Validation of GNSS Multipath Model for Space Proximity Operations Using the Hubble Servicing Mission 4 Experiment

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ABSTRACT

In the rendezvous and docking of spacecraft, GNSS signals can reflect off the target vehicle and cause large errors in the chaser vehicle receiver at ranges below a few hundred meters. It has been proposed that these additional ray paths, or multipath, be used as a source of information about the state of the target relative to the receiver. With Hubble Servicing Mission 4 as a case study, electromagnetic ray tracing has been used to construct a model of reflected signals from known geometry. Oscillations in the prompt correlator power due to multipath, known as multipath fading, are studied as a means of model validation. Agreement between the measured and simulated multipath fading serves to confirm the presence of signals reflected off the target spacecraft that might be used for relative navigation.
one right and one left-hand circularly polarized, were used to collect raw, sampled radio frequency (RF) data. HSM4 is used as a case study to explore the potential for relative ranging from reflected signals.

**BACKGROUND**

Although the concepts here are broadly applicable to all GNSS, the following discussion will use the specific architecture of the GPS C/A signal on the L1 carrier as an example. The direct signal received from the $i$-th GPS satellite can be written

$$y_i(t) = a_i^* d_i^*(t - \tau_{code,i}(t)) p_i^*(t - \tau_{code,i}(t)) \times \cos(2\pi f_{L1} t + \theta_i(t)) + v_i(t).$$  

(1)

Dropping the satellite superscripts when there is no risk of ambiguity, the signal essentially consists of two binary sequences, $d(t)$ and $p(t)$, modulated by an RF carrier with amplitude $a$. The $d(t)$ sequence represents the 12.5 minute, 50 Hz navigation data message, and the spreading code $p(t)$ is the 1.023 MHz Coarse Acquisition (C/A) or pseudo-random noise (PRN) code. The code terms arrive at the receive antenna delayed by $\tau_{code}(t)$ seconds due to the time of propagation.

The carrier frequency of the GPS L1 signal, $f_{L1}$, is 1.57542 GHz. The carrier has phase $\theta(t) = -2\pi f_{L1} \tau_{carr}(t) + \theta_0$, where $\tau_{carr}(t)$ is the propagation delay of the carrier terms and the initial carrier phase at some reference time $t_0$ is $\theta_0 = -2\pi f_{L1} t_0$. Random noise affecting the signal at reception is modeled by adding a circular symmetric Gaussian random variable $v(t)$ with zero mean and variance $\sigma_v^2$.

When a large object is located near the receiver, reflections of the GPS signals may also be received. These delayed, attenuated replicas of the direct signals contribute erroneous time of travel information to the receiver’s position calculation. Multipath modeling efforts fall into two broad categories: those that seek to model the receiver errors produced by multipath (e.g., [2], [3], and [4]), and those that seek to model properties of the multipath signals themselves (e.g., [5], [6], [7], and [8]). This latter category is most relevant to the work presented here; in order to estimate environmental features from multipath (such as the distance of a reflecting target spacecraft from a receiver), it is necessary to characterize the relationship between the environment features of interest and the observable multipath signal properties.

Ignoring the navigation message and satellite superscripts, the received GPS signal with multipath can be written

$$y(t) = \sum_{m=0}^{M} a_m p(t - \tau_{code,m}(t)) \times \cos(2\pi f_{L1} t + \theta_m(t)) + v_m(t).$$  

(2)

for $M$ multipath rays. The carrier phase of the $m$-th ray is $\theta_m(t) = -2\pi f_{L1} \tau_{carr,m}(t) + \theta_{0,m}$. The direct signal is assigned the index $m = 0$, while the reflected signals are indexed from $m = 1$ to $M$. The direct signal is right-hand circularly polarized (RCP) so, assuming an RCP antenna, the signal amplitude for $m = 0$ is a function of $G_R$, the antenna’s co-polarization gain,

$$a_0 = \sqrt{P_{T,0} G_R A^2_{11}/4\pi}$$  

(3)

where $P_{T,0}$ is the power spatial density produced by the GPS satellite at the receiver along the line of sight, and $G_R$ depends on the elevation and azimuth of the incoming signal. Notation indicating the time dependence of $a_0$, $P_{T,0}$, and $G_R$ is left off for simplicity. The signal amplitude for $m \neq 0$ is a function of $G_{R\times}$, the antenna’s cross polarization gain,

$$a_m = \sqrt{P_{T,m} G_{R\times} \lambda^2_{11}/4\pi}.$$  

(4)

