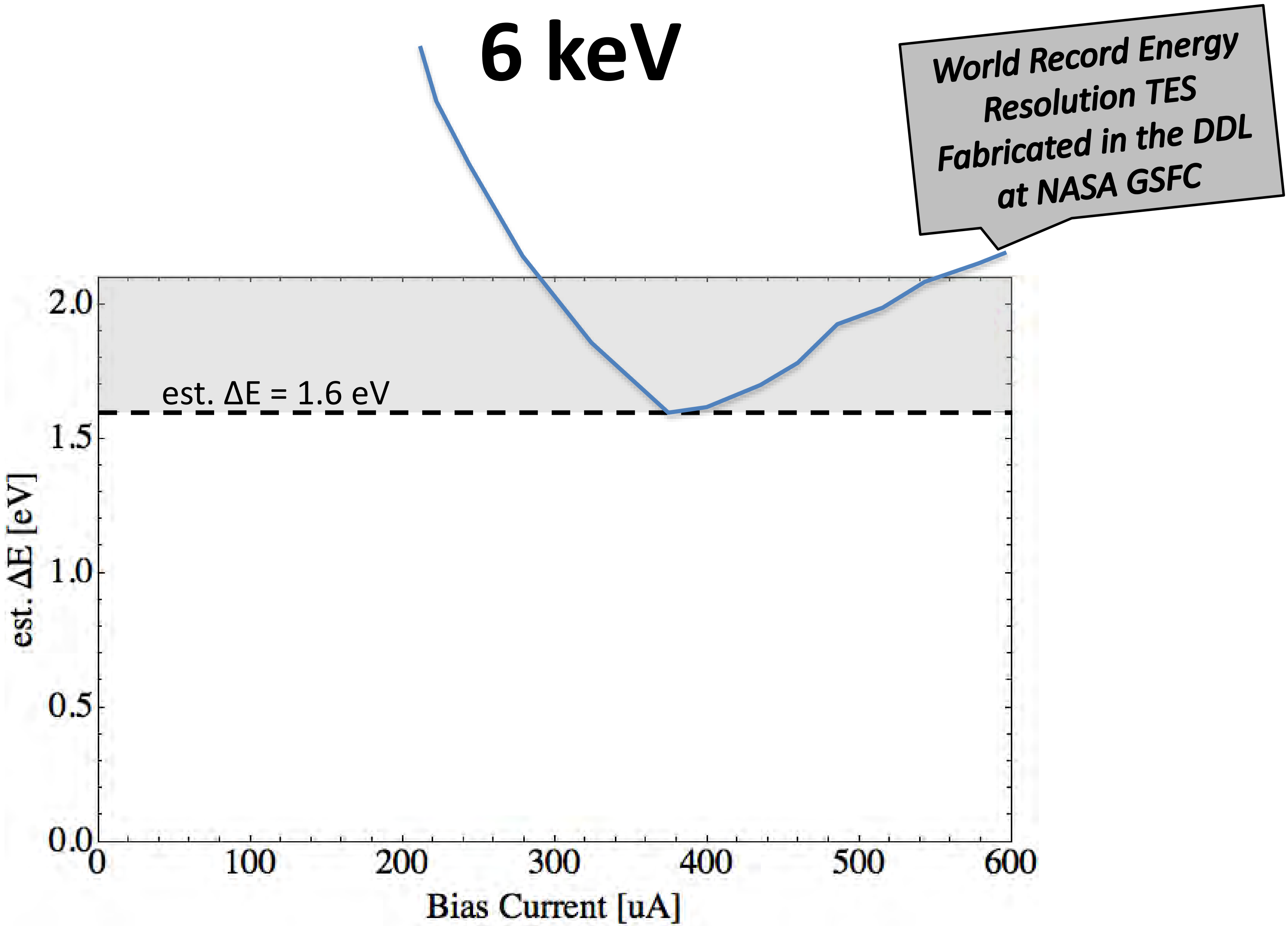
Unexpected Nonlinear Effects in Superconducting Transition-Edge Sensors

Jack Sadleir

Code 553

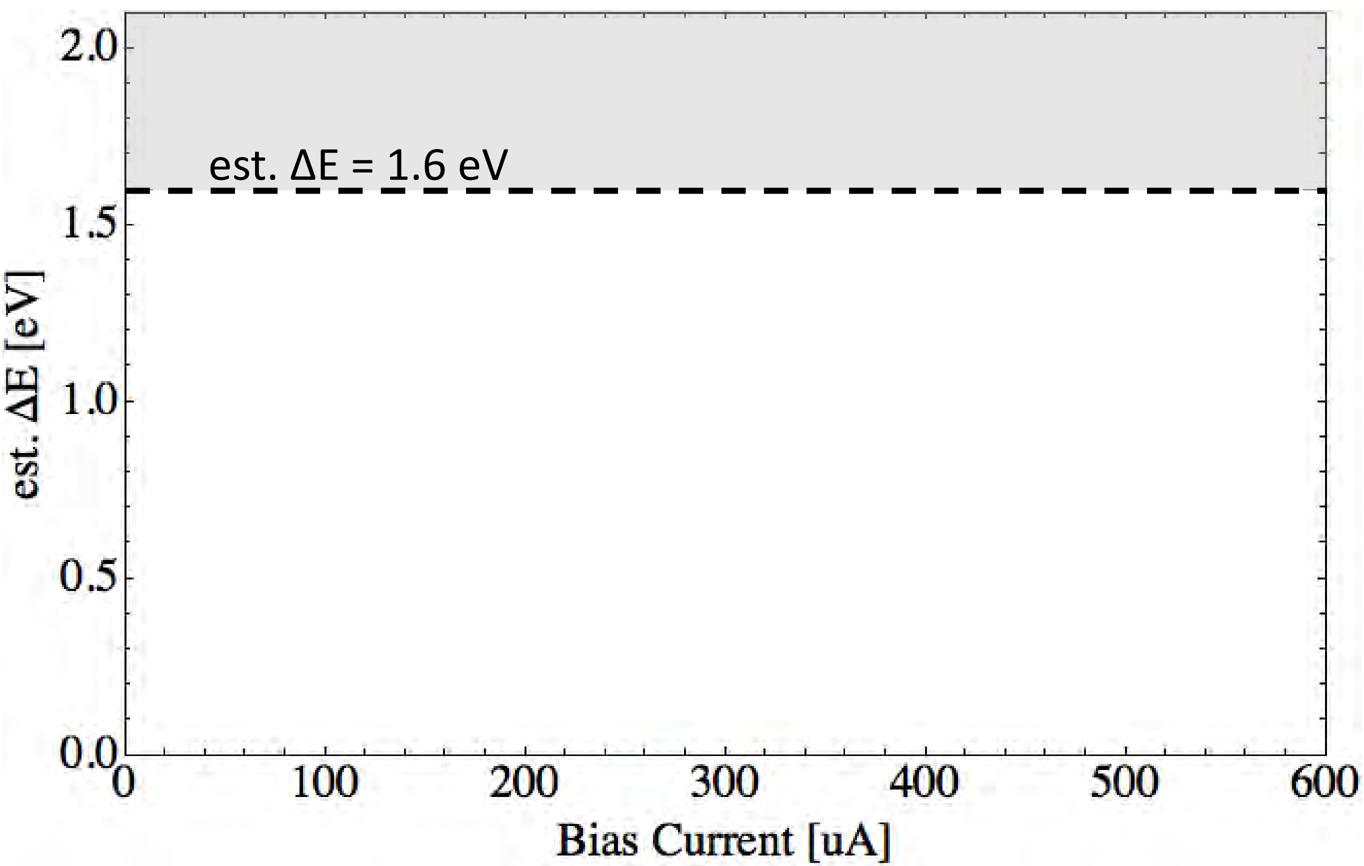
NASA Goddard Space Flight Center

6 keV



World Record Energy Resolution TES Fabricated in the DDL at NASA GSFC
est. ΔE = 1.6 eV
2.0
1.5
1.0
0.5
0.0
est. ΔE [eV]
0
100
200
300
400
500
600
Bias Current [uA]

6 keV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1.0
0.5
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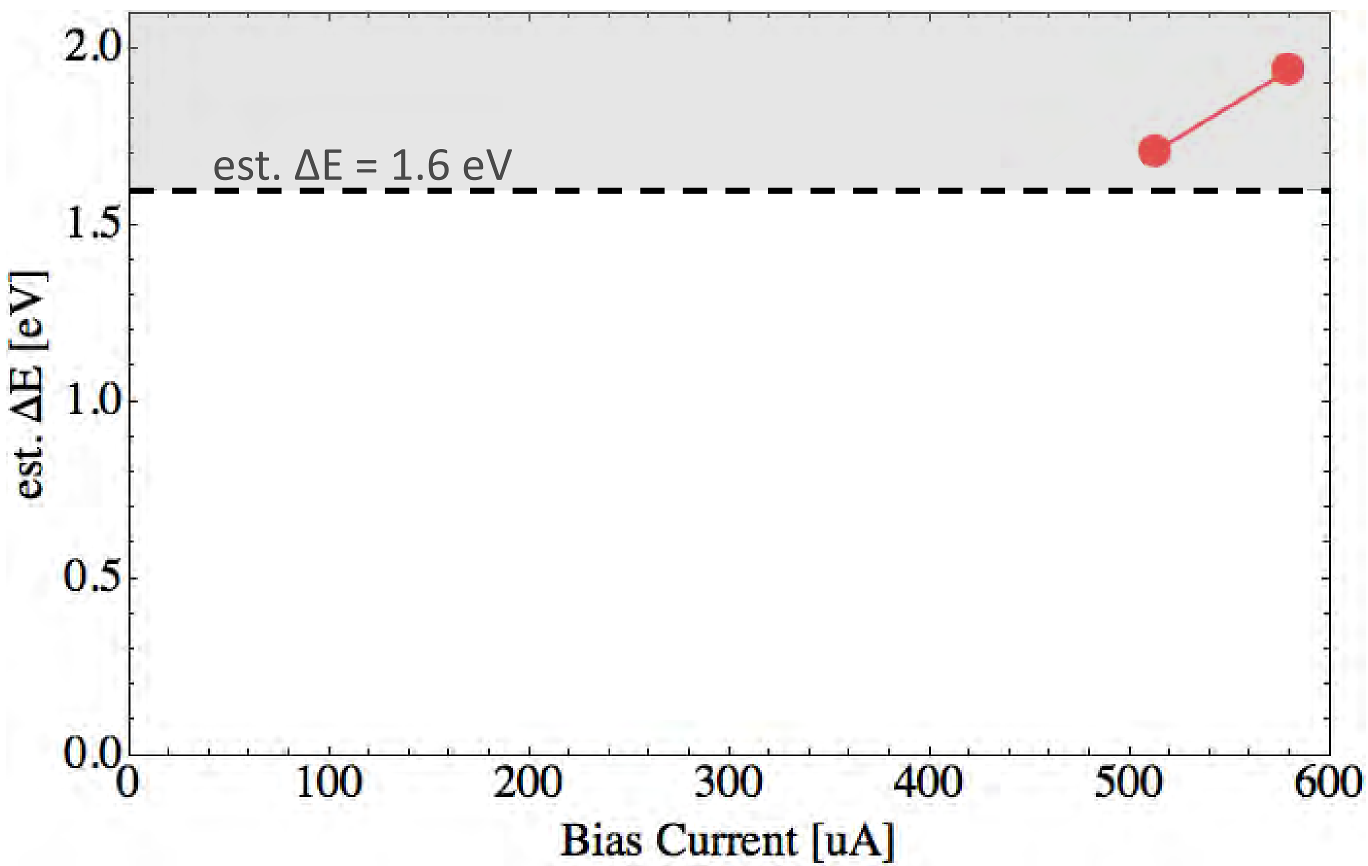
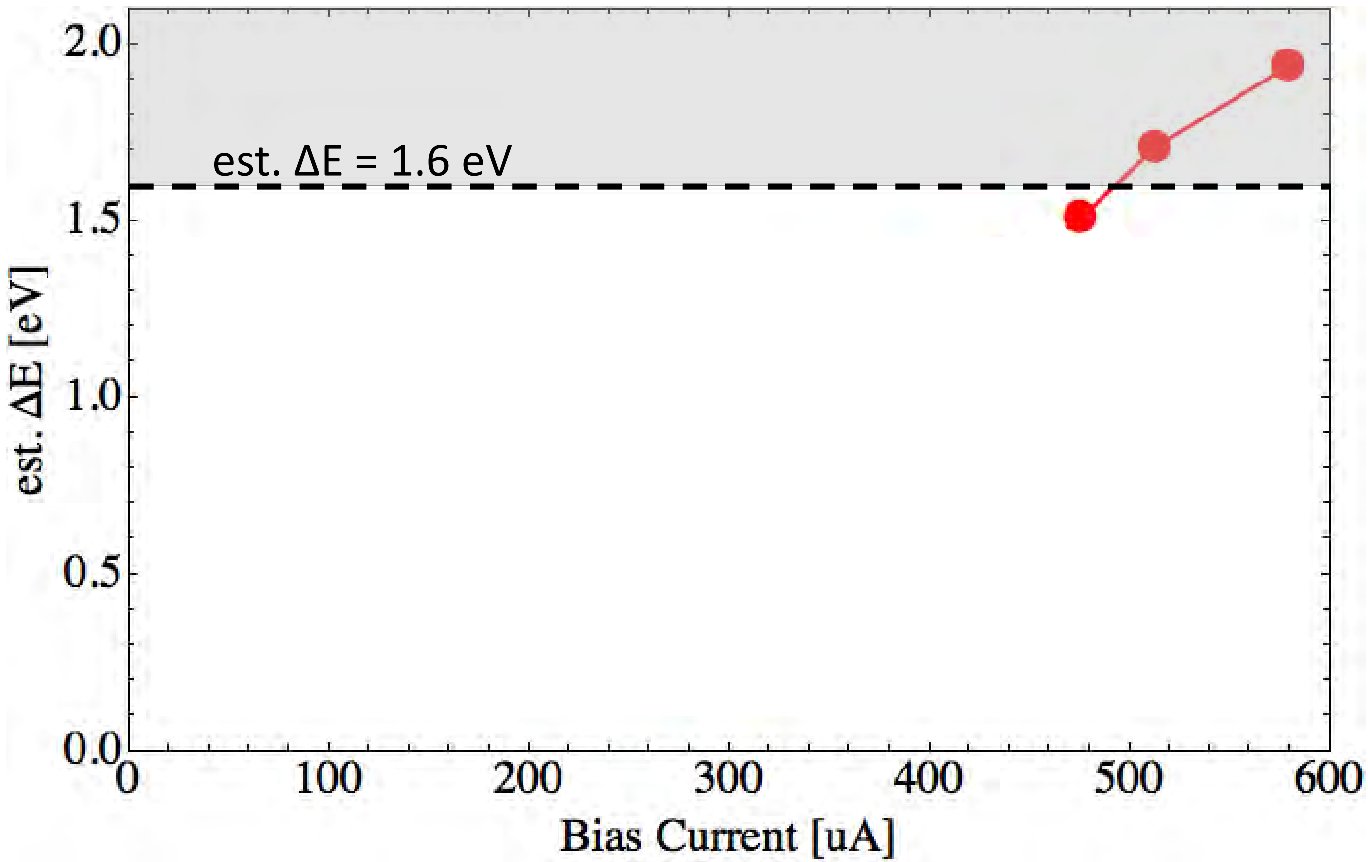
0
100
200
300
400
500
600
Bias Current [uA]

6 keV

TES Fabricated in the DDL at NASA GSFC (But not specifically designed for this application)



6 keV



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1.5
1.0
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100
200
300
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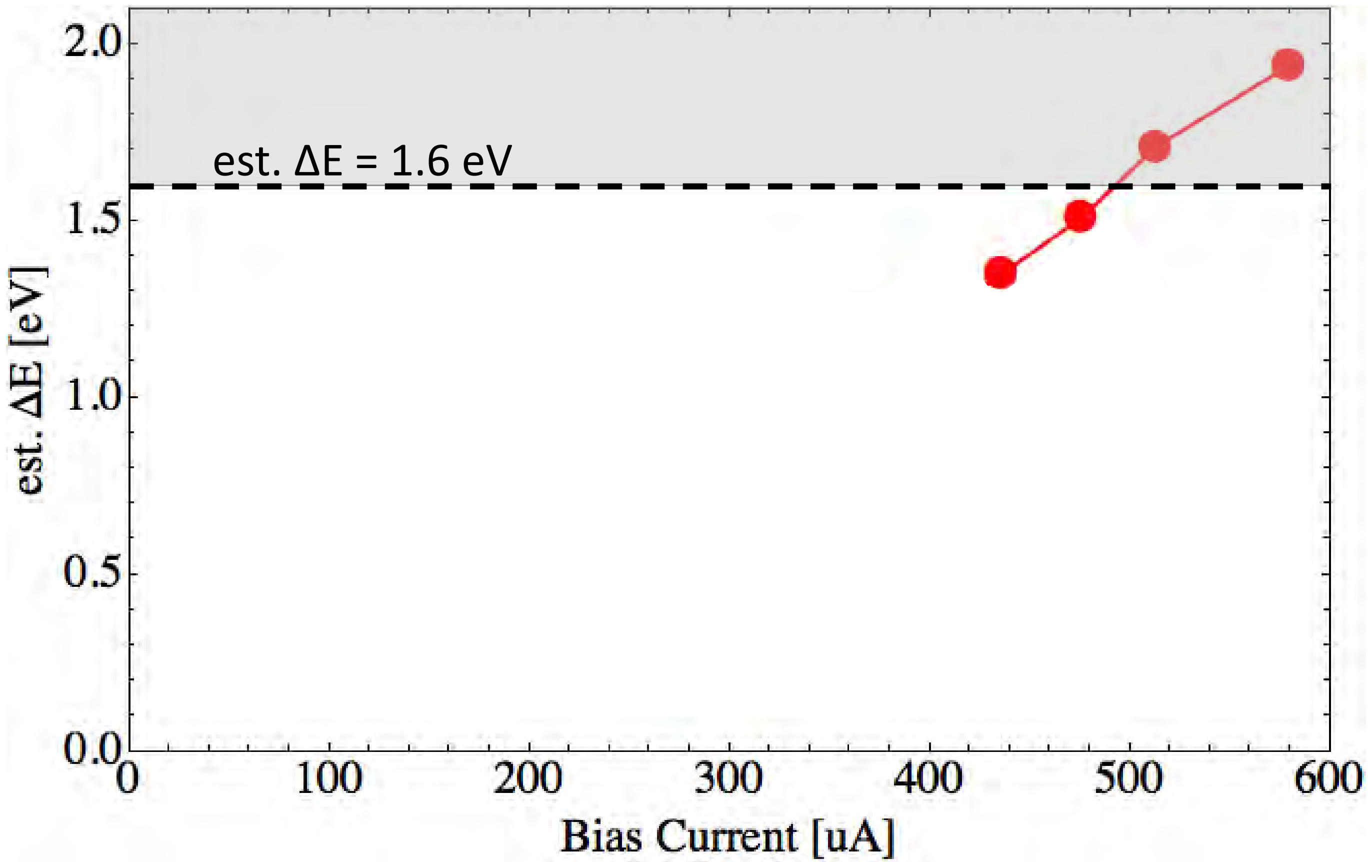
Bias Current [uA]

6 keV



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2.0
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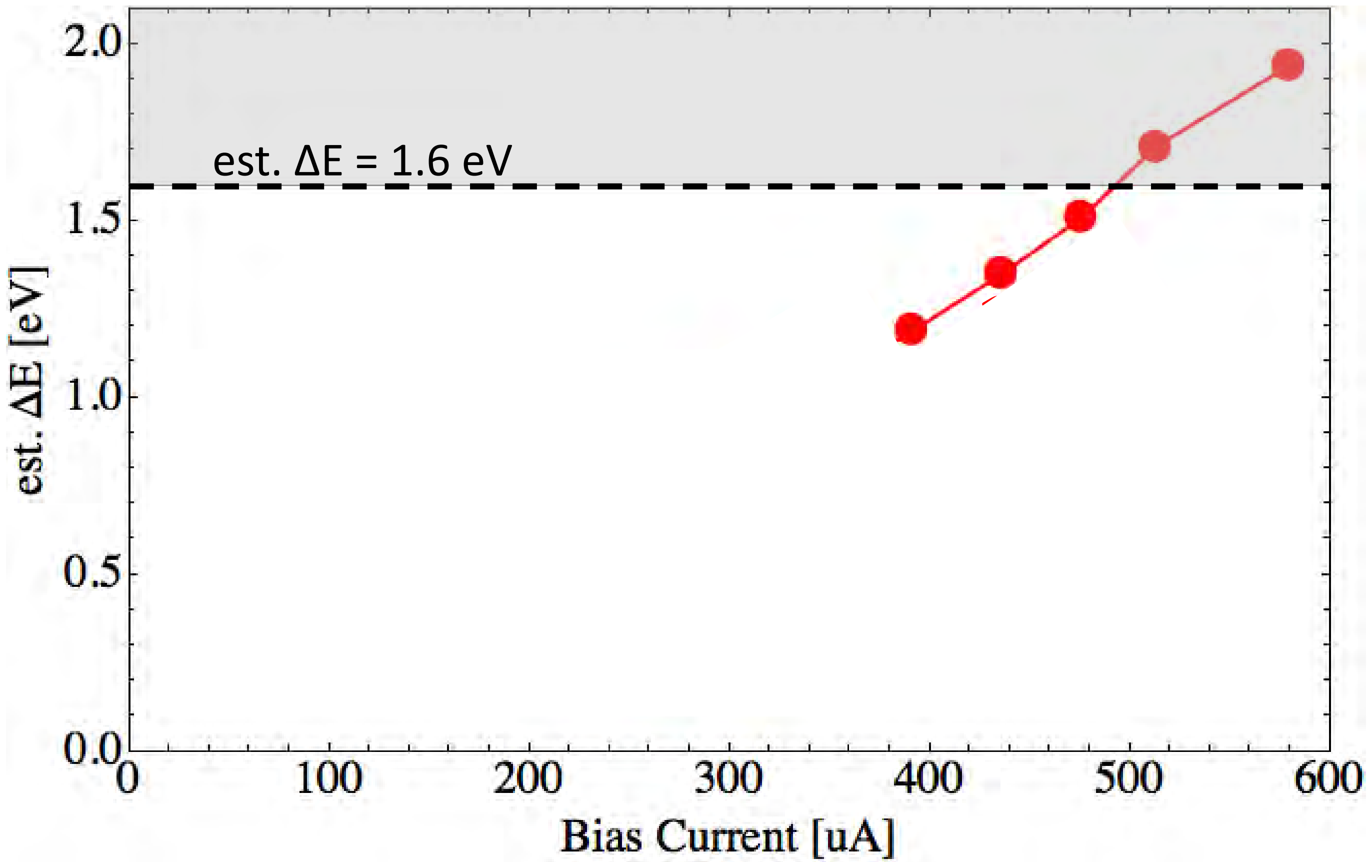

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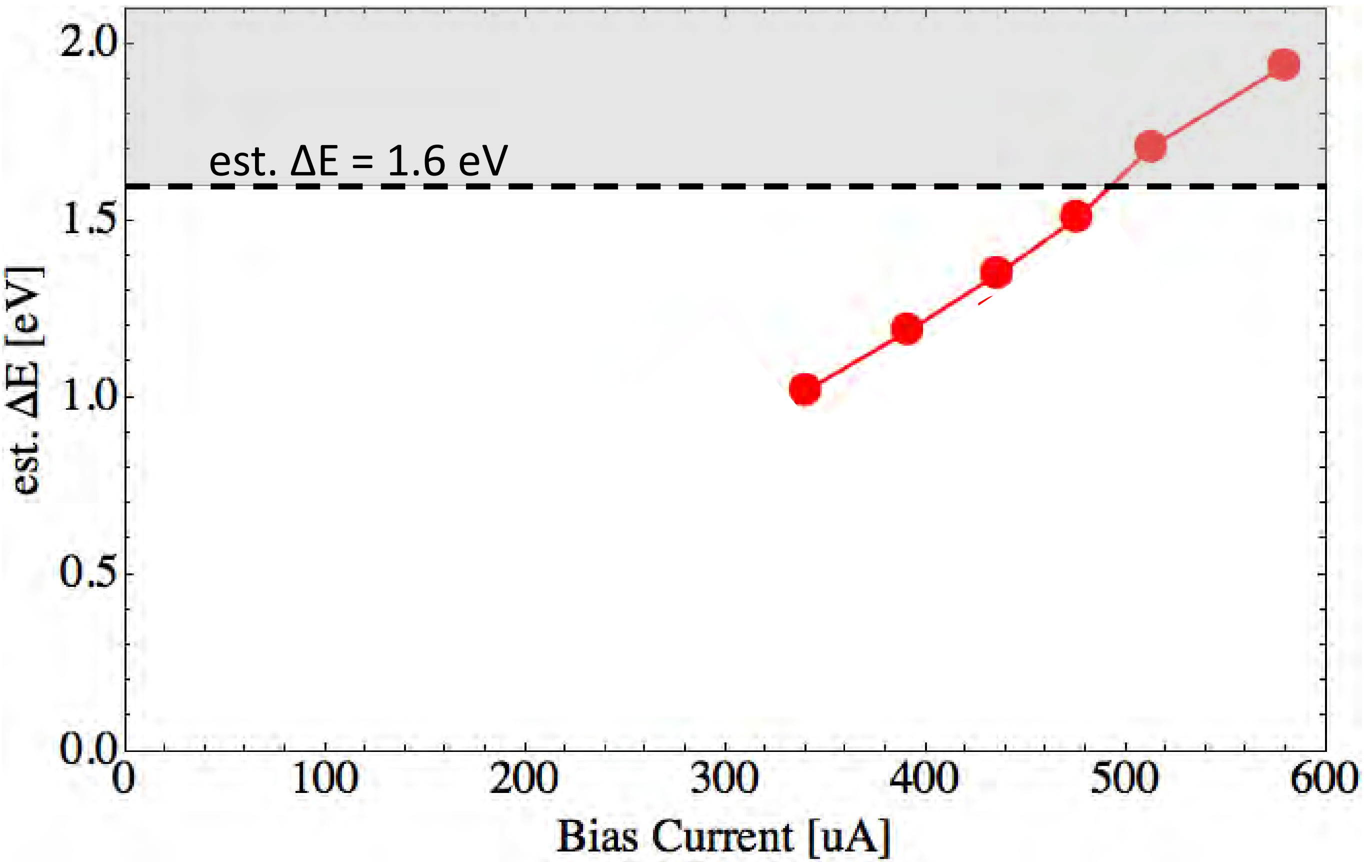
2.0
1.5
1.0
0.5
0.0
0
100
200
300
400
500
600
Bias Current [uA]

