

NASA/TM—2016-219120



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September 2016

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Beam Wave Considerations for Optical Link Budget Calculations

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Abstract

The bounded beam wave nature of electromagnetic radiation emanating from a finite size aperture is considered for diffraction-based link power budget calculations for an optical communications system. Unlike at radio frequency wavelengths, diffraction effects are very important at optical wavelengths. In the general case, the situation cannot be modeled by supposing isotropic radiating antennas and employing the concept of effective isotropic radiated power. It is shown here, however, that these considerations are no more difficult to treat than spherical-wave isotropic based calculations. From first principles, a general expression governing the power transfer for a collimated beam wave is derived and from this are defined the three regions of near-field, first Fresnel zone, and far-field behavior. Corresponding equations for the power transfer are given for each region. It is shown that although the well-known linear expressions for power transfer in the far-field hold for all distances between source and receiver in the radio frequency case, nonlinear behavior within the first Fresnel zone must be accounted for in the optical case at 1550 nm with typical aperture sizes at source/receiver separations less than 100 km.

What is ‘Isotropic’ About a Directed Laser Beam When Applying the Concept of EIRP to an Optical Satellite Link Budget?

The use of the concept of effective isotropic radiated power (EIRP) in the calculation of communication system performance has its roots in the consideration of power transfer from a transmitter to a receiver in which the receiver subtended a portion of the transmitted wave field that could be considered spherical. Such spherical field conditions are easily realized at radio frequencies due to the relatively large wavelengths as compared to the sizes and the distances of the antennas. As antenna configurations evolved that were able to shape the phase fronts of the transmitted waves, EIRP was still employed but with ‘correction factors’ to account for the non-isotropic fields. Such corrections are usually applied in the calculation of the receiver antenna gain parameter. In essence, what was lost in power by using the isotropic assumption was recaptured by appropriately adjusting the receiver gain (Ref. 1).

These concepts that have been historically inherent in the calculation of the radio frequency power connecting the transmitter to the receiver in a communications link evaluation have persisted into the case where short wavelength optical radiation is employed. However, the very aspect of directed optical radiation that makes its use desirable for free-space communication systems is diametrically opposed to that of ‘isotropic’ transmission. The phenomena occurring here at these shorter wavelengths are solely due to diffraction. Diffraction, being a characteristic of all wave propagation, also occurs at radio frequencies but does so in a much less noticeable fashion since the ratio of the sizes of the reflector antennas that are used to direct the radiation to the sizes of the wavelengths to are very much smaller for radio frequencies than for optical frequencies. As shown below, diffraction effects at radio frequencies occur very close to the transmitter aperture and are not seen at the distances a receiver aperture would be placed. The radio frequency wave field can always be treated as a spherical wave at the receiver locations. Optical radiation such as a laser beam emanating from a typical sized optical aperture is, because of its short wavelength, dominated by diffractive spatial broadening of its beam as it exits the aperture output

plane. For optical wavelengths, the situation is much different than the RF case and the diffractive effects at potential receiver locations need to be considered at the outset. The geometrical characteristics of the phase front of the optical beam can significantly change at very large distances from the transmitter aperture. Depending on the aperture sizes as well as their separation, diffractive beam wave aspects of the optical wave field must be included in the power transfer calculations.

The proper way to describe the power transfer characteristics of all such wave types, i.e., beam waves, at any wavelength and propagation distance is to derive equations for the power within the beam in terms of the wave equation for the propagating electric field based on the Helmholtz wave equation. This is the subject to the present work. General expressions will be derived and, through appropriate approximation, are shown to finally reduce to the form of power transfer equations derived using EIRP considerations for the optical case so long as the optical receiver aperture is on the order of 100 km from the transmitter. For intermediate distances, other forms of the power transfer expression will be obtained.

