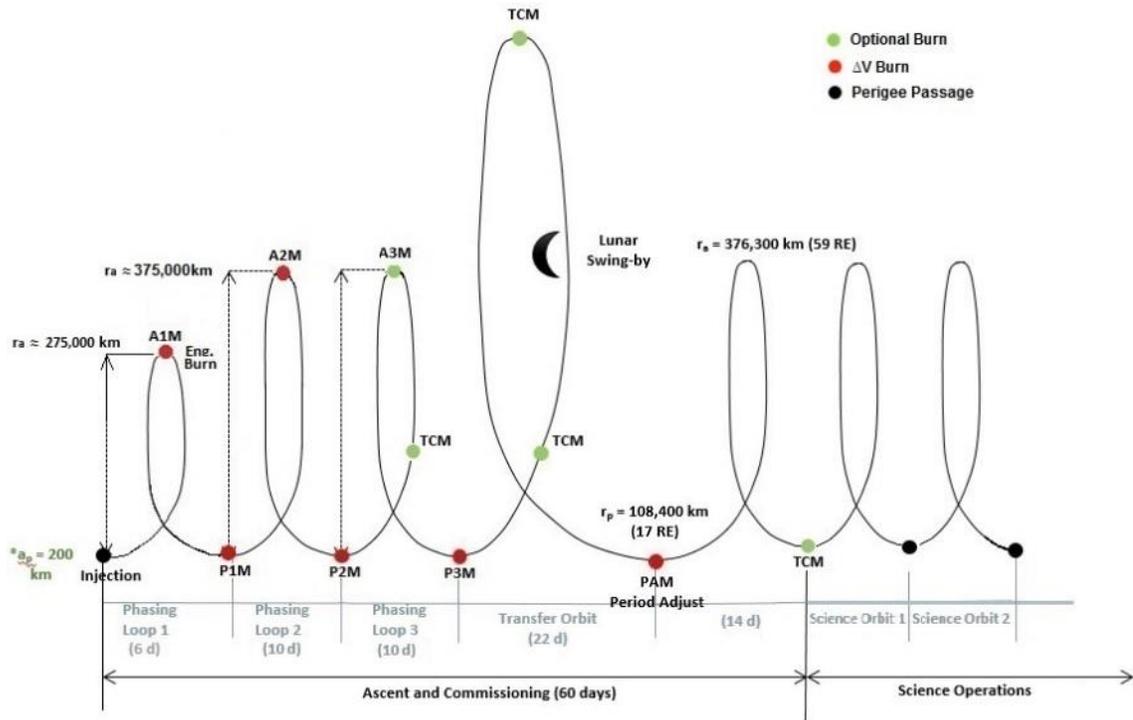

Figure 1. Notional phasing loop diagram. As illustrated in this figure, apogee radius at A2 and A3 will ideally be at the lunar orbit radius at flyby, so maneuvers P2M and P3M are small.

The TESS mission has limited propellant and is subject to several constraints on the mission orbit and on the phasing loops. No eclipse can be longer than five hours, and most must be no longer that 4 hours. For collision avoidance there is also a NASA requirement that the satellite remain above geostationary orbit (GEO) radius for decades. Based on the Kozai model, we have selected mission orbit parameters to mitigate risk of long eclipses and low perigee, but high-fidelity force modeling is required to confirm that these conditions are met. There is an additional constraint that the Sun remain outside the field of view of the instrument during each maneuver, to prevent damage to the instrument.

The phasing loops allow us to mitigate risk of errors in the flyby, in part by allowing time to calibrate the thrusters. However there still remains some risk due to maneuver execution errors that must be addressed. The satellite will be launched on a SpaceX Falcon 9 from Kennedy Space Center and injected into an orbit with perigee altitude of 200 km and apogee radius of 270,000 km. We need to be confident that the lunar and solar perturbations on the first loop do not lower the next perigee. To address this issue we include a maneuver at first apogee, called A1M. However, whenever practical, the phasing loop duration is chosen so that the Moon raises perigee around first apogee, so A1M is not needed to raise perigee. Analysis and simulation have demonstrated a potential mission risk if large maneuvers

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are required at the second and third perigees. Consequently we have selected the phasing loop duration so that the second and third perigee maneuvers are near zero.
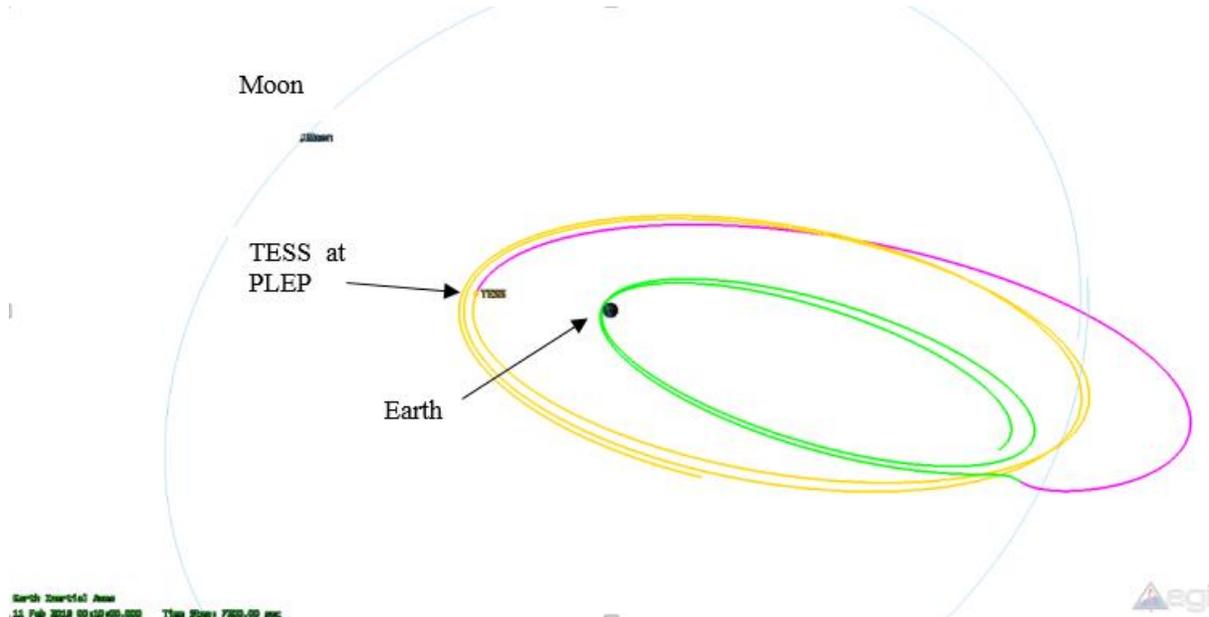

Figure 2. Example of a TESS trajectory, in the inertial frame. The phasing loops are in green, the transfer from flyby to mission orbit in purple and the mission orbit in yellow. The position of TESS is shown at the Post Lunar Encounter Perigee (PLEP), where it enters the mission orbit.

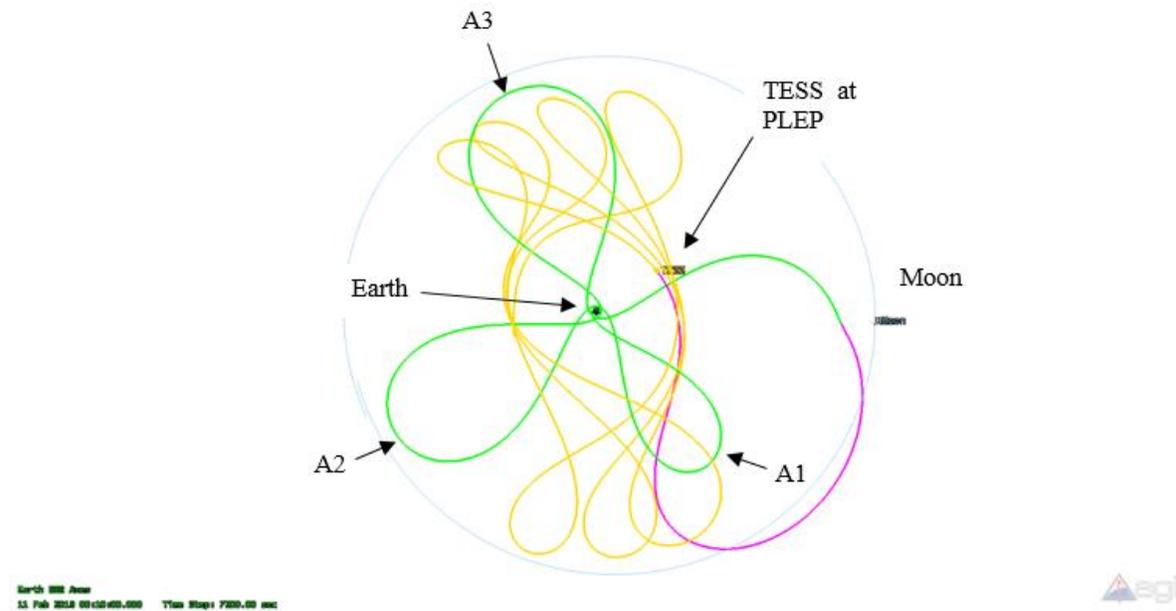American Institute of Aeronautics and Astronautics