6 keV



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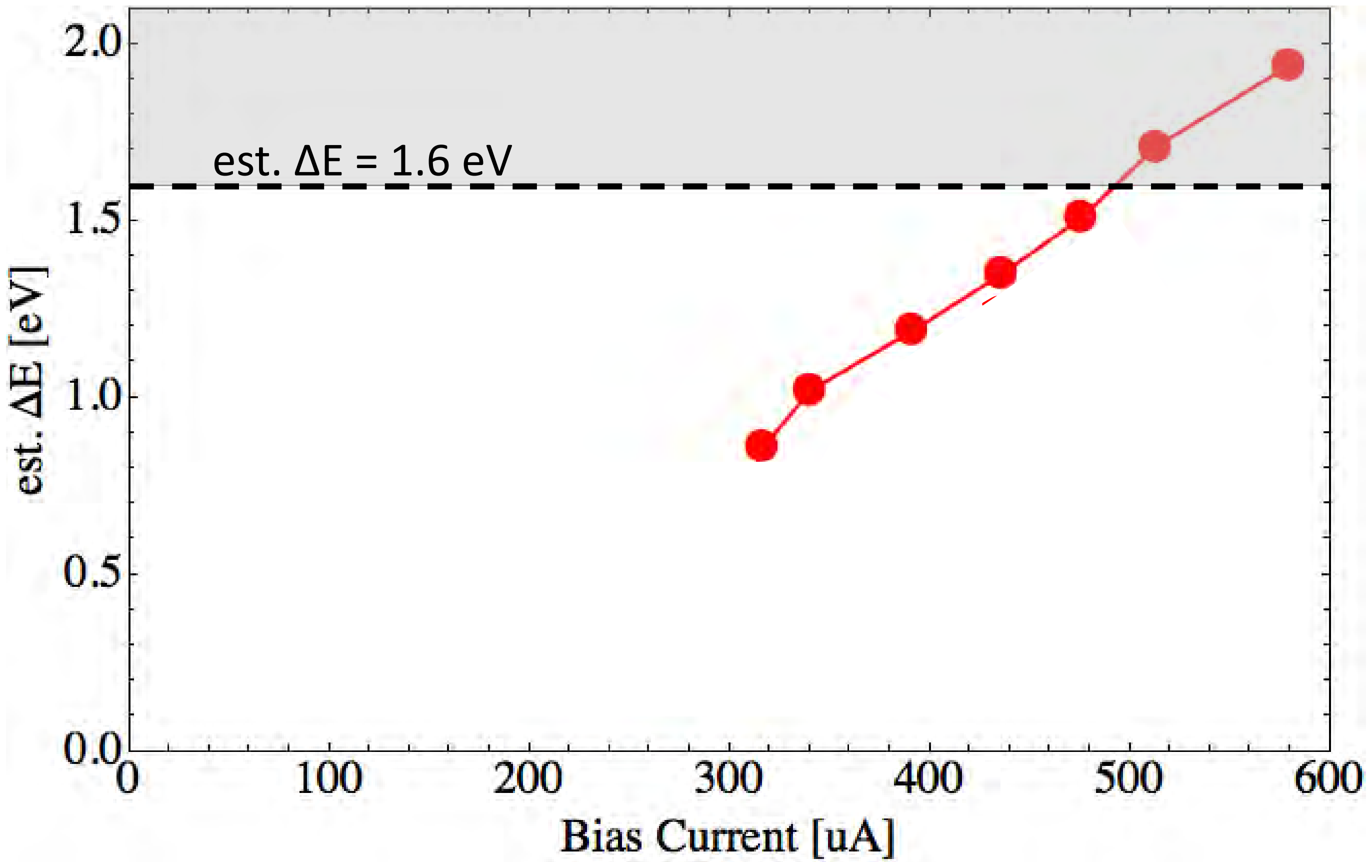
est. ΔE [eV]
2.0
1.5
1.0
0.5
0.0
0
100
200
300
400
500
600
Bias Current [uA]

6 keV



est. ΔE = 1.6 eV
2.0
1.5
1.0
0.5
0.0
est. ΔE [eV]
0
100
200
300
400
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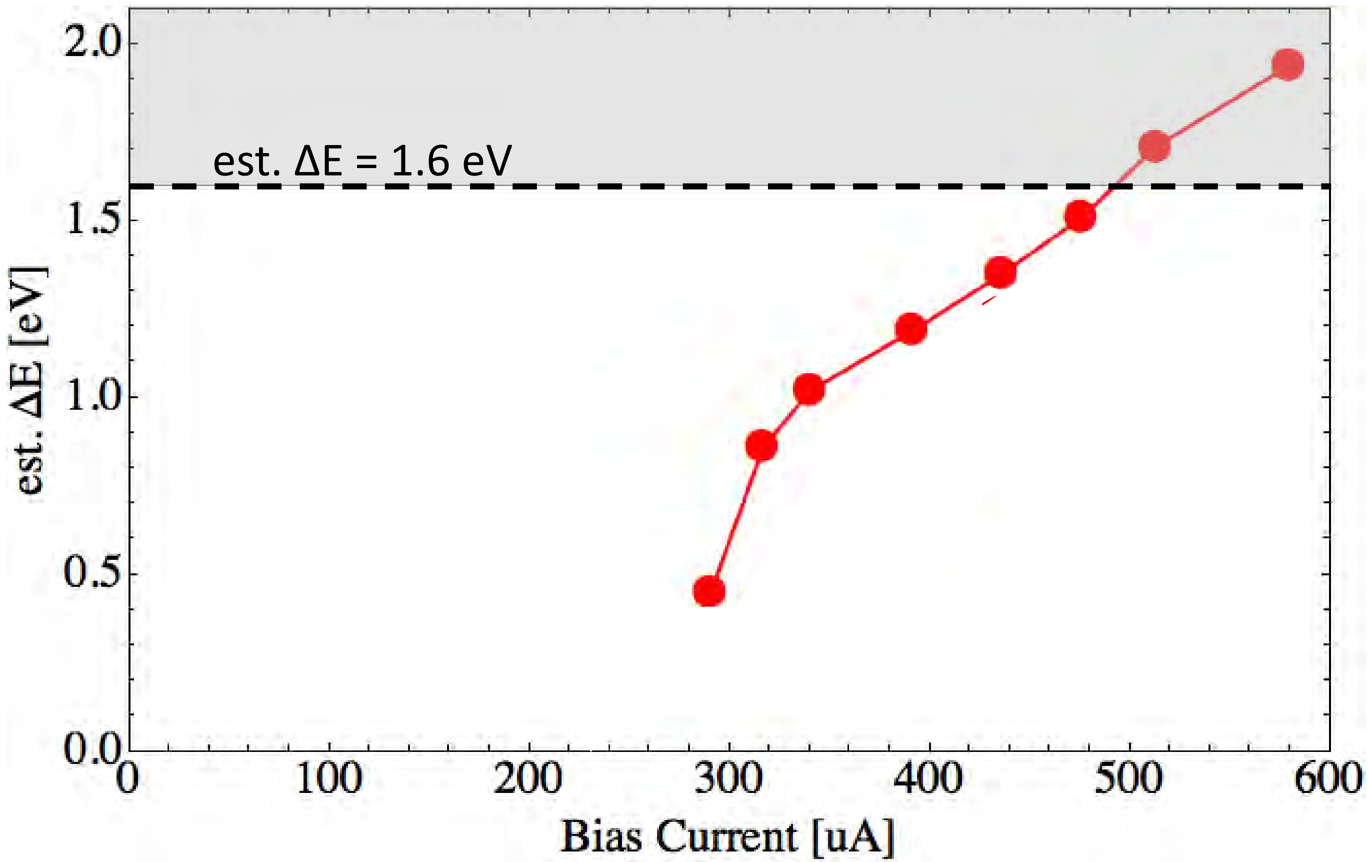
Bias Current [uA]

6 keV



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2.0
1.5
1.0
0.5
0.0
est. ΔE [eV]
0
100
200
300
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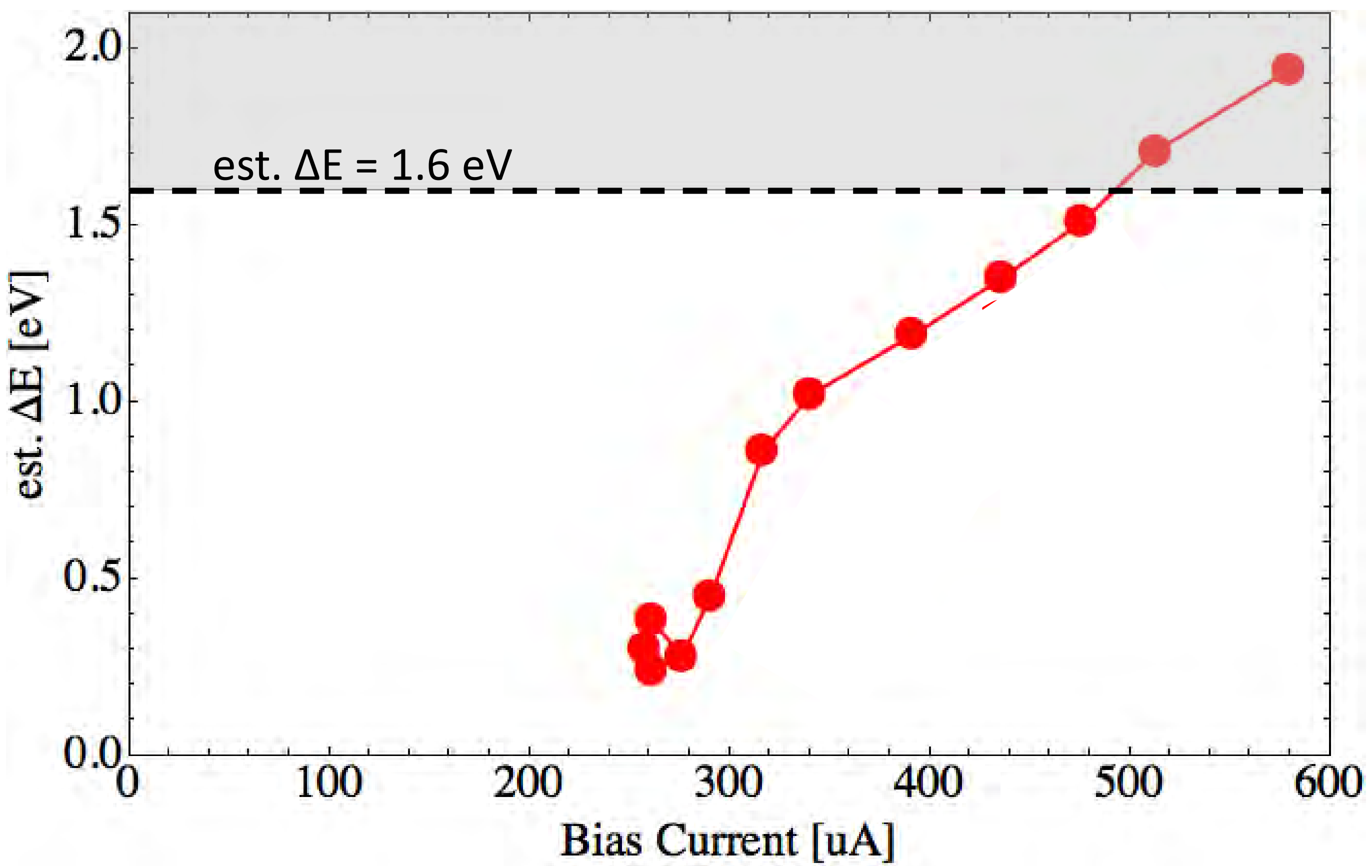
Bias Current [uA]

6 keV



est. ΔE = 1.6 eV
est. ΔE [eV]
2.0
1.5
1.0
0.5
0.0
0
100
200
300
400
500
600
Bias Current [uA]

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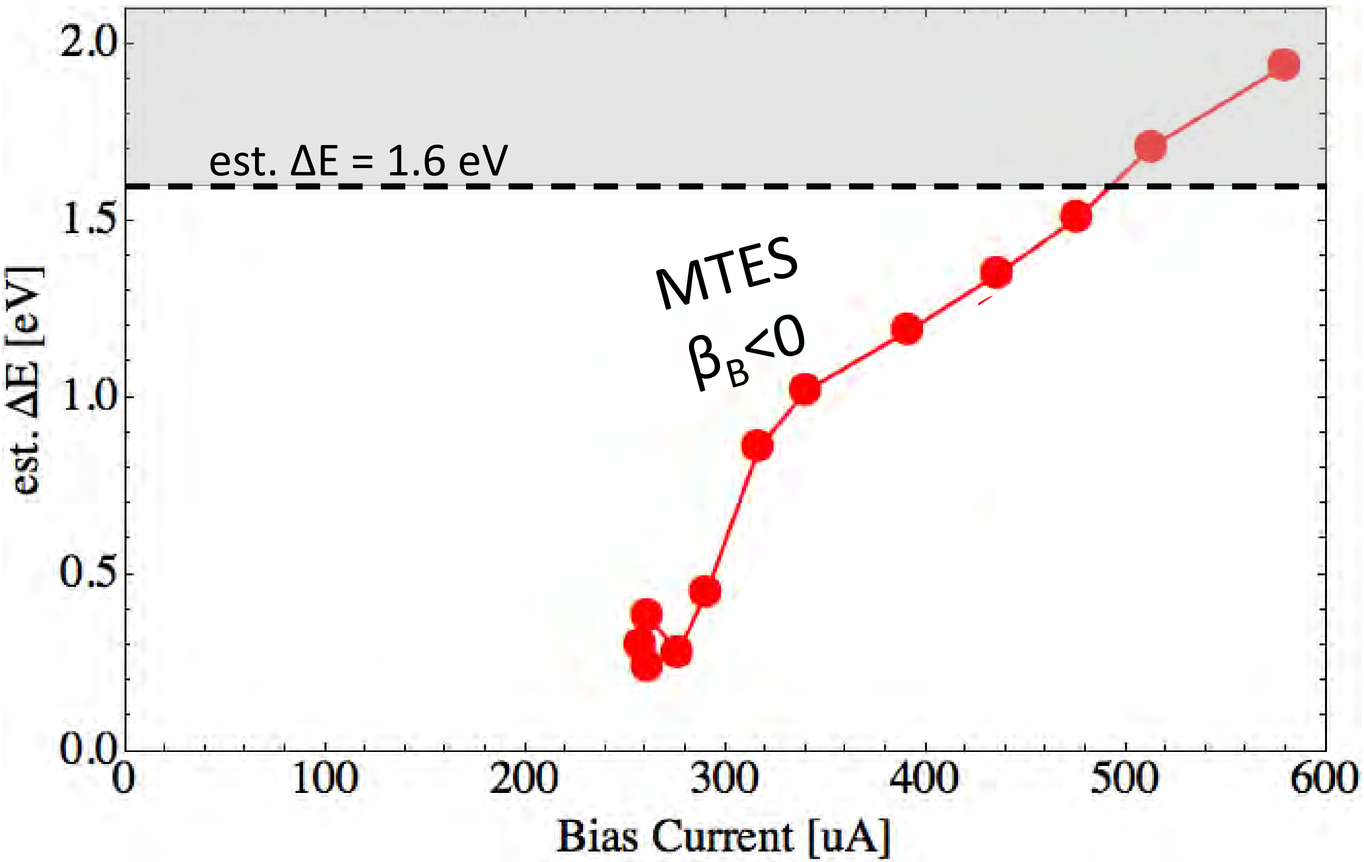


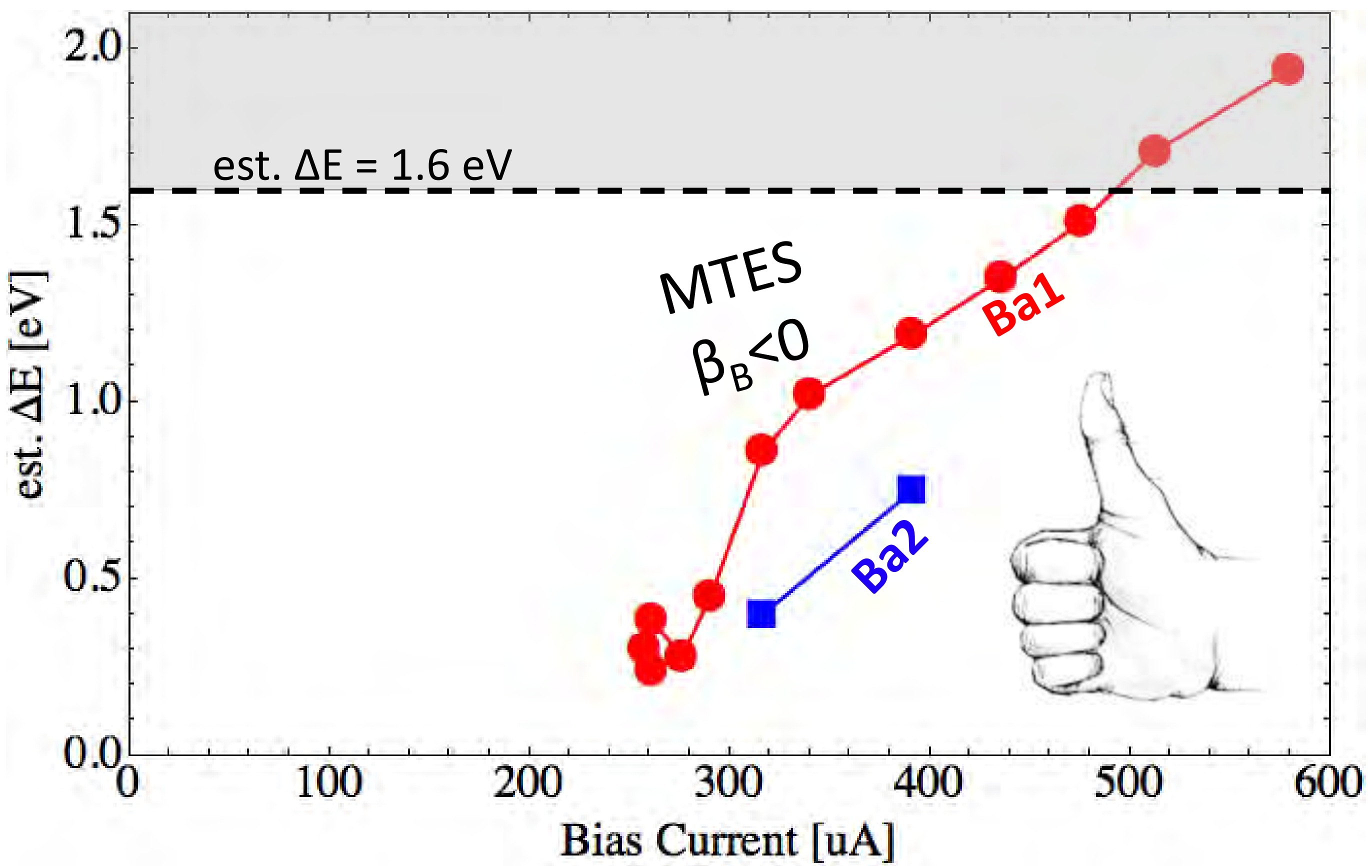
est. ΔE = 1.6 eV
MTES
βB<0
est. ΔE [eV]
2.0
1.5
1.0
0.5
0.0
0
100
200
300
400
500
600
Bias Current [uA]

6 keV

CAUTION:

- ***If*** *a* ***Linear*** *detector…*
- *(Estimated)*

est. ΔE = true ΔE



What is "nonlinearity"? the "nonlinear-model"… field "nonlinear dynamics"
Nonlinear → Everything Else *(a <u>very</u> ∞ set ;-)*

- **Nonlinearity ubiquitous** in nature:
  - e.g. 60 Hz harmonics pickup at 120 Hz, 180 Hz,.... requires nonlinearity in the system.
- **Linearity ubiquitous** in our mathematical description of nature:
  - often a good approximation to real physical systems.
  - It is mathematically easier
  - Linear Tools:
    - Superposition Principle
    - Transform methods, transfer functions etc.

What is "nonlinearity"? the "nonlinear-model"… field "nonlinear dynamics"
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Known or Expected Nonlinearities

- If our thermistor TES sensor…
  - If R(T) → R(T,J).
  - Then **Nonlinear** system of Diff Eqs.
  - (Can approximate by a linearized system of Diff Eqs.)
- "**Nonlinear**" if
  - R(T) → anything other than a single straight line
  - R(T,J) → " " " a single plane.
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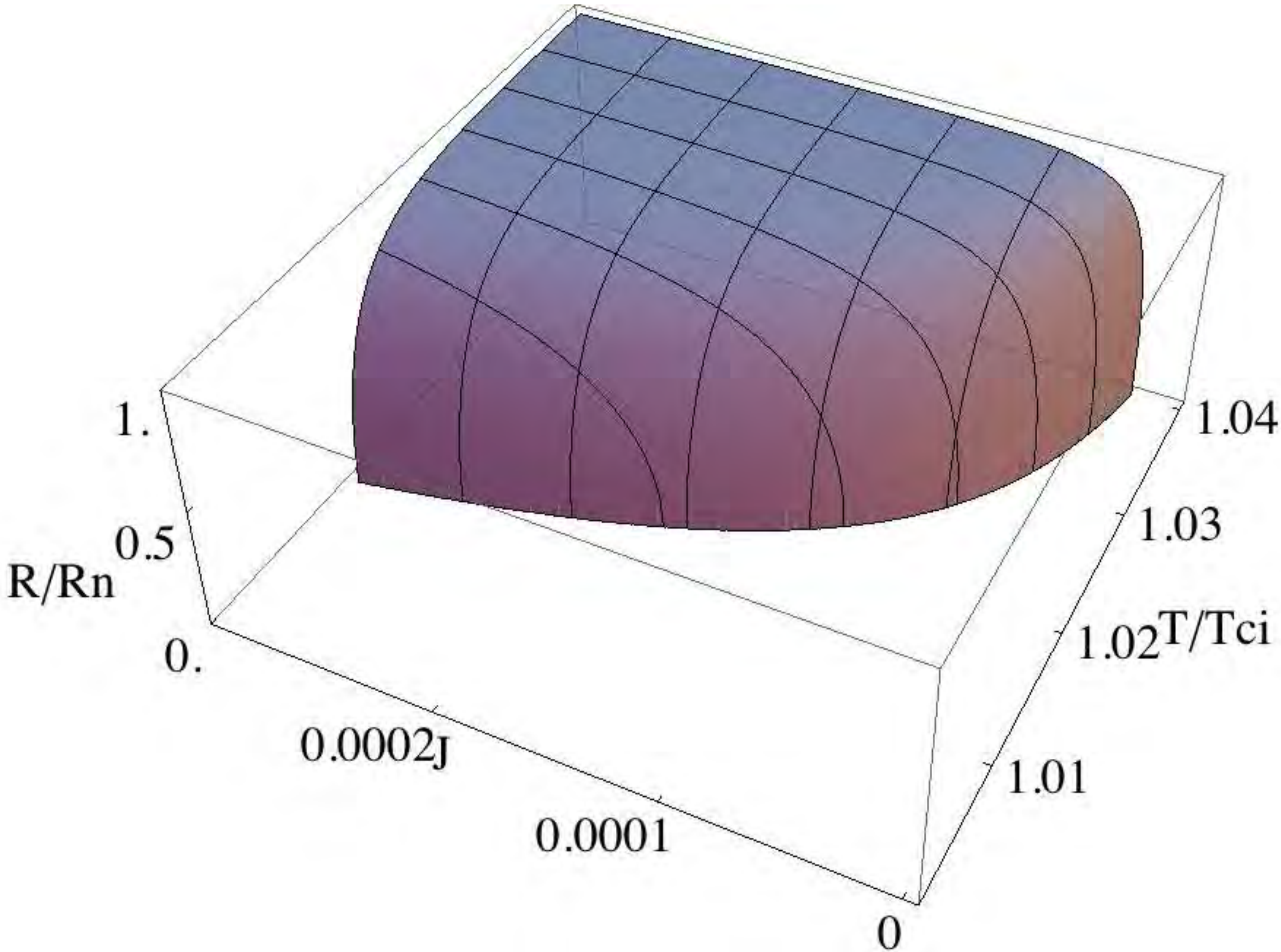
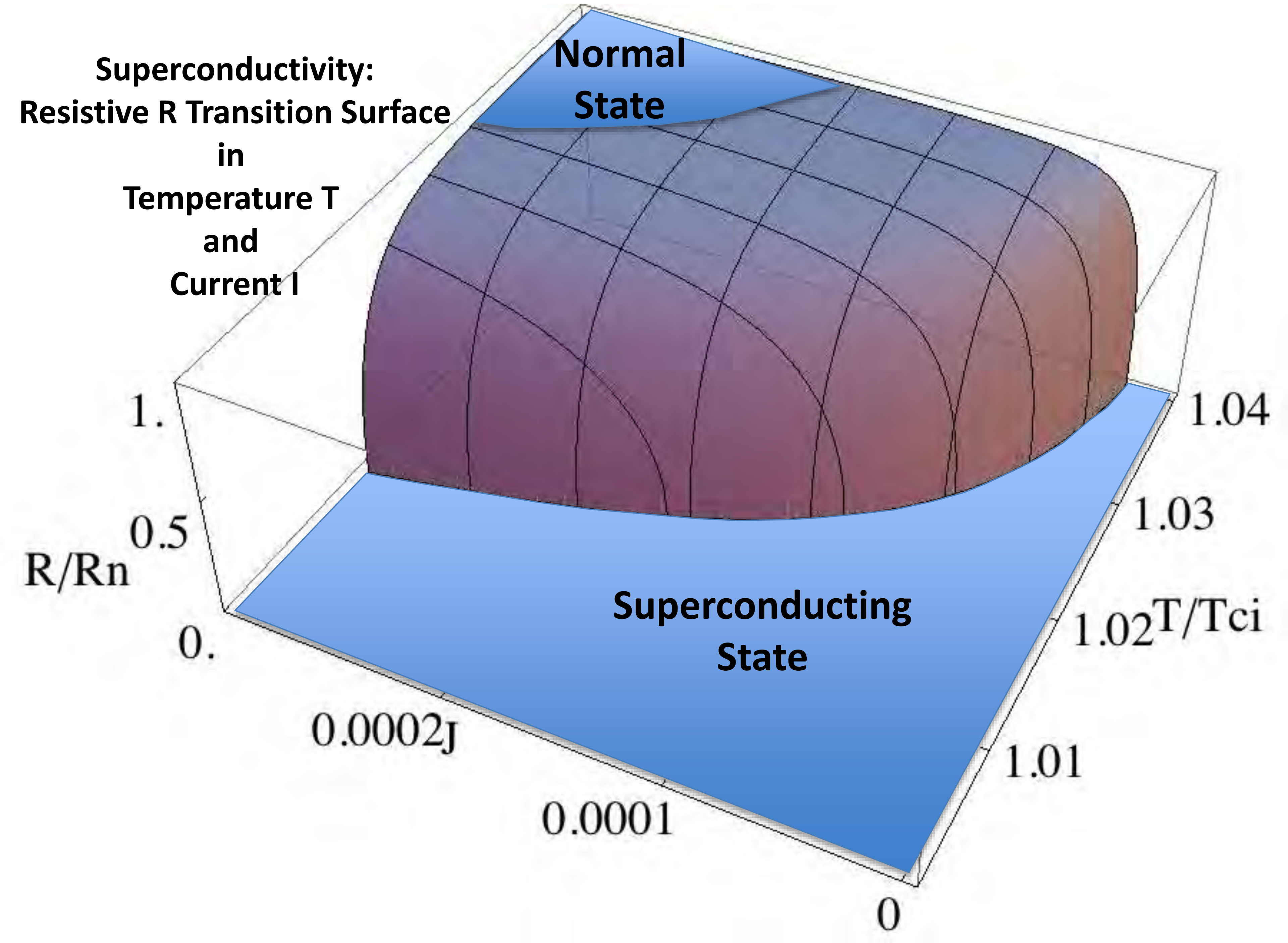
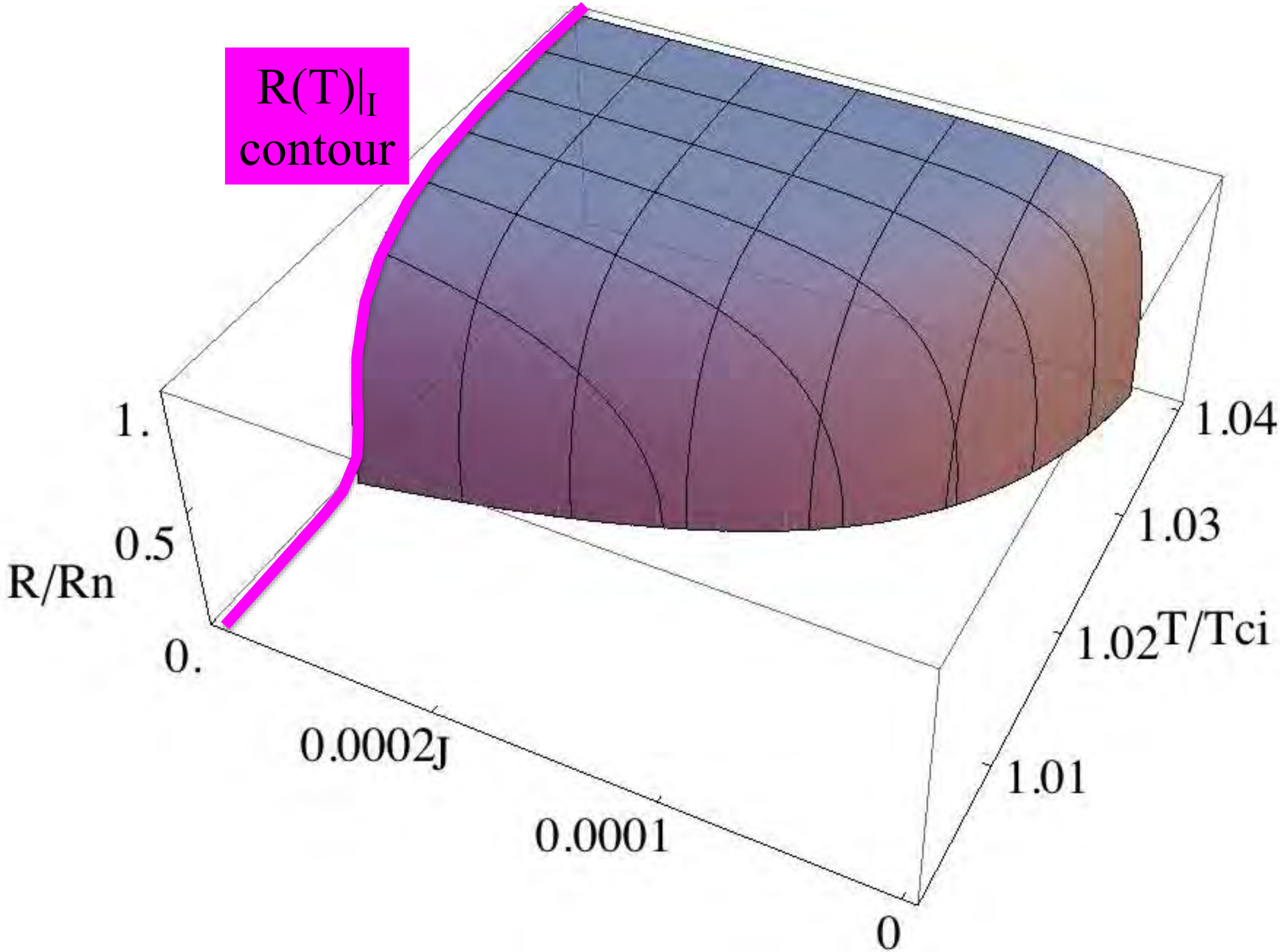


