A COLLISION AVOIDANCE STRATEGY FOR A POTENTIAL NATURAL SATELLITE AROUND THE ASTEROID BENNU FOR THE OSIRIS-REX MISSION

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AGENDA

- Introduction to the OSIRIS-REx mission
- Proximity Operations Concept
- Current status of Natural Satellites
- Approaches to Collision Avoidance
- Wald Sequential Probability Ratio Test (WSPRT)
- Conjunction analysis Example 1: 8 hr prediction
- Conjunction analysis Example 2: 3 hr prediction
- Summary

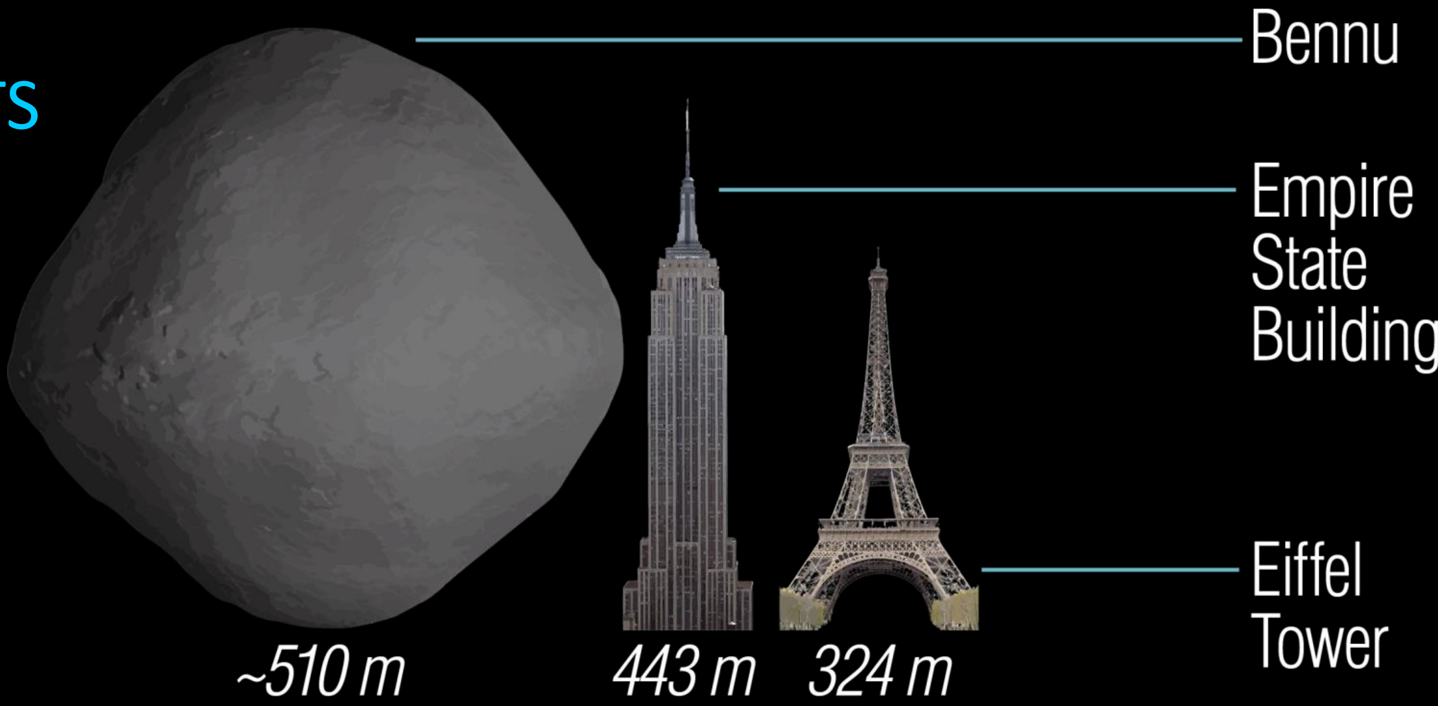
Introduction to the OSIRIS-REx mission

- The OSIRIS-REx mission launched on Sept 8th 2016 at Cape Canaveral, FL onwards to the asteroid Bennu.
- Bennu is a carbonaceous asteroid and a potentially hazardous asteroid with a probability of impacting the Earth in the late 22nd century. The determination of Bennu's physical and chemical properties are of key importance in the event an impact mitigation mission will be required.
- OSIRIS-REx's key science objectives include[1]:
  - Return and analyze a sample of Bennu's surface
  - Map the asteroid
  - Document the sample site
  - Measure the orbit deviation cause by non-gravitational forces (the Yarkovsky effect)
  - Compare observations at the asteroid to ground-based observations