Power Transfer Between Apertures by Bounded Beam Waves

As just mentioned, the proper place to begin the study is from the Helmholtz wave equation that, of course, is a solution of the Maxwell equations. Within an approximation that holds for all wave propagation problems met with in free-space communications, the wave equation is solved with prevailing boundary conditions that describe a finite output aperture (i.e., an antenna or lens) so as to capture all the diffraction phenomena that can occur (Refs. 2 and 3). In an effort not to deter from the major concepts to be discussed here, an appendix is provided in which an approximation known as the paraxial approximation is applied to the Helmholtz wave equation that yields the fundamental analytical description of such bounded beam wave modes. This material forms the basis for all optical propagation scenarios and should be considered for overall performance analyses. Exact expressions are given in the appendix for a specific beam wave mode, the Gaussian beam wave, which has the fidelity to describe five different types of wave propagation encountered in most applications: focused, divergent, and collimated beam waves. From the collimated beam wave, the special cases of plane and spherical waves can be obtained. For the purposes of this discussion, only collimated beam waves will be used due to their dominance in the applications. *Expressions shown below only hold for a collimated beam wave;* expressions for the other types of beam waves can similarly be developed.

Neglecting absorption by the atmosphere of the beam energy, Equation (A14) gives for the intensity of a collimated beam at a distance L from the transmitter

$$I(L, \rho) = A_0^2 \frac{R_T^2}{R^2(L)} e^{-\rho^2/R^2(L)} \quad (1)$$

where $R(L)$ is, in this case, the radius of a collimated beam at a distance L from the transmitter and is given by Equation (A15),

$$R^2(L) = R_T^2 (1 + \alpha_1^2 L^2) \quad (2)$$

in which

$$\alpha_1 \equiv \frac{1}{kR_T^2} \quad (3)$$

is the diffraction parameter involving the wave number k of the radiation, $k \equiv 2\pi/\lambda$ where λ is the wavelength, and the physical radius of the transmitter aperture is R_T . The quantity A_0^2 is square amplitude

of the radiation at the output aperture and will be related to the transmitted power in what is to follow. The total amount of power $P_R(L)$ received by an aperture of radius R_R placed at a distance L from the transmitter is given by

$$P_R(L) = 2\pi \int_0^{R_R} I(L, \rho) \rho d\rho = \pi A_0^2 R_T^2 \left(1 - e^{-R_R^2/R^2(L)}\right) \quad (4)$$

Defining the output power of the transmitter aperture by

$$P_T = \pi R_T^2 A_0^2 \quad (5)$$

and substituting Equations (2) and (3) into Equation (4) gives for the exact general equation for power transfer between a transmitting aperture and a receiving aperture

$$P_R(L) = P_T \left(1 - e^{-k^2 R_T^2 R_R^2 / (k^2 R_T^4 + L^2)}\right) \quad (6)$$

Three spatial regions along the propagation route can be identified using this general equation. Equation (6) holds for power transfer in the *near-field* of the transmitter. The first line of demarcation is obtained by considering at what distance L the denominator of the exponential function is such that

$$L^2 \gg k^2 R_T^4, \text{ i.e., } L > k R_T^2 \quad (7)$$

The region defined by the inequality of Equation (7) defines the case where the *first Fresnel zone* of the transmitter aperture is located. Within this approximation, Equation (6) achieves its first approximate form

$$P_R(L) \approx P_T \left(1 - e^{-k^2 R_T^2 R_R^2 / L^2}\right), \quad L > k R_T^2 \quad (8)$$

To get an idea of the various Fresnel zones for various wavelengths, consider for $R_T = 0.1$ m at an optical wavelength of $\lambda = 1550$ nm, i.e., for $k = 4 \times 10^6$ m⁻¹. In this case, one has the first Fresnel zone is located at $L_{Fresnel} = 40.5$ km far enough from the transmitter within which the receiver may exist. On the contrary, at a wavelength of $\lambda = 1.0$ cm (30 GHz), the position of the zone occurs at $L_{Fresnel} = 6.3$ km well before the point at which a receiver would be placed.