Each multipath ray can be characterized by its amplitude, code phase, and carrier phase relative to the direct signal, denoted $a_m$, $\delta_m$ and $\psi_m$ respectively. The EM ray tracing used in this work characterizes each multipath ray by its relative electric field strength $E_m = P_{T,m}/P_{T,0}$. The relationship between the simulated value and $\alpha_m$ is

$$\alpha_m = E_m \sqrt{G_{R\times}/G_R} = \frac{a_m}{a_0},$$  

(5)

where again the antenna gains are azimuth and elevation dependent. Relative code phase is $\delta_m(t) = c(\tau_{code,m}(t) - \tau_{code,0}(t))$ and relative carrier phase $\psi_m(t) = \theta_m(t) - \theta_0(t)$.

As the excess path length traveled by a reflected signal changes, the relative carrier phase will cycle through phases that add to the direct signal constructively and destructively, an effect known as fading. This results in a characteristic oscillation of received power, the frequency of which is determined by the rate of change of the excess path length. Neglecting code and carrier divergence, this relationship can be expressed

$$\psi_m(t) = (1/\lambda_{L1}) \dot{\delta}_m(t).$$  

(6)

The dot indicates a time derivative. In stationary terrestrial receivers where the reflecting surface is horizontal, this oscillation frequency is known to be a function of satellite elevation [9]. Ground reflections will cause a higher frequency oscillation from low elevation satellites, as the excess path length of the reflections is changing more rapidly than for satellites at higher elevations. In spacecraft docking, the oscillation frequency is driven by the relative geometry of the transmitting GNSS satellite, target, and receiver.

**HUBBLE SERVICING MISSION**

The fourth Hubble servicing mission, STS-125, took place in 2009. The crew of the space shuttle Atlantis docked with Hubble Space Telescope (HST) on May 13th and deployed the space telescope on May 18th. During
The Relative Navigation Sensor (RNS) experiment recorded camera imagery from the shuttle cargo bay and estimated the relative position and attitude of HST using several vision processing algorithms [10]. Two GPS antennas were included as part of this experiment and recorded data during the same time frame; one antenna was RCP, intended for receiving direct signals, the other left-hand circularly polarized (LCP), intended for receiving reflected signals. The position and attitude of HST relative to the shuttle was reconstructed for this research from a composite of United Space Alliance’s Relative Best Estimate Trajectory (RELBET) product (primarily derived from shuttle-based rendezvous radar measurements) [11] and RNS researcher’s image processing estimates. The absolute position of the shuttle was taken from United Space Alliance’s Postflight Attitude and Trajectory History (PATH) product [12]. Although the relative state of HST has a 3σ position accuracy on the order of centimeters in each dimension, the shuttle position 3σ position accuracy is hundreds of meters. Further details on the mission geometry and its reconstruction were given in [13]; new antenna testing results are presented in the following section.

**Antenna Properties**

Sensor Systems RCP (Model S67-1575-39) and LCP (Model S67-1575-139) antennas were flown on HSM4. The antennas were mounted in the shuttle cargo bay on top of the Multi-use Logistic Equipment carrier beside the RNS cameras. After initial tracking results suggested poor isolation of direct and reflected signals, it was important to determine the cross-pole discrimination of the antennas. The antennas were not measured prior to the mission, but information provided by Sensor Systems suggested only a 3 dB cross-pole attenuation [14]. More accurate measurements were needed. The antenna mounting plate was reconstructed according to HSM4 photographs, then the flight antenna gain patterns were measured in the Goddard ElectroMagnetic Anechoic Chamber (GEMAC), shown in Figure 2.

Azimuth cuts of the measured gain patterns are shown in Figure 1. Measurements were taken across antenna elevation from -179 to 179 degrees in increments of one degree at each antenna azimuth cut. Antenna azimuth was tested from 0 to 180 degrees in increments of 15 degrees. Both antennas are active antennas, but the 26 dB gain of the low noise amplifier has been subtracted. In each case, the co-pole gain is plotted with solid lines and the cross-pole gain with dotted lines. Average gain across the different azimuth cuts is plotted in black; the attenuation of an LCP signal at the RCP antenna boresight is 12 dB, while the attenuation of an RCP signal at the LCP antenna boresight is 7 dB. This is better than the cross-pole attenuation specified by the manufacturer, but the poorer polarization of the LCP antenna causes difficulty in the isolation of LCP signals [15].
Results

A. Simulation

Figures 5 and 6 show the prompt correlator power in the time and frequency domains (left and right plots, respectively). The expected oscillation from the simulated geometry (i.e., $\delta$) and Equation (6) is given in the $\psi$ column of Table I. Consider PRNs 1 and 30: the peak measured oscillation, determined through the first peak in the frequency domain, is 0.2594 Hz for PRN 1 and 0.3052 Hz for PRN 30. Due to the simulated signal length (165 seconds), the resolution of the Fast Fourier Transform (FFT) is 0.0061 Hz; both measured fading frequencies match the expected frequencies to within the FFT resolution.

