Center of Mass Estimation for a Spinning Spacecraft Using Doppler Shift of the GPS Carrier Frequency

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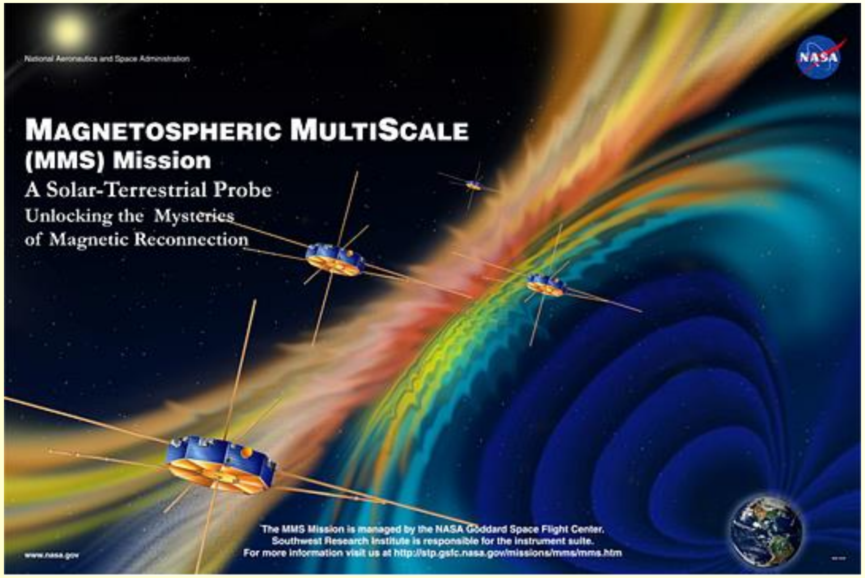


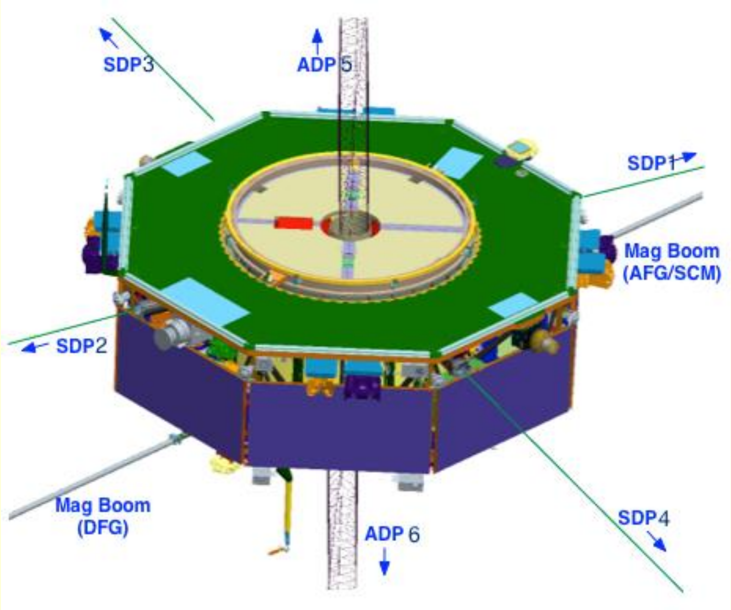
SPACE 2016, AIAA/AAS Astrodynamics Specialist Conference, Long Beach, CA

Abstract

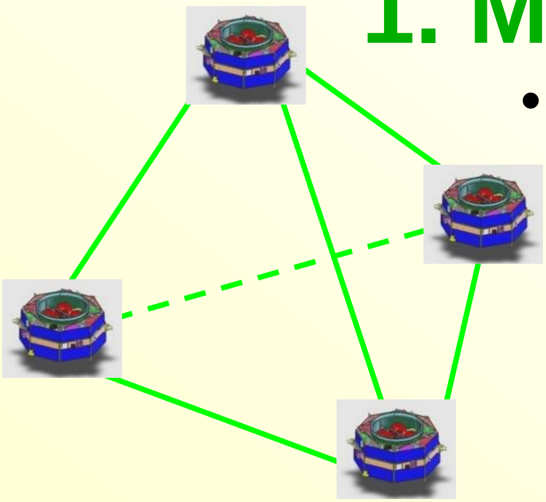
- Improved knowledge of spacecraft Center of Mass (CM) contributes to improved maneuver performance.
  - Error in CM knowledge leads to unneeded torques and angular momentum accumulation that must be unloaded.
- This paper presents a CM filter that uses Doppler shift of the GPS carrier frequency.
- Application to Magnetospheric Multiscale (MMS) mission:
  - Changes in CM and coning angle (that is, the spin axis in the body frame) both depend on mass asymmetry.
  - Comparison of CM estimate and mass asymmetry deduced from coning angle shows agreement, validating the new filter.