Figure 3. The TESS trajectory from Figure 1 shown in the Earth-Moon rotating frame, where the Earth is at the center and the Moon is to the right on the horizontal axis. The color scheme is the same as in Figure 2. The plot shows the 2:1 lunar resonant phasing condition, where the satellite at apogee is far from the Moon.
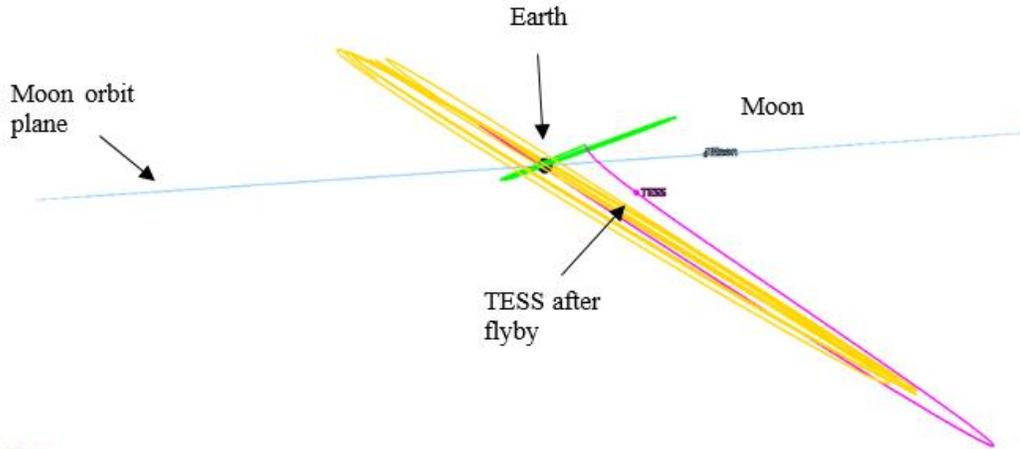

Figure 4. Illustration of the plane change produced by the lunar flyby. In this figure the TESS spacecraft is shown in the transfer orbit after flyby. The transfer orbit is inclined about 40 deg to the Moon's orbit plane (light blue).
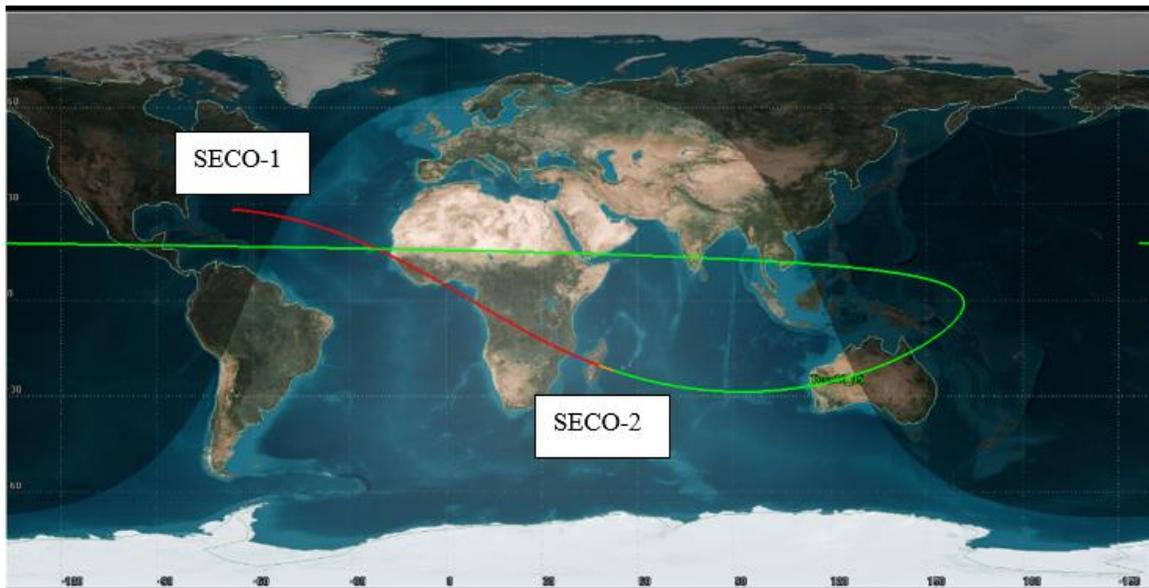

Figure 5. Ground track for launch date of 02 January 2018. Coast arc is in red, SECO-2 burn is in orange, phasing loops in green. For this launch date, injection occurs over Madagascar.

To analyze trajectories over a full year of launch opportunities, we have automated the trajectory design and analysis process and use distributed processing to generate and optimize nominal trajectories and check constraint satisfaction. We then run a Monte Carlo simulation for statistical DV, and perform a contingency analysis on each maneuver. Details about the Monte Carlo simulation and contingency analysis are described in a companion paper[2]. Using this

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assessment of each trajectory we are able to select the trajectory for each launch date that best meets the mission requirements.

The remainder of this paper is structured as follows. Section II contains a mission overview. This includes a spacecraft description together with the mission requirements and goals. Section III contains a Trajectory Overview, from SECO-1 through the phasing loops to flyby to mission orbit. Section IV reviews the relevant Dynamical Systems modeling, with particular attention to the long-term orbit predictability. In Section V we describe the orbit dynamics techniques that are applied to achieve Mission Requirements Satisfaction and Risk Mitigation. In Section VI we describe the scripting method used to automate the trajectory design and analysis. In Section VII we give an overview of the solutions for launch dates in January 2018. Finally Section VIII gives a summary of the paper and future work.

II. Mission Overview

TESS, scheduled to be launched in 2017, is a NASA Explorer-class mission that will perform a survey of the entire sky over its nominal two-year mission. TESS will monitor 500,000 stars for temporary drops in brightness caused by planetary transits, and it is expected to discover thousands of exoplanets in orbit around the brightest stars in the sky. This first-ever spaceborne all-sky transit survey will identify planets ranging from Earth-sized to gas giants, around a wide range of stellar types and orbital distances. TESS will provide prime targets for observation with the James Webb Space Telescope, as well as other large ground-based and space-based telescopes of the future[4].

A. Spacecraft Description
The TESS spacecraft bus, shown in Figure 6, is a variation of the Orbital ATK Leostar-2. There is one 22-N orbit maneuver thruster. TESS has a single instrument composed of an array of four cameras with boresight along the same axis as the orbit maneuver thrust direction. Each of the four cameras has a field of view of 24 deg by 24 deg. Together the four cameras have a field of view of 24 deg by 92 deg. The wide field of view makes it possible for TESS to perform its mission to survey the full sky in two years. However if thrust is in the direction of the Sun then the Sun will be in the instrument field of view.
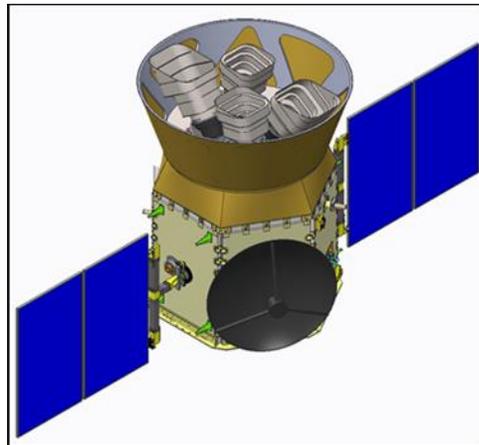

Figure 6. TESS spacecraft with solar arrays deployed, viewed along the instrument boresight (Z axis).

B. Mission Requirements and Mission Goals
The TESS mission is subject to several requirements that constrain the orbit design. Following injection, TESS will perform 3.5 phasing loops, leading up to a lunar flyby. The first phasing loop after injection has an apogee radius of 270,000 km. During the phasing loops there are five deterministic maneuvers. At first apogee A1 we perform maneuver A1M. A1M is designated as a calibration burn with duration of 54 sec, which achieves a DV of 4 m/s. At first perigee P1 we perform maneuver P1M to raise apogee radius. The other deterministic phasing loop maneuvers are A2M, P2M and P3M. Maneuvers P2M and P3M are used for relatively small adjustments in apogee radius. Maneuver A2M is used to raise perigee altitude at P2 and P3 if needed, and to adjust orbit period.