(1) www.asteroidmission.org

INTRODUCTION TO THE OSIRIS-REX MISSION

BENNU FACTS

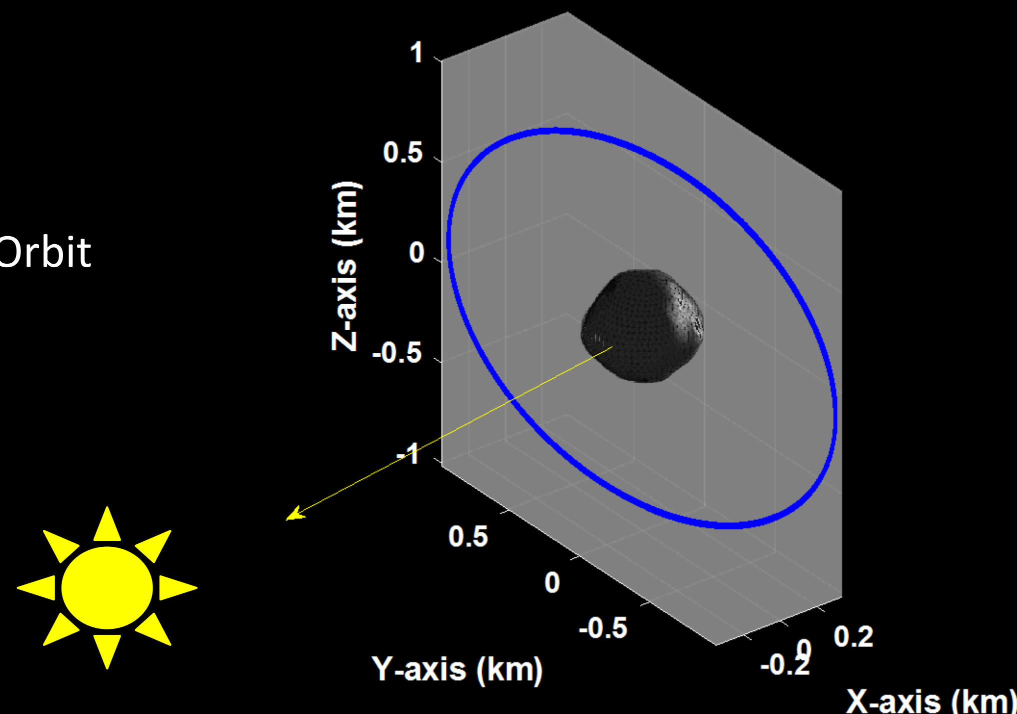


- *Equatorial Diameter:* ***~500m***
- *Polar Diameter:* ***~510 m***
- *Average Speed:* ***63,000 mph***
- *Rotation Period:* ***4.3 hrs***
- *Orbital Period:* ***1.2 yrs***
- *Orbital Inclination:* ***6 degrees***
- *Earth Approach:* ***Bennu comes close to Earth every 6 yrs***

PROXIMITY OPERATIONS

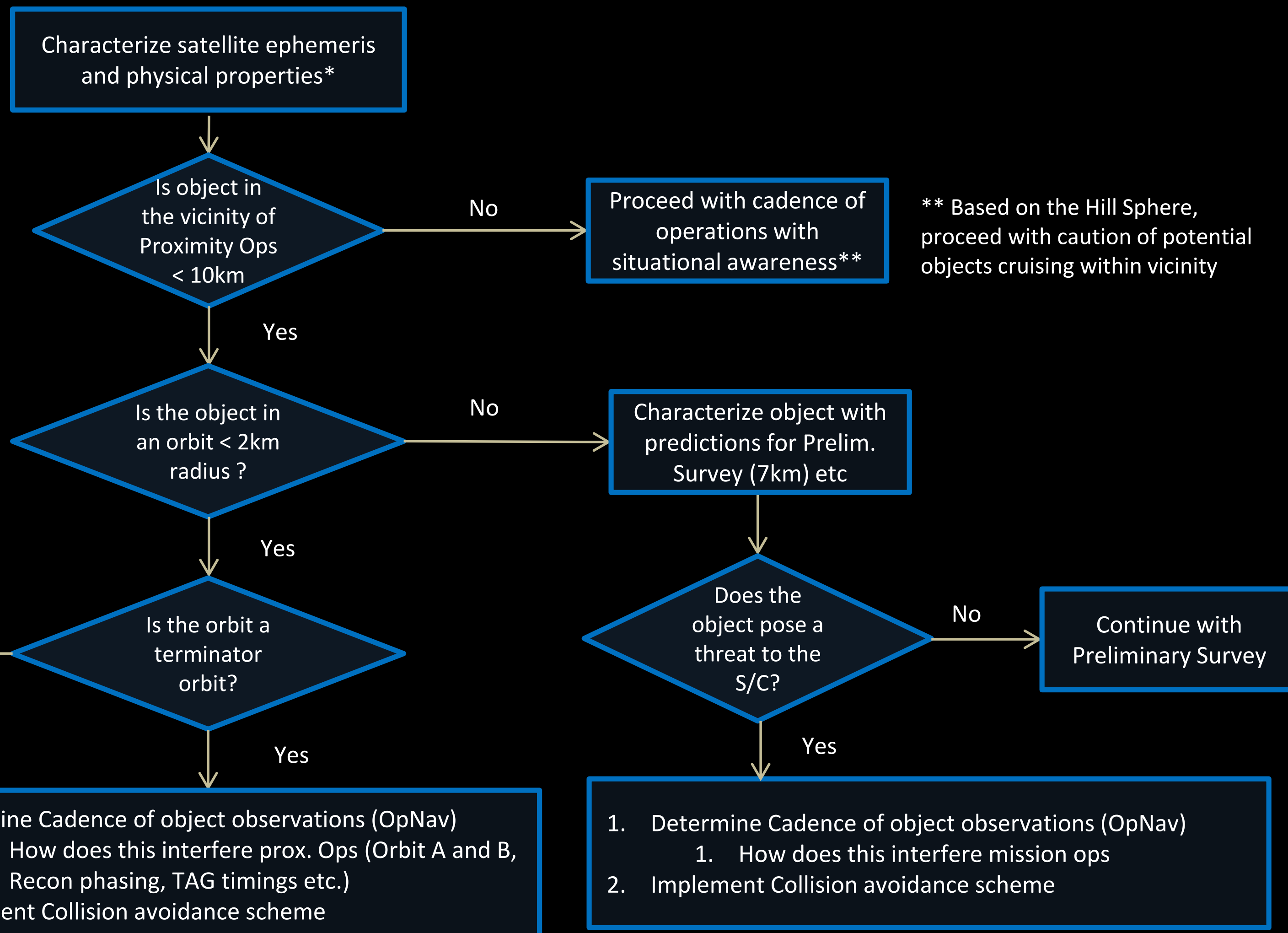
- There are various phases of the OSIRIS-REx mission proximity operations in which specific scientific campaigns at specified cadences are in place.
- Eventually, there is a safe home orbit in which OSIRIS-REx remains in a terminator orbit as the staging point for all subsequent activities.
- The terminator orbit is a plane that is perpendicular to the sun vector:
  - For OSIRIS-REx, minimizes solar radiation pressure (SRP) perturbations
  - Relatively large perturbation due to the small size of Bennu