Presentation Outline

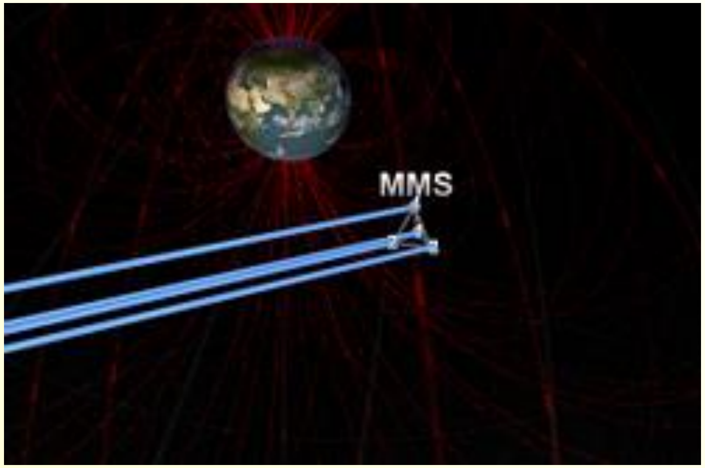
- Overview of Magnetospheric Multiscale (MMS) mission
- Overview of Attitude Ground System (AGS)
- Center of Mass (CM) and Inertia Tensor Calibration
- Discussion of mass asymmetry effects on CM and coning angle
- Results



1. MMS Mission Overview



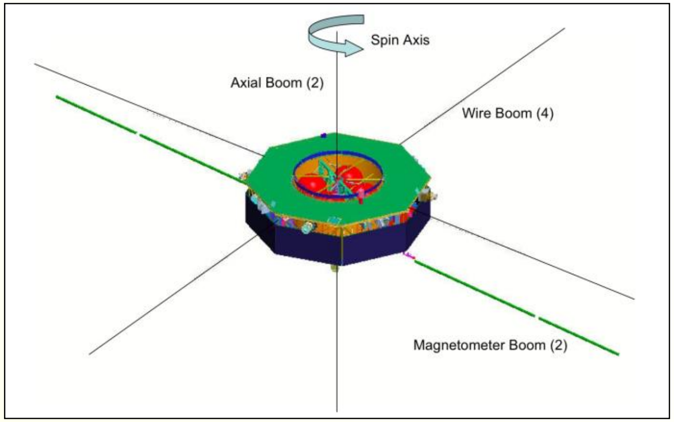
- Magnetospheric Multiscale (MMS) mission
  - Constellation of four identical spacecraft in Earth orbit
  - Tetrahedral formation with variable size (7 to 160 km)
  - CM estimation improves maneuver performance
- Orbit chosen to observe key regions of interest
  - 1.2 x 12 R_E in Phase 1, with apogee on dayside to observe bow shock
  - 1.2 x 25 R_E in Phase 2, with apogee on nightside to observe magnetotail
  - Orbit determined onboard with GPS; four antennas; 12-channel receiver; uses GEONS (see Ref. 1)



Spacecraft Description

Attitude:

Spin axis is near ecliptic pole
(tipped ~3.5 deg toward Sun)



Spin stabilized:

3.1 rpm = 18.6 deg/sec

Core radius: 1.7 m

Two radial mag booms: 5 m

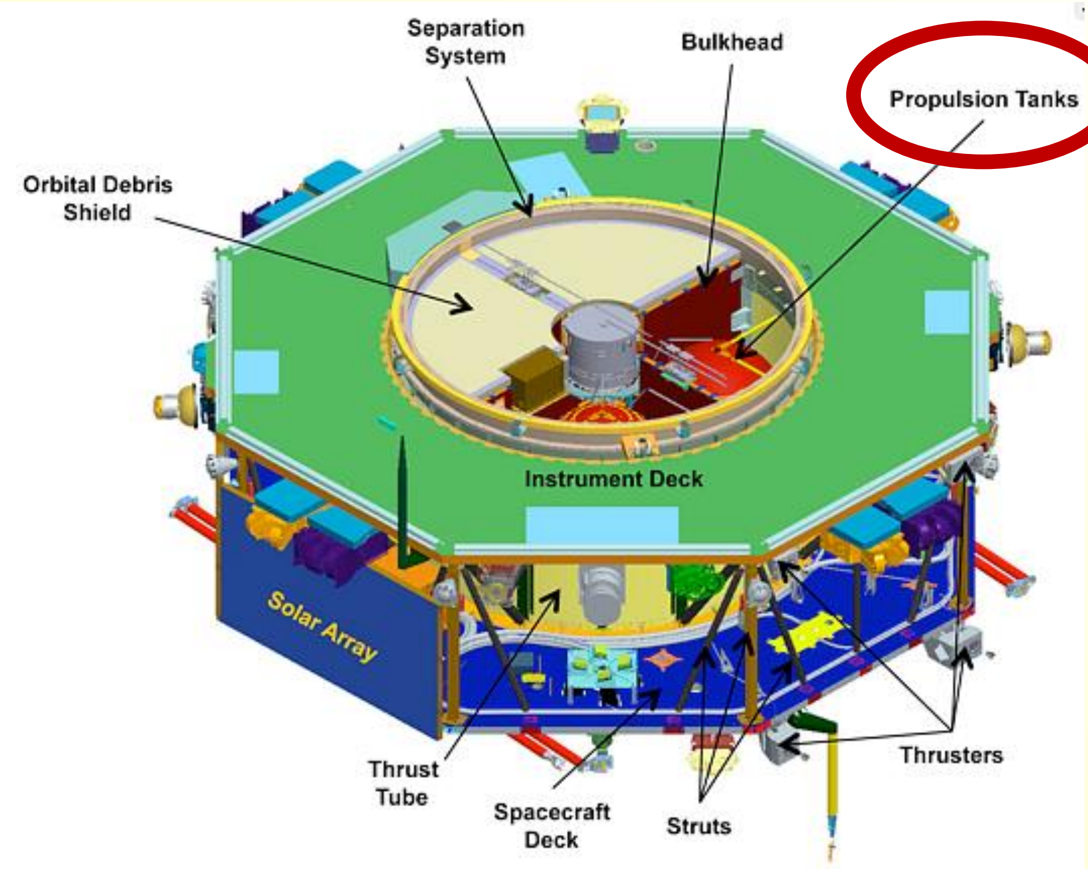
Two axial booms: ~12.5 m

Four wire booms: 60 m

Attitude sensors:

- Autonomous star tracker (4 heads)
  (4 independent quaternion
  measurements at 1 Hz)
- Digital Sun sensor
  (Sun pulse and elevation once per spin)

Fuel Tank Description



- Four hydrazine fuel tanks located symmetrically about spin axis (103 kg/tank)
- Geometric centers of tanks are each 0.468 meter from spin axis, and near the spacecraft CM in the *Z*-direction
- Diaphragms prevent most fuel slosh
  - Two tanks oriented up; two oriented down

2. Attitude Ground System (AGS)

- AGS overview
  - AGS is built using library of attitude software, known as the Multimission Three-Axis Stabilized Spacecraft (MTASS) library
  - AGS includes MTASS core utilities plus mission-specific code built using the MTASS library
  - MTASS AGS has been used for more than 30 previous missions at NASA/Goddard Space Flight Center
  - Originally in FORTRAN on mainframe; now entirely in MATLAB
- AGS functionality for MMS mission
  - **Generate** definitive attitude and angular momentum history
  - **Predict** spin axis precession and select targets for maneuvers
  - **Calibrate** attitude sensors, accelerometer, CM, and inertia tensor
  - **Validate** onboard estimates of attitude and angular momentum vector