The flyby raises the perigee radius to 17 Re or about 100,000 km. The flyby also raises the ecliptic inclination of the mission orbit, to avoid long eclipses. In addition the flyby raises transfer orbit apogee to achieve a desired phasing condition at PLEP: The Spacecraft-Earth-Moon (SEM) angle must be near zero at PLEP to accomplish the LRP condition. The transfer orbit apogee radius is required to be less than 90 Re. Following flyby, the final maneuver is performed at PLEP: The Period Adjust Maneuver (PAM) achieves the required 2:1 lunar resonance, with an orbit period of 27.3/2 = 13.65 days. The associated mission orbit semimajor axis is 38 Re. Because the mission orbit period is half the Moon orbit period, as the spacecraft moves through half of its orbit from perigee to apogee, and the Moon moves through one quarter of its orbit. Therefore, if the SEM angle is near zero at mission orbit perigee then the SEM angle is near 90 deg at mission orbit apogee. The 2:1 lunar resonance implies that the LRP condition will persist throughout the mission. The large angular separation from the Moon at spacecraft apogee has a stabilizing effect on the spacecraft orbit, which in turn eliminates the need for orbit maintenance maneuvers[5,7]. The mission duration is two years, with a possibility to extend another two years.

The total DV for orbit maneuvers is 215 m/s. This is roughly allocated as 150 m/s for deterministic maneuvers, 25 m/s for launch dispersion, and 40 m/s for statistical DV other than launch dispersion. No single maneuver magnitude can be larger than 95 m/s. The size of the batteries imposes a limited on the duration and number of eclipses. There can be at most 16 total eclipses. Of these, at most two eclipses of duration between 4 and 5 hours. The rest must be less than 4 hours. The Sun cannot be in view of the instrument for more than 15 minutes at a time. This constraint protected the Charge Couple Devices in the instrument from damage.

The lunisolar perturbations induce quasiperiodic oscillations in the mission orbit perigee radius, inclination and ecliptic AOP, as modeled by the Kozai mechanism. The mission orbit perigee radius must be less than 22 Re, to support communications. Phasing loop perigee altitude must be at least 600 km. We are required to identify at least five launch dates in each month that meet the mission requirements. One of the driving requirements on the TESS orbit is designed to protect assets in the GEO belt after end of mission operations. According to NASA Standard 8719.14a, the TESS satellite is required to remain at least 300 km above the GEO radius of 42164 km for decades after commissioning[3]. The TESS mission has come to an agreement with NASA Headquarters regarding how satisfaction of the GEO requirement is to be demonstrated, as is discussed in Section IV.C.

In addition to the mission requirements, there are several mission goals that we seek to satisfy. We want the mission orbit to be operationally stable, meaning that no orbit maneuvers are needed after commissioning. There is a goal that none of the maneuvers be critical. A maneuver is considered critical if the failure to perform the maneuver as planned, within expected statistical error, could mean that the mission cannot be completed. The criticality of the phasing loop perigee maneuvers P1M, P2M and P3M was a particular concern because the spacecraft is only near perigee for a few hours. Because the engine is being calibrated at A1M, there is a goal that the maneuver duration not be any longer that the minimum required. The TESS project also seeks to limit the set of injection points so that ground tracking can be performed without excessive cost or risk. Injection is not allowed far from the African coast. Specifically, as illustrated in Figure 7, the injection equatorial AOP must be between 150 and 235 deg., inclusive.
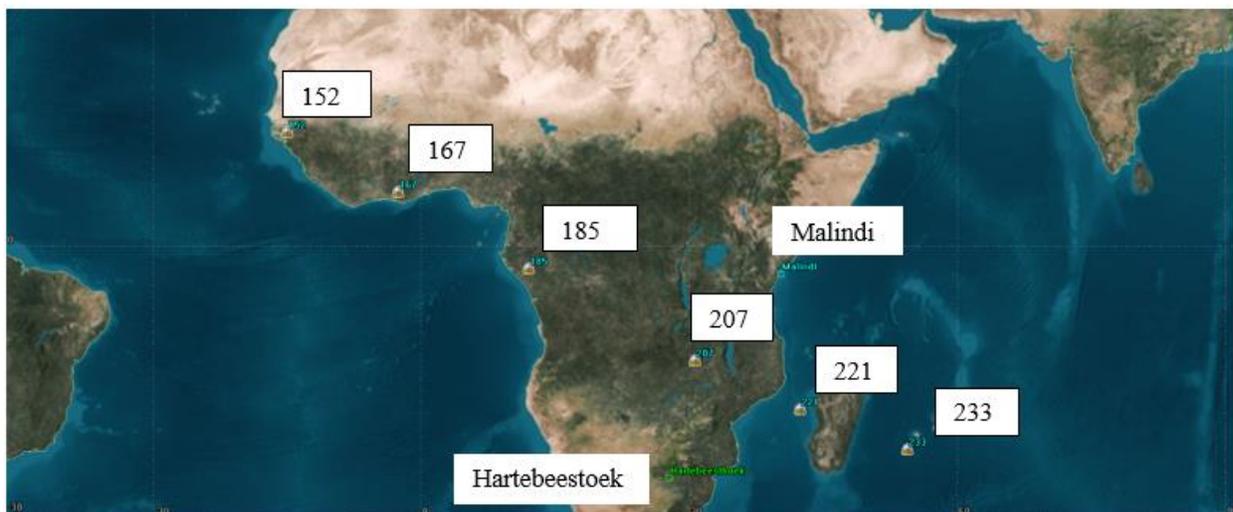
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Figure 7. Locations of injection points following SECO-2 for a range of injection AOP values, from AOP = 152 deg over the west coast of Africa to AOP = 233 deg over the Indian Ocean. For reference the plot also shows the locations of the tracking station at Hartebeesthoek in South Africa and Malindi in Kenya.

III. Trajectory Overview

Because the lunar flyby is critical to the TESS trajectory, it is the focus of our design. The direction of the Moon at flyby is the primary factor in determining the injection orbit plane. The distance of the Moon at flyby is the primary factor in determining the phasing loop duration. And the flyby geometry makes it possible to achieve the orbit plane, the perigee radius and the LRP condition for the mission orbit.

A key factor in the mission orbit design is the eclipse constraint. To keep eclipses from becoming too long, the mission orbit apogee must remain well outside the ecliptic plane. That means the ecliptic inclination must remain well above zero. It also means ecliptic AOP must remain away from 0 and 180 deg.

The phasing loop line of apsides must be oriented to point toward the Moon at the time of flyby. The phasing loop duration must be selected so we reach the Moon at the desired epoch. As described in Ref. 1 and in Section VI below, we begin with the design of the mission orbit and the flyby that leads to it. We then develop the phasing loops to connect with the launch. Specifically, we design the trajectory to begin at the SECO-1 state provided by SpaceX in Earth-Centered Earth-Fixed coordinates. As shown in Figure 5, SECO-1 is followed by a coast arc and the SECO-2 burn, which injects TESS into the phasing loops.

IV. Dynamical Systems Modeling

The section provides a brief overview of the dynamical systems modeling related to lunar resonant orbits for missions like TESS. We look at three aspects of the dynamics: the Tisserand Criterion, the Kozai Mechanism and long-term predictability. For further details, see Ref. 1, 5, 6, 7 and 8.

A. Tisserand Criterion at Flyby
In the three-body problem, when an orbital object has a close approach with one of the primary bodies, the orbit elements can change significantly yet the following Tisserand parameter T is nearly conserved:

$$(1) \qquad T = \frac{1}{2a} + \cos(i)\sqrt{a(1-e^2)}$$

Here a is the semimajor axis (scaled by the distance between the primaries), e is the eccentricity and i is the inclination relative to the orbit plane of the primary bodies. Because a is unitless, the parameter T is unitless. The conservation of T was noted by the French astronomer Tisserand for comets that have close approaches with Jupiter. Note that the Tisserand parameter used by astronomers is typically two times the quantity T in the Equation (1). In terms of the CR3BP, the Tisserand parameter is closely related to the Jacobi parameter[10]. For the TESS mission we generally use 1.15 as the initial guess for T, then allow the trajectory design algorithm to vary T as needed. For the TESS mission orbit, the selection of the value of T is driven by the requirement following the flyby to achieve the required apogee radius, perigee radius and inclination for the transfer orbit. The perigee radius and inclination for the transfer orbit are the same as those required for the mission orbit. The apogee radius for the transfer orbit is chosen to achieve the proper timing for the LRP condition.

B. Kozai Mechanism in Mission Orbit
The Kozai mechanism (also called the Lidov-Kozai mechanism) describes the long-term behavior of a highly eccentric, highly inclined orbit[11,12]. The Kozai model is obtained by performing long-term averaging in the CR3BP, to remove oscillations on the time scale of the orbit period of the primary bodies. Here we apply the Kozai model to the case of the TESS spacecraft orbiting the Earth and perturbed by the Moon. The Kozai model has several implications regarding the long-term evolution of orbit elements. The semimajor axis is constant. (This means that in the CR3BP the semimajor axis oscillates around a constant value.) Another constant is the Kozai parameter:

$$(2) \qquad K = \cos(i)\sqrt{1-e^2}$$

Here, e is the eccentricity and i is the inclination to the Moon's orbit plane. The parameter K is proportional to the orbital angular momentum about the normal to the Moon's orbit plane. While K remains constant in the CR3BP, the inclination i, the eccentricity e and the perigee radius oscillate with a period around 10 years. See Figure 8. Another prediction of the Kozai model is that the AOP relative the Moon's orbit plane oscillates about 90 deg or 270 deg, for i below the critical inclination of about 40 deg. See Figure 9.