Terminator Orbit



OPERATIONS CONCEPT FLOWCHART

- Hill Sphere: 31.7 +3.3/-4.2 km (2)



(2) D.S. Lauretta et al.,"The OSIRIS-Rex target asteroid (101955) Bennu: constraints on its physical, geological, and dynamical nature from astronomical observations" Meteoritics & Planteary Science, Vol 50, No 4, pg 834-849

Current status of Bennu natural satellites

- The presence of natural satellites depend on the rotation rates of the primary body. Bennu's rotation rate is 4.29 hrs for a Bennu sidereal day [3]
- Most NEA of spheroidal shapes and rapid rotation rates have been found to be primaries of a binary system.
- About 16% of Near Earth Asteroids (NEA) with diameters larger than 200m may belong to binary systems.[4]

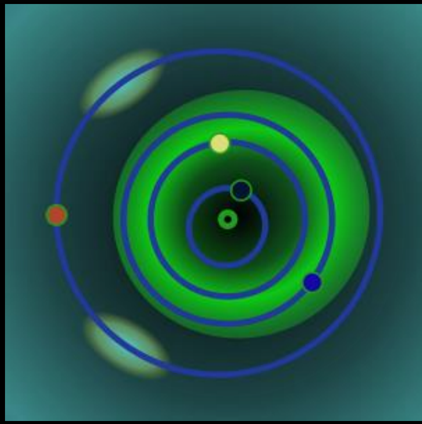
Potential Stable Natural Satellites size	Bennu's Hill Sphere
Diameters 1m	Out to 26 km
Diameters 10cm	Out to 16 km
Diameters 1cm	Out to 5km

- Based on radar albedo of Bennu and a tidally locked rotation period, the largest undetected satellite within 300km of Bennu is 2m.[1]

(1) D.S. Lauretta et al.,"The OSIRIS-Rex target asteroid (101955) Bennu: constraints on its physical, geological, and dynamical nature from astronomical observations" Meteoritics & Planetary Science, Vol 50, No 4, pg 834-849
(3) M.C. Nolan, "Shape model and surface properties of the OSIRIS-Rex target Asteroid (101955) Bennu from radar and lightcurve observations," Icarus, Vol.226, 2013, pp.629-640
(4) J.Margot et al.,"Binary asteroids in the Near-Earth object population," Science, Vol.296, No. 5572, 2002.

APOLLO ASTEROIDS SIMILAR TO BENNU

- Apollo asteroids are Earth-crossing asteroids with
  - Semi-major axes, a > Earth's semi major axis (1 AU)
  - Perihelion distances < Earth's aphelion (1.017 AU)



http://en.wikipedia.org/wiki/List_of_Apollo_asteroids (5)

- There are 55 known NEAs with moons
  - (14 Amor, 34 Apollo, and 7 Aten) with a total of 57 moons

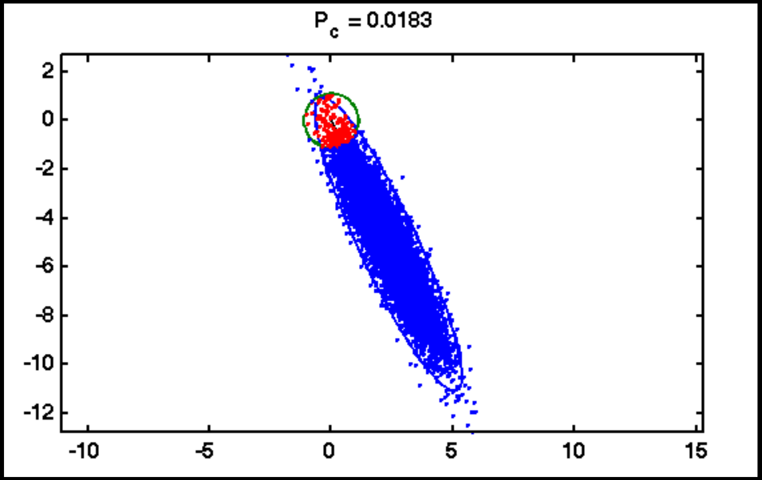
Name of Asteroid	Diameter (km)	Name of Moon	Diameter (km)	Separation
1999 DJ4	0.43+/- 0.08	S/2004	0.21+/- 0.05	0.8
2002 AM31	0.45+/- 0.05	S/2012	0.11	1.5
2004 DC	0.36	S/2006	0.07	0.75+/- 0.045

(5) Wikipedia. "Minor-planet moon" 2015

* Note: No rotation rate information is included here

STANDARD APPROACH TO COLLISION AVOIDANCE (CA)

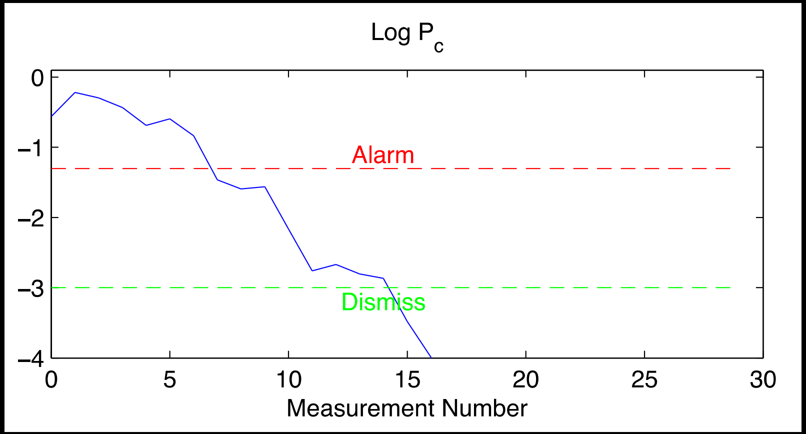
Collision Probability, Pc



- Compute P_c (might be hard)
- Compare to some threshold

Potential Issues with P_c [5]

- Often integrating PDF in the tail region
- Must project PDF into future
- Is PDF even Gaussian?
- P_c might "roll-off" – when to decide?