Proceeding on to the next spatial region defined by approximation, consider the exponential function once again and consider distances L for which

$$\frac{k^2 R_T^2 R_R^2}{L^2} \ll 1, \text{ i.e., } L > k R_T R_R \quad (9)$$

This condition defines the *far-field* of the transmitter in which Equation (8) further approximates to

$$P_R(L) \approx P_T \left(\frac{k^2 R_T^2 R_R^2}{L^2}\right), \quad L > k R_T R_R \quad (10)$$

Using the same numerical example used above with a receiver radius of $R_R = 0.25$ m, one has for $\lambda = 1550$ nm, $L_{Far} = 101$ km whereas for $\lambda = 1.0$ cm, $L_{Far} = 15.7$ m. Again, an optical receiver must be placed beyond 101 km from the transmitter to be in the far-field and described by Equation (10) but a RF receiver must only be beyond 15.7 m from the transmitter. Equation (10) is the same as the classical result

obtained using EIRP considerations. Hence, Equation (10) will always suffice for power transfer calculations for radio frequencies but not for optical link calculations unless the optical receiver is beyond the far-field threshold defined for the particular problem.

Connection to Transmitter and Receiver Gain and Free Space Path Loss of a Traditional Link Budget

Equation (6) is the most general expression that describes the spatial power transfer for a collimated beam wave through free-space at distances which satisfy $kR_T^2 > L$ within the Fresnel zone of the transmitter. To make contact with the traditional transmitter and receiver system gains used in RF link budgets as well as the all-important free-space path loss, one simply considers the dimensionless combinations of parameters of the problem that are proportional to power (such as areas of the apertures) and the operating wavelength (such as k). Thus, if one defines the dimensionless combination involving the area of the transmitter aperture

$$G_T = (kR_T)^2 \quad (11)$$

i.e., the classical definition of transmitter antenna gain, and similarly for the receiver aperture

$$G_R = (kR_R)^2 \quad (12)$$

i.e., the classical definition of receiver antenna gain, the exponential function occurring in Equation (6) can be written

$$\frac{k^2 R_T^2 R_R^2}{k^2 R_T^4 + L^2} = \frac{G_T L_{FS} G_R}{G_T^2 + k^2 L^2} \quad (13)$$

The remaining term in the denominator is, of course, the inverse of the classically defined free-space path loss

$$L_{FS} \equiv \left(\frac{1}{kL} \right)^2 \quad (14)$$

(These gain and loss factors are to within a normalization factor of 2 that is sometimes included in some formulations.) Thus, the general expression of Equation (6) becomes

$$P_R(L_{FS}) = P_T \left(1 - e^{-G_T L_{FS} G_R / (1 + G_T^2 L_{FS})} \right) \quad (\text{near-field optical case}) \quad (15)$$

Given the results obtained earlier, in the event that $L > kR_T^2$, i.e., $G_T \sqrt{L_{FS}} < 1$, this reduces, as shown by Equation (8), to the first Fresnel zone expression,

$$P_R(L_{FS}) \approx P_T \left(1 - e^{-G_T L_{FS} G_R} \right) \quad (\text{first Fresnel zone optical case}) \quad (16)$$

Finally, when $L > kR_T R_R$, i.e., $\sqrt{G_T L_{FS} G_R} < 1$, this far-field condition allows Equation (16) to further simplify to

$$P_R(L_{FS}) \approx P_T G_T L_{FS} G_R \quad (\text{far-field optical case and all RF cases}) \quad (17)$$