When employing the Kozai model we must be aware that it is a variation of the CR3BP. In reality the Moon's orbit is not circular and there are other perturbing bodies, especially the Sun. As a result the predictions of the Kozai model are not exact. Nevertheless, like the CR3BP, the Kozai model can be a valuable guide to the qualitative long-term dynamics. Simulations as well as observations from the IBEX mission show that the Keplerian orbit period oscillates with amplitude of about 1 day, and oscillation period of about ten months. See Figure 10.

A seminal analysis performed by Gangestad et al. demonstrated that a careful selection of the value of K, near 0.64, is a significant step toward meeting TESS mission requirements on eclipses and on the mission orbit perigee radius[6]. This topic is discussion further in Section V. As with the Tisserand value T, we make a first guess at the value of K to start the solver, then allow the solver to vary K as needed but within certain limits to help us achieve operational stability and keep mission orbit perigee above the GEO limit.
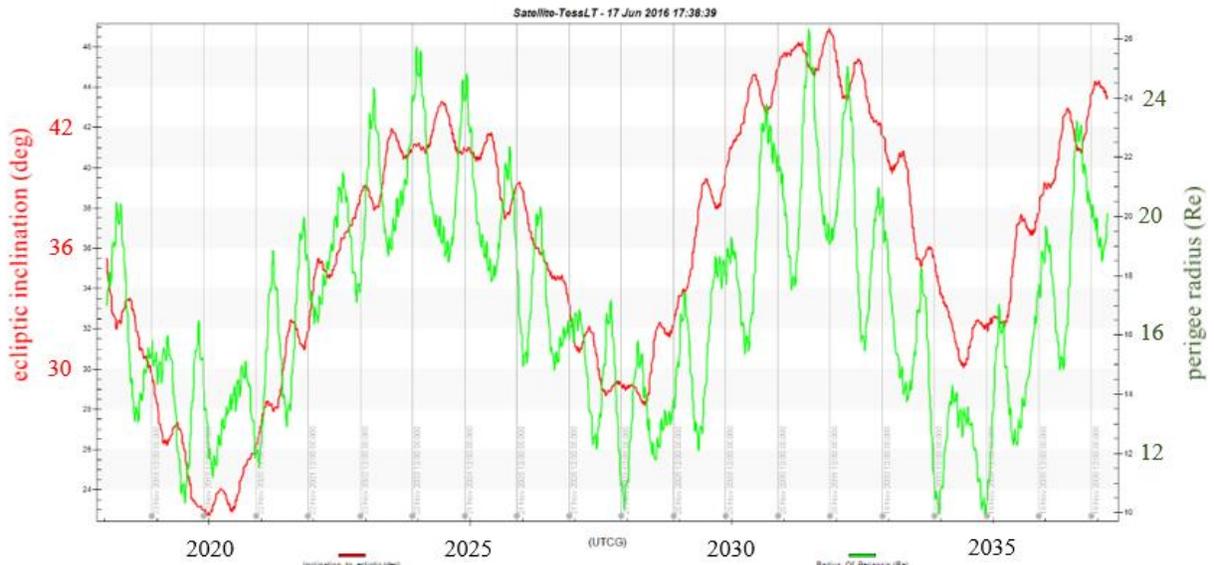

Figure 8. Ecliptic Inclination (red) and Perigee Radius (green) vs time, over 20 years starting in 2018. The oscillation period is about 10 years for each curve.
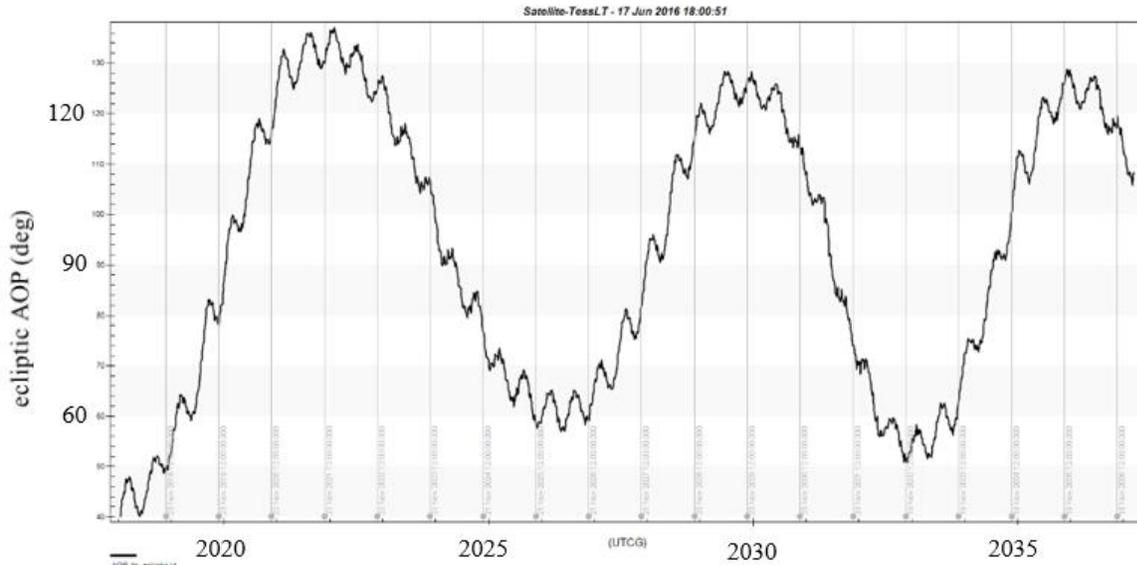
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Figure 9. Ecliptic AOP vs time over 20 years. The oscillation period is about 10 years.
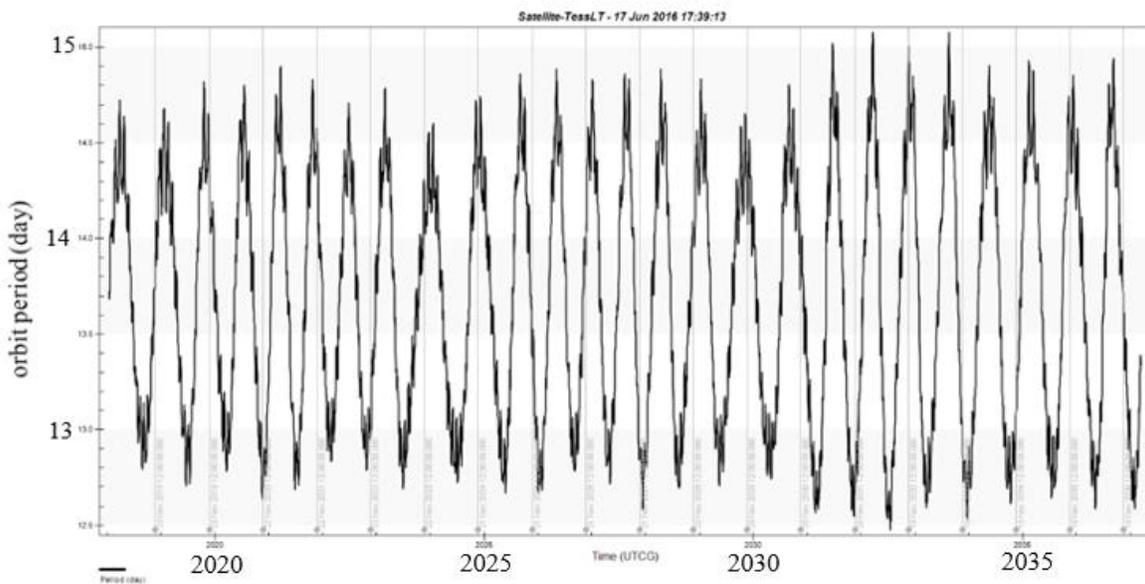

Figure 10. Orbit period vs time, over 20 years. Orbit period varies by as much at 1 day from the ideal 2:1 resonant orbit period of 13.65 days. Oscillation period is about ten months.

C. Operational Stability with Long-term Uncertainty

In Ref. 1, 3:1 lunar resonant orbits were analyzed from a dynamical systems viewpoint. It was shown for one orbit that, when Sun gravity perturbations are included, the largest Floquet multiplier is about 1.01 per obit period of 9.1 days. The value 1.01 for the Floquet multiplier indicates that there is some error growth but the growth is slow on the time scale of the mission. Over the 5 years since IBEX was placed in its resonant orbit, there have been approximately 200 orbits and an error grows by a factor of $1.01^{200} = 7.3$. This indicates that an initial position error of 1 km would grow to about 7 km, which suggests that the orbit is predictable.

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However over 25 years the error growth factor is approximately $1.01^{1000} = 21{,}000$, which suggests that the orbit becomes unpredictable after about 25 years. Note that these calculations do not delineate a clear point where chaos begins, but rather a time interval around 25 years when the orbit state becomes sensitive to small changes in initial conditions or to the addition of small maneuvers. Short et al. also have demonstrated chaos for 2:1 resonant orbit[8].