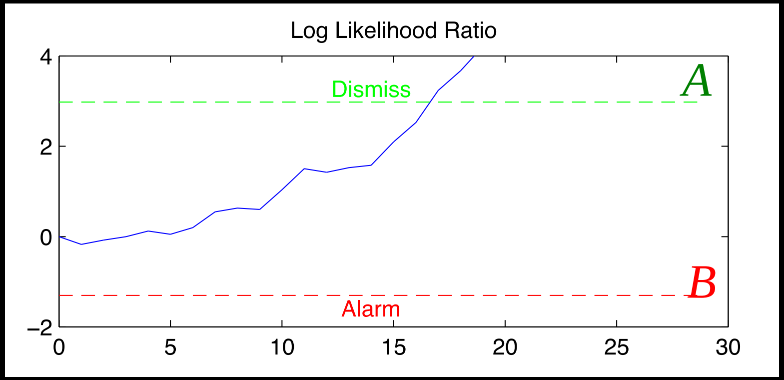


(5) R.Carpenter, L. Markley and D. Gold, "Wald Sequential Probability Ratio Test for Analysis of Orbital Conjunction Data", GNC , August 19-22 2013

WALD SEQUENTIAL PROBABILITY RATIO TEST (WSPRT) FOR CA

- Test for true miss distance at time of closest approach: $r_* = r_{\text{ca}}$
- Given set of observations at times t_k prior to $t_* = t_{\text{ca}}$: $\mathbf{Y}_{1:k}$
  - $\mathcal{H}_0$: fixed* hypothesis that true miss distance is unsafe ($\|r_*\| \le \mathcal{R}$, hard body radius)
  - $\mathcal{H}_1$: fixed* hypothesis that true miss distance is safe
- Form ratio of conditional PDF's:

$$\Lambda_k = \frac{p\left(\mathbb{Y}_{1:k}|\mathcal{H}_1\right)}{p\left(\mathbb{Y}_{1:k}|\mathcal{H}_0\right)} = \frac{p\left(\mathbb{Y}_{1:k}|\,\|r_*\| > \mathcal{R}\right)}{p\left(\mathbb{Y}_{1:k}|\,\|r_*\| \le \mathcal{R}\right)}$$



- Compare ratio to decision limits A & B:
  - If $\Lambda_k \ge A$, reject $\mathcal{H}_0$ and dismiss conjunction
  - If $\Lambda_k \le B$, accept $\mathcal{H}_0$ and maneuver
  - Otherwise, if possible, seek another observation

** Fixed hypotheses imply that there are no random disturbances, e.g. process noise, that can change a hit into a miss; if this can occur, must use a different test, such as Shirayeyev SPRT*

$$A = \frac{1 - \bar{P}_{fa}}{\bar{P}_{md}} \quad \text{and} \quad B = \frac{\bar{P}_{fa}}{1 - \bar{P}_{md}}$$

  - Targeted P_{md} (missed detection) and targeted P_{fa} (false alarm) are values that need to be pre-determined based on apriori statistics and/or Monte Carlo analysis

(6) R.Carpenter, L. Markley and D. Gold, "Wald Sequential Probability Ratio Test for Analysis of Orbital Conjunction Data", GNC , August 19-22 2013

WSPRT Algorithm

Likelihood ratio of WSPRT:

$$\Lambda_k = \frac{p\left(\mathbb{Y}_{1:k}|\mathcal{H}_1\right)}{p\left(\mathbb{Y}_{1:k}|\mathcal{H}_0\right)} = \frac{p\left(\mathbb{Y}_{1:k}|\ \|r_*\| > \mathcal{R}\right)}{p\left(\mathbb{Y}_{1:k}|\ \|r_*\| \leq \mathcal{R}\right)}$$

The conditional probabilities are calculated as:

$$\mathrm{p}\left(\mathbb{Y}_{k:1}|\,\boldsymbol{r}_* \in \mathbb{B}\right) = \frac{\prod_{i=1}^{k}\left(\frac{1}{(2\pi)^{\frac{n}{2}}\sqrt{|\tilde{\boldsymbol{P}}_{*|i}|}}\right)\sqrt{\frac{|\hat{\boldsymbol{P}}_{*|k}|}{|\hat{\boldsymbol{P}}_{*|o}|}}\,\mathrm{e}^{-\frac{1}{2}\alpha}\,P_{c|k}}{P_{c|o}}$$

$$\mathrm{p}\left(\mathbb{Y}_{k:1}|\,\boldsymbol{r}_* \notin \mathbb{B}\right) = \frac{\prod_{i=1}^{k}\left(\frac{1}{(2\pi)^{\frac{n}{2}}\sqrt{|\tilde{\boldsymbol{P}}_{*|i}|}}\right)\sqrt{\frac{|\hat{\boldsymbol{P}}_{*|k}|}{|\hat{\boldsymbol{P}}_{*|o}|}}\,\mathrm{e}^{-\frac{1}{2}\alpha}(1-P_{c|k})}{1-P_{c|o}}$$