The multipath oscillations of PRNs 26 and 27 are an
However, the similarity between the results of tracking quite as closely (Frequencies in the filtered RCP and LCP data do not agree the spectrum of the filtered power measurement at domain on the right, a secondary peak may be present in simulated data, agree with these values. In the frequency domain on the right, a secondary peak may be present in the spectrum of the filtered power measurement at 0.0244 Hz, but the relatively coarse resolution of the FFT makes this difficult to discern.

B. Experiment

Multipath-induced oscillation of the prompt correlator power can also be measured in the experimental data from HSM4. The measured frequency of oscillation is generally consistent with the rate of change of the reflected signal’s relative delay in Table I.

The conclusion that this oscillation is due to multipath is further supported by the LCP tracking results, shown below the RCP results in both figures. The depth of the oscillation is greater, resulting in a larger frequency domain peak. While the frequency of multipath fading is determined by the rate of change of the multipath delay, the magnitude is a consequence of multipath strength. In the LCP data, reflected signals are expected to be stronger.

The experimentally measured prompt correlator power for PRN 1 is shown in Figure 8. On the top left is the RCP time domain, where a coherent integration time of 1 ms is used. A 20 ms non-coherent average is overlaid in orange. The spectra of these unfiltered and filtered RCP power measurements are shown on the top right. The measured frequency of the unfiltered LCP power is 0.2594 Hz - within the FFT resolution of the expected $\psi = 0.2595$ Hz. Frequencies in the filtered RCP and LCP data do not agree quite as closely (0.2441 Hz and 0.2686 Hz respectively). However, the similarity between the results of tracking the simulated and experimental data suggest that the ray specified in Table I for PRN 1 is accurate. PRN 30 exhibits close agreement with ray tracing results as well, shown in Figure 9, with a measured $\psi = 0.3418$ Hz (from the filtered LCP peak) and calculated $\psi = 0.3012$ Hz.

The slower oscillations of PRNs 26 and 27 are more difficult to detect. A peak of 0.0244 Hz was measured in the filtered RCP spectra of both PRN 26 and 27, shown in Figures 10 and Figures 11 respectively. LCP tracking fails for PRN 27 and is not shown. While this $\psi$ is within the FFT resolution of the calculated frequency for both (0.0230 Hz for PRN 27 and 0.0244 Hz for PRN 26), the peak is not pronounced and the coarseness of the FFT leaves some doubt as to whether this measurement is physically meaningful. The time domain results are inconclusive.

Although the multipath offset rate of change is simulated as constant, this is not the case in reality. Unmodeled variation in the power oscillation frequency over time causes the wider frequency peak observed in PRNs 1 and 30, as compared to the simulated cases, and contributes to the difficulty in measuring $\psi$ experimentally for PRNs 26 and 27.

Conclusions

Multipath fading confirms the presence of reflected signals in the HSM4 data. The general agreement of these features with the effects of simulated Hubble-reflected signals provides validation that the multipath model correctly represents the GNSS reflections received during docking. Consequently, ray tracing results support the hypothesis that there are reflections off Hubble that may be useful for relative navigation. Although the multipath fading frequency provides only a range rate measurement, other measurements can be used to extract range from reflected signals. For example, multipath parameters can be estimated from deformation of the code correlation shape (see [5] and [8]) or the reflected signals may be tracked independently. Such measurements rely on sufficiently wide receiver bandwidth and reflected signal isolation (e.g., an LCP antenna with good cross-polarization rejection) respectively [15]. The study of these techniques is ongoing.

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[10] Bo Naasz and John Van Eepoel and Steve Queen and Michael
Fig. 7: Simulation: PRN 26 prompt power oscillation

![Simulation: PRN 26 prompt power oscillation]

Fig. 8: Experiment: PRN 1 prompt power oscillation

![Experiment: PRN 1 prompt power oscillation]
Fig. 9: Experiment: PRN 30 prompt power oscillation

Fig. 10: Experiment: PRN 26 prompt power oscillation