3. CM and Inertia Tensor Calibration

- CM and inertia tensor are determined first from analytic models
  - These models use prelaunch measurements with booms stowed, combined with measurements of the separate boom inertias
  - Models interpolate on remaining fuel fraction
    - Assume fuel usage is symmetric among the four tanks
  - AGS corrects inertia tensor for asymmetry (coning angle correction)
    - (See Ref. 2 for description of inertia tensor calibration)
    - Coning angle is the angle between body *Z*-axis and major principal axis (MPA)
      - MPA is the eigenvector of inertia tensor having largest eigenvalue
      - MPA is direction of spin vector and angular momentum vector in body frame after all nutation and boom vibrations have damped out
  - CM was not corrected for mass asymmetry; new CM filter adds this capability

CM Filter: Fractional Doppler Shift

- Spacecraft relative velocity (MMS to GPS) causes a baseline Doppler shift of the GPS carrier frequency, *f*
- MMS spin adds a small extra velocity ($\boldsymbol{V}_{ant} = \boldsymbol{V}_{MMS} + \omega\times\boldsymbol{r}_{CM}$)
  - Where $\boldsymbol{r}_{CM}$ is vector from CM to GPS antenna on MMS
  - Spin causes oscillation in Doppler shift on top of baseline shift

$$f' = \gamma\left(1 - \frac{\vec{V}\cdot\vec{R}}{c\left|\vec{R}\right|}\right)f$$

- Amplitude of that oscillation is proportional to true $\boldsymbol{r}_{CM}$
- Fractional Doppler shift, D, is the effective measurement for the CM filter

$$D \equiv \frac{f' - f}{f} \approx -\frac{\vec{V}\cdot\vec{R}}{c\left|\vec{R}\right|}$$

CM Filter: Sensitivity Matrix

- Define the Doppler residual to be difference between observed and predicted values for fractional shift, D

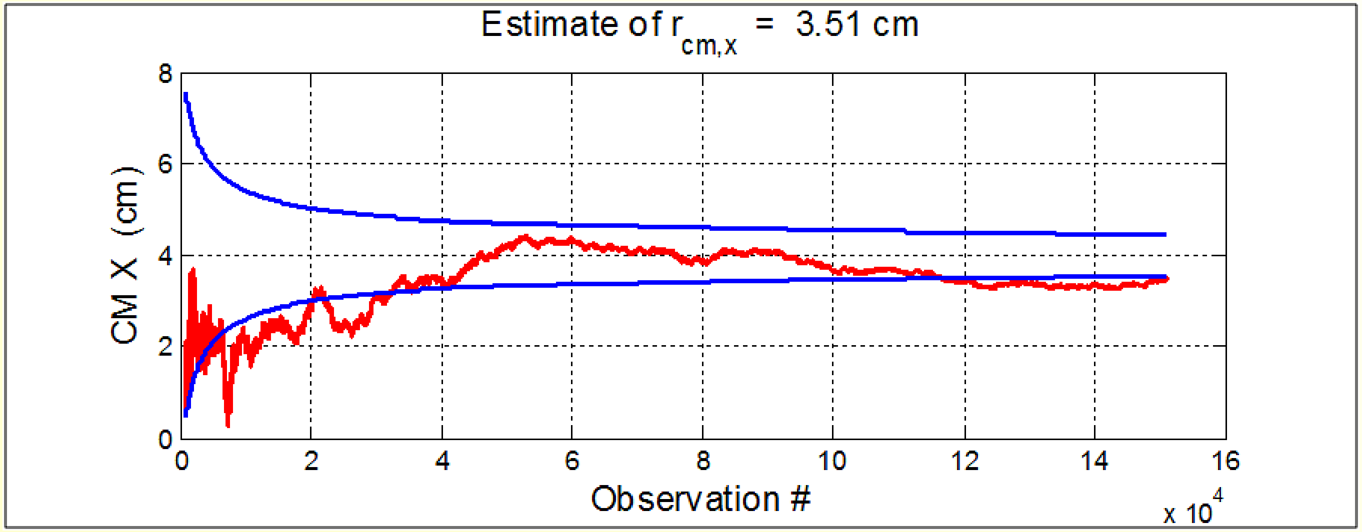
$$D \equiv \frac{f' - f}{f} \approx -\frac{\vec{V} \cdot \vec{R}}{c\left|\vec{R}\right|}$$