To pursue the question of orbit predictability further, we took a state for the mission orbit at the end of operations and propagated it in three ways: (a) propagate using the Prince-Dormand propagator with a maximum step size of 1 day; (b) propagate using Runge-Kutta 8/9 using a maximum step size of 1 day; (c) propagate using Prince-Dormand with a maximum step of 1 hour. All three are accurate orbit propagators. The results are shown in Figure 11. We see that for 70 years the three solutions remain close together, but afterwards they diverge. In particular we see that the solution for case (a) stays above GEO radius within 100 years, while the other two solutions do not. These results make it clear that we cannot say with certainty whether a solution remains above the GEO radius for 100 years.

To further investigate orbit predictability, we made small changes in the PAM maneuver – multiplying by 0.99 in one case, and by 0.999 in another – and propagated for 100 years. The resulting perigee radius evolution is shown in Figure 12. Here we see the solutions are nearly the same for 30 years, but afterwards diverge. We obtained similar divergence after about 30 years when we propagated the mission orbit with and without the expected Momentum Unloads.

Based on these results, we conclude that the TESS orbit is predictable for approximately 30 years. The TESS project does not plan to perform any orbit maintenance maneuvers after insertion into mission orbit with PAM. Accordingly, the TESS project plans to meet the GEO-avoidance constraint by demonstrating that the orbit remains above 6.66 Re for 30 years after PAM.
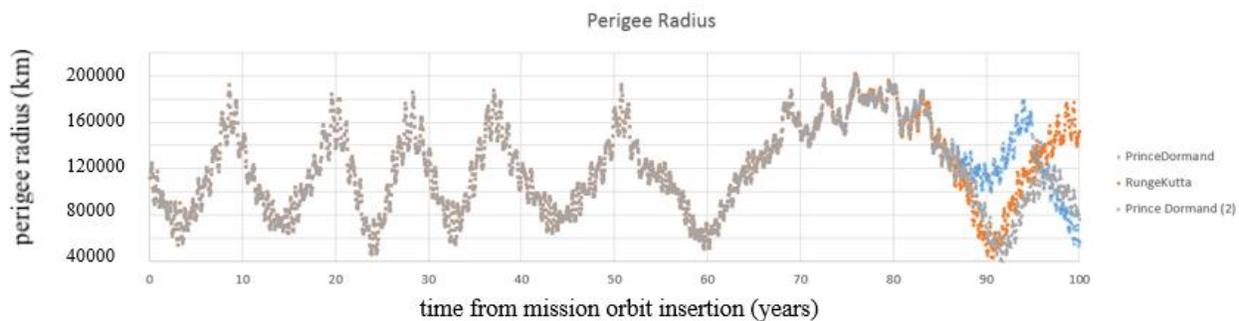

Figure 11. Effect of change in orbit propagator. The blue curve labeled "PrinceDormand" is case (a) with maximum time step of 1 day. The orange curve labeled "RungeKutta" is case (b) with maximum time step of a 1 day. The gray curve labeled "PrinceDormand (2)" is case (c) with maximum time step of 1 hour. Case (a) remains above the GEO radius for 100 years. Cases (b) and (c) do not.

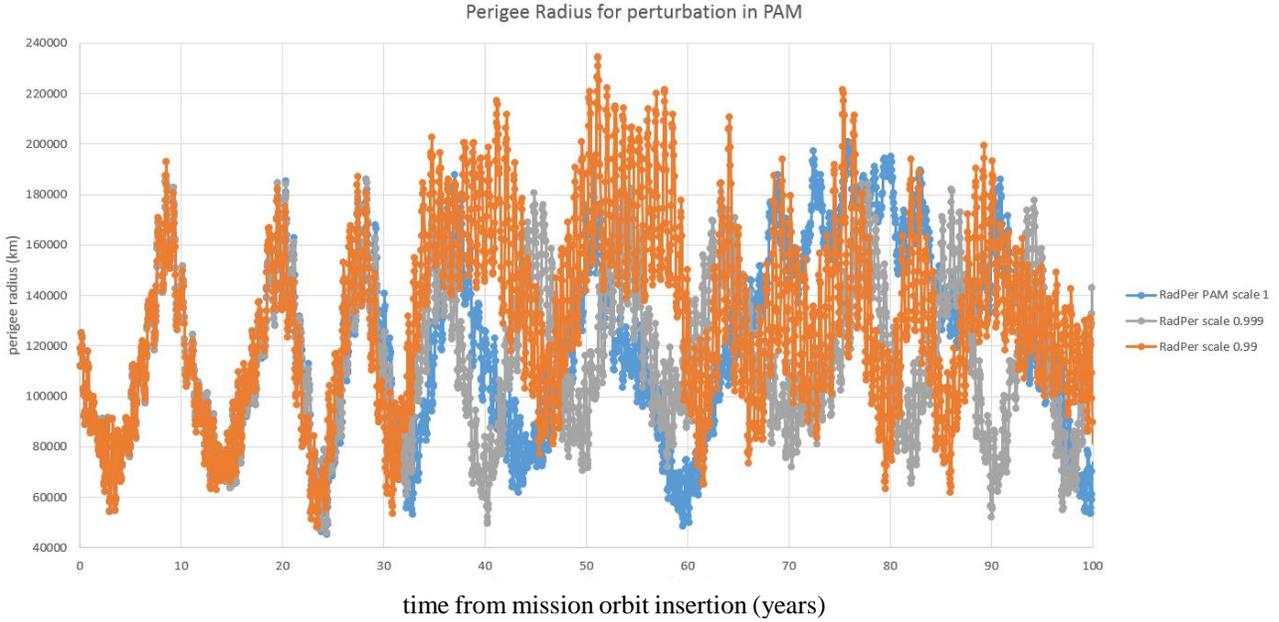American Institute of Aeronautics and Astronautics

Figure 12. Effect of error in PAM. Nominal PAM magnitude is 54 m/s, so a scale factor 0.99 implies an error of 0.54 m/s, and scale factor 0.999 implies an error of 0.054 m/s = 5.4 cm/s. Solutions diverge after about 30 years.

V. Requirements Satisfaction and Risk Mitigation

In this section we describe the methods we used to achieve requirements satisfaction and to mitigate risk. Wherever possible we use a feature of trajectory design to satisfy more than one requirement or goal. In particular, as shown below, we select the value of *K* near 0.64 to achieve the eclipse constraint and to keep mission orbit perigee within an acceptable range.

A. No Critical Maneuvers

It is important for mission success that we achieve the lunar flyby with nearly the same geometry as planned in the nominal trajectory. If the nominal P3M is large and it is missed then it may not be possible to achieve the planned flyby. For this reason we plan A3 to have apogee radius equal to the lunar orbit radius at flyby, so P3M is nominally near zero.

A key element in our effort to eliminate critical maneuvers is the careful selection of the phasing loop duration. If the Moon is behind the spacecraft in its orbit at A1, the gravity of the Moon could lower perigee and perhaps cause TESS to re-enter at P1. In that case, the A1M maneuver to raise perigee at P1 becomes critical. Alternatively, suppose that in the phasing loops the apogee radius at A2 is well below the lunar flyby orbit radius. In that case maneuver P2M would need to be relatively large (more than about 10 m/s). If that occurs, we find in some cases that it was not possible to recover from a missed or partial burn at P2M with the available DV budget. If the nominal value of P2M is 10 m/s, and P2M is missed then the period of the third phasing loop may be reduced by 1.5 days and TESS may not have sufficient DV to correct for that timing error at P3.

The goals to assure the non-criticality of A1M and P2M impose different conditions on the phasing loop duration, as shown in Figure 13. To keep P2M close to zero we need the apogee radii at both A2 and A3 to be close to the lunar orbit radius at flyby. That ideal condition determines the phasing loop duration as an increasing function of lunar orbit radius at flyby, shown by the blue curve in Figure 13.

On the other hand, to prevent A1M from becoming critical we need to get a perigee raise from the Moon at A1, which means the Moon must be ahead of TESS in its orbit at A1. This implies the time interval from A1 to flyby should be no longer than one lunar orbit period of 27.3 days. Because the time from injection to A1 is about 3 days, the phasing loop duration from injection to flyby should be no longer than 27.3 + 3 = 30.3 days. This bound is represented by the

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horizontal orange line in Figure 13. Both conditions are met when the lunar radius at flyby is less than 59.8 Re or 380,000 km, which is less than the Moon's semimajor axis. For lunar radius at flyby above 59.8 Re, we choose to bound the phasing loop duration above by 30.3 days. This has the effect of forcing P2M to be larger. Because we do not want P2M to become too large, this constraint on phasing loop duration can eliminate some solutions with flyby near lunar apogee. We say more about this later in the discussion of the AOP bound in Section V.G.
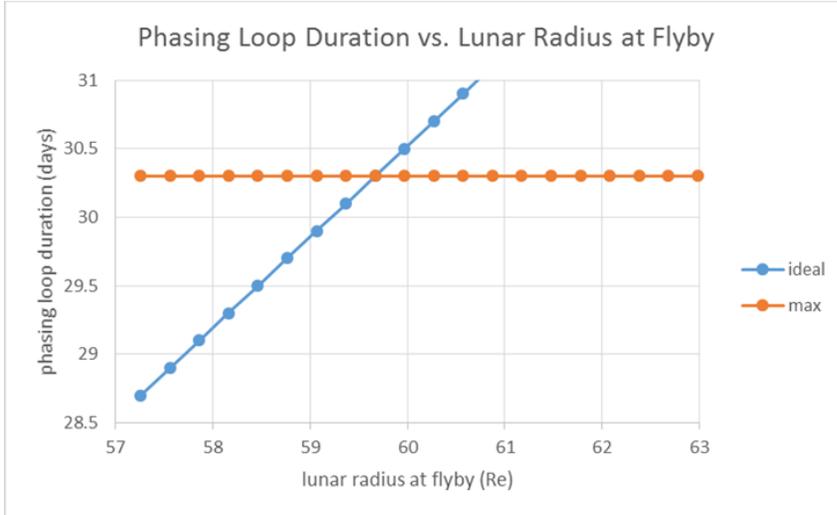

Figure 13. Plots of the ideal phasing loop duration (blue line) and maximum allowed (horizontal orange line).