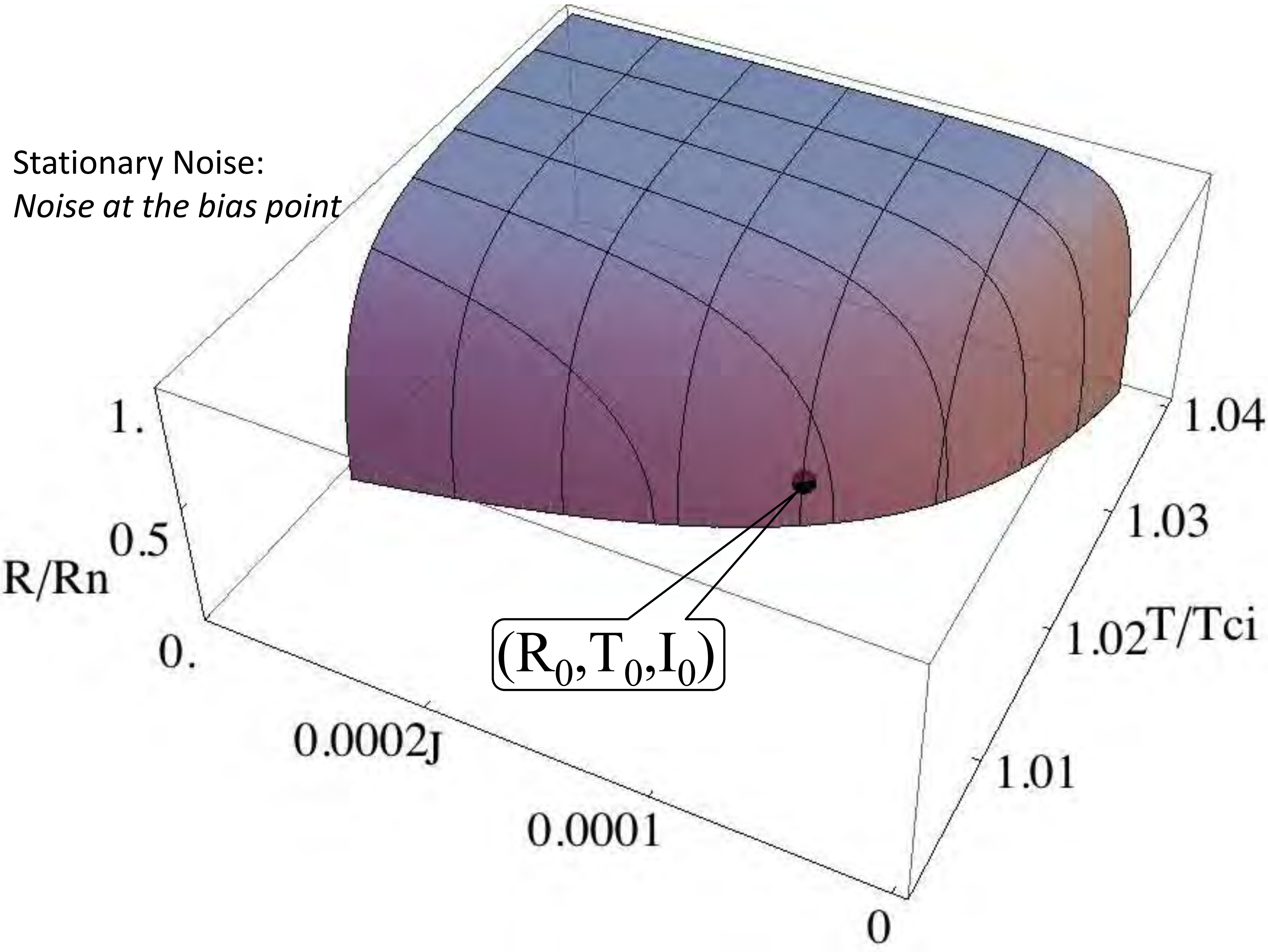
1.
0.5
R/Rn
0.
0.0002
J
0.0001
0
1.01
1.02
T/Tci
1.03
1.04

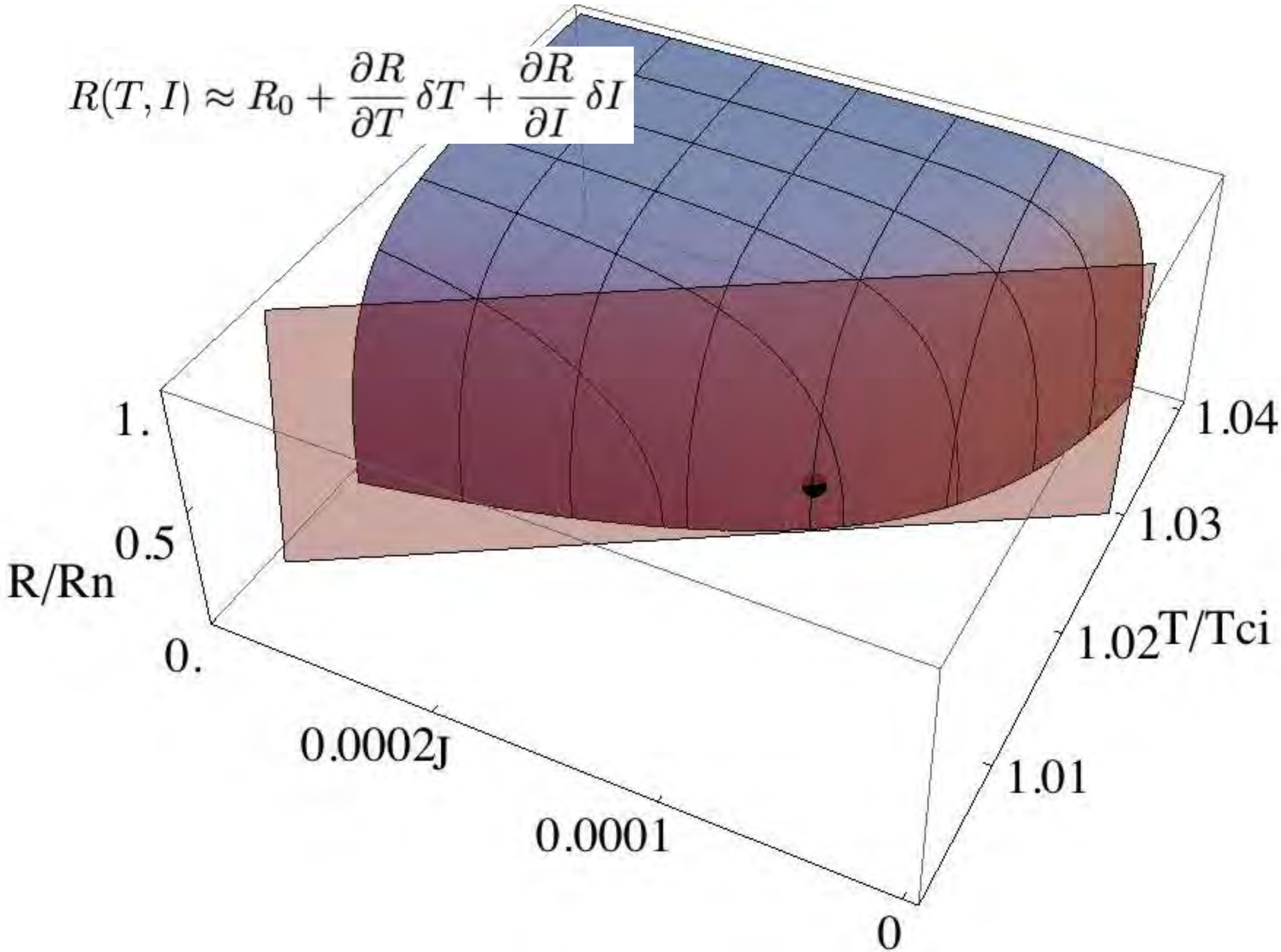

Superconductivity:
Resistive R Transition Surface
in
Temperature T
and
Current I
Normal
State
Superconducting
State
R/Rn
1.
0.5
0.
0.0002
J
0.0001
0
T/Tci
1.04
1.03
1.02
1.01



R(T)|I
contour
R/Rn
1.
0.5
0.
0.0002
J
0.0001
0
1.04
1.03
1.02
T/Tci
1.01

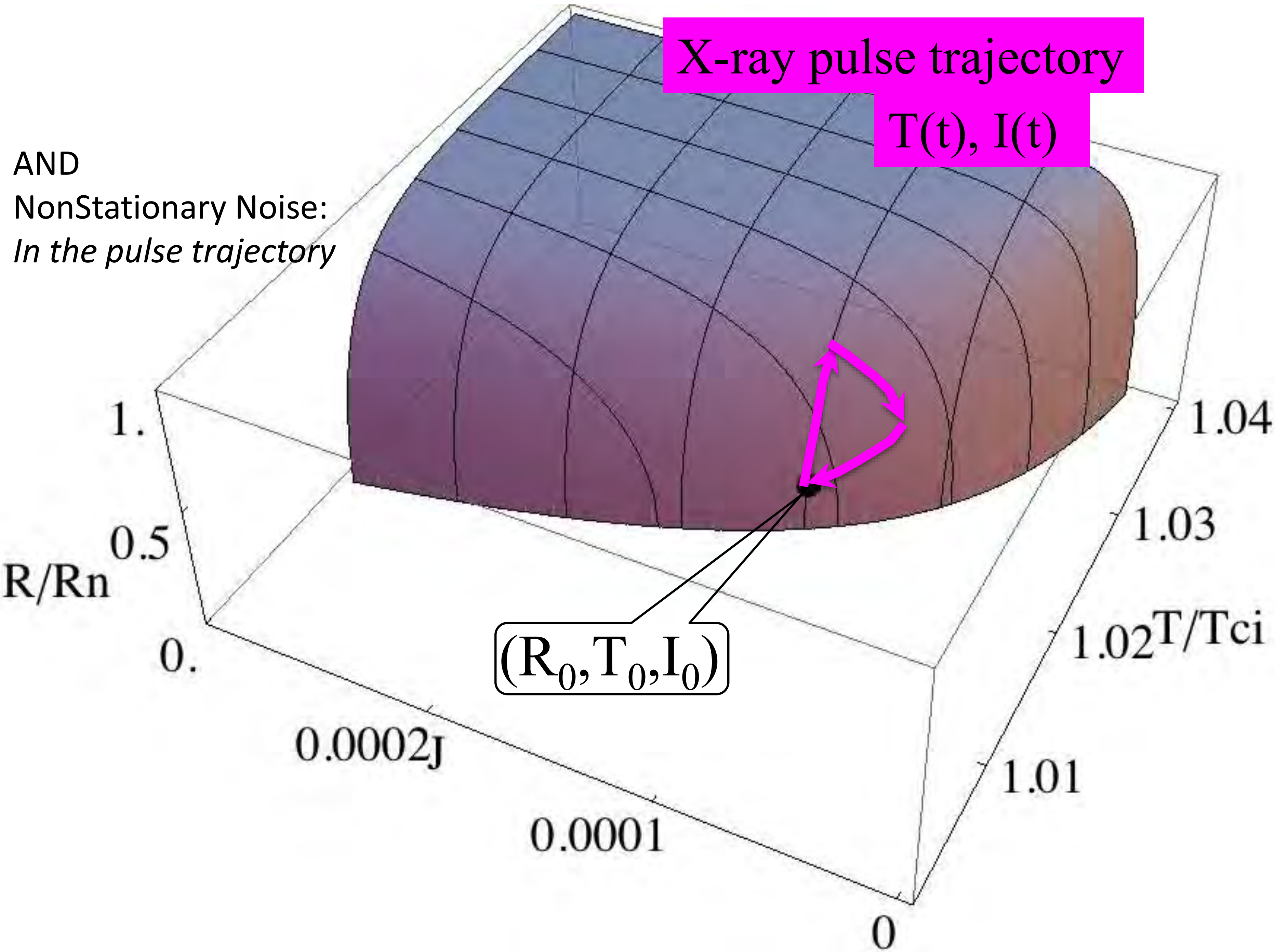
Stationary Noise:
Noise at the bias point



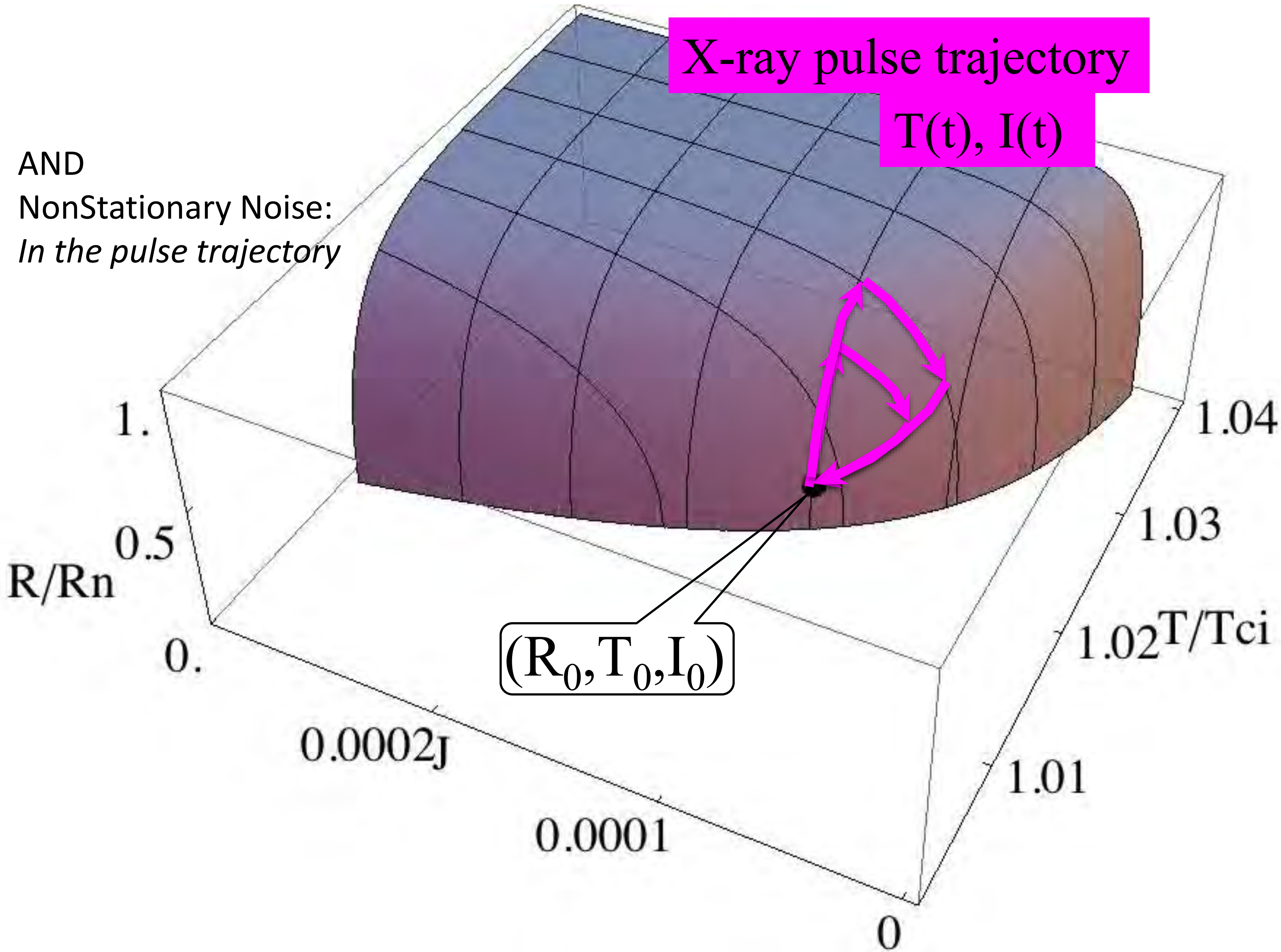


$R(T,I) \approx R_0 + \frac{\partial R}{\partial T}\,\delta T + \frac{\partial R}{\partial I}\,\delta I$
R/Rn
1.
0.5
0.
0.0002
J
0.0001
0
1.01
1.02
T/Tci
1.03
1.04

AND
NonStationary Noise:
In the pulse trajectory



AND
NonStationary Noise:
In the pulse trajectory



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"Unexpected Nonlinearities"

In steady state with standard DC biased TES operation. The TES current is wildly oscillating in time at high frequencies and we only measure *(and are only aware of)* the time averaged current in the TES.

Conclusions: *…Paradigm Shifting*

- In steady state with standard DC biased TES operation. The TES current is wildly oscillating in time at high frequencies and we only measure *(and are only aware of)* the time averaged current in the TES.
- The equations governing this time response is **nonlinear** <u>time-dependent</u> diff eqs.
  - **KEY POINT**: This time dependent current exists when the TES is only DC biased, NO time dependent inputs what so ever. It comes about from the intrinsic physics governing the superconducting state of the TES.

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  - **KEY POINT**: This time dependent current exists when the TES is only DC biased, NO time dependent inputs what so ever. In comes about from the intrinsic superconductivity physics governing TESs.
- The measured resistive transition surface is not simply a function of R(T,J,B). But is also a function of the electric circuit. R(T,J,B,L,Rsh).
  - → Problem compartmentalizing: → *Same TES measured in slightly different circuits will appear to have a different resistive transition surface. TEST in real setup early!!!*

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  - → Problem compartmentalizing: → *Same TES measured in slightly different circuits will appear to have a different resistive transition surface. TEST in real setup early!!!*
- Circuit and TES parameters can have values such that these time dependent solutions become multivalued in time.
  - *What does that mean?* Can mean excess noise or fine structure of the time averaged resistive transition surface.

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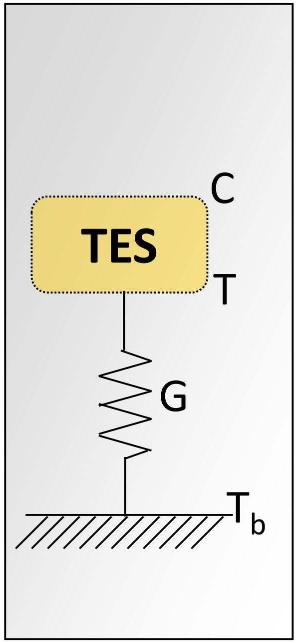
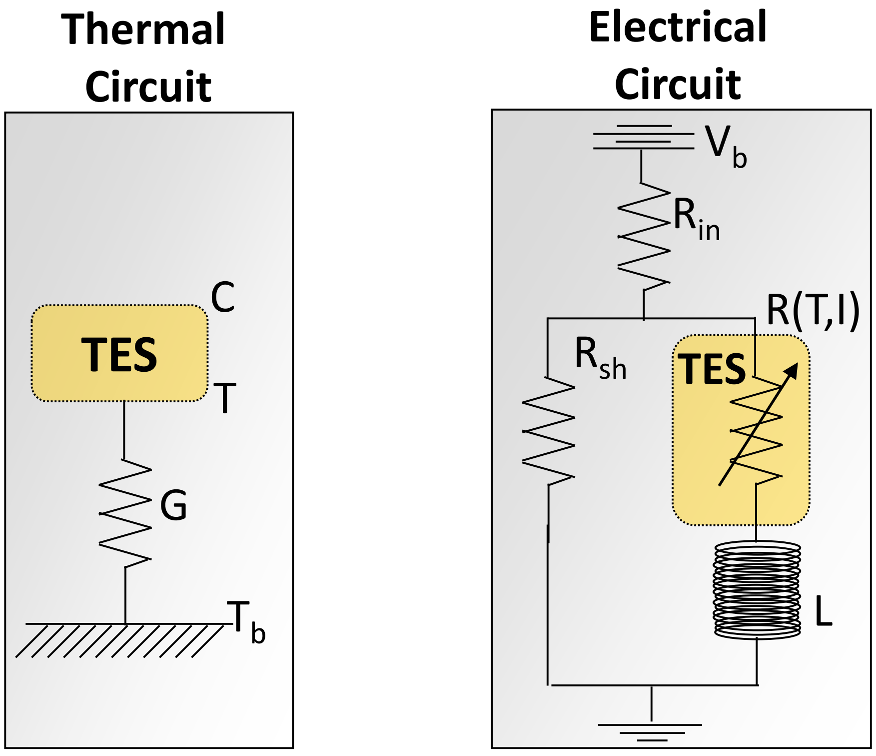
- Circuit and TES parameters can have values such that these time dependent solutions become multivalued in time.
  - *What does that mean?* Can mean excess noise or fine structure of the time averaged resistive transition surface.
- The biased TES waiting to detect a photon (μ-calorimeter) or flux of photons (μ-bolometers) can itself act as a radiation source.
  - Tricky when an array of exquisitely sensitive micro-wave radiation detectors are themselves sources of microwave radiation.
  - This can lead to radiation resistance and fine structure in the resistive transition.
  - Possible cross talk between pixels in an array.
  - The time dependence of the current can take on pure sin waves and also very nonsinusoidal forms (variable harmonic content)
  - The fundamental frequency of these oscillations changes with bias voltage.
  - Makes the prospects of FDM TES arrays with an AC-biased TES challenging. Structure, sensitivity, and cross-talk.