- Doppler residual is proportional to the CM correction
  - Solve for this CM correction using a sequential batch-least-squares filter
- Key part of the filter is the sensitivity matrix, H
  - H is the partial derivative of the measurement with respect to the state vector, $\boldsymbol{r}_{CM}$

$$H \equiv \frac{\partial D}{\partial \vec{r}_{CM}} \approx \frac{-1}{c\Delta R}\left[\Delta\vec{R}^T[\vec{\omega}\,\times] + \Delta\vec{V}^T - \left(\Delta\vec{V} \cdot \Delta\hat{R}\right)\Delta\hat{R}^T\right]$$

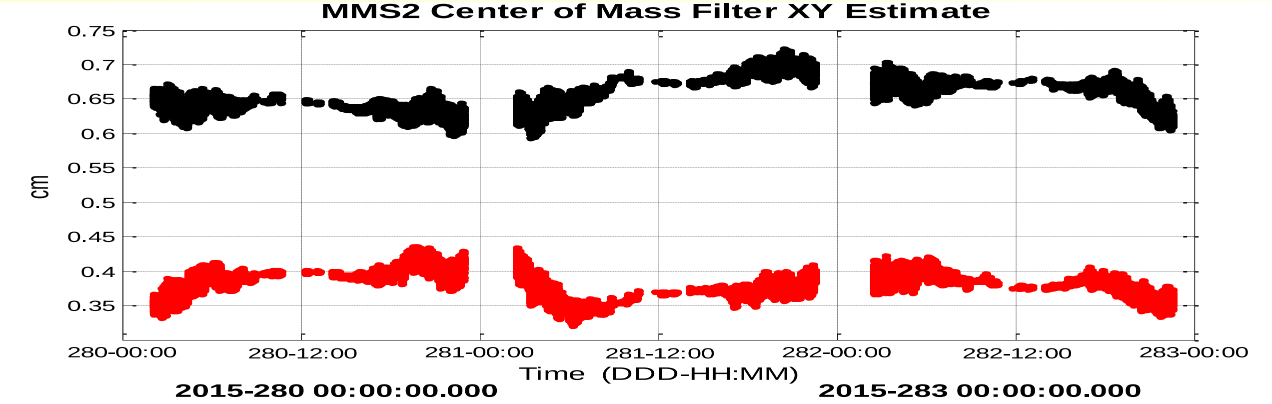
CM Filter: Test Results

- Assume avg. of 7 GPS vehicles tracked for 6 hrs at 1 Hz
- Error in fractional Doppler shift, D:
  zero-mean, Gaussian-distributed, white noise of 10^{-9}
- Test with CM offsets of 4 cm on *X*, and -4 cm on *Y*



