Unexpected Nonlinear Effects in Superconducting Transition-Edge Sensors

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Code 553

NASA Goddard Space Flight Center
6 keV

est. $\Delta E = 1.6$ eV

World Record Energy Resolution TES Fabricated in the DDL at NASA GSFC
6 keV

est. $\Delta E = 1.6$ eV
6 keV

est. $\Delta E = 1.6$ eV

TES Fabricated in the DDL at NASA GSFC (But not specifically designed for this application)
6 keV

est. $\Delta E = 1.6$ eV
est. $\Delta E = 1.6 \text{ eV}$

$6 \text{ keV}$
6 keV

est. $\Delta E = 1.6 \text{ eV}$
$6 \text{ keV}$
6 keV

**Graph:**
- **Y-axis:** est. $\Delta E$ [eV]
- **X-axis:** Bias Current [uA]
- **Dotted line:** est. $\Delta E = 1.6$ eV
- **Red line and dots:** Data points indicating a linear trend with increasing bias current.
6 keV

est. $\Delta E = 1.6$ eV
6 keV

est. $\Delta E = 1.6$ eV
6 keV

Est. $\Delta E = 1.6 \, \text{eV}$

MTES

$\beta_b < 0$
6 keV

CAUTION:
- If a Linear detector...
- (Estimated)
  
est. ΔE = true ΔE

![Graph showing bias current vs. estimated ΔE with two linear regions labeled MTES and β_B < 0.]
What is “nonlinearity”? the “nonlinear-model”... field “nonlinear dynamics”
Nonlinear $\rightarrow$ Everything Else  (*a very $\infty$ set ;-* )

• **Nonlinearity ubiquitous** in nature:
  – e.g. 60 Hz harmonics pickup at 120 Hz, 180 Hz,... requires nonlinearity in the system.

• **Linearity ubiquitous** in our mathematical description of nature:
  – often a good approximation to real physical systems.
  – It is mathematically easier
  – Linear Tools:
    • Superposition Principle
    • Transform methods, transfer functions etc.
What is “nonlinearity”? the “nonlinear-model”... field “nonlinear dynamics”

Nonlinear $\rightarrow$ Everything Else (a very $\infty$ set ;-

**Known or Expected Nonlinearities**

• If our thermistor TES sensor...
  – If $R(T) \rightarrow R(T,J)$.
  – Then **Nonlinear** system of Diff Eqs.
  – (Can approximate by a linearized system of Diff Eqs.)

• “**Nonlinear**” if
  – $R(T) \rightarrow$ anything other than a single straight line
Superconductivity: Resistive R Transition Surface in Temperature T and Current I

Normal State

Superconducting State
R(T)|_I
contour
Stationary Noise: 
Noise at the bias point

\[(R_0, T_0, I_0)\]
\[ R(T, I) \approx R_0 + \frac{\partial R}{\partial T} \delta T + \frac{\partial R}{\partial I} \delta I \]
AND
NonStationary Noise:
*In the pulse trajectory*
AND NonStationary Noise: 

In the pulse trajectory

X-ray pulse trajectory $T(t), I(t)$

$(R_0, T_0, I_0)$
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Known or Expected Nonlinearities

“Unexpected Nonlinearities”
In steady state with standard DC biased TES operation. The TES current is wildly oscillating in time at high frequencies and we only measure (and are only aware of) the time averaged current in the TES.
Conclusions: ...*Paradigm Shifting*

- In steady state with standard DC biased TES operation. The TES current is wildly oscillating in time at high frequencies and we only measure *(and are only aware of)* the time averaged current in the TES.

- The equations governing this time response is **nonlinear time-dependent** diff eqs.
  - **KEY POINT**: This time dependent current exists when the TES is only DC biased, NO time dependent inputs what so ever. It comes about from the intrinsic physics governing the superconducting state of the TES.
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• The measured resistive transition surface is not simply a function of $R(T,J,B)$. But is also a function of the electric circuit. $R(T,J,B,L,R_{sh})$.
  – → Problem compartmentalizing: → *Same TES measured in slightly different circuits will appear to have a different resistive transition surface.*  TEST in real setup early!!!