where

$$P_{c|k} = \frac{1}{(2\pi)^{\frac{n}{2}}\sqrt{|\hat{\boldsymbol{P}}_{*|k}|}}\int_{\mathbb{B}}\exp\left(-\frac{1}{2}(\hat{\boldsymbol{r}}_{*|k}-\boldsymbol{r}_*)^\top\hat{\boldsymbol{P}}_{*|k}^{-1}(\hat{\boldsymbol{r}}_{*|k}-\boldsymbol{r}_*)\right)\mathrm{d}\boldsymbol{r}_*$$

$$P_{c|o} = \frac{1}{(2\pi)^{\frac{n}{2}}\sqrt{|\hat{\boldsymbol{P}}_{*|o}|}}\int_{\mathbb{B}}\exp\left(-\frac{1}{2}(\boldsymbol{r}_*-\hat{\boldsymbol{r}}_{*|o})^\top\hat{\boldsymbol{P}}_{*|o}^{-1}(\boldsymbol{r}_*-\hat{\boldsymbol{r}}_{*|o})\right)\mathrm{d}\boldsymbol{r}_*$$

The time series relative position vector and its covariance:

$$\hat{\boldsymbol{P}}_{*|k} = \left(\hat{\boldsymbol{P}}_{*|o}^{-1} + \sum_{i=1}^{k}\tilde{\boldsymbol{P}}_{*|i}^{-1}\right)^{-1}$$

$$\hat{\boldsymbol{r}}_{*|k} = \hat{\boldsymbol{P}}_{*|k}\left(\hat{\boldsymbol{P}}_{*|o}^{-1}\hat{\boldsymbol{r}}_{*|o} + \sum_{i=1}^{k}\tilde{\boldsymbol{P}}_{*|i}^{-1}\tilde{\boldsymbol{r}}_{*|i}\right)$$

$$\alpha = \hat{\boldsymbol{r}}_{*|o}^\top\hat{\boldsymbol{P}}_{*|o}^{-1}\hat{\boldsymbol{r}}_{*|o} + \sum_{i=1}^{k}\tilde{\boldsymbol{r}}_{*|i}^\top\tilde{\boldsymbol{P}}_{*|i}^{-1}\tilde{\boldsymbol{r}}_{*|i} - \hat{\boldsymbol{r}}_{*|k}^\top\hat{\boldsymbol{P}}_{*|k}^{-1}\hat{\boldsymbol{r}}_{*|k}$$

Therefore:

$$\Lambda_k = \frac{p\left(\mathbb{Y}_{1:k}|\mathcal{H}_1\right)}{p\left(\mathbb{Y}_{1:k}|\mathcal{H}_0\right)} = \frac{p\left(\mathbb{Y}_{1:k}|\ \|r_*\| > \mathcal{R}\right)}{p\left(\mathbb{Y}_{1:k}|\ \|r_*\| \leq \mathcal{R}\right)}$$

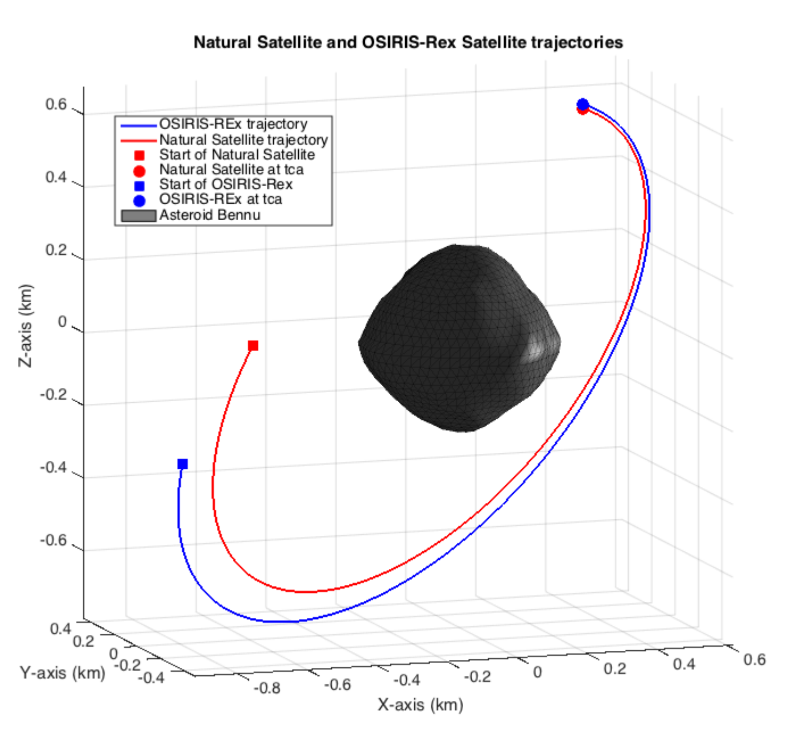
Simplifies to :

$$\Lambda_k = \frac{1-P_{c|k}}{P_{c|k}}\frac{P_{c|o}}{1-P_{c|o}}$$

Conjunction analysis Example 1: 8 hr prediction

- Simulated range, azimuth and elevation measurements for 4 hrs
- Epoch 18 Feb 2019 12:00:00.000 UTC
- 500 Monte Carlo runs
- Natural Satellite
  - Po = [σ^2_{xx}=(2/3 HBR)^{2}km^2, σ^2_{vv}=(10e-6)^2 km^2/s^2]
  - R = 0.01km
  - $P_{c|0}$ = 0.052075

FAR (False Alarm Rate)
MDR (Missed Detection Rate)