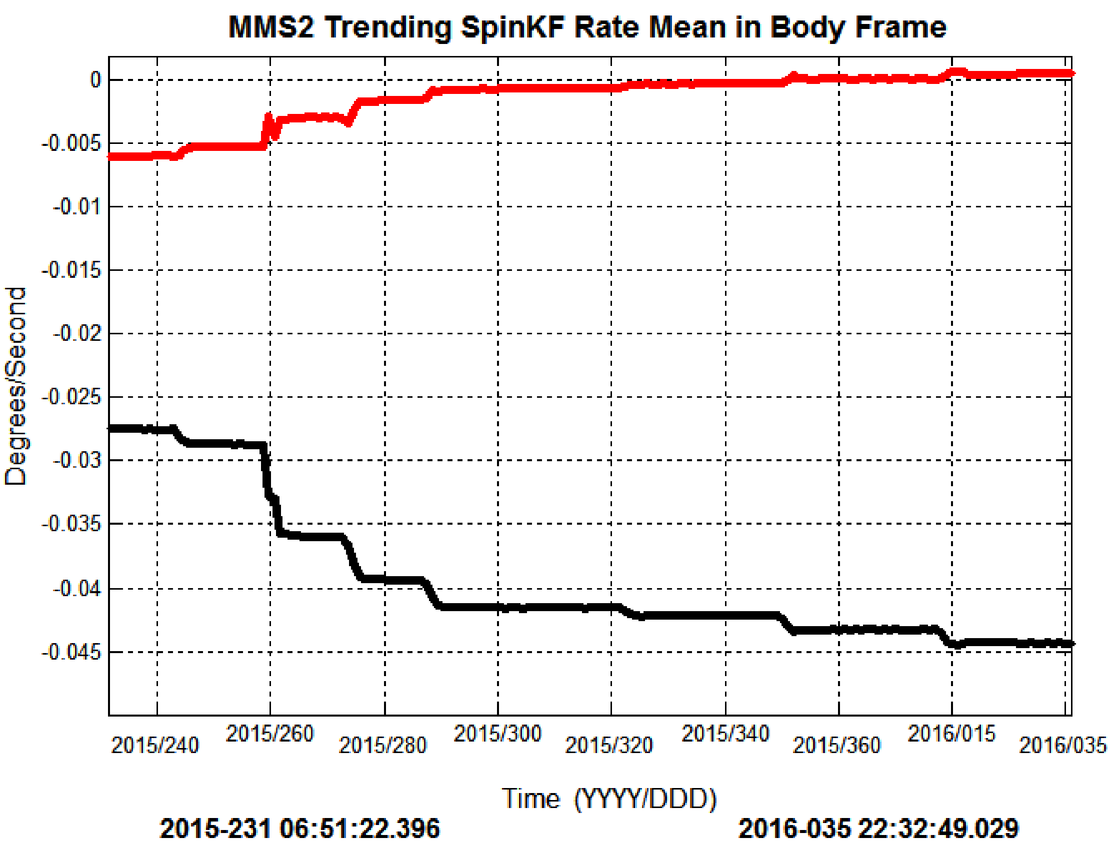
CM Filter: MMS Results

- On MMS mission, Doppler noise was much larger than expected and data frequency much lower
- Needed to improve results by doing the following:
  - Use large multi-orbit datasets
  - Discard points from distant GPS that may have weak signals
  - Discard points near perigee where attitude is perturbed by g-g torque
  - Discard outlying points with a tight sigma-edit test
  - Use unphysical Doppler noise parameter
  - Solve for trend of CM change over several data sets



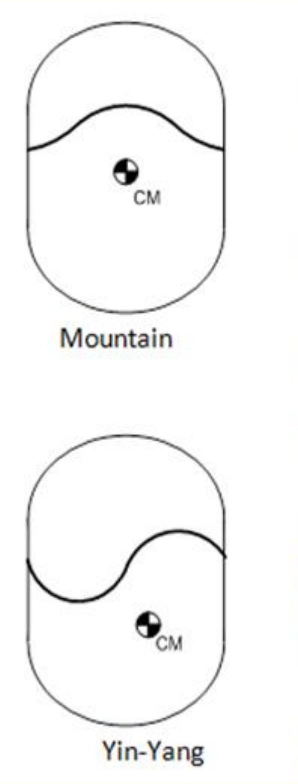
4. Mass Asymmetry & Coning Angle

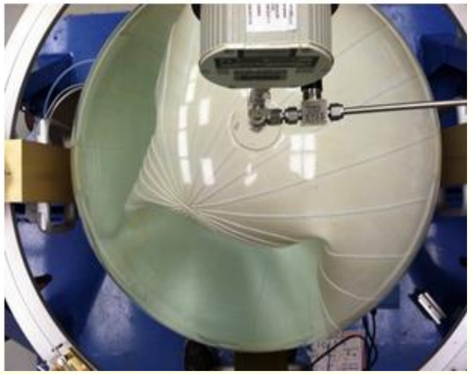
- On MMS, daily definitive attitude and rate estimates show shift of transverse components of spin rate with each burn
- Deviation of mean spin vector from body *Z*-axis is the coning angle
- Change of coning angle can be related to off-diagonal elements of inertia tensor caused by mass asymmetry



Sources of Mass Asymmetry

- Propulsion system is designed to draw fuel equally from all 4 tanks
  - Some small error in fuel usage will accumulate with each burn
  - Tanks are 0.468 m from spin axis
  - Changes in fuel mass occur at locations of the diaphragms
    - Diaphragms are between 0.06 and 0.12 m from spacecraft CM in *Z*-direction
- Mass asymmetry can also come from change in shape of diaphragms
  - Two diaphragms were "mountain" shape and two were "yin-yang" shape at launch (see Ref. 7)