While we design the trajectory to keep A1M from being critical, we also recognize that TESS will be near apogee for about a day in each phasing loop. Thus if A1M cannot be performed at the planned time, we have the option to delay A1M by several hours.

B. Eclipse Constraint
To limit eclipse duration in the mission orbit we select K near 0.64 so that apogee remains outside ecliptic plane, as shown in Section IV.B. Inclination starts near 40 deg and remains above 10 deg. Ecliptic AOP starts near 30 deg and increases toward 90 deg, where the line of apsides is orthogonal to the line of nodes in the ecliptic plane.

The phasing loop orbit plane for each launch date is determined by the launch inclination and the flyby epoch. Our primary way to avoid eclipses in the phasing loops is through careful selection of the flyby epoch. Simulations have demonstrated that long eclipses in the phasing loops occur most often when the flyby takes place near a full Moon. In that case the Moon is near or in the shadow of the Earth, and the spacecraft is moving nearly along the line from the Earth to the Moon. Of particular concern is a total lunar eclipse that will take place on 31 Jan 2018. However, we can have a flyby on the same day as a total lunar eclipse provided that the flyby does not occur during the lunar eclipse.

C. Perigee Constraints
To keep the mission orbit perigee above the GEO radius near 6.7 Re, we require that the initial mission orbit perigee radius of 17 Re. That together with setting K near 0.64 would keep the perigee radius well above the GEO radius if the orbit dynamics were governed by the idealized CR3BP dynamics. Because the Sun also affects the orbit and the Moon's orbit is not circular, additional measures are required. We draw on a technique from IBEX to adjust the initial mission orbit period[5]. For IBEX the orbit period was adjusted to avoid long eclipses in the mission orbit. In this case we adjust the orbit period to keep perigee above GEO. As shown in Section IV.B, we can vary the initial mission orbit period by as much as 1 day from the ideal value of 13.65 days and still achieve a resonant orbit. So we scan through a range of mission orbit periods between 12.65 and 14.65 days. To achieve an eligible orbit period we adjust the PAM maneuver accordingly, then propagate for 30 years to determine if the orbit remains above the GEO radius. Among the PAM values that achieve the GEO constraint (if any), and we choose the one with the smallest PAM magnitude.

There is also a constraint that the mission orbit perigee remain below 22 Re, to facilitate communications. We find that constraint can be met simply by starting perigee radius and 17 Re, and starting with a K near 0.64.

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D. Total Delta-V

We employ a variety of techniques to keep the total DV within the budget of 215 m/s. One fundamental step is to utilize a lunar flyby to achieve the transfer orbit inclination, perigee radius and apogee radius that delivers TESS to its mission orbit with a feasible LRP angle. We also employ optimization with the goal to minimize total DV (deterministic plus statistical). It was noted above that P2M and P3M can become critical maneuvers if they are too large. We also know from Monte Carlo simulations that it can be expensive in DV to correct for errors in P2M and P3M when those maneuvers have large magnitude. However it is not practical to perform Monte Carlo simulations while designing the nominal trajectory. Each Monte Carlo trial takes at least five minutes. To run hundreds of Monte Carlo trials during each iteration of the nominal trajectory design would mean each iteration would take hours instead of minutes. As a proxy for the total DV we minimize a cost function

$$(3) \qquad cost = DV_{det} + k_{P2}\,|P2M| + k_{P3}\,|P3M|$$

where DV_{det} represents the deterministic DV. In Equation (3) the terms after DV_{det} are used to approximate the statistical DV due to correcting errors at P2 and P3, respectively. To obtain values for the unitless coefficients k_{P2} and k_{P3} we used several previously computed trajectories, we ran a set of Monte Carlo trials, and for each trial we computed the DV to correct for a maneuver error divided by the nominal maneuver magnitude. Based on these results we chose the coefficients $k_{P2} = 4$ and $k_{P3} = 4$. By minimizing cost in (3) we tend to drive down P2M and P3M magnitudes. In addition, we explicitly bound P2M and P3M from above. Finally, for each nominal trajectory we accurately compute the statistical DV using a Monte Carlo simulation as described in Ref. 2.

E. Operational Stability

To achieve operational stability we place TESS in a lunar resonant orbit that meets the Lunar Resonance Phasing condition. Specifically, based on numerous simulation, in the design process we require that the RLP angle at PAM be below 37 deg.

F. Sun Angle Constraint

Our method for handling the Sun angle constraint at each maneuver is represented in Figure 14. If the maneuver at perigee would have the Sun in the field of view, we shift the maneuver before or after perigee, with the burn still along the velocity vector, so the Sun is not in the field of view. The magnitude of the maneuver is adjusted to achieve the planned orbit energy. With this approach there can be a DV penalty for performing the maneuver away from perigee, as much as 21% of the nominal maneuver magnitude for phasing loop perigee maneuvers and as much as 13% for PAM. We may also need to shift apogee maneuvers. However in that case there can be a DV savings because orbit velocity is higher away from apogee.
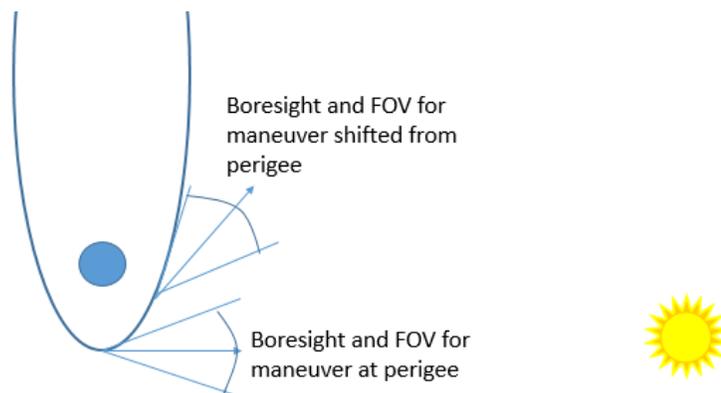

Figure 14. Concept for handling Sun angle constraint: Shift the maneuver so the Sun is no longer in the instrument field of view.

G. Bounds on Injection AOP

The position of the Moon at flyby largely determines the RAAN and AOP of the injection state: The Moon direction at flyby is close to the apogee direction at injection. (The Moon direction at flyby is not exactly the apogee direction, and lunisolar perturbations during the phasing loops also have some effect.) Figure 15 shows the relationship between the injection AOP and the lunar argument of latitude at flyby, based on a set of trajectories for launches in Dec 2017 through Feb 2018. When the argument of latitude is 0 or 180 deg, the Moon is in the equatorial plane, and so on either the ascending node or descending node. This implies the TESS orbit perigee must be nearly aligned with either the ascending node or descending node. Because we choose the TESS orbit to be ascending before flyby, perigee is in the opposite direction from the ascending node, so the injection AOP must be close to 180 deg. The injection inclination is 28.5 deg, and the inclination of the Moon's orbit to the equatorial plane is about 23.5 deg. A geometric analysis shows that, when the lunar argument of latitude is +/- 90 deg, the deviation of AOP from 180 deg is

$$(4) \quad \text{AOP amplitude} = \text{asin}\left(\frac{\tan(23.5 \text{ deg})}{\tan(28.5)}\right) = 54 \text{ deg}$$

Consequently, injection AOP can vary between about 126 and 234 deg during a lunar cycle.
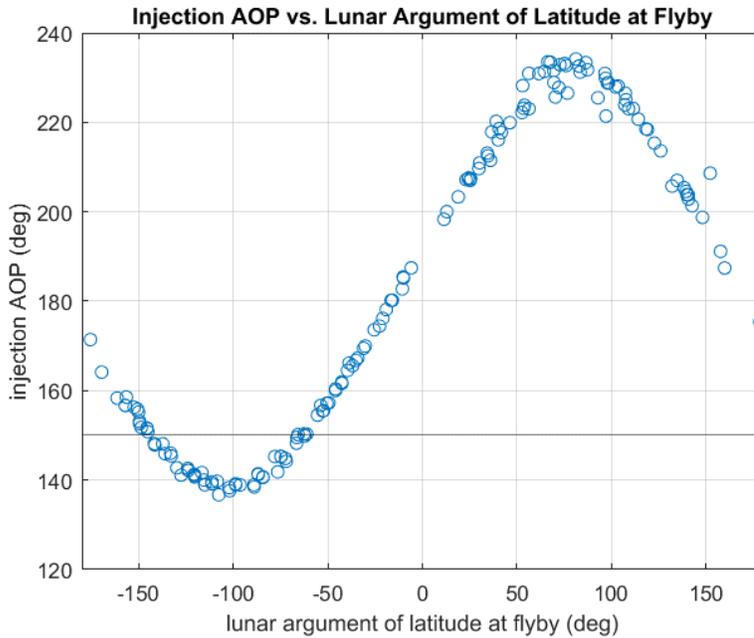

Figure 15. Plot of Injection AOP vs lunar arg lat at flyby. The location of the lunar flyby in the lunar orbit is a key parameter in the trajectory design: It is the primary factor that determines the RAAN and AOP at injection. The injection RAAN and AOP together with injection epoch determine the location of injection over the Earth.