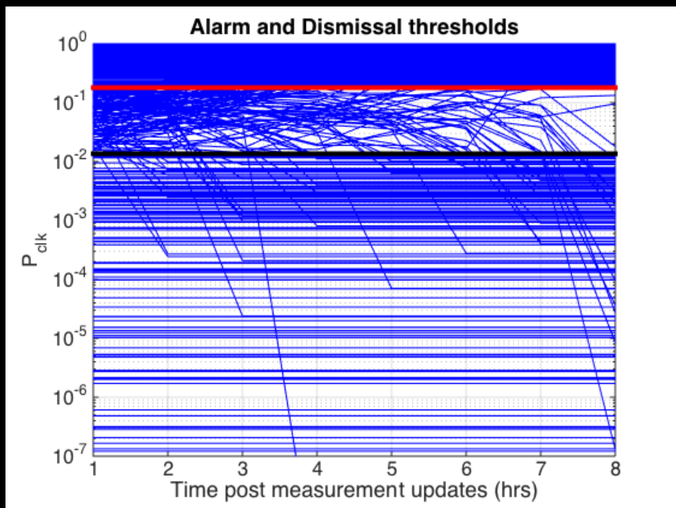
Parameter	(a) $\bar{P}_{fa_{0.2}}, \bar{P}_{md_{0.2}}$	(b) $\bar{P}_{fa_{0.01}}, \bar{P}_{md_{0.01}}$	(c) $\bar{P}_{fa_{0.2}}, \bar{P}_{md_{0.01}}$	(d) $\bar{P}_{fa_{0.01}}, \bar{P}_{md_{0.2}}$
Alarm Limit P_c^{Alarm}	0.180155	0.844687	0.213794	0.814638
Dismissal Limit $P_c^{Dismiss}$	0.013548	0.000555	0.000686	0.010976
Alarms	234	77	242	75
Dismissals	263	286	234	344
Hits	19	19	19	19
Misses	481	481	481	481
True Alarms	17	11	17	11
False Alarms	217	66	225	64
True Dismissals	261	284	232	341
False Dismissals	2	2	2	3
No Decisions	3	137	24	81
False Alarm Rate	45.11%	13.72%	46.78%	13.31%
Missed Detection Rate	10.53%	10.53%	10.53%	15.79%

Low P_{fa}
- High Alarm Limit
- Low Alarms
- Low False Alarms

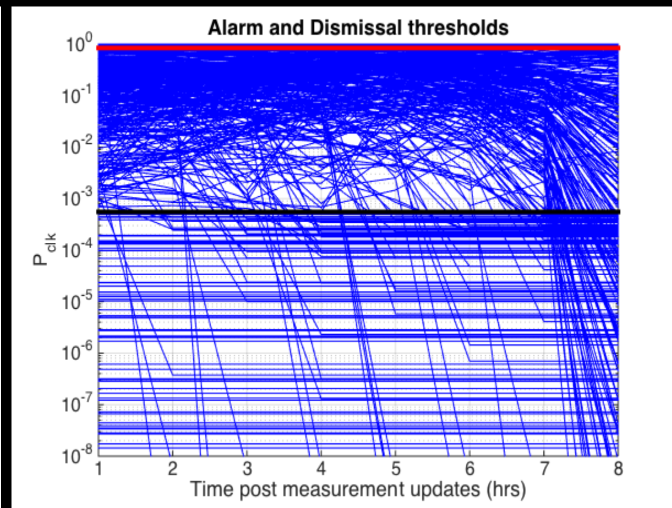
Hi P_{md}
- Hi Dismissal Limit
- Affects No Decisions (f(P_{fa}))

How to decide based on one parameter only or two?

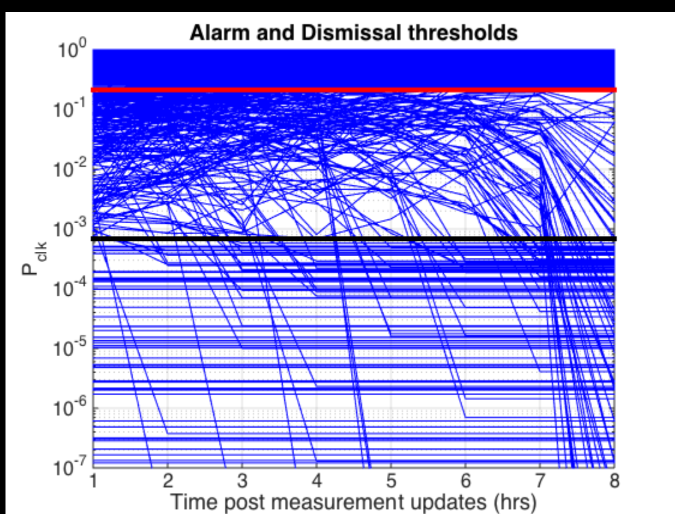
CONJUNCTION ANALYSIS EXAMPLE 1: 8 HR PREDICTION