Appendix—The Parabolic Approximation of the Helmholtz Equation

Without going into too much detail, there exists the paraxial solution (Refs. 2 and 3) to the wave equation that describes the propagation of a short wavelength beam waves with a Gaussian distribution of complex amplitude (i.e., by ‘short wavelength’ one implies that the wavelength λ is very much smaller than any other spatial quantity that enters the problem, such as the aperture size or the propagation distance). The usual Helmholtz wave equation for a scalar electric field $E(\vec{r})$ (as opposed to the more general vector electric field which incorporates potential depolarization effects) is given by

$$\nabla^2 E(\vec{r}) + k^2 E(\vec{r}) = 0 \quad (\text{A1})$$

where, as usual, $k \equiv 2\pi/\lambda$ is the wave number associated with the wavelength λ and the usual three-dimensional Laplacian $\nabla^2 E(\vec{r})$ of the electric field. The first step is to specialize this equation for a bounded wave field (a beam wave) propagating in the \vec{x} direction by writing

$$E(\vec{r}) = U(\vec{r}) e^{ikx} \quad (\text{A2})$$

where the newly introduced function $U(\vec{r})$ still describes the evolution of the electric field but devoid of its oscillatory wave component e^{ikx} . Putting Equation (A2) into Equation (A1) and remembering that the Laplacian of the electric field shown in Equation (A1) really means

$$\nabla^2 E(\vec{r}) \equiv \frac{\partial^2 E(\vec{r})}{\partial x^2} + \frac{\partial^2 E(\vec{r})}{\partial y^2} + \frac{\partial^2 E(\vec{r})}{\partial z^2} \quad (\text{A3})$$

one can get the equation for $U(\vec{r})$

$$\nabla^2 U(\vec{r}) + 2ik \frac{\partial U(\vec{r})}{\partial x} = 0 \quad (\text{A4})$$

In an effort to continually isolate the \vec{x} direction in which the beam wave is propagating from the transverse directions (along which diffractive phenomena exist), one can decompose $\nabla^2 U(\vec{r})$ into its longitudinal part (along the direction of propagation which is taken to be along the \vec{x} axis) and its transverse part (along the remaining \vec{y} and \vec{z} axes which are collectively written as $\vec{\rho} \equiv y\hat{y} + z\hat{z}$ where \hat{y} and \hat{z} are unit vectors)

$$\nabla^2 U(\vec{r}) = \nabla^2 U(x, \vec{\rho}) \equiv \frac{\partial^2 U(x, \vec{\rho})}{\partial x^2} + \nabla_{\vec{\rho}}^2 U(x, \vec{\rho}) \quad (\text{A5})$$

Putting this into Equation (A4) gives the expression

$$\frac{\partial^2 U(x, \vec{\rho})}{\partial x^2} + \nabla_{\vec{\rho}}^2 U(x, \vec{\rho}) + 2ik \frac{\partial U(\vec{r})}{\partial x} = 0 \quad (\text{A6})$$

Now, here is where the famous paraxial approximation comes into play: by construction, the field $U(x, \vec{\rho})$ is devoid of any spatially-driven effects except for the oscillatory variation e^{ikx} . Thus, the value of $U(x, \vec{\rho})$ only changes over a distance of a wavelength λ which means that the second spatial derivative

$\partial^2 U(x, \bar{\rho}) / \partial^2 x$ can only change by the amount on the order of $U(x, \bar{\rho}) / \lambda^2$. If the similar spatial variations that are represented by $\nabla_{\bar{\rho}}^2 U(x, \bar{\rho})$ are much larger (i.e., these transverse variation will be affected by other spatial quantities in the transverse direction such as the initial radius W_0 of the transmitter aperture), that is, if $W_0 \gg \lambda$, then the first term of Equation (A6) is much more smaller than the second or third term, i.e.,

$$\frac{\partial^2 U(x, \bar{\rho})}{\partial x^2} \ll \nabla_{\bar{\rho}}^2 U(x, \bar{\rho}), \left| 2ik \frac{\partial U(\bar{r})}{\partial x} \right| \quad (\text{A7})$$

and Equation (A7) becomes

$$\nabla_{\bar{\rho}}^2 U(x, \bar{\rho}) + 2ik \frac{\partial U(\bar{r})}{\partial x} = 0 \quad (\text{A8})$$

(There is also the implicit constraint on the total distance of propagation L and the wavelength, i.e., $L \gg \lambda$ in addition to $L \gg W_0$.) This is the paraxial approximation of the original wave equation Equation (A1). It reduces the full three-dimensional form of Equation (A1) to a much more easily handled ‘diffusion’ equation form of Equation (A8).