• The biased TES waiting to detect a photon ($\mu$-calorimeter) or flux of photons ($\mu$-bolometers) can itself act as a radiation source.
  – → T**ricky when an array of exquisitely sensitive micro-wave radiation detectors are themselves sources of microwave radiation.**
  – → This can lead to radiation resistance and fine structure in the resistive transition.
  – → Possible cross talk between pixels in an array.
  – → The time dependence of the current can take on pure sin waves and also very nonsinusoidal forms (variable harmonic content)
  – → The fundamental frequency of these oscillations changes with bias voltage.
  – → Makes the prospects of FDM TES arrays with an AC-biased TES challenging. Structure, sensitivity, and cross-talk.
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- Circuit and TES parameters can have values such that these time dependent solutions become multivalued in time.
  - **What does that mean?** Can mean excess noise or fine structure of the time averaged resistive transition surface. Possible cross talk between pixels in an array. The time dependence of the current can take on pure sin waves and also very nonsinusoidal forms (variable harmonic content). The fundamental frequency of these oscillations changes with bias voltage. Makes the prospects of FDM TES arrays with an AC biased TES challenging. Structure, sensitivity, and cross talk.
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DONE
Thermal Circuit

ELECTROTHERMAL MODEL
ELECTROThERMAL MODEL
ELECTROTHEMAL MODEL
ELECTROTHERMAL MODEL
ELECTROTHERMAL MODEL

Thermal Circuit

Independent Variables: $T_b, V_b$

Electrical Circuit

Measure: I
TES on a: ...
(1) solid substrate
(2) thin isolated membrane
(3) thin perforated membrane
(4) island suspended with long thin legs

**ELECTROTHERMAL MODEL**
TES on a: ...
(1) solid substrate
(2) thin isolated membrane
(3) thin perforated membrane
(4) island suspended with long thin legs

Thermal power balance
\[ I^2 R = \frac{G}{nT^{n-1}} (T^n - T_b^n) \]

Electrical steady state
\[ I = \frac{V_b}{R_{in}} \frac{R_{sh}}{R + R_{sh}} \]

IN STEADY STATE

Measure: I
TES on a: ...

1. solid substrate
2. thin isolated membrane
3. thin perforated membrane
4. island suspended with long thin legs

Thermal power balance:
\[ I^2 R = \frac{G}{nT(n-1)} (T^n - T_b^n) \]

Electrical steady state:
\[ I = \frac{V_b}{R_{in}} - \frac{R_{sh}}{R + R_{sh}} \]

Independent Variables: \( T_b, V_b \)
TES on a: ...
(1) solid substrate
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Thermal power balance

\[ I^2 R = \frac{G}{nT^{(n-1)}} (T^n - T_b^n) \]

\[ R = \frac{G}{I^2 nT^{(n-1)}} (T^n - T_b^n) \]

Electrical steady state

\[ I = \frac{V_b}{R_{in}} \frac{R_{sh}}{R + R_{sh}} \]

\[ R = \frac{V_b}{R_{in}} \frac{R_{sh}}{I} - R_{sh} \]
Different G’s

Thermal Surfaces
Intersection point = “bias point”
Different Vb’s
Bias Circuit

Separable?

TES

$R(T,j,B)$

$R_{sh}, L$
Does changing $R_{sh}$ and $L$ have any direct impact on the TESs $R(T,j,B)$?

Of course changing $R_{sh}$ and $L$ impacts:
- bias stability conditions
- time constants
- the jin needed to get the the same $R$ etc.

Can we separate the TES transition from the bias circuit?
Does changing $R_{sh}$ and $L$ have no direct impact on the TESs $R(T,j,B)$?

Of course changing $R_{sh}$ and $L$ impacts:
- bias stability conditions
- time constants
- the $j_{in}$ needed to get the the same $R$ etc.

Can we separate the TES transition from the bias circuit?

**ANSWER:** generally NO.
ELECTROTHERMAL MODEL

Thermal Circuit

Independent Variables: \( T_b, V_b \)

Electrical Circuit

Measure: \( I \)

Simply drop in TESs \( R(T,I) \)
Can we separate the TES transition from the bias circuit?