DONE

Thermal Circuit



ELECTROTHERMAL MODEL

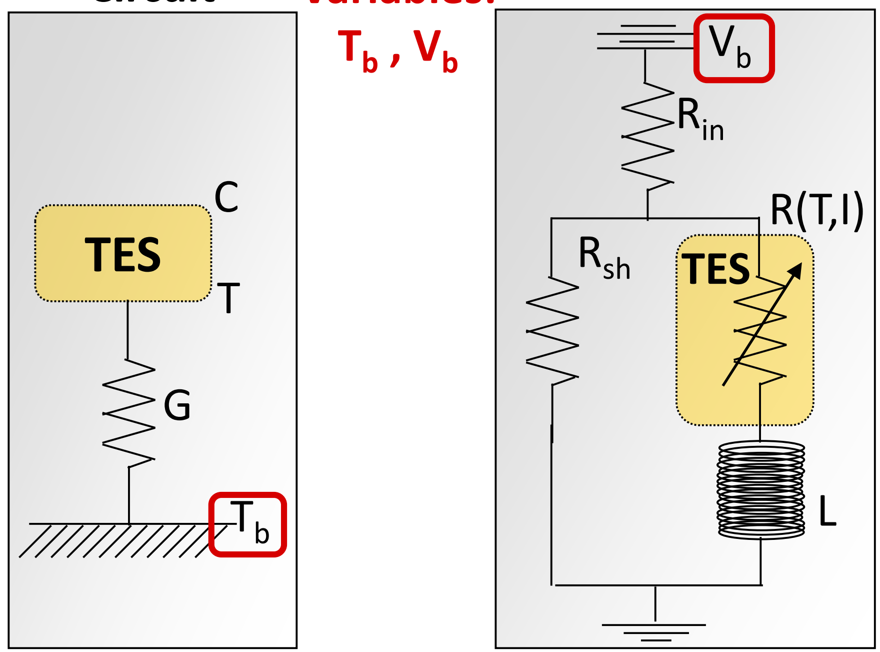


ELECTROTHERMAL MODEL

Thermal Circuit

Independent Variables: T_b , V_b

Electrical Circuit



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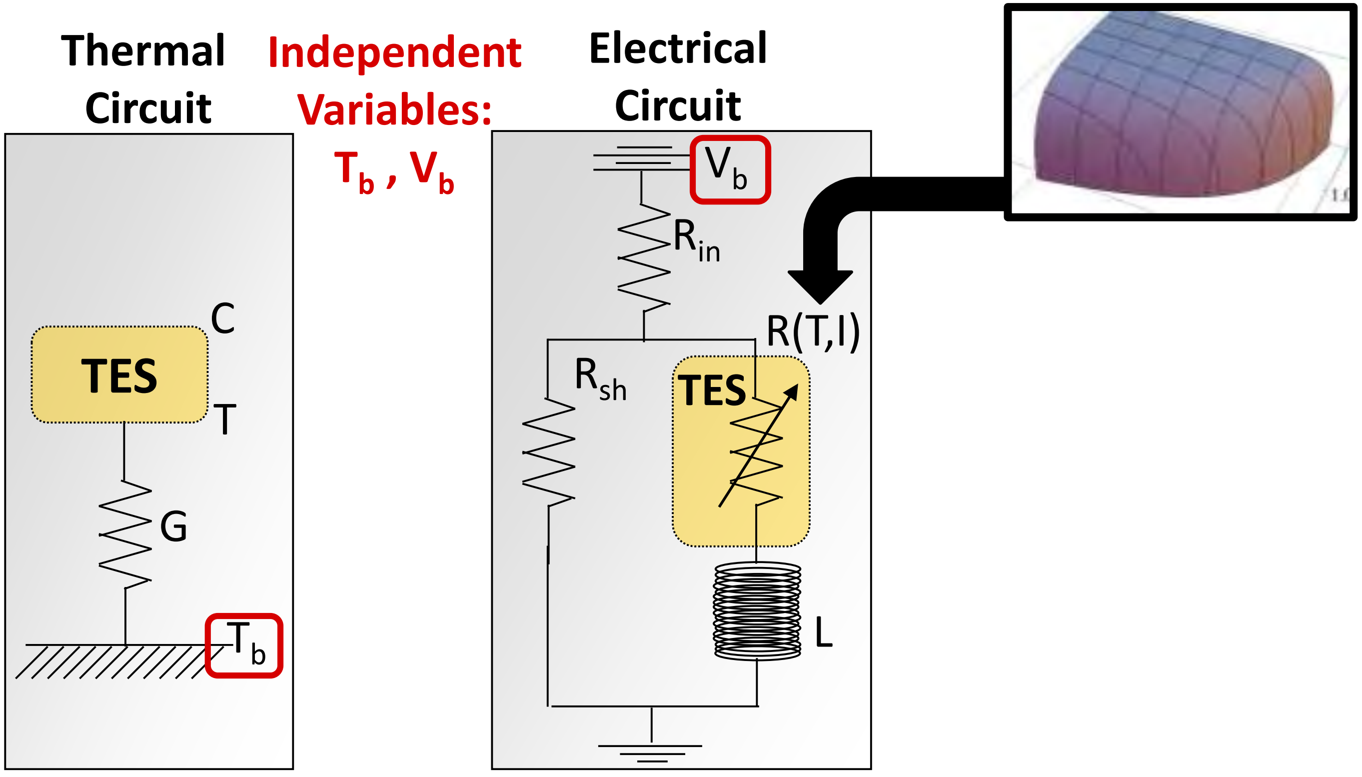
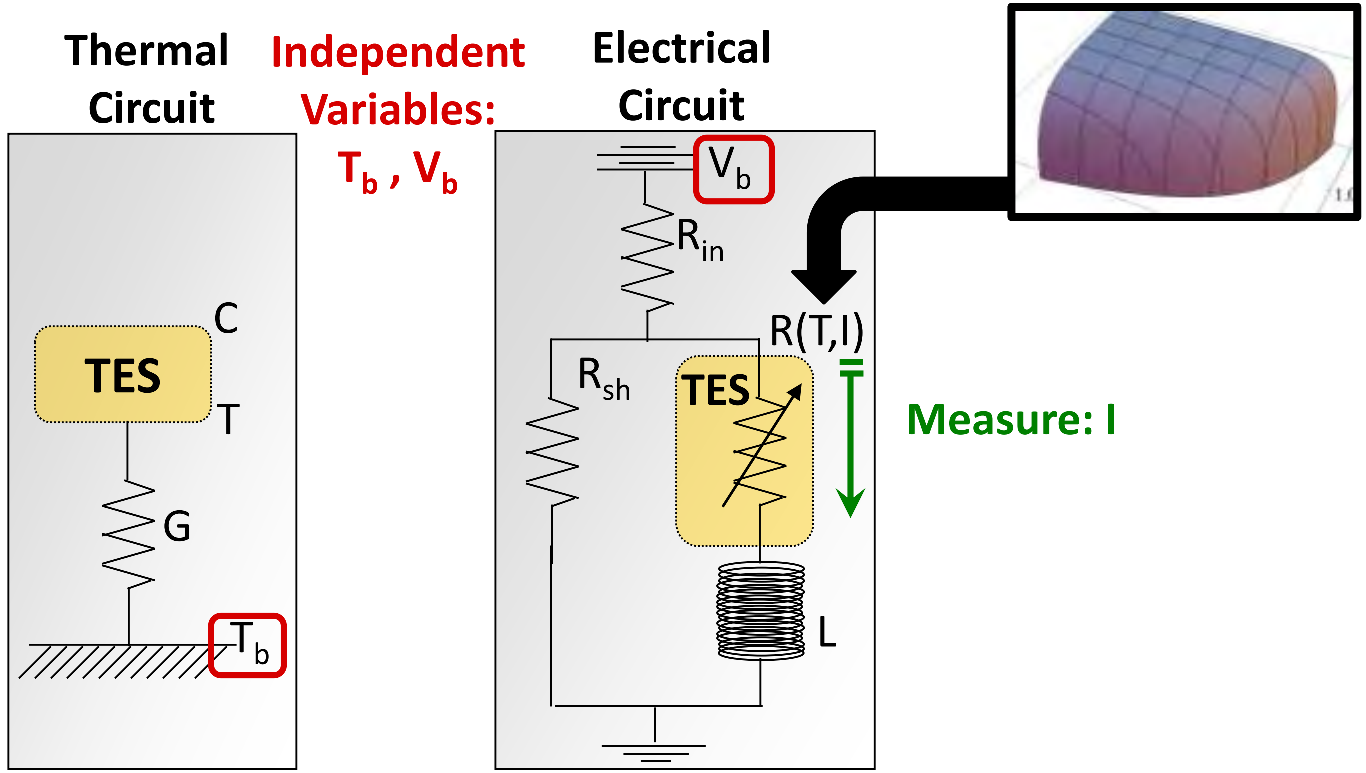
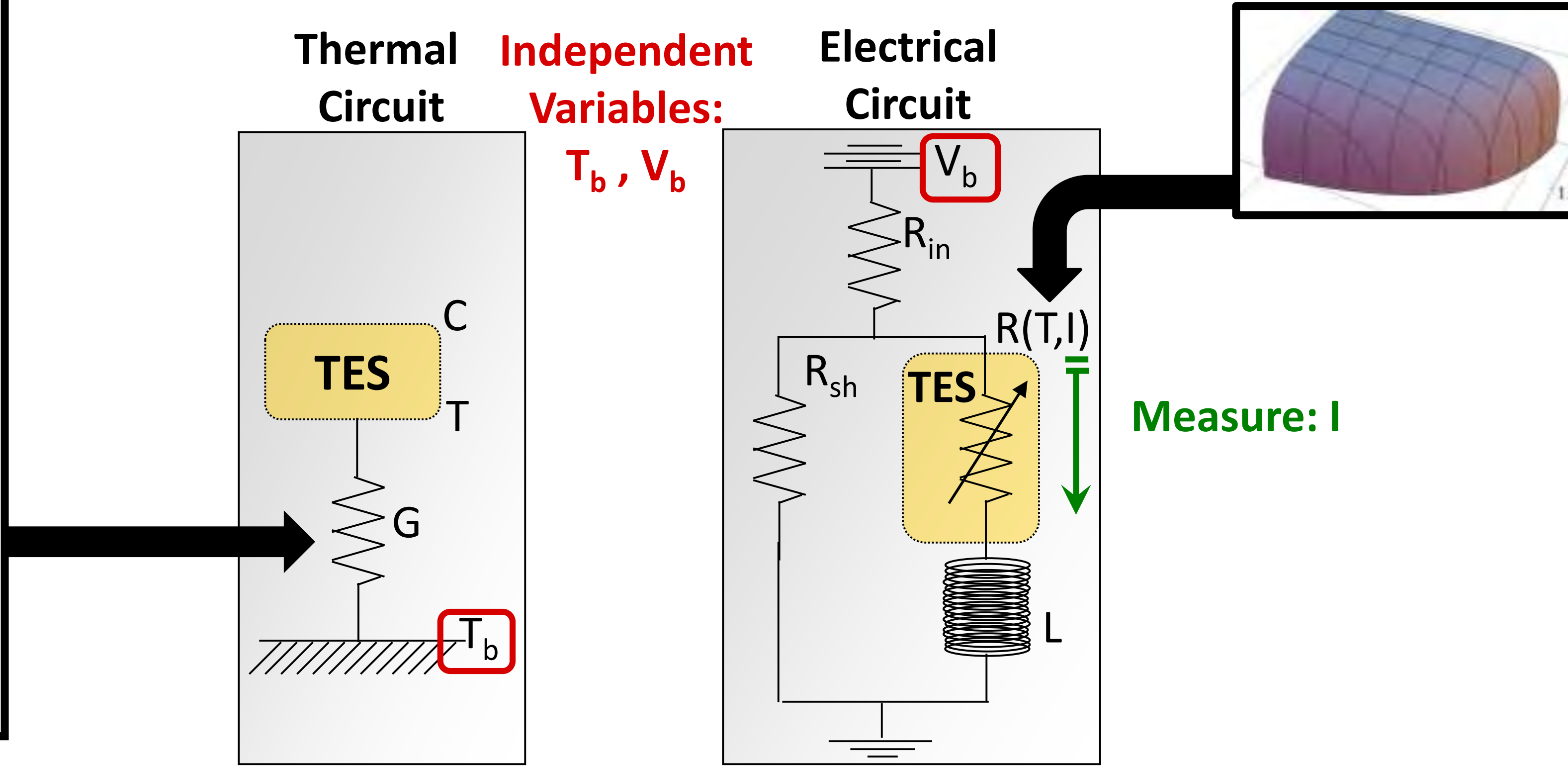


ELECTROTHERMAL MODEL



ELECTROTHERMAL MODEL

TES on a: …
(1) solid substrate
(2) thin isolated membrane
(3) thin perforated membrane
(4) island suspended with long thin legs



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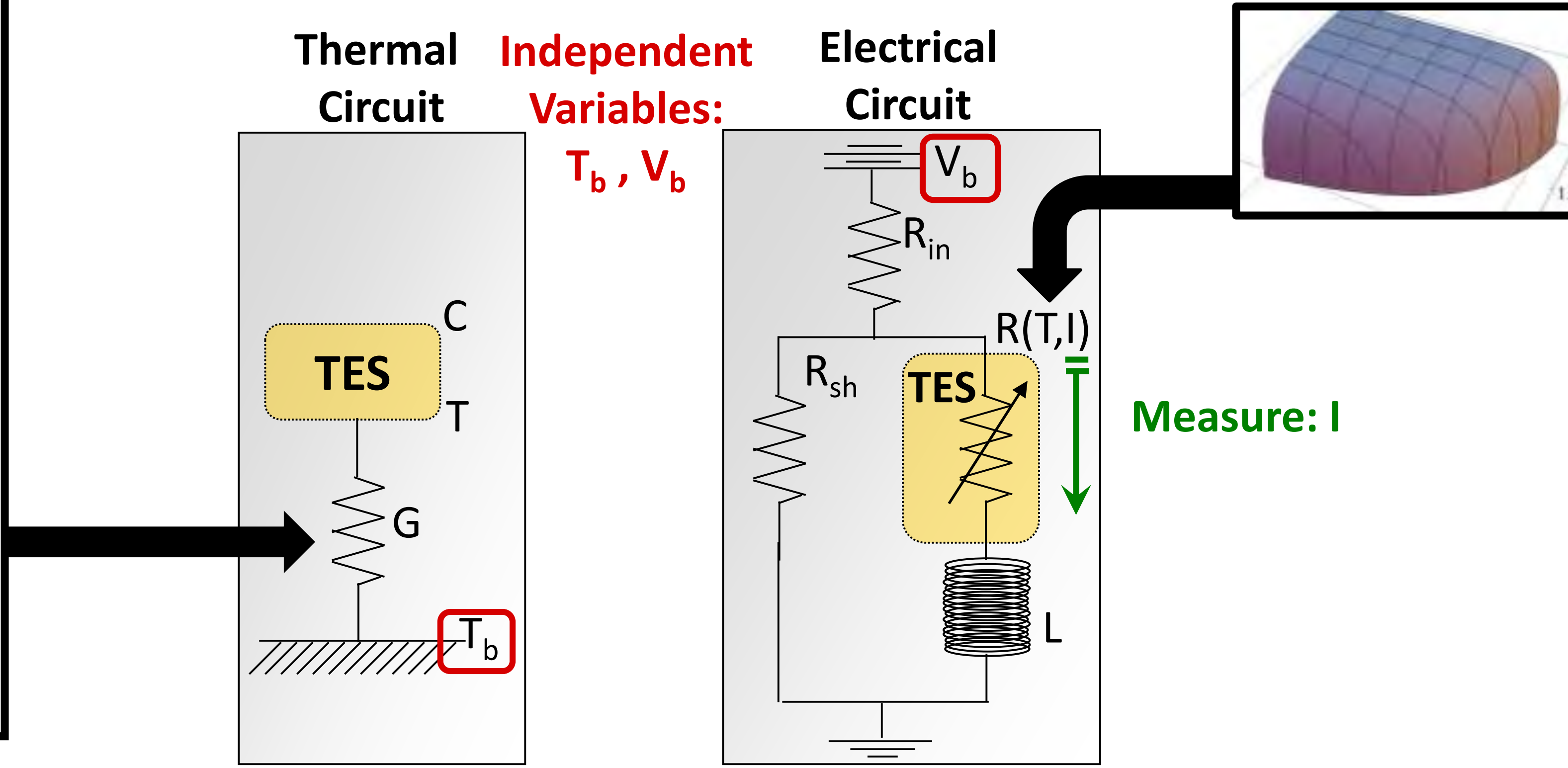


Thermal power balance

$$I^2 R = \frac{G}{nT^{(n-1)}}\left(T^n - T_b^n\right)$$

Electrical steady state

$$I = \frac{V_b}{R_{in}} \frac{R_{sh}}{R + R_{sh}}$$

IN STEADY STATE

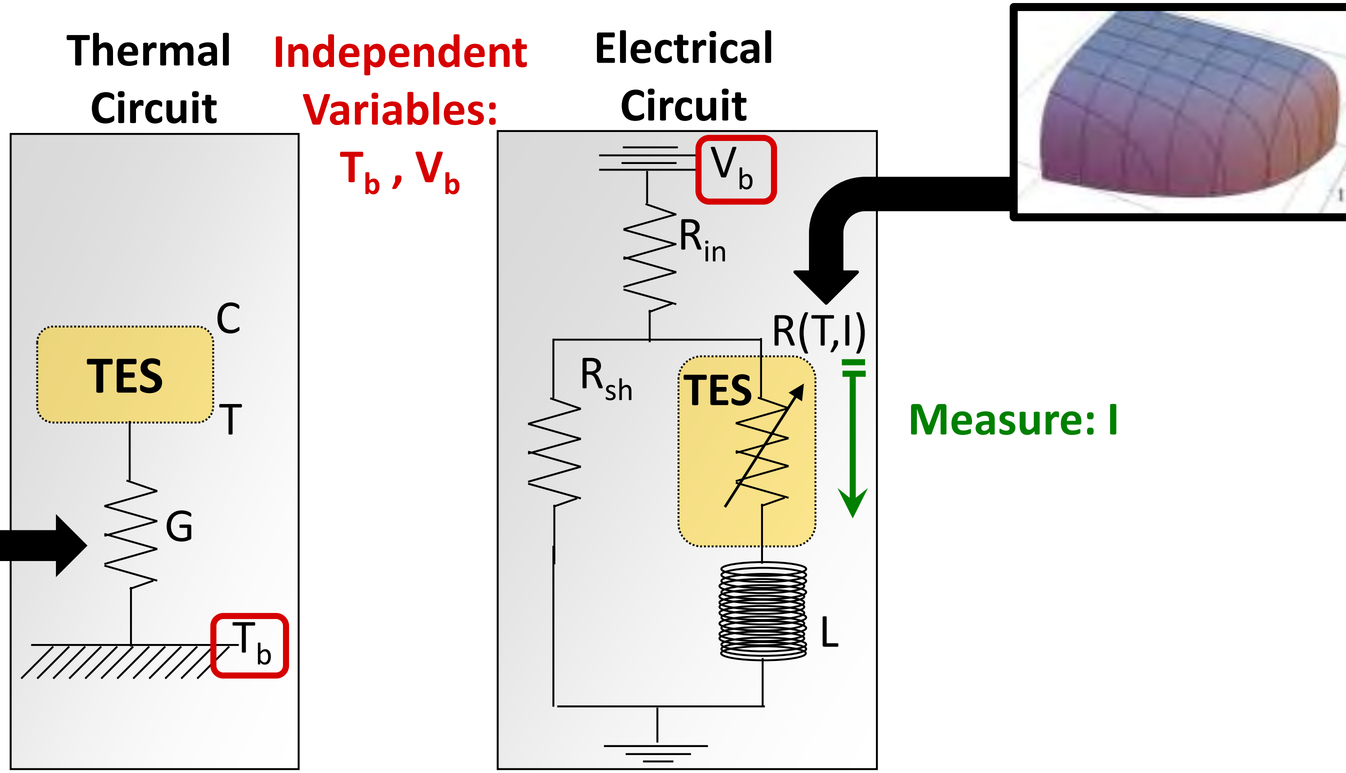
TES on a: …

(1) solid substrate

(2) thin isolated membrane

(3) thin perforated membrane

(4) island suspended with long thin legs



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TES on a: …
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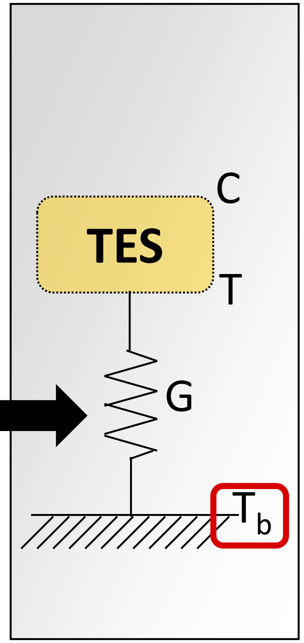
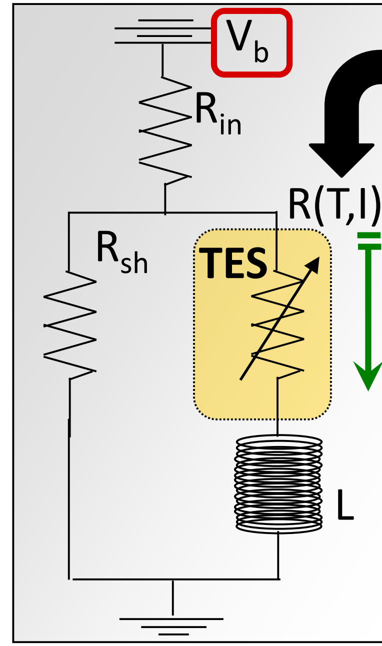
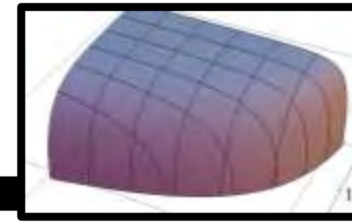


Independent Variables:
T_b , V_b

Electrical Circuit





Measure: I

Thermal power balance

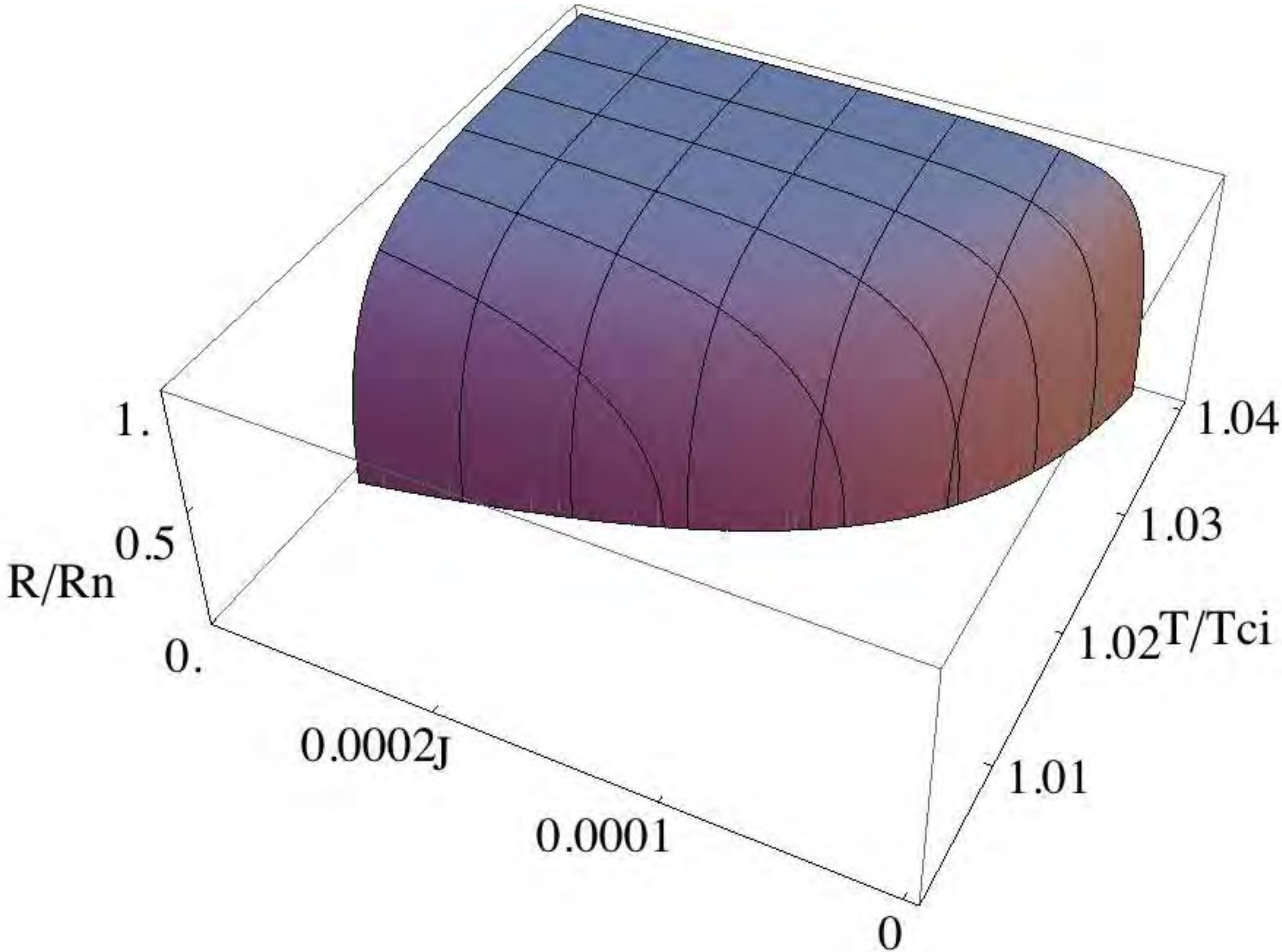
$$I^2R = \frac{G}{nT^{(n-1)}}(T^n - T_b^n)$$

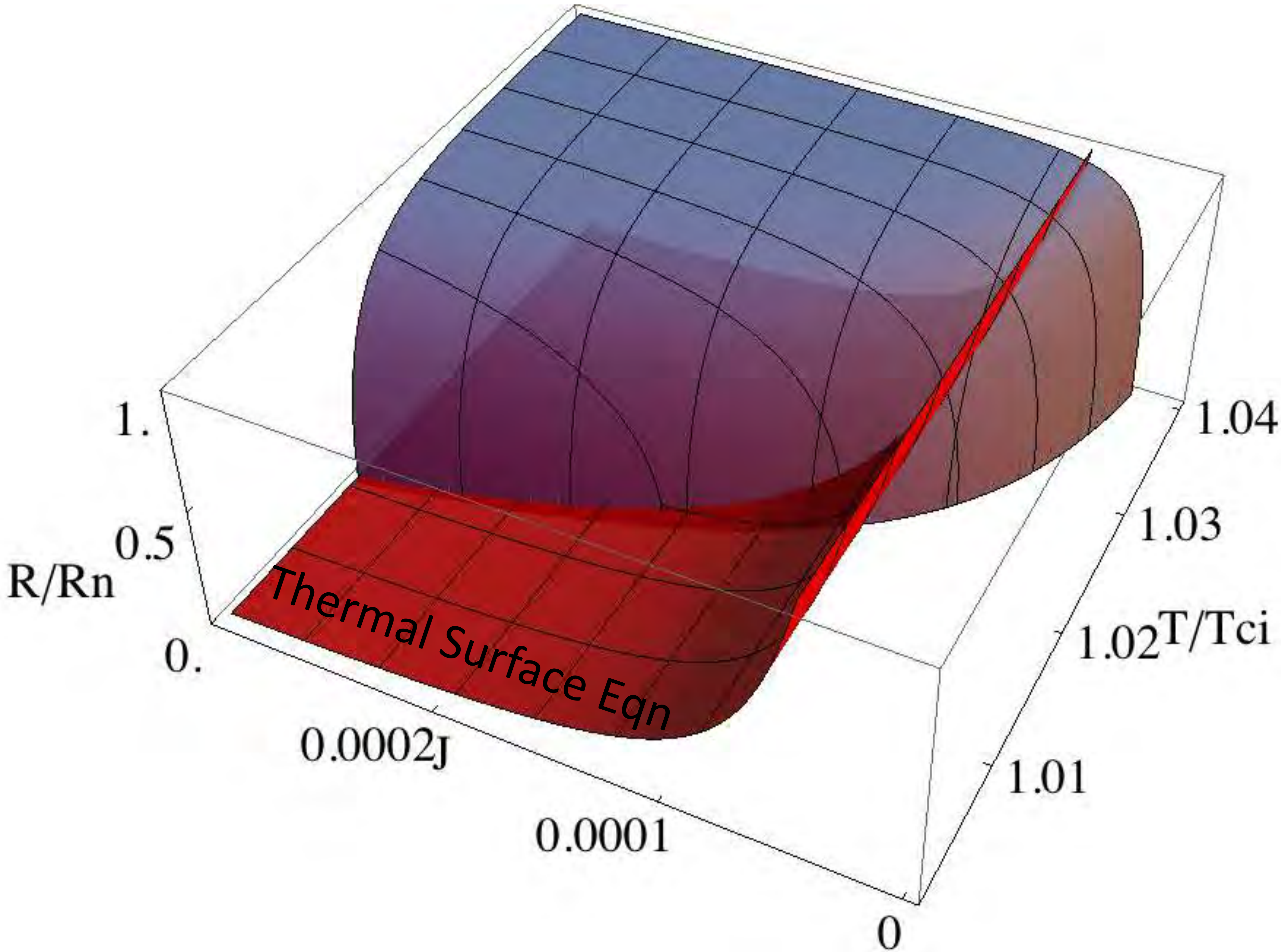
$$R = \frac{G}{I^2nT^{(n-1)}}(T^n - T_b^n)$$