As seen in Figure 15, the condition that injection AOP be no more than 235 deg is satisfied naturally. As indicated in Figure 7, for values of AOP below 150 deg the injection state is off the west coast of Africa. The TESS project has determined that launch vehicle tracking is not practical for injections over the Atlantic, so injection AOP less than 150 deg is not allowed. This eliminates about 8 days from each month as possible launch dates.

VI. GMAT Trajectory Design Script

We perform the trajectory design and analysis using the GMAT software tool[9]. A discussion of an earlier implementation of the trajectory design GMAT script is in Ref. 1. We have since enhanced that script to include further trajectory design improvements. The GMAT trajectory design script now uses a sequence of four solvers to gradually build the trajectory, with each solver increasing the fidelity of the trajectory design. Each solver employs a constrained optimization algorithm to enforce both equality and inequality constraints. Some of the solvers are also confirmed to optimize the cost function in Equation (3).

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Solver 1 begins the trajectory design, from Translunar Injection (TLI) to mission orbit insertion using the PAM. For this solver the TESS Flight Dynamics team chose to approximate the trajectory by several segments, each based on the appropriate requirements for that segment. We then employ multiple shooting to blend the segments into a single continuous trajectory. The segment starting at TLI is approximated to achieve the equatorial inclination at launch and the required line of nodes to achieve flyby at a given epoch. The segment near PAM is approximated based on the mission orbit requirements and the Kozai mechanism. The transfer orbit segment is approximated to achieve the timing condition to achieve the LRP condition at PLEP. The segment around flyby is approximated based on the orbit segments before and after flyby. The multiple shooting uses an optimizer to achieve continuity, enforce some inequality constraints on the orbit, and optimize PAM magnitude.

Solver 2 develops a complete trajectory from SECO-1 to mission orbit. In this solver we employ multiple shooting with three segments: SECO-1 to injection at the end of SECO-2; injection to a patch point between TLI flyby; and a segment from the patch point to mission orbit apogee. Solver 2 enforces mission orbit constraints and minimize the cost function in Equation (3).

The remaining solvers refine the trajectory to achieve higher fidelity force modeling and to enforce further constraints. In Solver 3 we include atmospheric drag in the phasing loop force model. We elected not to model drag until the phasing loops were well-defined in solver 2. Otherwise, intermediate estimates of the phasing loop could go deep into the atmosphere at high speed, generating large drag forces that make convergence problematic. In Solver 4 we enforce the Sun angle constraint. We also perform long-term year propagation, and scale PAM appropriately to enforce the GEO constraint. The process of running the GMAT script for a range of dates is automated using a driver written in Matlab. See our companion paper, Ref. 2, for further details.

VII. Summary of January 2018 Launch Solutions

We have applied the techniques described above to generate three months of launch opportunities, for launches in December 2107 through February 2018. Figure 16 shows a stacked bar chart of the total deterministic DV for launch opportunities that meet the deterministic mission requirements. Figure 17 shows the results for January 2018 only. Each bar shows the total deterministic DV, partitioned into the six maneuvers A1M through PAM.

We can see from Figure 16 that the deterministic DV remains below the desired value of 150 m/s in all cases. For most launch dates, the PAM and P1M maneuvers are largest. In some cases, the A2M maneuver is also fairly large, between 20 and 40 m/s. In the original design, A2M was intended primarily to correct for lunisolar perturbations and keep the perigee altitude at P2 and P3 above 600 km. As a result A2M was expected to be small, and it usually is. However, as the trajectory design has progressed, additional design constraints have been placed on A1M, P2M and P3M as described above. As a result A2M has grown larger in some cases to achieve design objectives that would have been handled by other maneuvers.

Figure 18 shows the eclipse durations for January solutions, where blue dots denote mission orbit eclipses and green dots denote phasing loop eclipses. For these solutions the eclipse durations remain well below the 5-hour limit. We noted above that long phasing loop eclipses tend to occur for a flyby near a full Moon, but whether an eclipse occurs depends on the orbit geometry. In fact there will be a total lunar eclipse on 31 January 2018, and that is the date when the 04 January solution has its flyby. Nevertheless we see no long phasing loop eclipses for that solution, because the flyby does not occur during the eclipse.

Figure 19 shows the minimum mission orbit perigee radius over 30 years for each January solutions. We see that the minimum perigee radius remains above the GEO radius boundary for all solutions, but in a few cases the minimum radius is just above the boundary. In those cases we typically have chosen the PAM maneuver to adjust the initial mission orbit period to meet the GEO constraint.

For the launch opportunities shown in the Figure 16 through Figure 19, we have not yet filtered to remove trajectories that do not meet the DV99 constraint. It turns out after applying the DV99 constraint that we can retain most of the January opportunities, but perhaps not all. For further details see the companion paper[2].
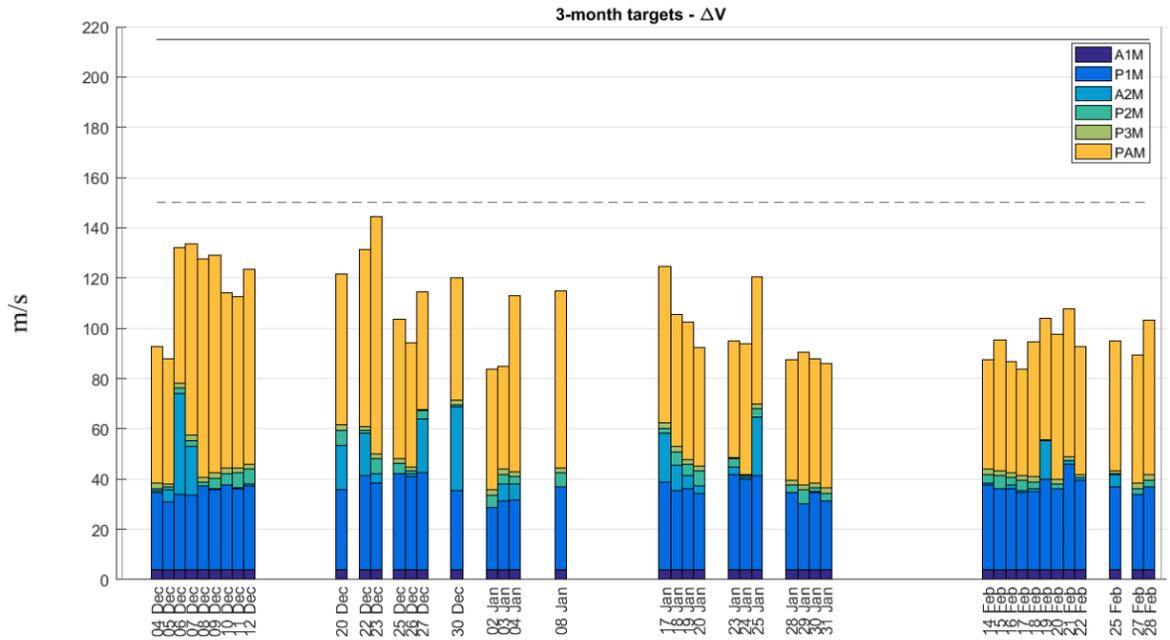
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Figure 16. Stacked bar chart of the deterministic DV for TESS launch opportunities in December 2017 through February 2018. The black horizontal line at 215 m/s indicates the hard limit of total DV. The dashed line at 150 m/s indicates the desired bound on the total deterministic DV.
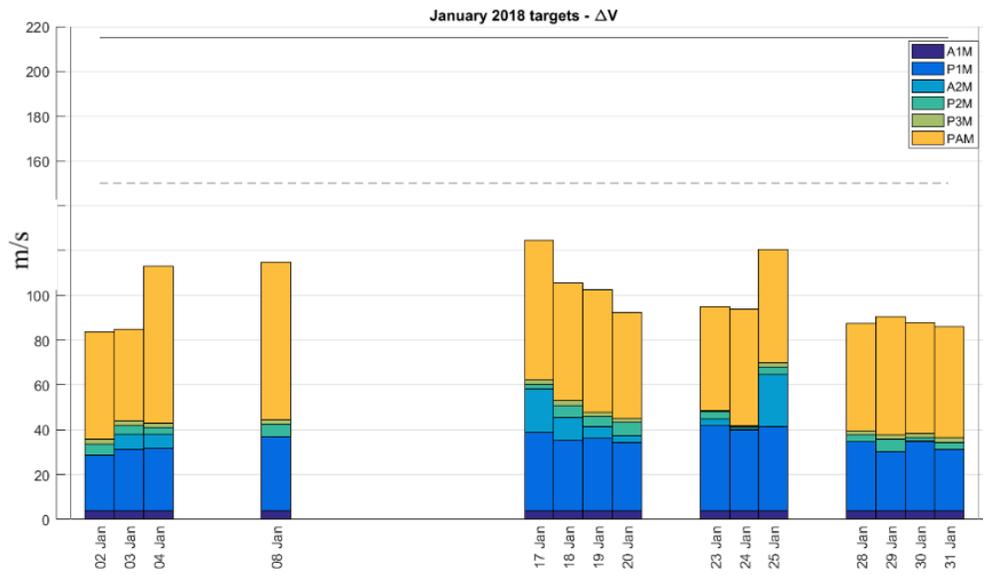