ANSWER: generally NO.
ELECTROTHERMAL MODEL

Can we separate the TES transition from the bias circuit?

ANSWER: generally NO.
ELECTROTHERMAL MODEL

Can we separate the TES transition from the bias circuit?

ANSWER: generally NO.
(1) $j(t) =$?
What is the time dependence of the current $j$?

(2) $\langle j(t) \rangle$ vs $j_{in} =$?
What is the shape of the time averaged IV curve?
J-TES model

Time rescaled so we can compare shape in time (harmonic content).

$L = 0$ limit

Increasing bias $jjin = jin/jc$
J-TES model

Green: TES current versus time
Red Dashed: time averaged TES current

Movie evolves with increasing bias current \( jj_{in} \).
Movie rescales time as the bias is increased so two periods are contained in time

Bias \( jj_{in} \):

Small:
slow spikes \((+jc \text{ to } -jc)\)

Large:
fast sinusoidal \((+jc \text{ to } -jc)\)
J-TES model
Finite Inductance Effect

*Closed form solution found!

<table>
<thead>
<tr>
<th>jjin = ( \frac{j_{in}}{j_c} )</th>
<th>3.33333</th>
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<tbody>
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<td>{L_{min},L_{max},L_{step}}</td>
<td>{0, 0.7, 0.1}</td>
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J-TES model
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Large $jjin$

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Increasing $L$

Small $jjin$

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Increasing $L$
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Finite Inductance Effect

Large $jj_{in}$

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Multivalued $j[t]$
Current becomes multivalued with sufficiently large $L$
J-TES model
Large L behavior

Large L $\rightarrow \sim$ Sawtooth $j[t]$

Allowed current states become very close together in the large L limit. Current jump between levels $\rightarrow$ 2 level system noise
Test my claim

Quantum wave function of the superconducting condensate

\[ \Psi = |\Psi| e^{i \phi(r,t)} \]

First Josephson Equation: \[ J = J_c \sin \phi(t) \]

Second Josephson Equation: \[ \phi'(t) = \frac{2\pi V}{\Phi_0} \]

Shapiro voltage steps:
\[ V_n = n \Phi_0 f \]

(voltage to frequency transducer)

Fundamental:
1. Gauge Invariance
2. Energy Conservation
\[ V_b = V_{b\,DC} \]
V_b = V_{b\,DC} + v \sin(2\pi f t)

Phase Locking

Voltage Steps Observed from
f=200 kHz to f=30 MHz
\[ V_n = n \Phi_0 f \]

- \( f = 200kHZ \)
- \( T_b = 102mK \)

- Green circle: measured IV curve
- Red cross: Shapiro step position
- Light purple dashed line: fit to the superconducting state

\[ V_{fb} \text{ [V]} \]
\[ V_{b} \text{ [mV]} \]
Measuring a 200 μOhm resistor to better than 1/1000!

$R_{sh}$ vs current is flat down to 5μA $\rightarrow$ Ohmic!
Time Averaged IV curves
VRSJ-TES model

Time averaged IV curve.

Only changing the circuit Inductor value
Everything else constant.
Time Averaged IV curves
VRSJ-TES model

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Time averaged IV curve.

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Everything else constant.

Measure: I
jj oscillation amplitude

jj oscillation range at $v\nu=0.01$ for each $\beta_L$ value.

All similar in size
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THE
END
\[ \beta_L \]

\[ \text{vv} = \{0.01, 0.1, 1, 5\} \]

\[ \text{sawtooth} \]

\[ \text{Sin} \]

\[ jj(t) \]
Power for relationship holds for: \( \lambda \gg \text{antenna length} \)

**Blue:** \( \Delta \ell / \lambda < 0.1 \) (satisfied)

**Red:** \( \Delta \ell / \lambda > 0.1 \)
Actual radiated power is larger than red surface

<table>
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<tr>
<th>( R_{\text{wsh}} )</th>
<th>( \log [ L ] )</th>
<th>( \log [ j_c ] { \text{min}, \text{max}, \text{step} } )</th>
</tr>
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<td>( \frac{1}{100} )</td>
<td>(-8)</td>
<td>{(-9, -3, 3)}</td>
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approximating the waveform as a pure sinusoid for the purposes of this