Electrical steady state

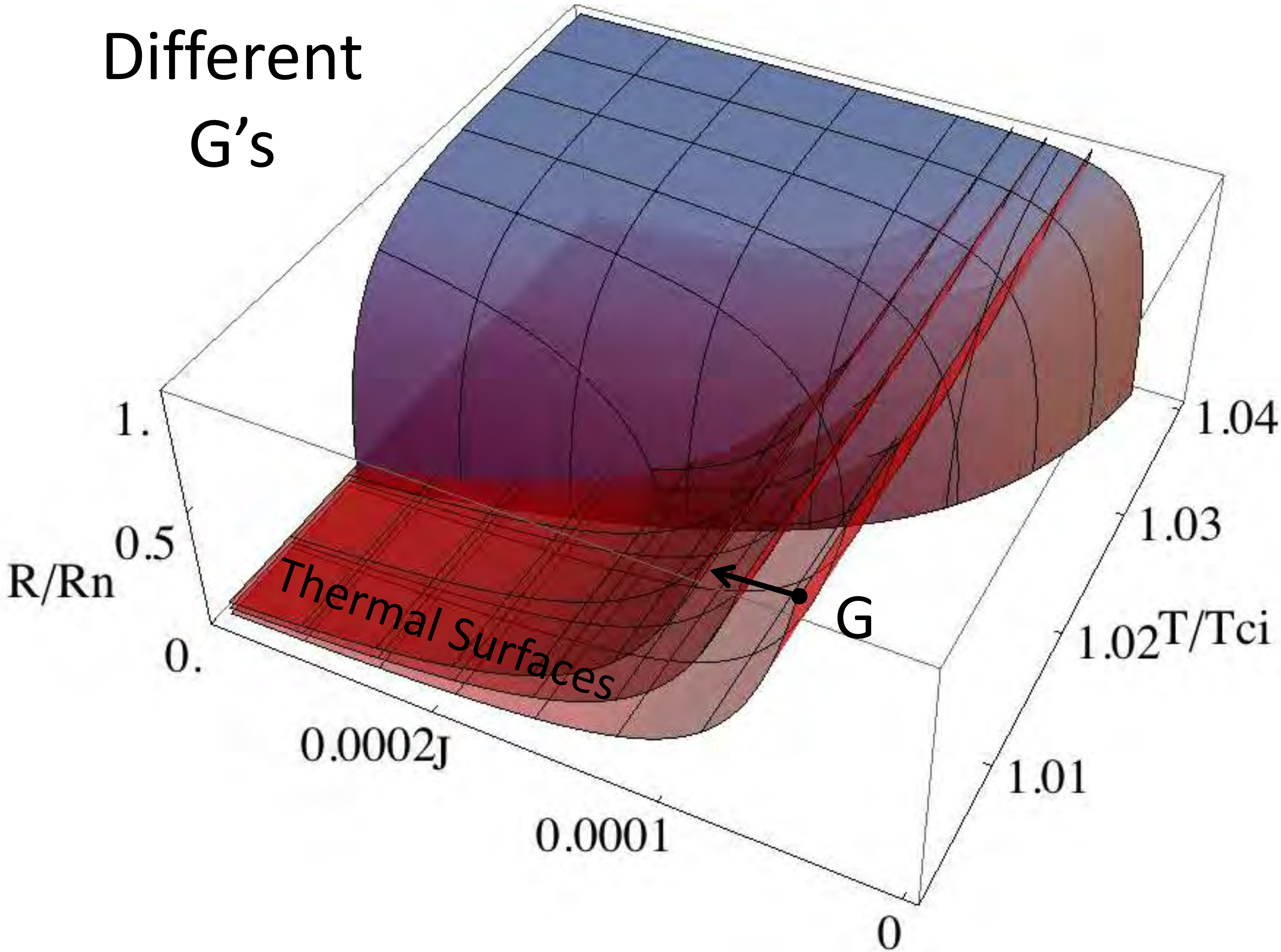
$$I = \frac{V_b}{R_{in}}\frac{R_{sh}}{R + R_{sh}}$$

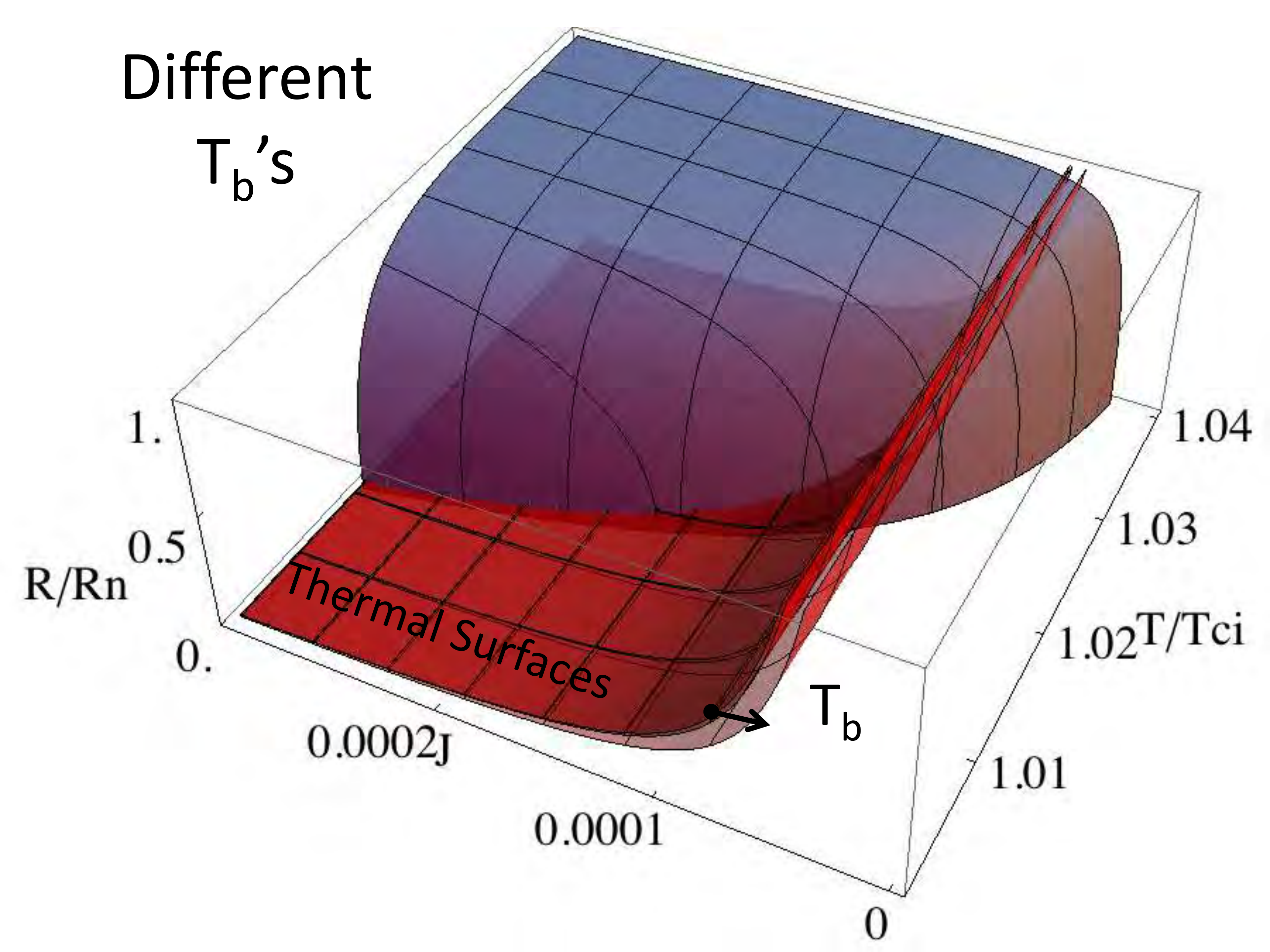
$$R = \frac{V_b}{R_{in}}\frac{R_{sh}}{I} - R_{sh}$$


R/Rn
1.
0.5
0.
0.0002
J
0.0001
0
1.01
1.02
1.03
1.04
T/Tci



1.
0.5
R/Rn
0.
Thermal Surface Eqn
0.0002J
0.0001
0
1.04
1.03
1.02T/Tci
1.01


Different
G's
Thermal Surfaces
G
R/Rn
1.
0.5
0.
0.0002
J
0.0001
0
1.04
1.03
1.02
T/Tci
1.01


Different
T_b's
Thermal Surfaces
T_b
R/Rn
1.
0.5
0.
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0.0001
0
T/Tci
1.04
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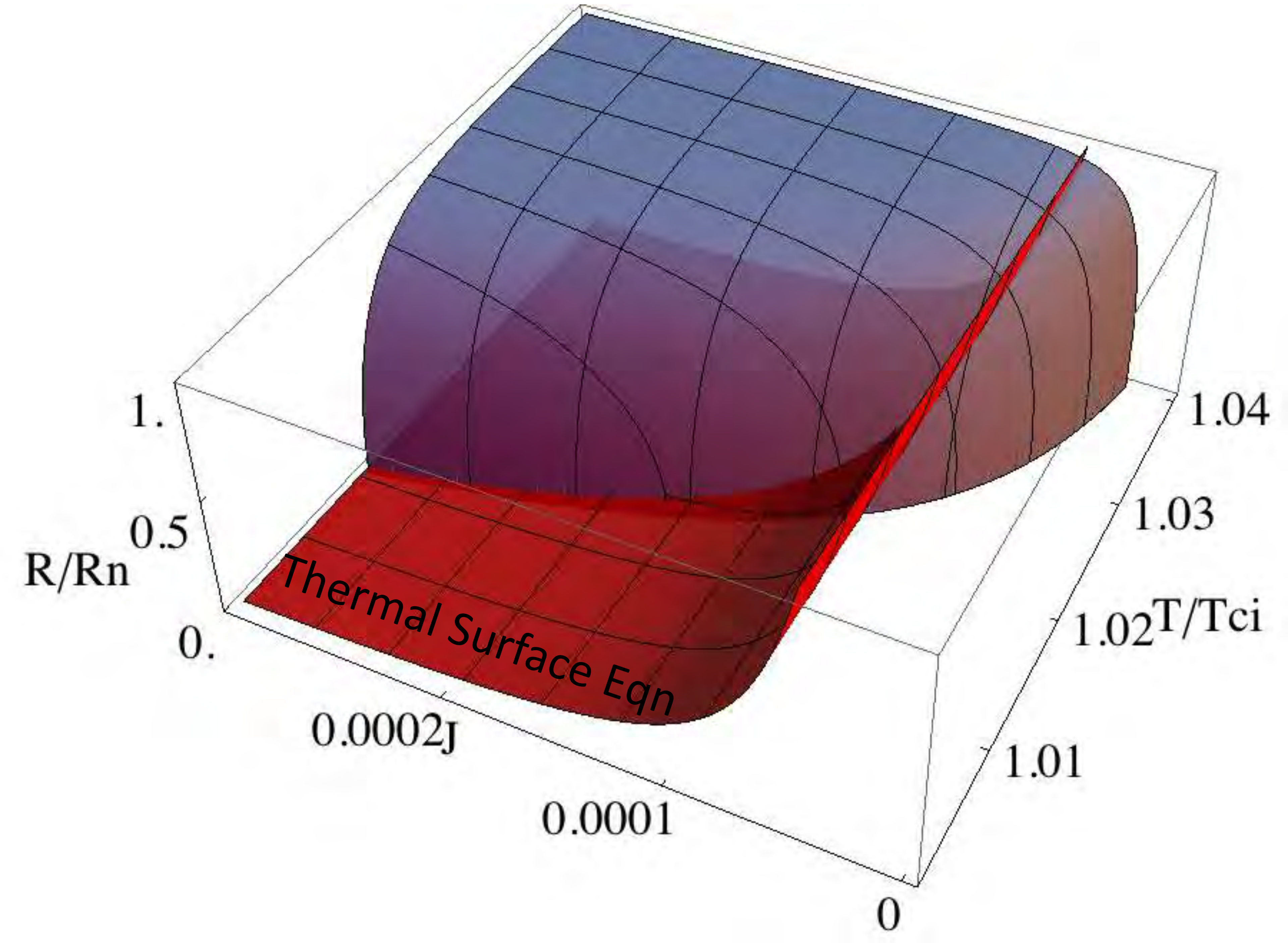
1.01



1.
0.5
R/Rn
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Thermal Surface Eqn
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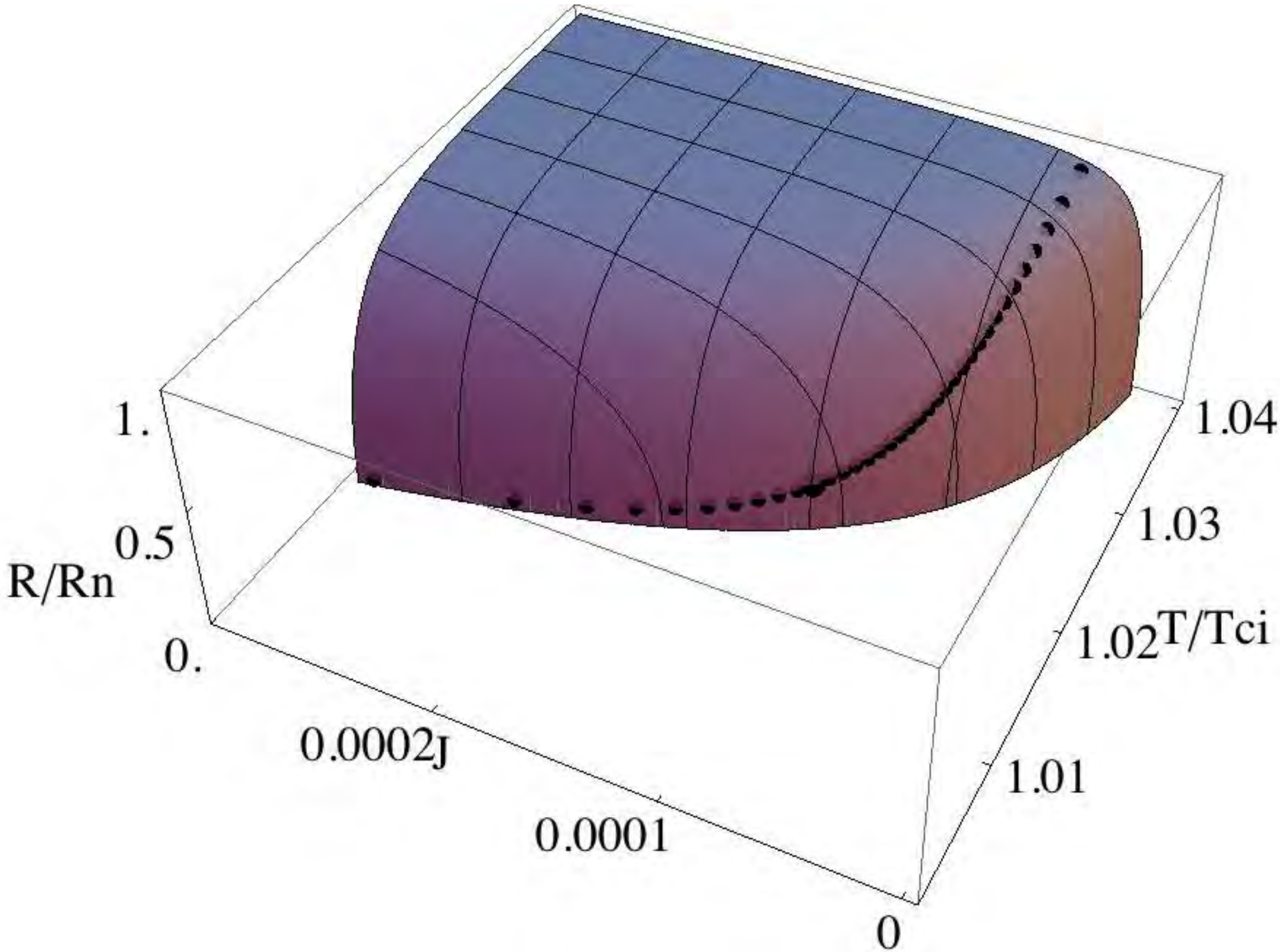

1.
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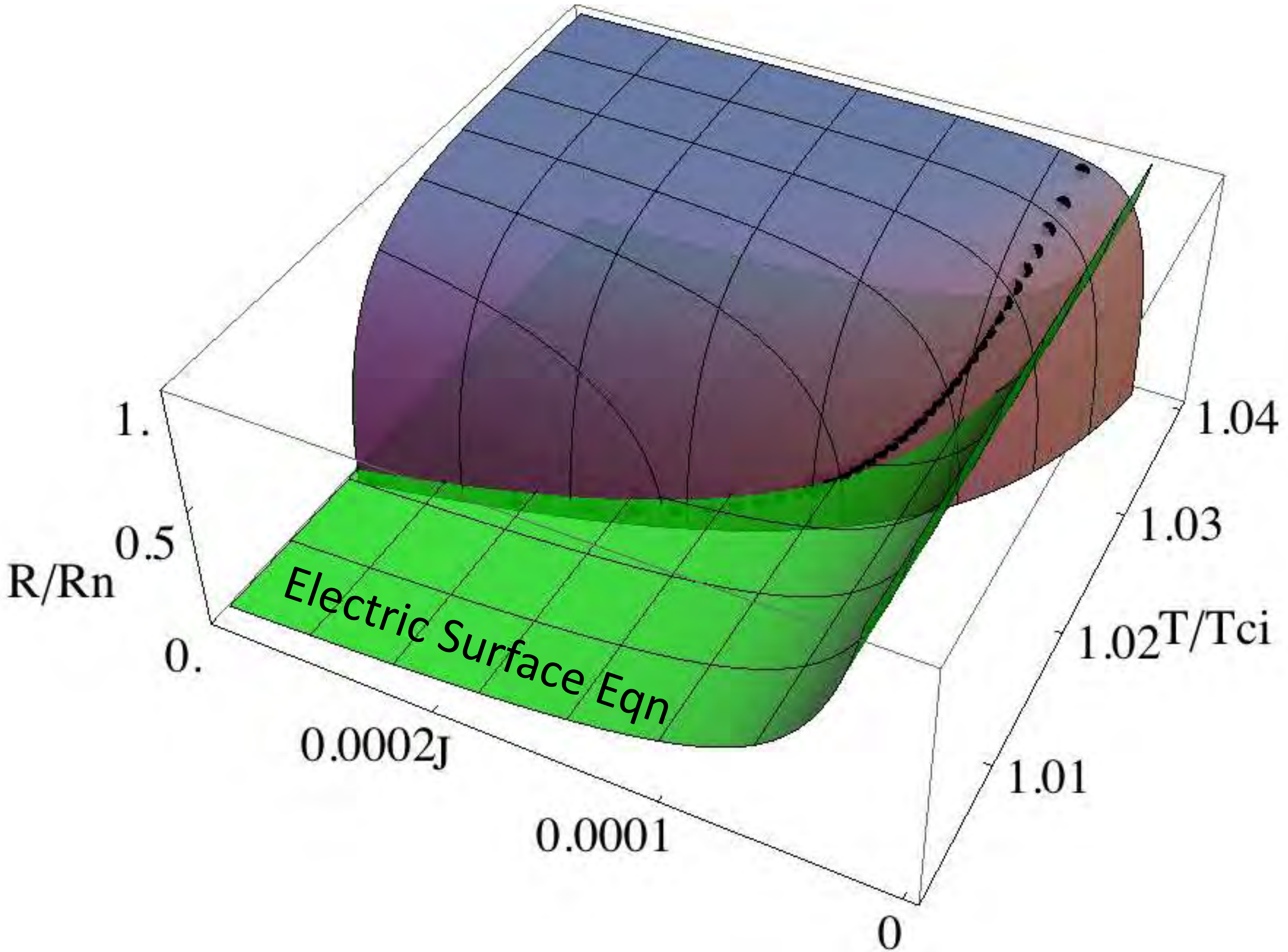

1.
0.5
0.
R/Rn
0.0002
J
0.0001
0
1.04
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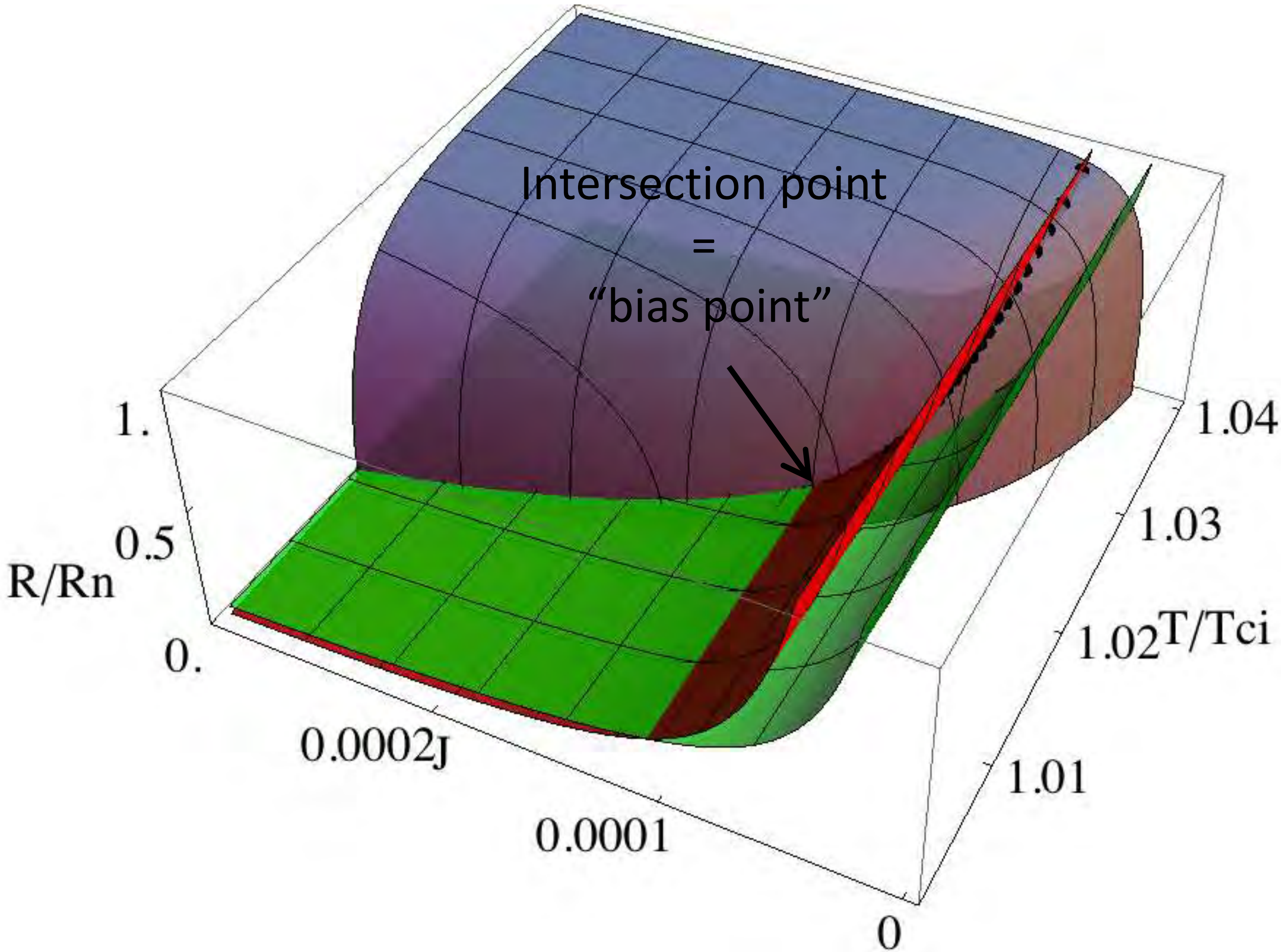
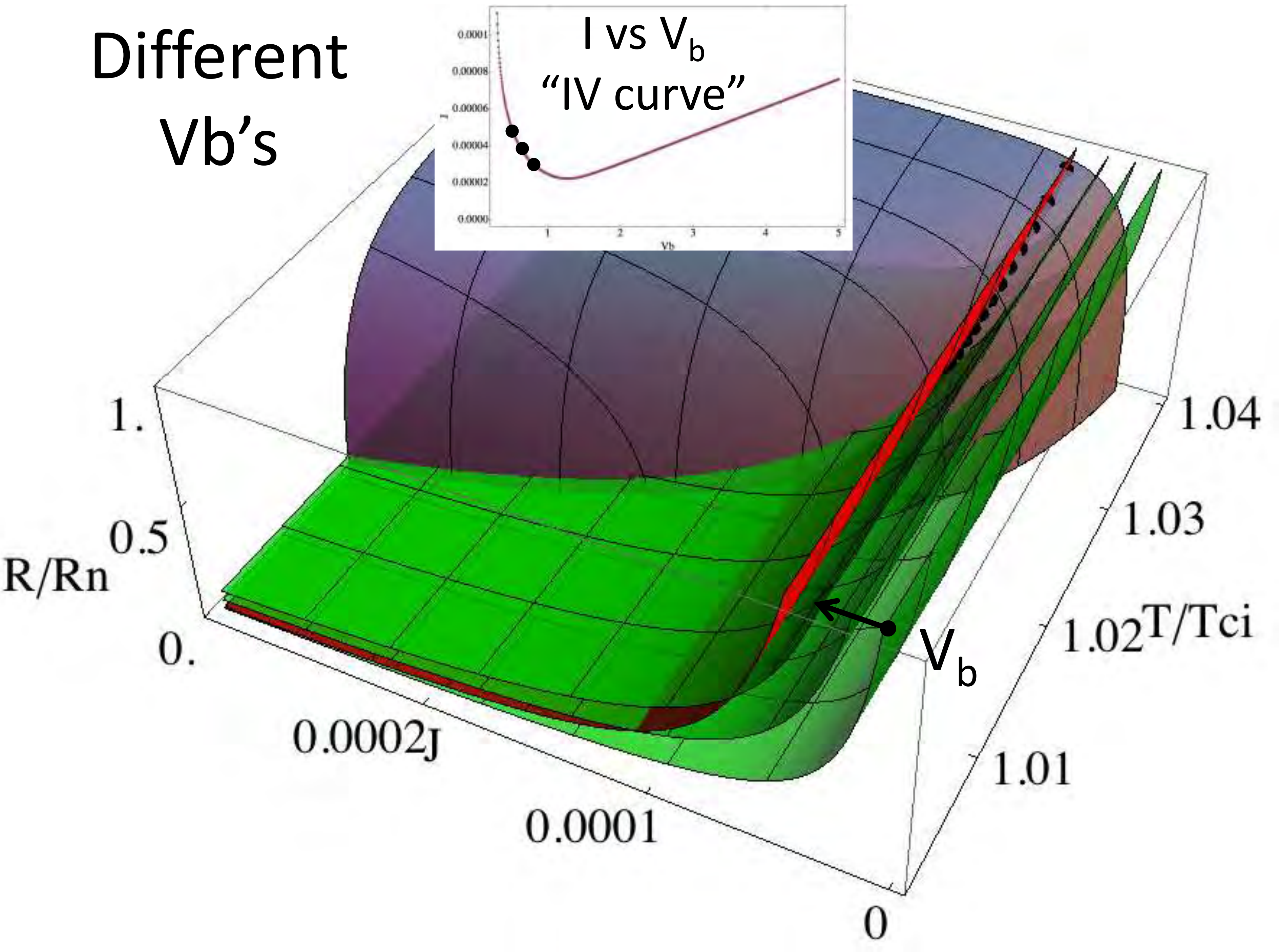
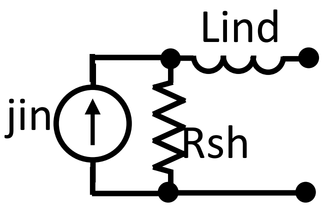
1.01
Electric Surface Eqn


Intersection point
=
“bias point”
R/Rn
1.
0.5
0.
0.0002
J
0.0001
0
1.04
1.03
1.02
T/Tci
1.01

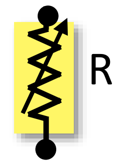
Different Vb's



Bias Circuit



TES

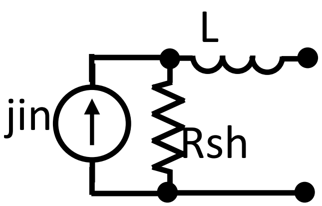


Separable?