Photos of diaphragms during test tank filling (diaphragm above fuel and below fuel)

Changes in CM and Coning Angle

- If the observed changes in coning angle are due to mass asymmetry, we can model the asymmetry as a shift of mass Δm between fuel tanks
  - Then, CM and inertia tensor both change

$$\Delta CM = 2\Delta y \Delta m / M_{tot} = 0.069 \text{ cm for each kg}$$

$$I_{yz} = 2\Delta m \Delta y \Delta z \approx 0.0842\,\Delta m \;\; \text{kg-m}^2 \quad (\text{with } \Delta m \text{ in kg})$$

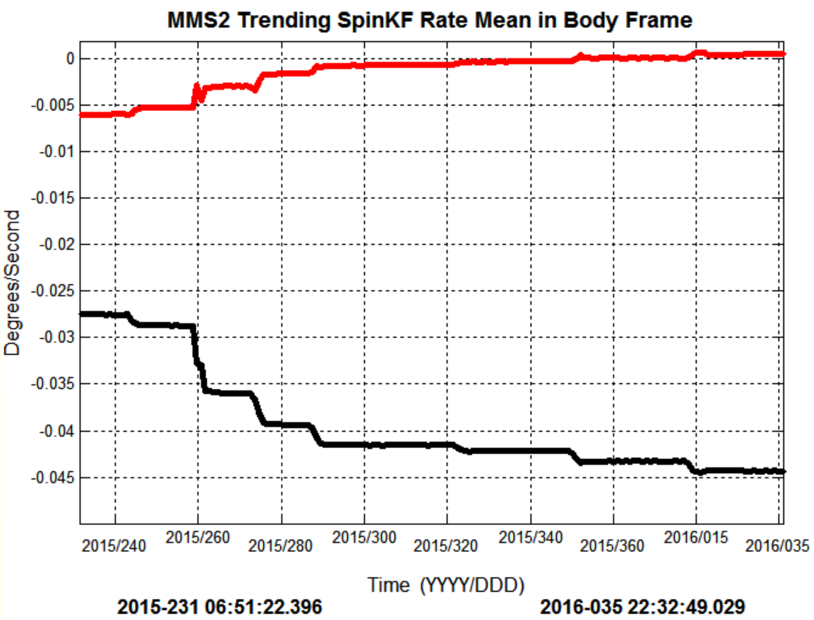
  - Solve for coning angle, ϑ, by diagonalizing inertia tensor (c=cos(ϑ), s=sin(ϑ))

$$\begin{bmatrix} I_t & 0 & 0 \\ 0 & \tilde{I}_t & 0 \\ 0 & 0 & \tilde{I}_z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c & s \\ 0 & -s & c \end{bmatrix}^T \begin{bmatrix} I_t & 0 & 0 \\ 0 & I_t & I_{yz} \\ 0 & I_{yz} & I_z \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & c & s \\ 0 & -s & c \end{bmatrix}$$

  - Result is $\vartheta \approx \dfrac{I_{yz}}{I_Z - I_t}$
  - So, CM shift and coning angle are both proportional to mass asymmetry

5. Results

- Solved for CM at time spans between maneuvers using new filter
- Obtained changes in CM for each maneuver
- Solved for overall CM change by linear fit over all maneuvers
- Result was -0.04 cm in *X*-direction and -0.16 cm in *Y*-direction
- Transverse rates changed by 0.016 deg/sec (see plot)
- Spin rate is 18.6 deg/sec, so this is a coning angle change of 0.050 deg
  - Implies 3.5 kg mass asymmetry
- This mass implies a CM change of -0.25 cm (see previous slide)



6. Summary

- We observed changes in transverse components of spin rate
- Related these changes to off-diagonal elements of inertia tensor
- Modeled off-diagonal elements as mass asymmetries between tanks
- Solved for relationship between transverse spin rate and mass asymmetry
- Used the deduced mass asymmetry to solve for change in CM
  - Changed by -0.25 cm
  - Similar result can be obtained by assuming fuel diaphragm shape change, so actual asymmetry may be combination of both types
- Estimated shift in CM from the new filter using GPS Doppler shift data
  - Changed by -0.16 cm
- Approximate agreement validates the new CM filter
  - Conclusion is weakened by large uncertainties in the Doppler data

Questions?