Figure 17. Stacked bar chart of the deterministic DV for TESS launch opportunities in January 2018, based on Figure 16.
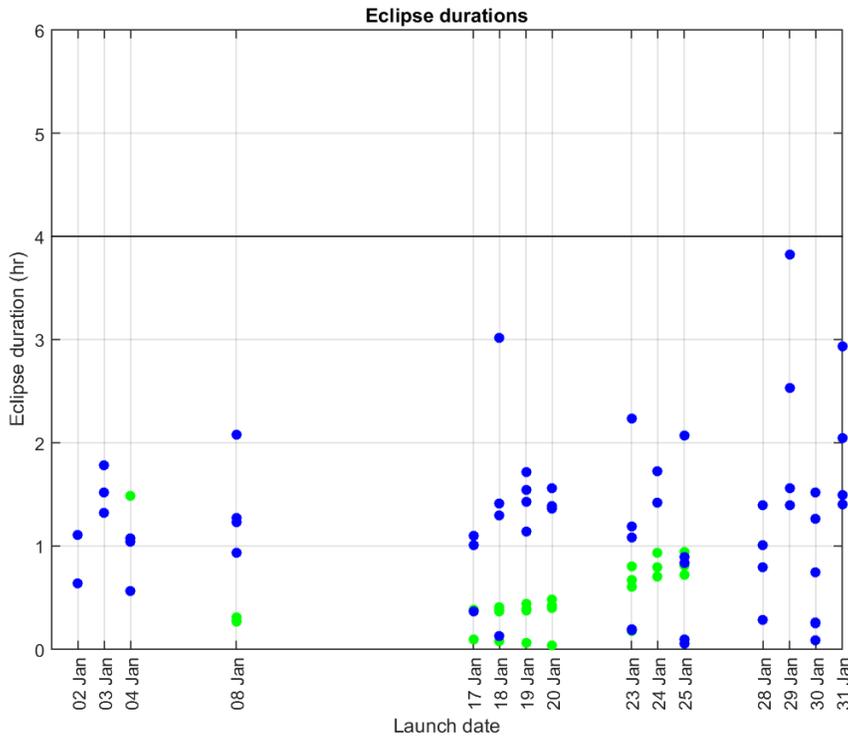
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Figure 18. Plot of eclipse durations for solutions with launch in January 2018. Green represents phasing loop eclipses, blue represents mission orbit eclipses. The dates with very long phasing loop eclipses are close to full Moon.
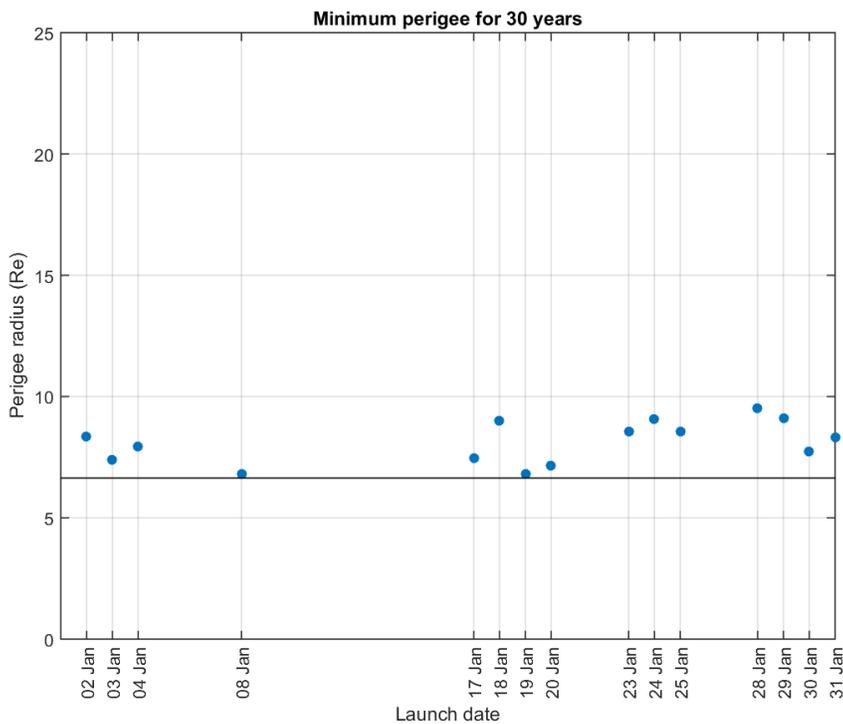

Figure 19. Plot of minimum mission orbit perigee radius over 30 years for January 2018 solutions. The black line at 6.7 Re indicates the boundary for the GEO radius constraint.

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VIII. Conclusion

For the TESS mission, trajectory design is driven by risk mitigation. In particular we mitigate the risk of a critical maneuver, the risk of close approach in the GEO belt, and the risk of damage to the instrument from Sun exposure. We are also constrained by limited DV. To meet these challenges we employ insights from orbit mechanics and we take advantage of features of Dynamical Systems modeling including the Tisserand criterion, the Kozai mechanism and our study of the long-term orbit uncertainty for lunar resonant orbits. We have also integrated the trajectory design process with a Monte Carlo simulation to facilitate a thorough assessment of prospective trajectories. Using these methods, we have demonstrated for the first full month of launch opportunities that we can meet the requirement of at least five launch dates in the month, all with DV99 less than 215 m/s. We are in the process of applying the same techniques to assess a full year of potential launch opportunities. The techniques we have developed with be valuable for future lunar resonant orbit missions and for missions that require phasing loops before a lunar flyby.

Acknowledgments

References

[1]Dichmann, D.J., Parker, J., Williams, T., Mendelsohn, C. "Trajectory Design for the Transiting Exoplanet Survey Satellite" in International Symposium on Spacecraft Flight Dynamics, Laurel MD, May 2014.

[2]Nickel, C., Dichmann, D.J., Lebois, R., Lutz, S. "Monte Carlo Analysis as a Trajectory Design Driver for the TESS Mission" in AIAA/AAS Astrodynamics Specialist Conference, Long Beach CA, Sept. 2016.

[3]National Aeronautical and Space Administration. "Process to Limit Orbital Debris", NASA-STD-8719.14A, 25 May 2012.

[4]Ricker, G., "Transiting Exoplanet Survey Satellite (TESS) Concept Study Report (CSR)," MIT, 2012.

[5]Carrico, J., Dichmann, D., Policastri, L., "Lunar-Resonant Trajectory Design for the Interstellar Boundary Explorer (IBEX) Extended Mission," in AAS/AIAA Astrodynamics Specialist Conference, Girdwood AK, 2011.

[6]Gangestad, J., Hennings, G., Ricker, G. and Persinger, R., "Design of a Resonant Orbit," in AAS/AIAA Astrodynamics Specialist Conference, Savannah GA, 2013.

[7]Dichmann, D.J., Lebois, R. and Carrico J., "Dynamics of Orbits near 2:1 Resonance in the Earth-Moon System," *Journal of Astronautical Sciences*, vol 60, no. 1, 2015

[8]Short, C., Howell, K., Haapala, A., and Dichmann, D., "Mode Analysis for Long-term Behavior in a Resonant Earth-Moon Trajectory" in AAS/AIAA Astrodynamics Specialist Conference, Vail, Colorado, August 9-13, 2015.

[9]General Mission Analysis Tool, NASA Goddard Space Flight Center, version R2015a, http://gmatcentral.org.

[10]Danby, J.M.A. Fundamentals of Celestial Mechanics, Second Edition, Willmann-Bell, 2003.

[11]Kozai, Y., "Secular perturbations of asteroids with high inclination and high eccentricity," Astron. J., vol. 67, p. 591, 1962.

[12]Lidov, M., "The Evolution of Orbits of Artificial Satellites of Planets under the Action of Gravitational Perturbations of External Bodies," Planetary and Space Science, vol. 9, pp. 719-759, 1962.

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