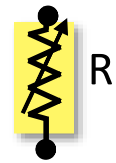
Rsh , L

R(T,j,B)

Bias Circuit



TES



Separable?

Rsh , L

R(T,j,B)

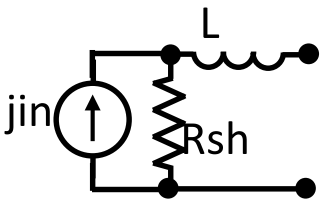
Does changing Rsh and L have any direct impact on the TESs R(T,j,B)?

Of course changing Rsh and L impacts:

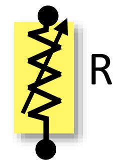
- bias stability conditions
- time constants
- the jin needed to get the the same R etc.

Can we separate the TES transition from the bias circuit?

Bias Circuit



TES



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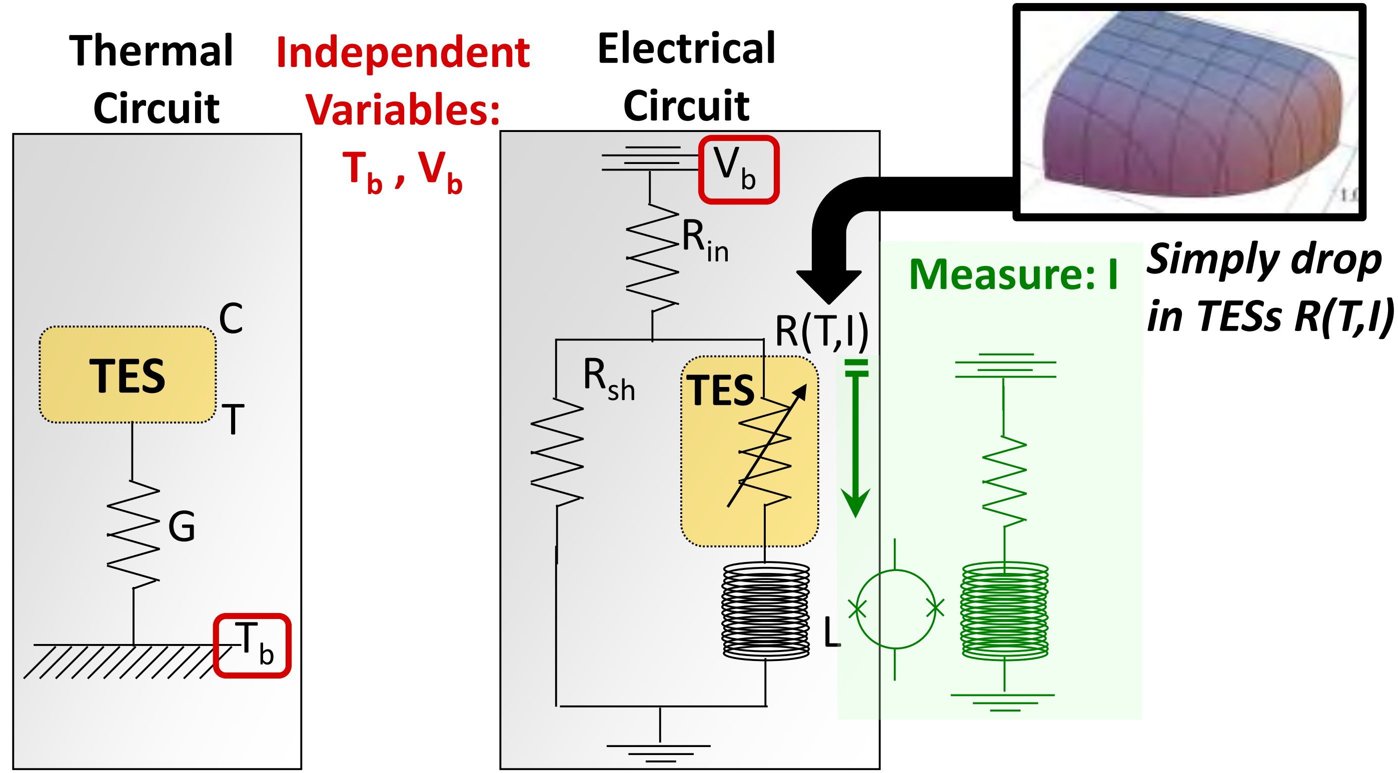
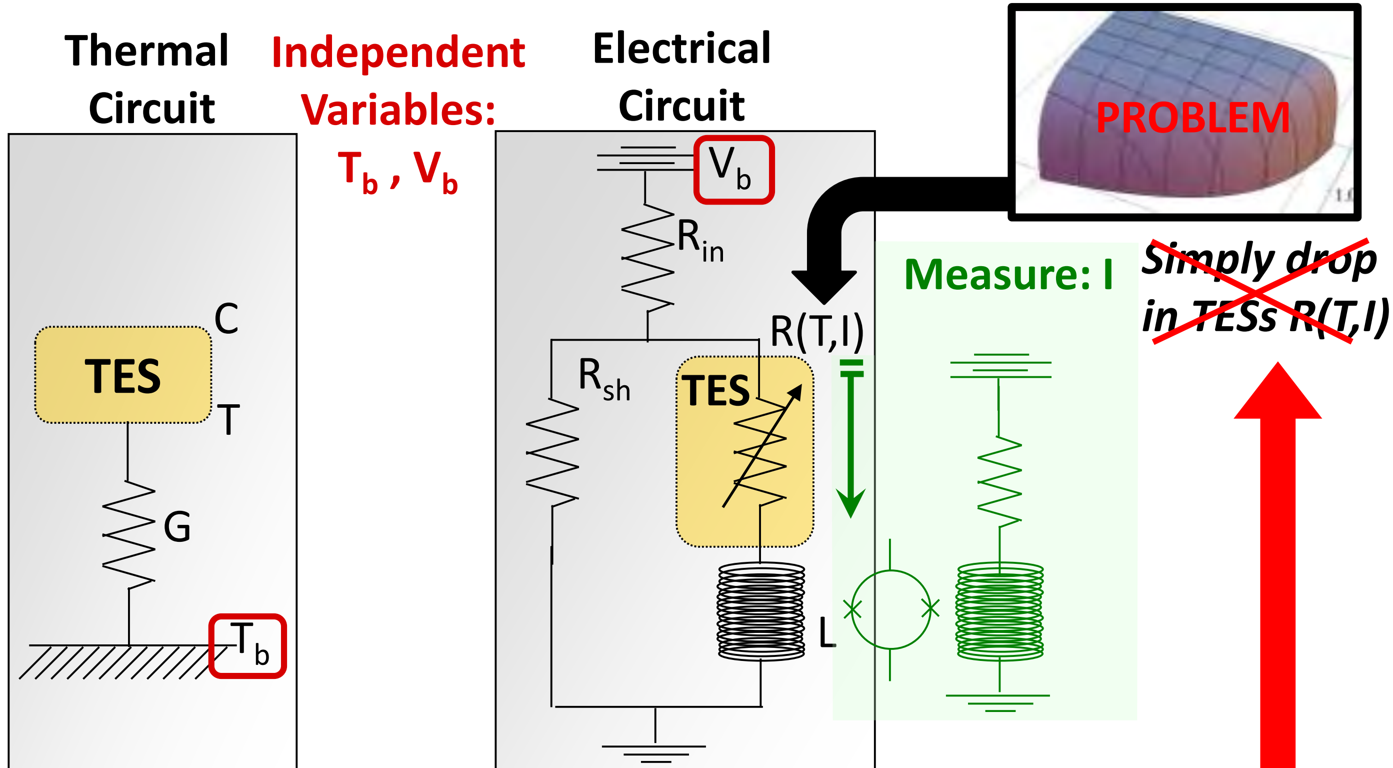
Can we separate the TES transition from the bias circuit?

ANSWER: generally NO.



ELECTROTHERMAL MODEL



ELECTROTHERMAL MODEL

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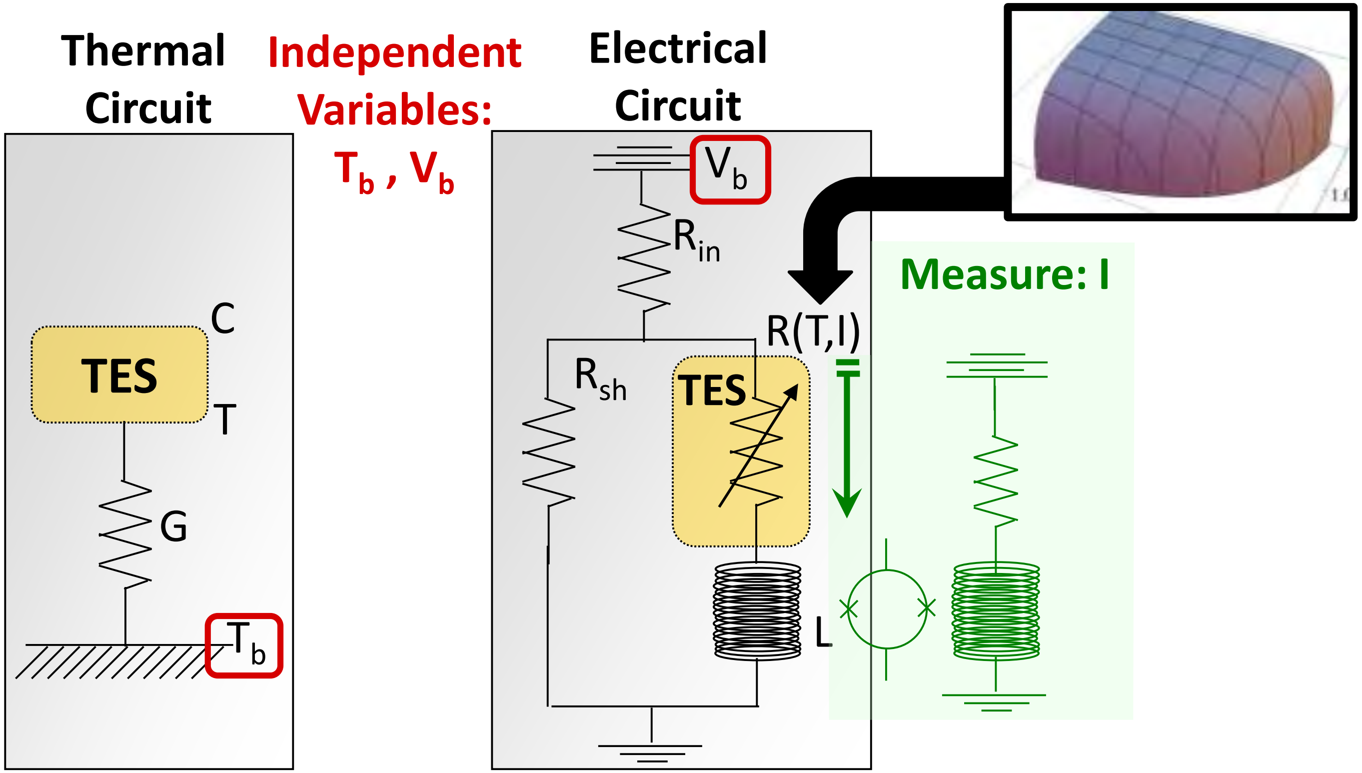
ANSWER: generally NO.



ELECTROTHERMAL MODEL

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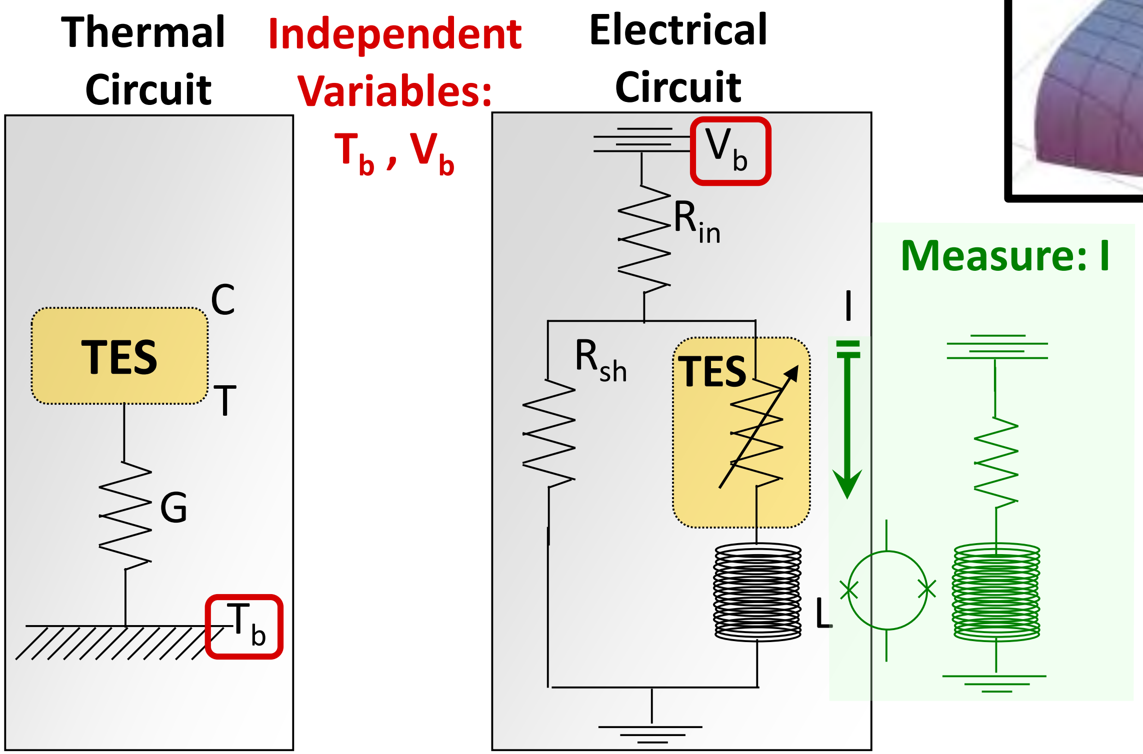
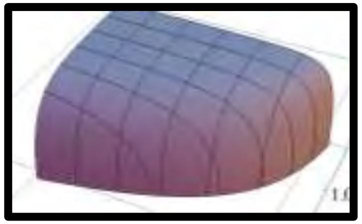




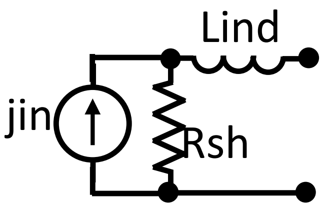
R(T,I,L,Rsh)

ELECTROTHERMAL MODEL

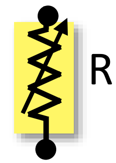
Can we separate the TES transition from the bias circuit?

ANSWER: generally NO.

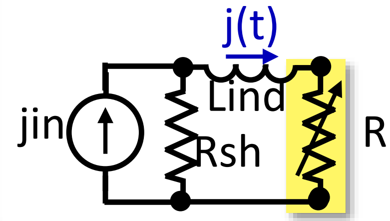
Bias Circuit



TES



Bias Circuit + TES



(1) j(t)=?

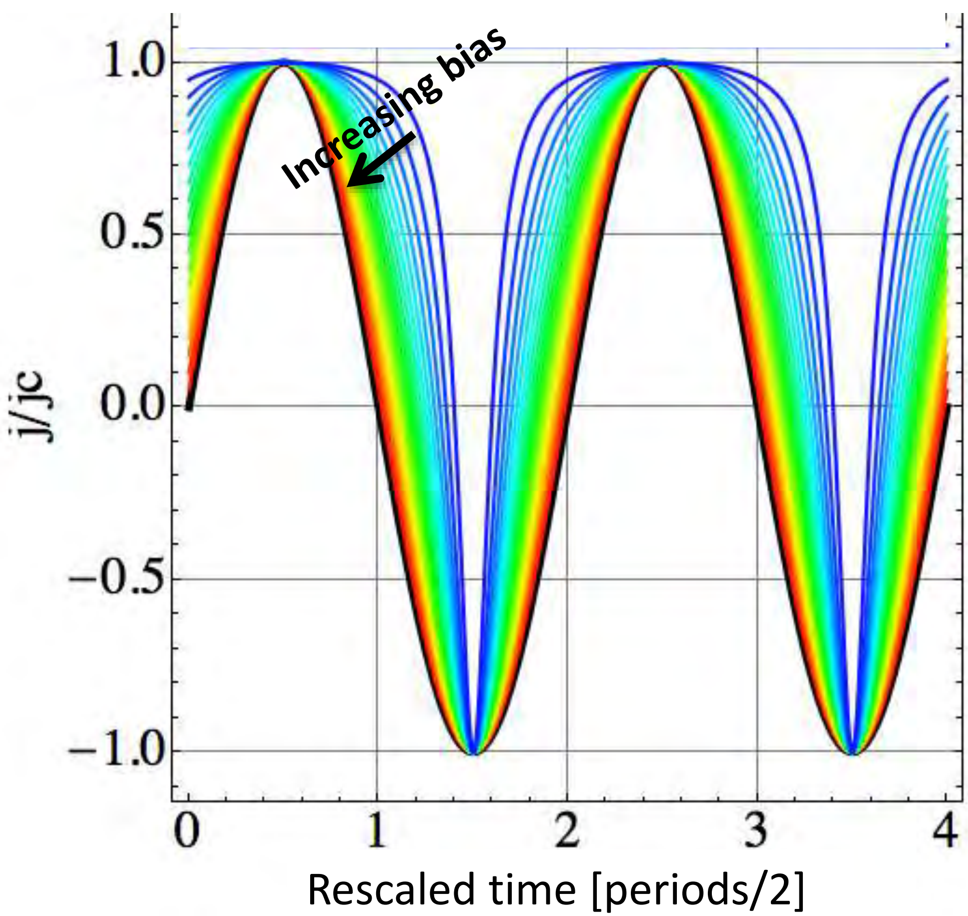
What is the time dependence of the current j?

(2) <j(t)> vs jin =?

What is the shape of the time averaged IV curve?

J-TES model

Time rescaled so we can compare shape in time (harmonic content).

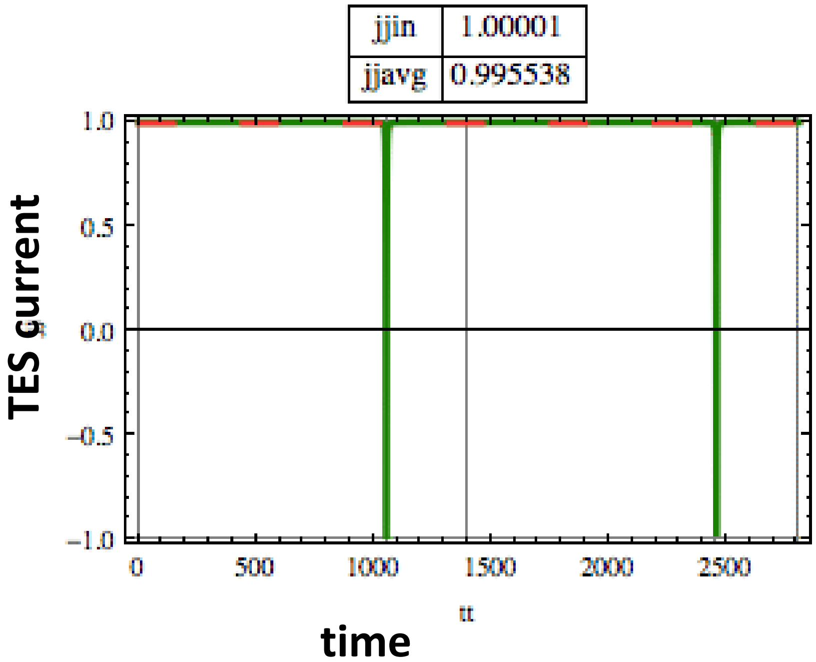


L = 0 limit

Increasing bias jjin=jin/jc

J-TES model

L = 0 limit



Green: TES current versus time

Red Dashed: time averaged TES current

Movie evolves with increasing bias current jjin.

Movie rescales time as the bias is increased so two periods are contained in time

Bias jjin:

<u>Small</u>:

slow spikes (+jc to -jc)

<u>Large</u>:

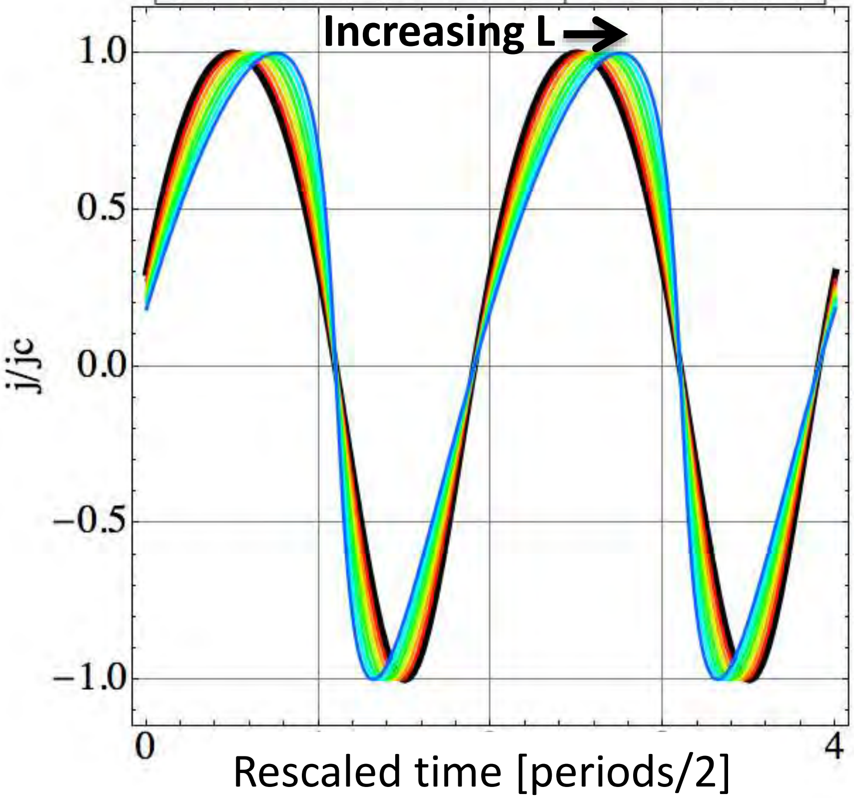
fast sinusoidal (+jc to -jc)

J-TES model
Finite Inductance Effect

**Closed form solution found!*

Large jjin

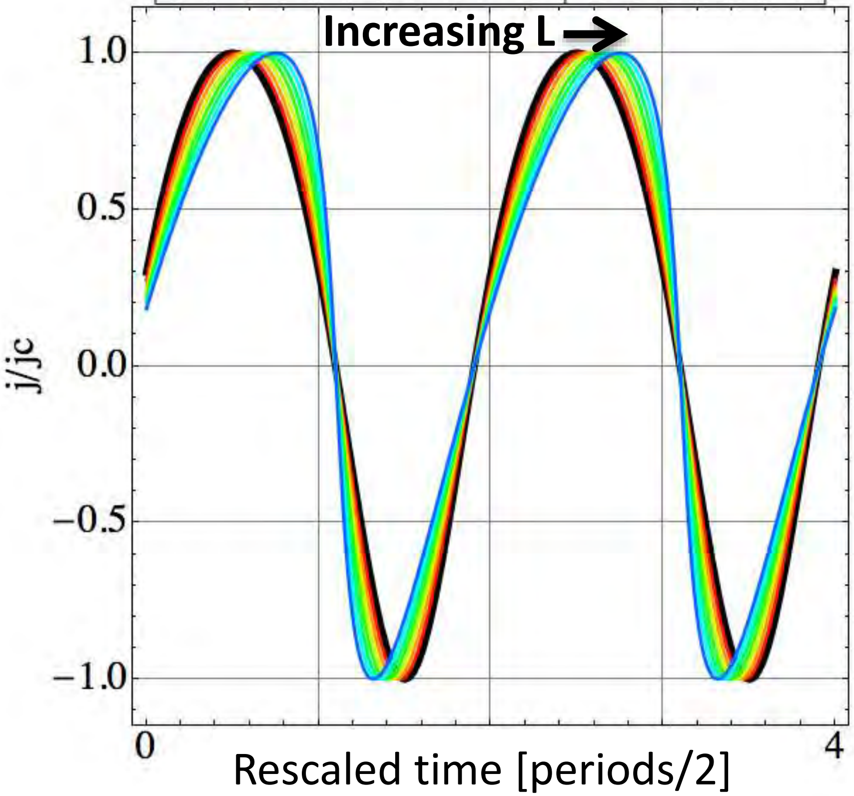
$jjin = \frac{jin}{jc}$	3.33333
{Lmin,Lmax,Lstep}	{0, 0.7, 0.1}