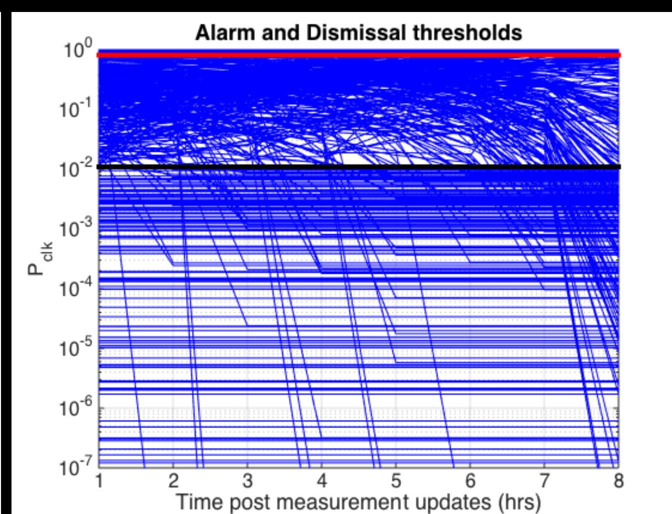
(a) P_{fa} = 0.2 and P_{md} = 0.2



(b) P_{fa} =0.01 and P_{md} = 0.01



(c) P_{fa} = 0.2 and P_{md} = 0.01



(d) P_{fa} = 0.01 and P_{md} = 0.2

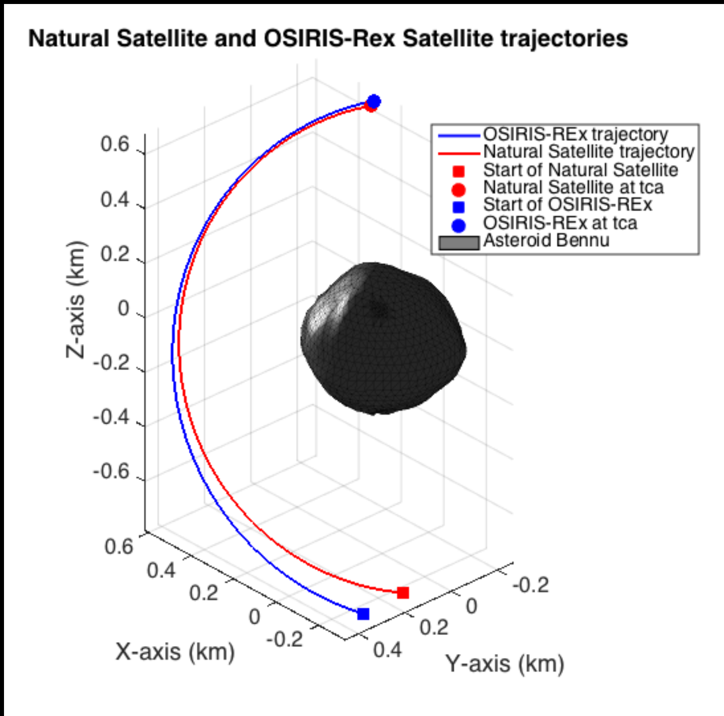
$P_{c|k} \geq P_c$ Alarm
Pc Alarm = $P_{c|0}(B + (1-B)P_{c|0})^{-1}$

$P_{c|k} < P_c$ Dismiss
Pc Dismiss = $P_{c|0}(A + (1-A)P_{c|0})^{-1}$

- The decision is made depending on the values of the targeted P_{fa} and P_{md} and calculated $P_{c|k}$.
- The required balance is a quick decision vs accuracy requirements.
- This can be implemented dependent on the mission phase.

Conjunction analysis Example 2: 3 hr prediction

- Simulated range, azimuth and elevation measurements for 4 hrs + Prediction for 3 hrs
- Epoch 18 Feb 2019 12:00:00.000 UTC
- 500 Monte Carlo runs
- Natural Satellite
  - Po = [σ^2_{xx}=(2/3 HBR)^{2}km^2, σ^2_{vv}=(10e-6)^2 km^2/s^2]
  - R = 0.01km
  - $P_{c|0}$ = 0.052075



Parameter	(a) $\bar{P}_{fa_{0.2}}, \bar{P}_{md_{0.2}}$	(b) $\bar{P}_{fa_{0.01}}, \bar{P}_{md_{0.01}}$	(c) $\bar{P}_{fa_{0.2}}, \bar{P}_{md_{0.01}}$	(d) $\bar{P}_{fa_{0.01}}, \bar{P}_{md_{0.2}}$
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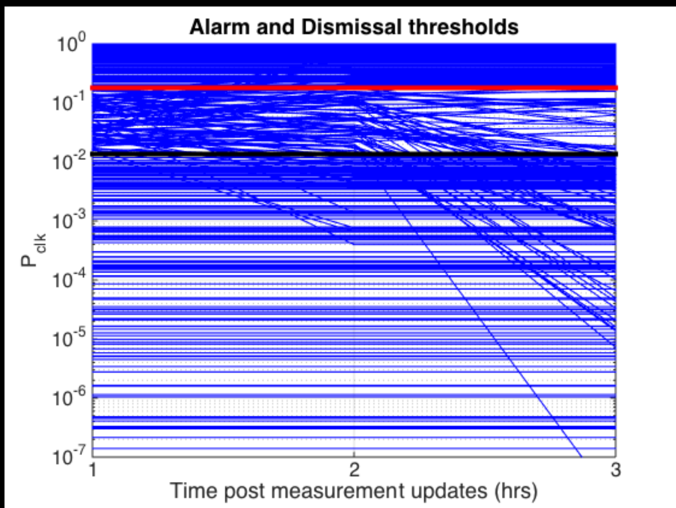
Dismissal Limit ($P_c^{Dismiss}$)	0.013548	0.000555	0.000686	0.010976
Alarms	198	91	195	96
Dismissals	273	236	229	282
Hits	19	19	19	19
Misses	481	481	481	481
True Alarms	18	14	18	15
False Alarms	180	77	177	81
True Dismissals	272	235	228	281
False Dismissals	1	1	1	1
No Decisions	29	173	76	122
False Alarm Rate	37.42%	16.01%	36.80%	16.84%
Missed Detection Rate	5.26%	5.26%	5.26%	5.26%