A Solution for the Paraxial Approximation—Beam Waves

In order to obtain an analytical expression that describes the subsequent propagation of a bounded wave beam that is controlled by Equation (A8), one simply begins with a ‘wish-list’ of what parameters should go into such a description. The two most important parameters that enter are the radius of the transmitter aperture W_0 as well as the curvature R_0 of the initially transmitted phase front of the beam. Assuming a single-mode of operation of the transmitter, than it can be expected that the electric field distribution of the beam as it exits the output aperture will be of a Gaussian function of transverse position across the beam. Given these constraints, than one can show (Refs. 2 and 3), after a bit of work on Equation (A8), that the $U(x, \bar{\rho})$ field distribution is given by the functional relationship

$$U(x, \bar{\rho}) = \frac{A_0}{1 + i\alpha x} e^{-k\alpha \rho^2 / (2(1 + i\alpha x))} \quad (\text{A9})$$

where A_0 is the initial amplitude of the beam field as it leaves the transmitter and the defining beam parameters enter into the problem through

$$\alpha \equiv \alpha_1 + i\alpha_2, \quad \alpha_1 \equiv \frac{2}{kW_0^2}, \quad \alpha_2 \equiv \frac{1}{R_0} \quad (\text{A10})$$

The corresponding electric field is found using Equation (A2). (Eq. (A9) is one of several beam modes that can be describes within the paraxial approximation, others being Laguerre beam waves and Bessel beam waves.)

The resulting intensity (i.e., the power density) of the beam at a distance L from the transmitter is given by Equation (A9) is

$$I(L, \rho) \equiv E(L, \rho) E^*(L, \rho) = |E(L, \rho)|^2 = A_0^2 \frac{W_0^2}{W^2(L)} e^{-2\rho^2/W^2(L)} \quad (\text{A11})$$

where

$$W^2(L) = W_0^2 \left((1 - \alpha_2 L)^2 + \alpha_1^2 L^2 \right) \quad (\text{A12})$$

is the square of the beam radius at L .

It is very important to note from the form of the exponential of Equation (A11) that at the output aperture of the transmitter placed at $L = 0$, the actual physical size of the radius R_T of the transmitter aperture is defined by the diffractive waist radius W_0 by

$$R_T \equiv W_0 / \sqrt{2} \quad (\text{A13})$$

i.e., the radius W_0 used to describe diffraction from an aperture of physical radius R_T is, due to diffraction, larger than R_T . Thus, any other diffractive radius such as $W(L)$ that can occur along the propagation path is connected to the prevailing physical radius by $R(L) = W(L) / \sqrt{2}$. Hence, the intensity of the beam given by Equation (A11) can be written in terms of the physical radius of the transmitter aperture

$$I(L, \rho) = A_0^2 \frac{R_T^2}{R^2(L)} e^{-\rho^2 / R^2(L)}, \quad R^2(L) = R_T^2 \left((1 - \alpha_2 L)^2 + \alpha_1^2 L^2 \right), \quad \alpha_1 \equiv \frac{1}{kR_T^2} \quad (\text{A14})$$

A Collimated Beam Wave

The model given above for a beam wave can describe five different wave types: divergent ($R_0 > 0$), focused ($R_0 < 0$), and collimated ($R_0 \rightarrow \infty$) beams as well as plane waves ($R_T \rightarrow \infty, R_0 \rightarrow \infty$) and spherical waves ($R_T \rightarrow 0, R_0 \rightarrow \infty$). For any space-based optical transmitter, a collimated beam wave is employed. Thus, the appropriate expression to use for intensity calculations involving collimated beams is Equation (A14) where, from Equation (A12) with $\alpha_2 = 0$,

$$R^2(L) \equiv R_{\text{coll}}^2(L) = R_T^2 (1 + \alpha_1^2 L^2) \quad (\text{A15})$$

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