J-TES model
Finite Inductance Effect

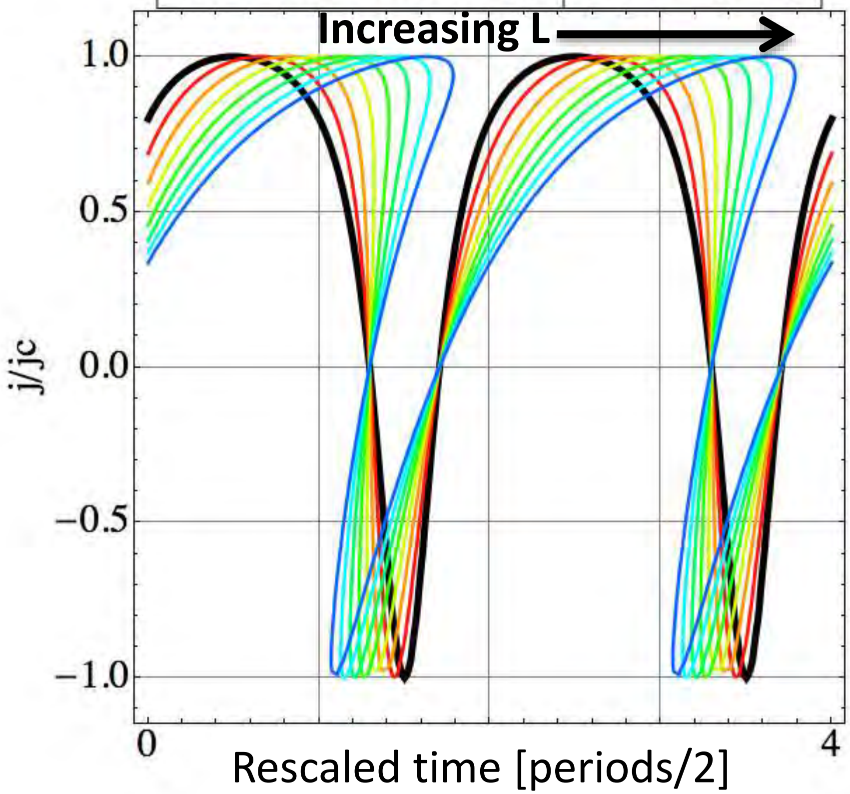
Large jjin

$jjin=\frac{jin}{jc}$	3.33333
{Lmin,Lmax,Lstep}	{0, 0.7, 0.1}



Small jjin

$jjin=\frac{jin}{jc}$	1.25
{Lmin,Lmax,Lstep}	{0, 0.7, 0.1}



J-TES model
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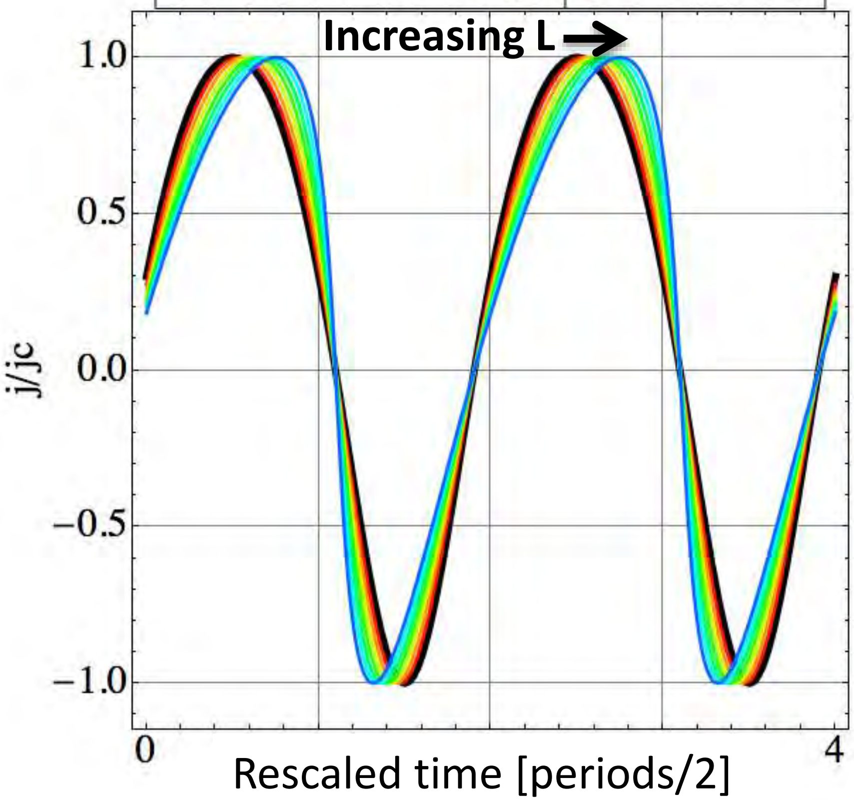

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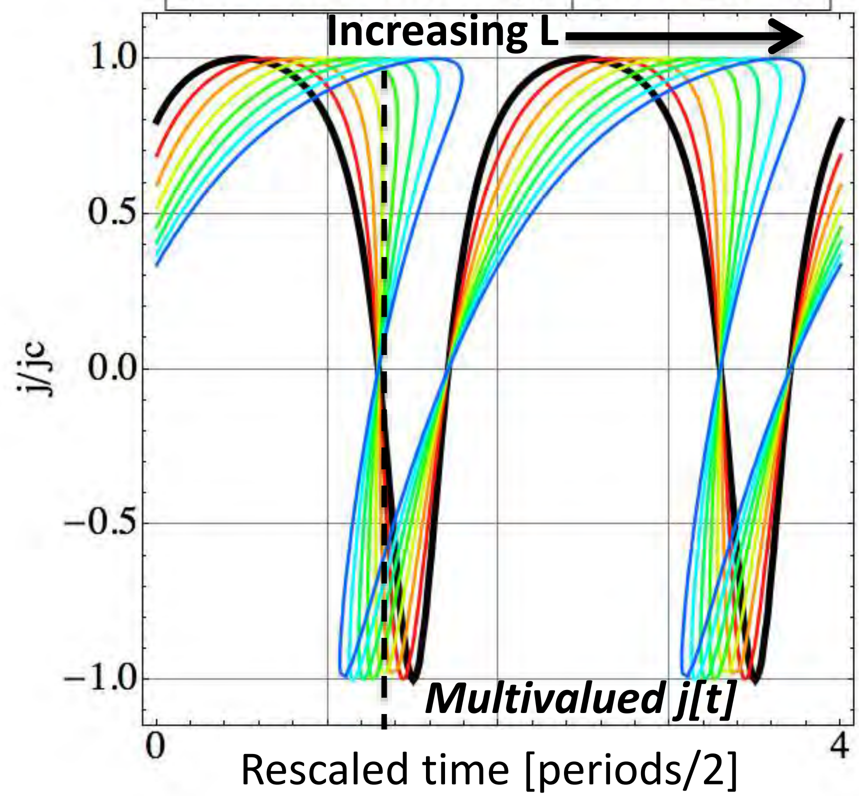
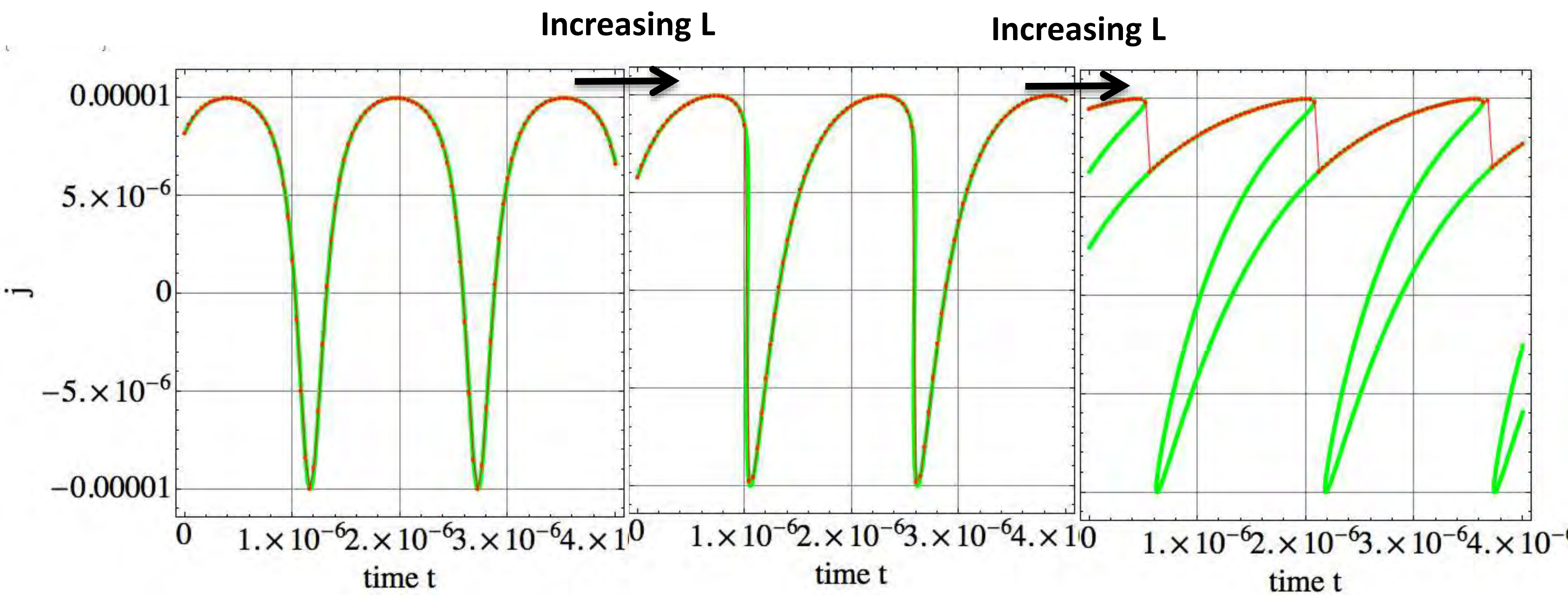


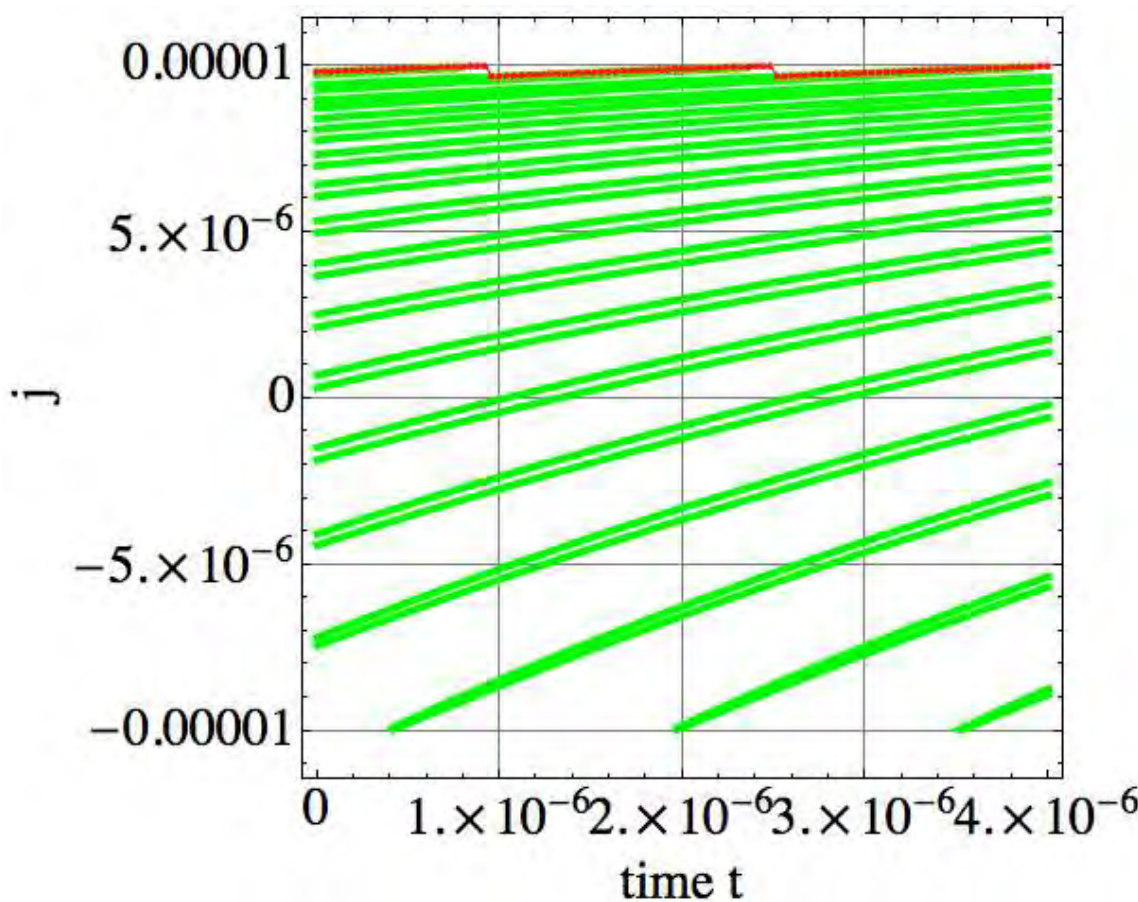
Current becomes multivalued with sufficiently large L



Red: solution

J-TES model
Large L behavior

Large L → ~ Sawtooth j[t]



Allowed current states become very close together in the large L limit.
Current jump between levels
→
2 level system noise

Test my claim

Quantum wave function of the superconducting condensate

$$\Psi = |\Psi|\, e^{i\,\phi(r,t)}$$

First Josephson Equation: $J = J_c \sin \phi(t)$

Second Josephson Equation: $\phi'(t) = 2\pi\, V / \Phi_0$

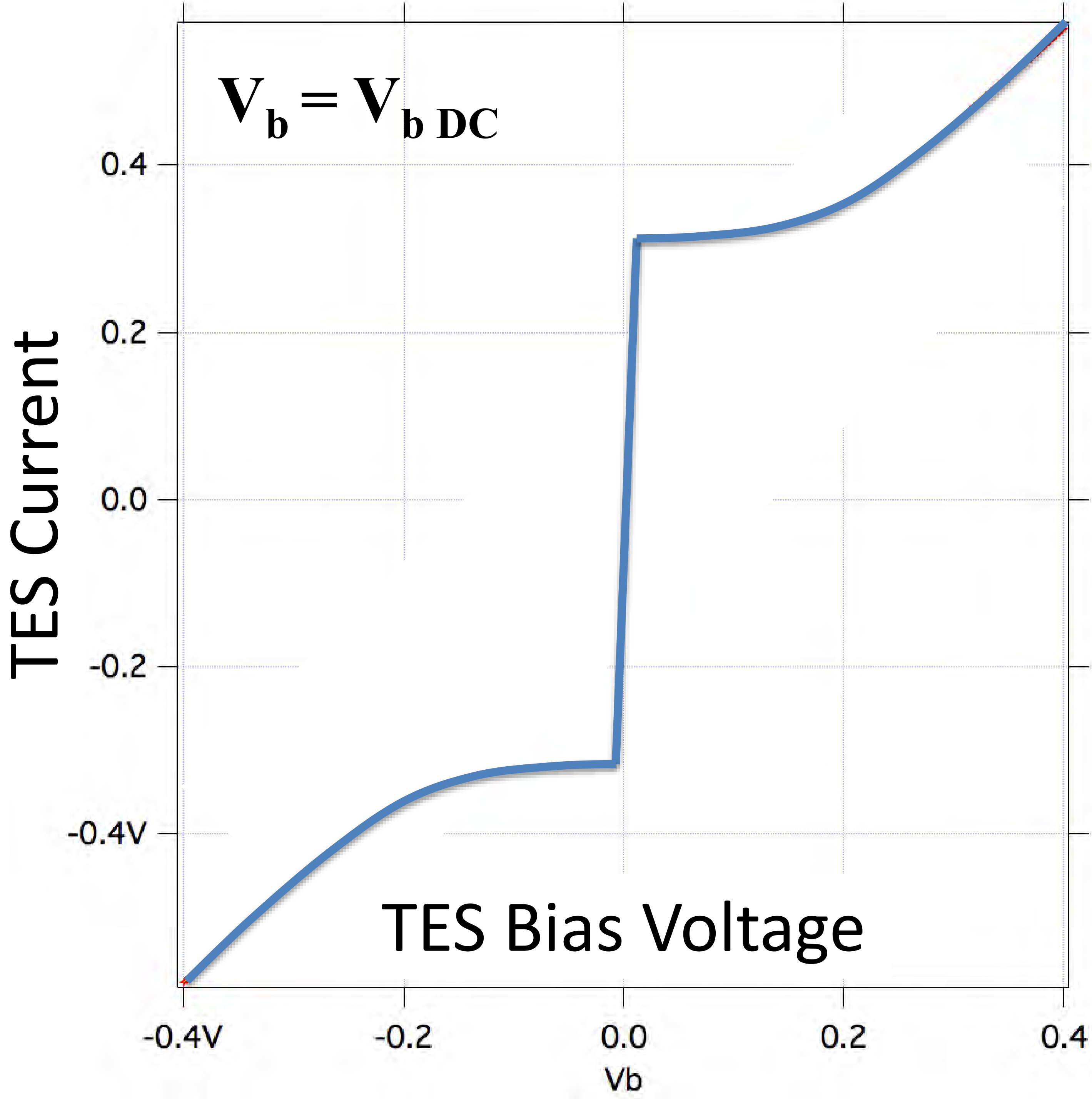
$$V_b = V_{b\,DC} + \underline{v \sin (2\pi\, f\, t)}$$

Shapiro voltage steps: $\underline{V_n = n\;\; \Phi_0\, f}$

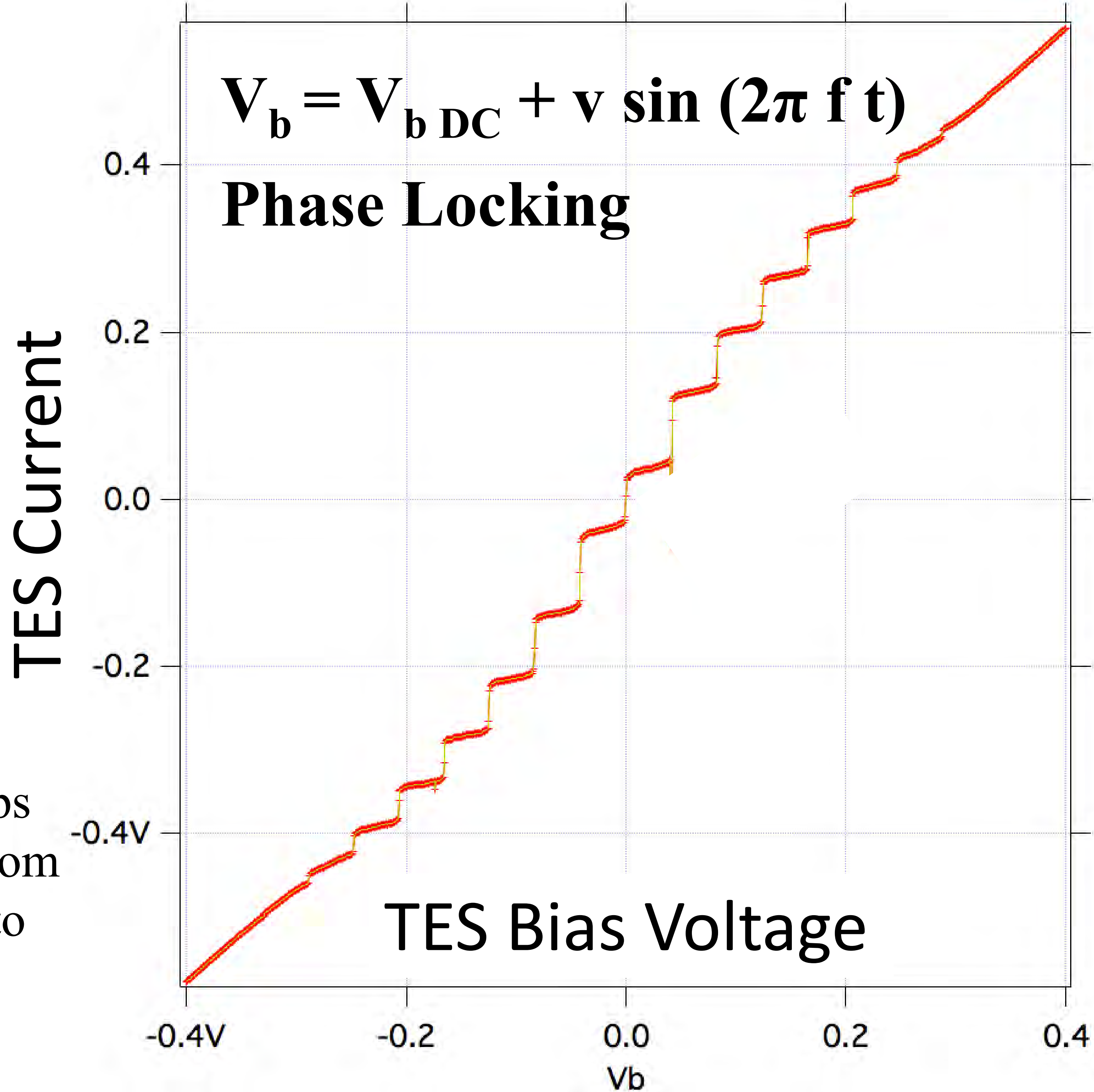
(voltage to frequency transducer)

Fundamental:

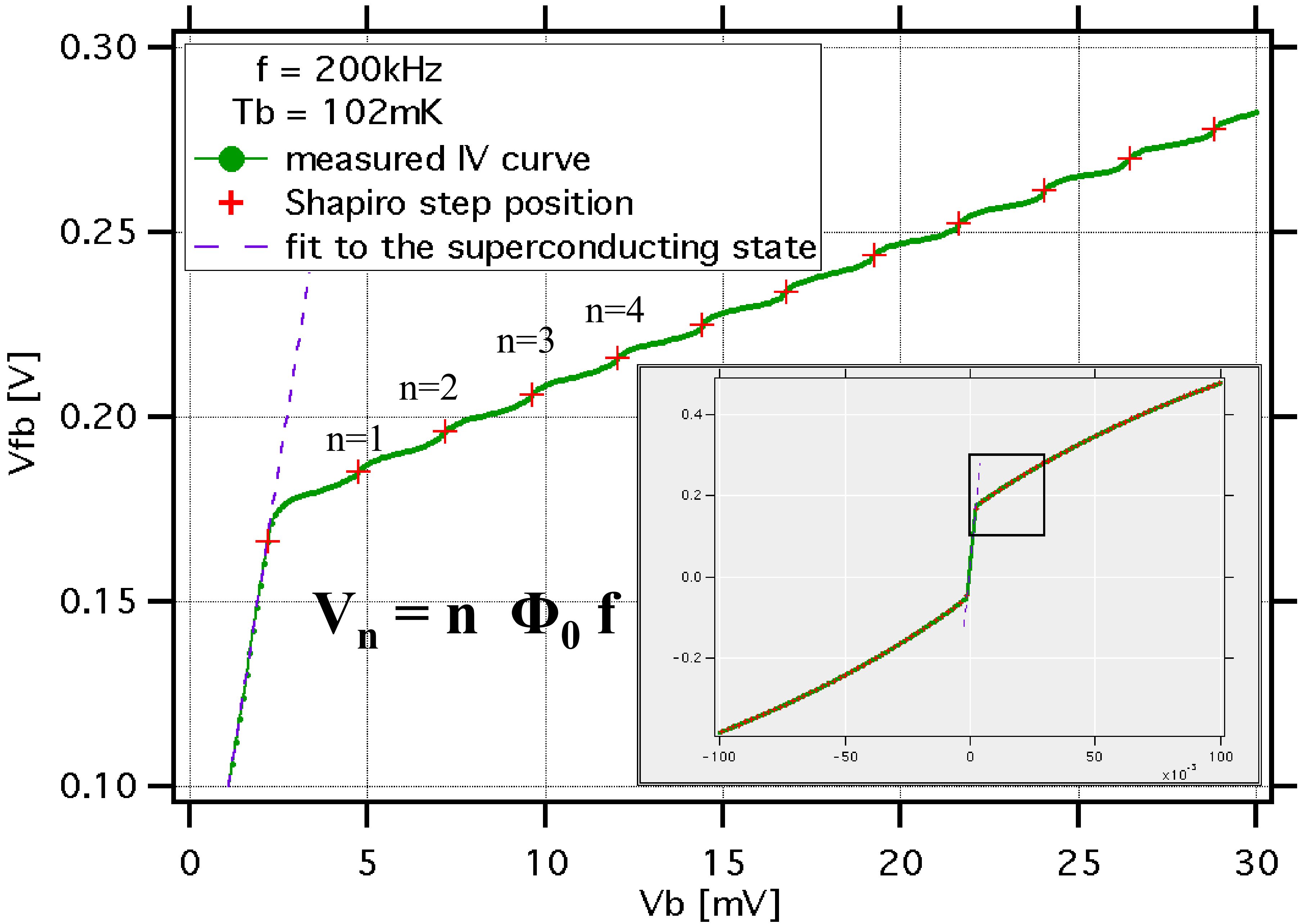
1. Gauge Invariance
2. Energy Conservation.