Low P_{fa}
- High Alarm Limit
- Low Alarms
- Low False Alarms

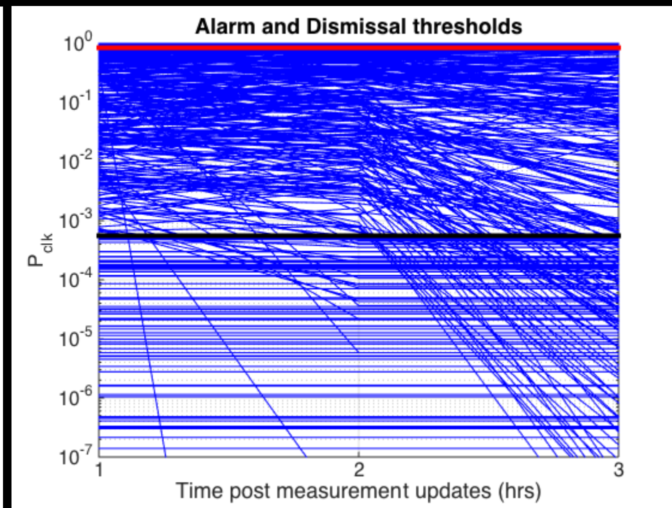
Hi P_{md}
- Hi Dismissal Limit
- Affects No Decisions (f(P_{fa}))

-Similar value patterns to 8 hr
-Noticeably reduced FAR and MDR

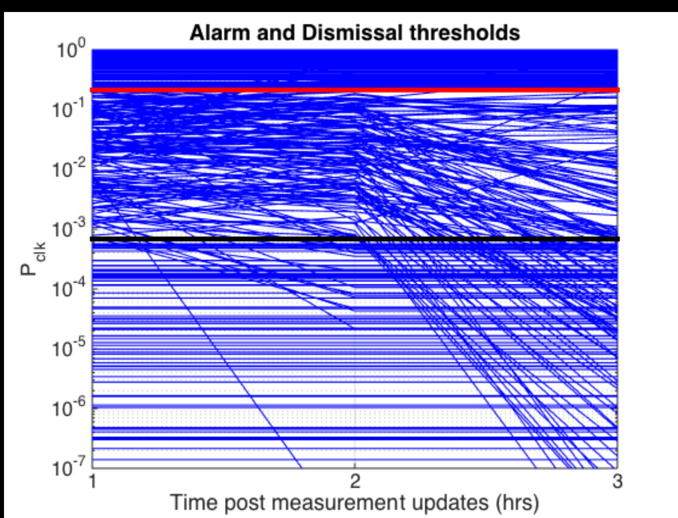
Conjunction analysis Example 2: 3 hr prediction



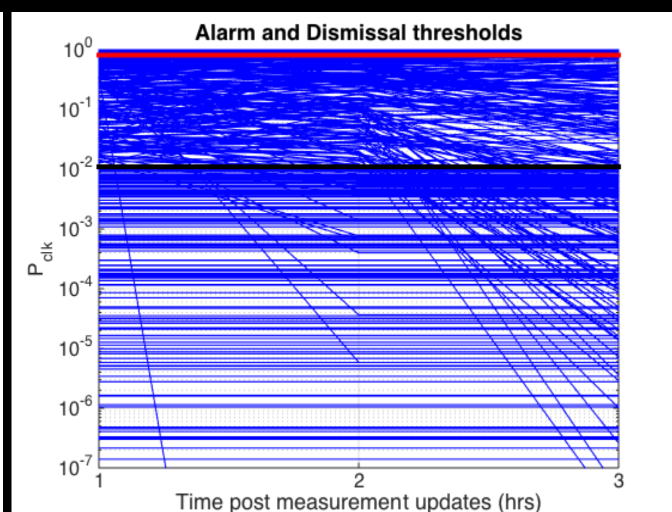
(a) $P_{fa} = 0.2$ and $P_{md} = 0.2$



(b) $P_{fa} = 0.01$ and $P_{md} = 0.01$



(c) $P_{fa} = 0.2$ and $P_{md} = 0.01$



(d) $P_{fa} = 0.01$ and $P_{md} = 0.2$

$P_{c|k} \geq P_c$ Alarm
Pc Alarm $= P_{c|0}\,(B + (1-B)P_{c|0})^{-1}$

$P_{c|k} < P_c$ Dismiss
Pc Dismiss $= P_{c|0}\,(A + (1-A)P_{c|0})^{-1}$

- The decision is made depending on the values of the targeted P_{fa} and P_{md} and calculated $P_{c|k}$.
- With 19 hits, still notice $P_{c|k} <$ Pc Dismiss overtime .

SUMMARY

8 hrs Prediction to Close approach				
	Hi P_{fa} /Hi P_{md}	Low P_{fa} /Low P_{md}	Hi P_{fa} /Low P_{md}	Low P_{fa} /Hi P_{md}
FAR	45.11%	13.72%	46.78%	13.31%
MDR	10.53%	10.53%	10.53%	15.79%

3 hrs Prediction to Close approach				
	Hi P_{fa} /Hi P_{md}	Low P_{fa} /Low P_{md}	Hi P_{fa} /Low P_{md}	Low P_{fa} /Hi P_{md}
FAR	37.42%	16.01%	36.80%	16.84%
MDR	5.26%	5.26%	5.26%	5.26%

- There exists a trade in False Alarm Rates and Missed Detection Rates accuracies with the prediction duration to the time of Closest approach (TCA).
- Desired P_{fa} and P_{md} can be tailored based on the mission phase and the available time of prediction to TCA.
- This preliminary study will be useful in determining the correct approaches for each mission phase during proximity operations

WORK TO GO:

- Complete build of generating a range of targeted P_{fa} and P_{md} using a representative number of Monte Carlo runs.
- Run examples of specific mission phase scenarios.

OSIRIS-REx™
ASTEROID SAMPLE RETURN MISSION

THANK YOU

THE UNIVERSITY OF ARIZONA – NASA GODDARD SPACE FLIGHT CENTER – LOCKHEED MARTIN - KINETX