$V_b = V_{b\ DC}$
TES Current
TES Bias Voltage
0.4
0.2
0.0
-0.2
-0.4V
-0.4V
-0.2
0.0
0.2
0.4
Vb

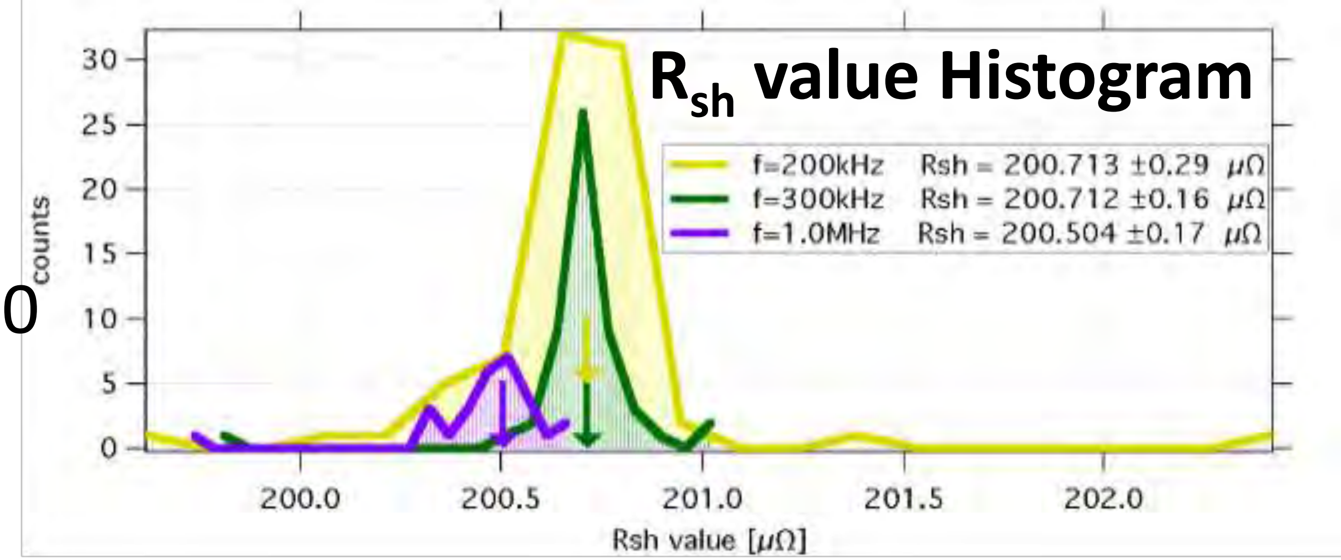


Voltage Steps Observed from f=200 kHz to f=30 MHz

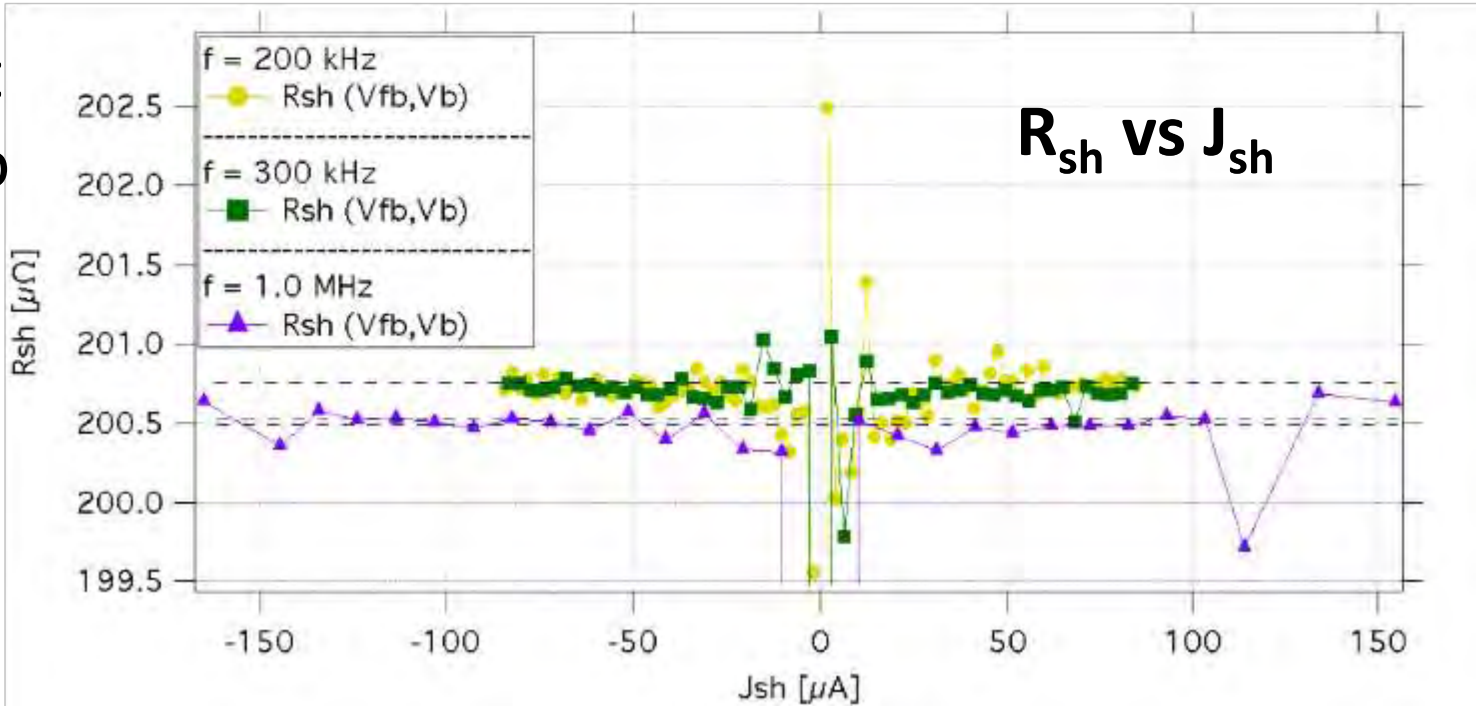


f = 200kHz
Tb = 102mK
measured IV curve
Shapiro step position
fit to the superconducting state
n=1
n=2
n=3
n=4
$V_n = n \Phi_0 f$
Vfb [V]
Vb [mV]
0.30
0.25
0.20
0.15
0.10
0
5
10
15
20
25
30
0.4
0.2
0.0
-0.2
-100
-50
0
50
100
x10^-3

Measuring a 200 µOhm resistor to better than 1/1000 !

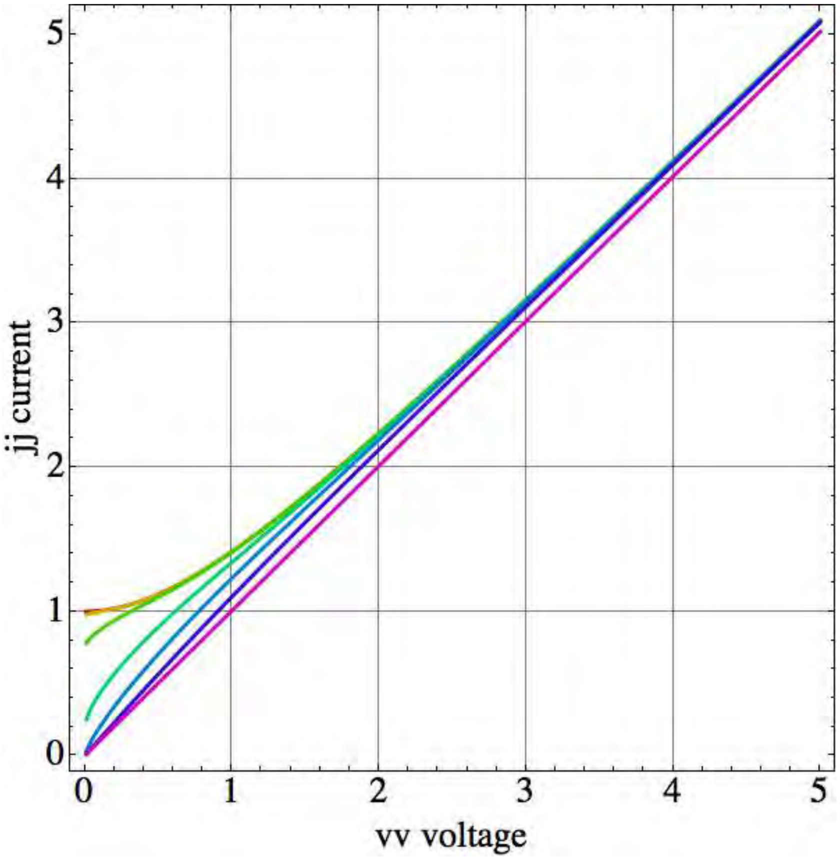


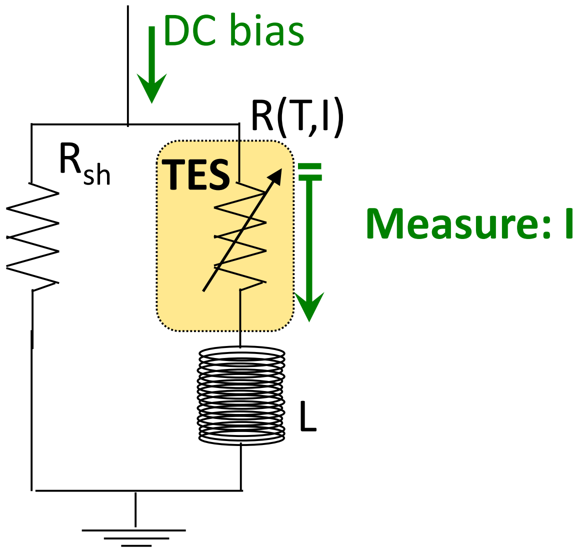
Rsh vs current is *flat* down to 5µA→**Ohmic**!



Time Averaged IV curves
VRSJ-TES model

Time averaged IV curve.

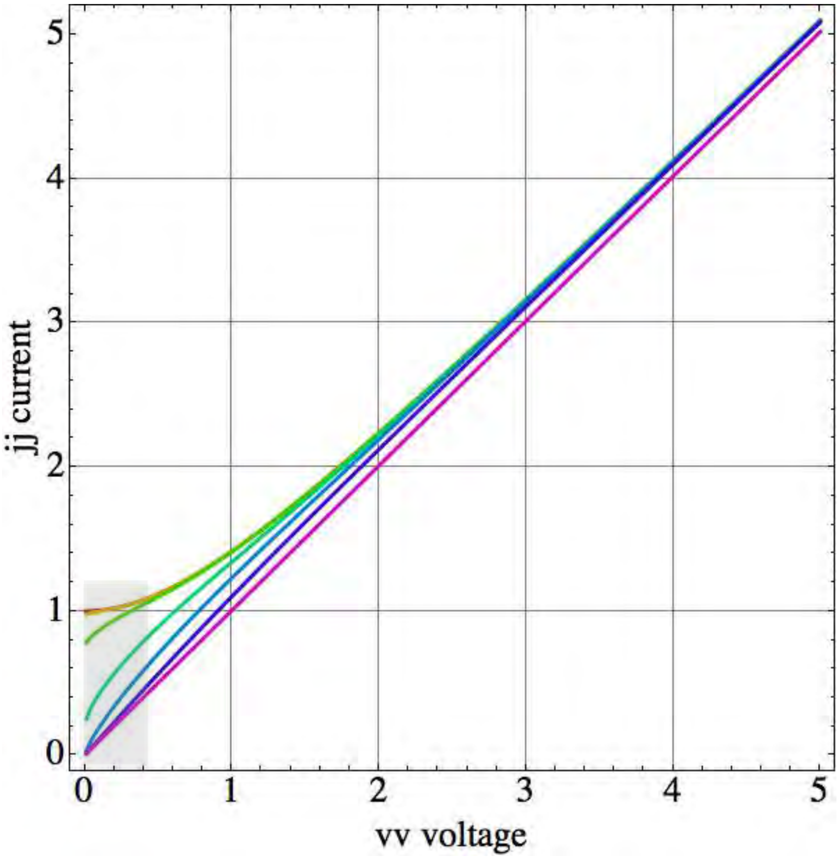


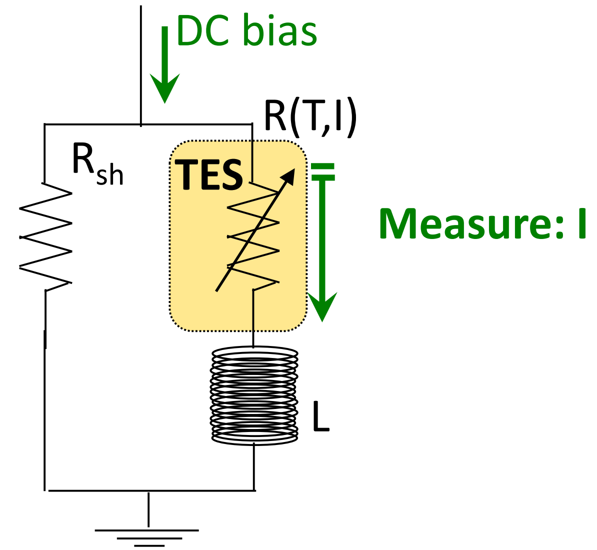


Only changing the circuit Inductor value
Everything else constant.

Time Averaged IV curves
VRSJ-TES model

Time averaged IV curve.





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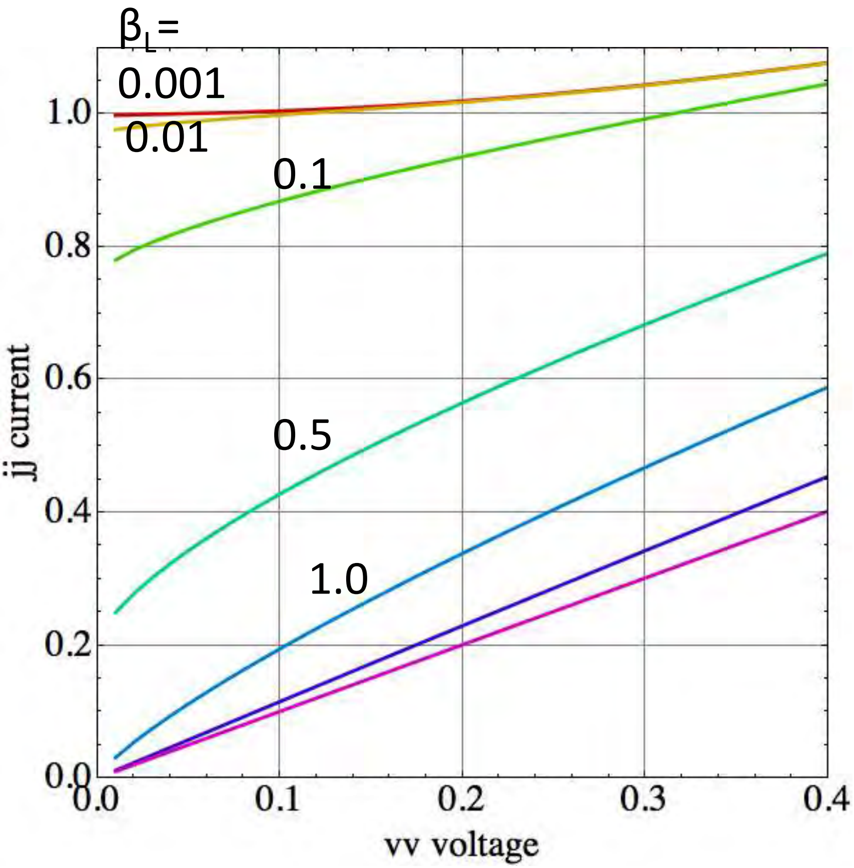
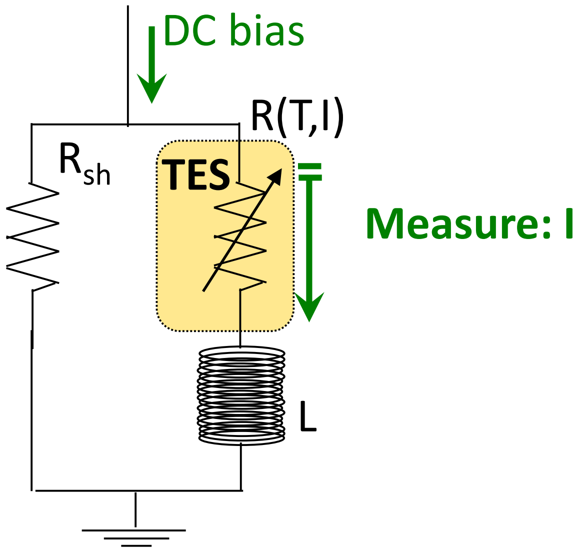
Time Averaged IV curves
VRSJ-TES model

Time averaged IV curve.

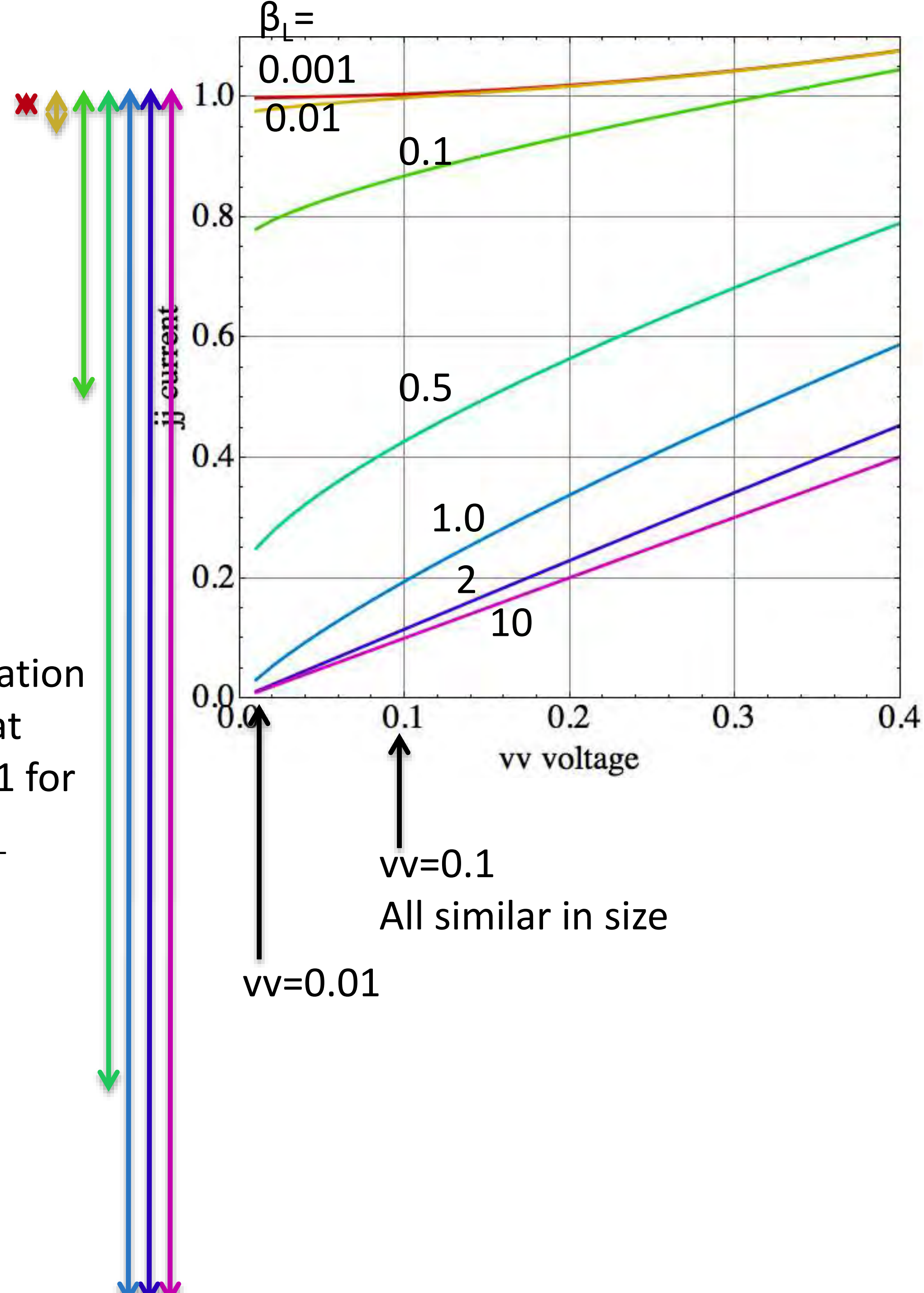




Only changing the circuit Inductor value
Everything else constant.

jj oscillation amplitude

jj oscillation range at vv=0.01 for each β_L value.

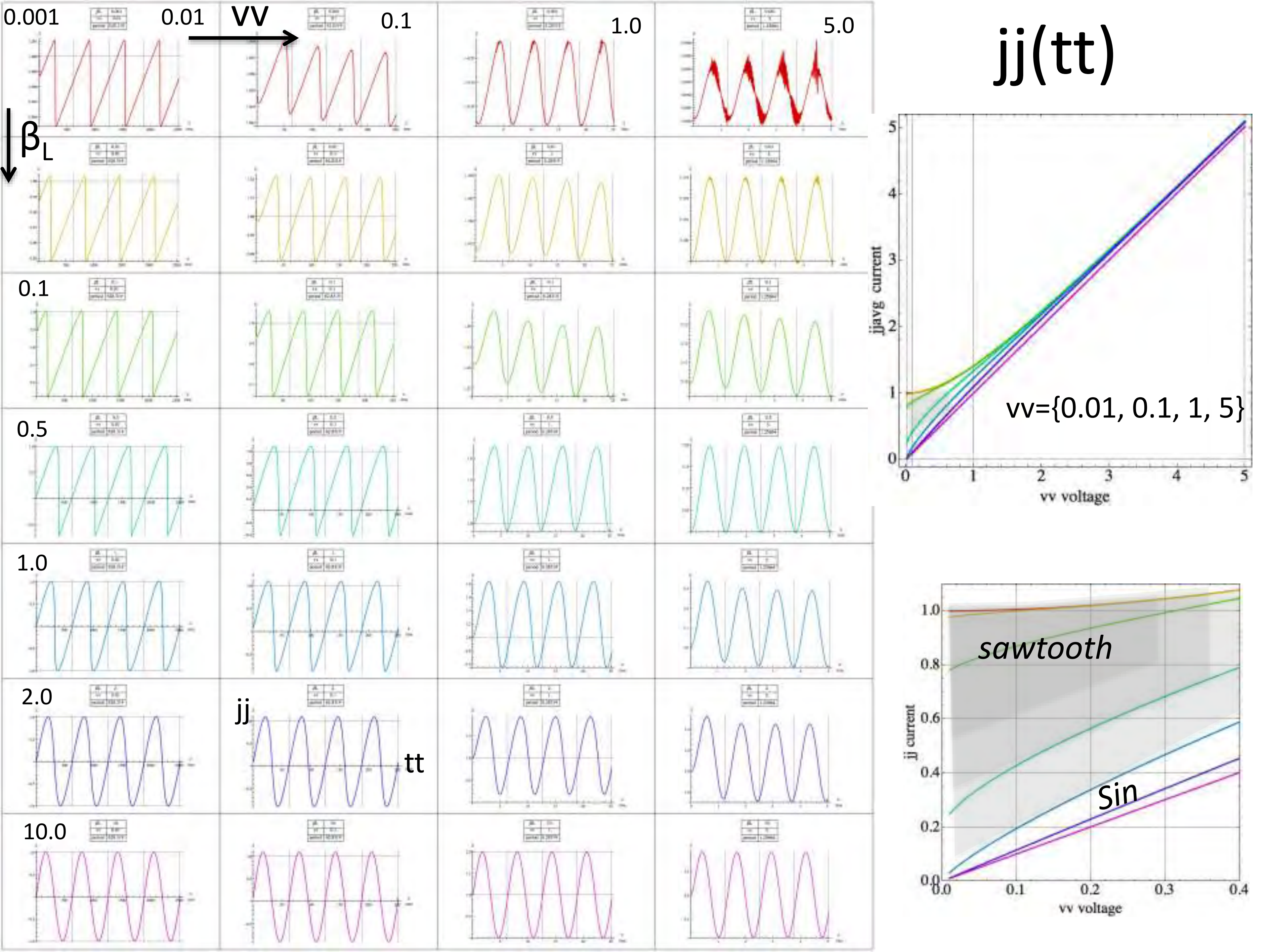


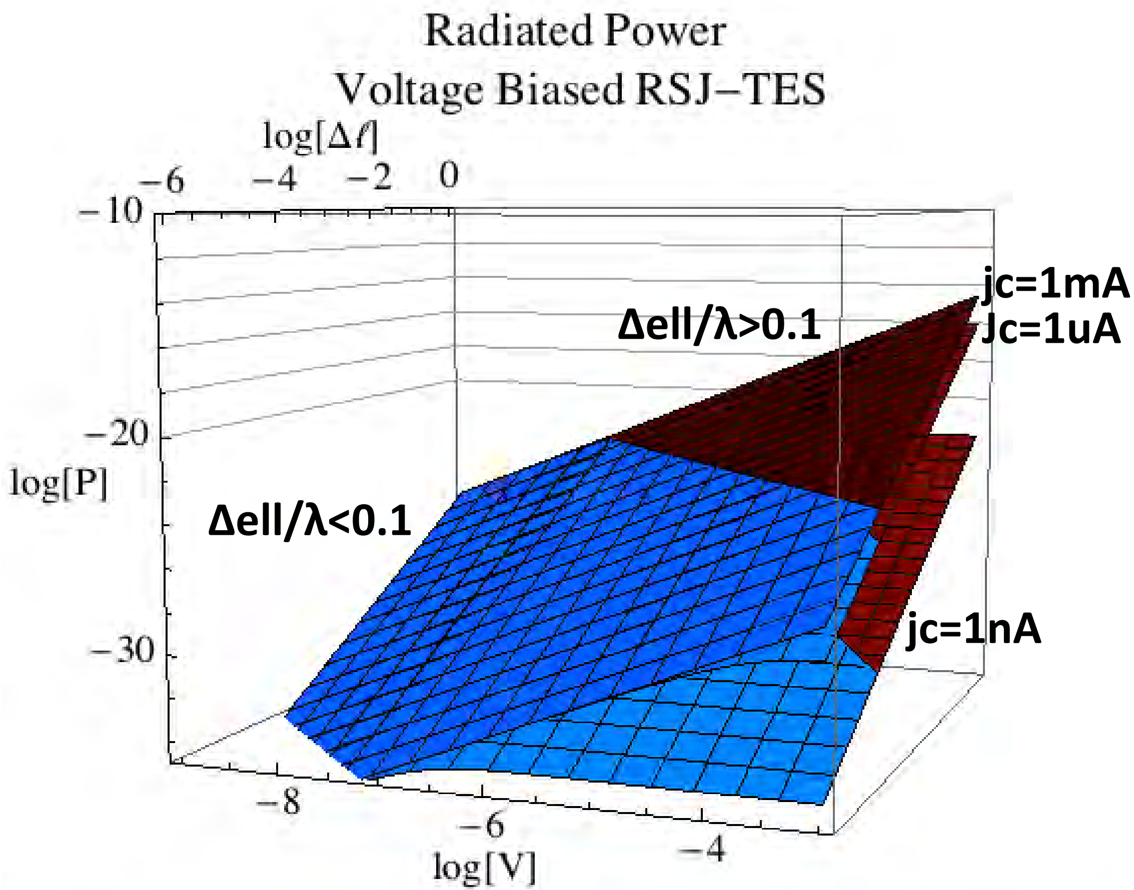
Conclusions: *…Paradigm Shifting*

- In steady state with standard DC biased TES operation. The TES current is wildly oscillating in time at high frequencies and we only measure *(and are only aware of)* the time averaged current in the TES.
- The equations governing this time response is **nonlinear** time-dependent diff eqs.
  - **KEY POINT**: This time dependent current exists when the TES is only DC biased, NO time dependent inputs what so ever. In comes about from the intrinsic superconductivity physics governing TESs.
- The measured resistive transition surface is not simply a function of R(T,J,B). But is also a function of the electric circuit. R(T,J,B,L,Rsh).
  - → Problem compartmentalizing: → *Same TES measured in slightly different circuits will appear to have a different resistive transition surface. TEST in real setup early!!!*
- Circuit and TES parameters can have values such that these time dependent solutions become multivalued in time.
  - *What does that mean?* Can mean excess noise or fine structure of the time averaged resistive transition surface.
- The biased TES waiting to detect a photon (μ-calorimeter) or flux of photons (μ-bolometers) can itself act as a radiation source.
  - Tricky when an array of exquisitely sensitive micro-wave radiation detectors are themselves sources of microwave radiation.
  - This can lead to radiation resistance and fine structure in the resistive transition.
  - Possible cross talk between pixels in an array.
  - The time dependence of the current can take on pure sin waves and also very nonsinusoidal forms (variable harmonic content)
  - The fundamental frequency of these oscillations changes with bias voltage.
  - Makes the prospects of FDM TES arrays with an AC-biased TES challenging. Structure, sensitivity, and cross-talk.

THE

END


jj(tt)
0.001
0.01
VV
0.1
1.0
5.0
βL
0.1
0.5
1.0
2.0
10.0
jj
tt
jjavg current
vv voltage
vv={0.01, 0.1, 1, 5}
sawtooth
Sin
jj current
vv voltage



Rwsh	$\frac{1}{100}$
log[L]	−8
log[jc]{min,max,step}	{−9, −3, 3}

approximating the waveform as a pure sinusoid for the purposes of this

Power for relationship holds for: λ>> antenna length

Blue: Δell/λ<0.1 (satisfied)

Red: Δell/λ>0.1 Actual radiated power is larger than red surface