Trajectory Design Employing Convex Optimization for Landing on Irregularly Shaped Asteroids

Robin M. Pinson[*]
NASA MSFC, EV42 MSFC, AL 35812, USA

Ping Lu[†]
Iowa State University, Ames, IA 50011, USA

Mission proposals that land on asteroids are becoming popular. However, in order to have a successful mission the spacecraft must reliably and softly land at the intended landing site. The problem under investigation is how to design a fuel-optimal powered descent trajectory that can be quickly computed onboard the spacecraft, without interaction from ground control. An optimal trajectory designed immediately prior to the descent burn has many advantages. These advantages include the ability to use the actual vehicle starting state as the initial condition in the trajectory design and the ease of updating the landing target site if the original landing site is no longer viable. For long trajectories, the trajectory can be updated periodically by a redesign of the optimal trajectory based on current vehicle conditions to improve the guidance performance. One of the key drivers for being completely autonomous is the infrequent and delayed communication between ground control and the vehicle. Challenges that arise from designing an asteroid powered descent trajectory include complicated nonlinear gravity fields, small rotating bodies and low thrust vehicles.

There are two previous studies that form the background to the current investigation. The first set looked in-depth at applying convex optimization to a powered descent trajectory on Mars with promising results.[1,2] This showed that the powered descent equations of motion can be relaxed and formed into a convex optimization problem and that the optimal solution of the relaxed problem is indeed a feasible solution to the original problem. This analysis used a constant gravity field. The second area applied a successive solution process to formulate a second order cone program that designs rendezvous and proximity operations trajectories.[3,4] These trajectories included a Newtonian gravity model. The equivalence of the solutions between the relaxed and the original problem is theoretically established.

The proposed solution for designing the asteroid powered descent trajectory is to use convex optimization, a gravity model with higher fidelity than Newtonian, and an iterative solution process to design the fuel optimal trajectory. The solution to the convex optimization problem is the thrust profile, magnitude and direction, that will yield the minimum fuel trajectory for a soft landing at the target site, subject to various mission and operational constraints. The equations of motion are formulated in a rotating coordinate system and includes a high fidelity gravity model. The vehicle's thrust magnitude can vary between maximum and minimum bounds during the burn. Also, constraints are included to ensure that the vehicle does not run out of propellant, or go below the asteroid's surface, and any vehicle pointing requirements. The equations of motion are discretized and propagated with the trapezoidal rule in order to produce equality constraints for the optimization problem. These equality constraints allow the optimization algorithm to solve the entire problem, without including a propagator inside the optimization algorithm.

The previous paper published by the authors[5] focuses on how to relax and manipulate a non-convex optimization problem into a convex problem whose solution will be the solution of the original problem. The gravity model is the most challenging aspect to manipulate into a convex form. The previous research assumed a perfect triaxial ellipsoid with a 2x2 spherical harmonic gravitational potential model. Once the spacecraft is inside the Brillouin sphere, in other words close to the surface, the 2x2 model is no longer accurate for the nonellipsoidal shaped asteroids, especially binary asteroids. The current research combines two gravity models to create a high fidelity model that is applicable to any asteroid shape and incorporates them in the fuel optimal convex optimization process and successive solution methodology. Outside the Brillouin sphere a full 4x4 spherical harmonic model[6] is applied, thus removing all triaxial ellipsoid assumptions. Inside

---

[*] Aerospace Engineer, Guidance, Navigation and Mission Analysis Branch, non-member.

[†] Professor, Department of Aerospace Engineering, AIAA Member.

the Brillouin sphere a model called interior spherical Bessel gravity field will be used. This model was published by Takahashi and Scheeres[7] and is chosen due to the balance between high fidelity, computational complexity and similarities to the 4x4 spherical harmonic expansions used outside the Brillouin sphere. The similarities in the expansion allow a clean and simple switch between the models without the need to change the equations of motion formulation at the Brillouin sphere. A Matlab based convex optimization package, cvx with the SDPT3 solver,[8] will be used to solve the optimization problem and highlight the success of the methodology.

Trajectories will be designed to land on the binary asteroid Castalia, thus showing the success of the optimization method including these higher fidelity gravity models. Figure 1 contains the shape model of Castalia along with a representative box that bounds the shape of the asteroid and depicts the three landing sites under investigation. A fuel optimal trajectory designed using the above optimization process and gravity model for a 400 second trajectory landing at site LS1 is shown in figure 2. The corresponding thrust profile, both magnitude and individual components, is depicted in figure 3. The shape of the thrust magnitude is maximum-minimum-maximum known as bang-bang, which is the profile corresponding to the solution from optimal control theory.

The previous analysis solved the fuel optimal problem assuming a fixed flight time. This was also assumed in the previous work.[5] During the analysis of triaxial ellipsoids, parameter sweeps varying the flight time proved that propellant usage is unimodal with respect to flight time. This allows the ability to optimize the flight time over a range of flight times by creating an outer optimization loop using Brent's method,[9] which will be discussed in this paper.

Additional references for this paper are listed in the reference section.[10, 11]

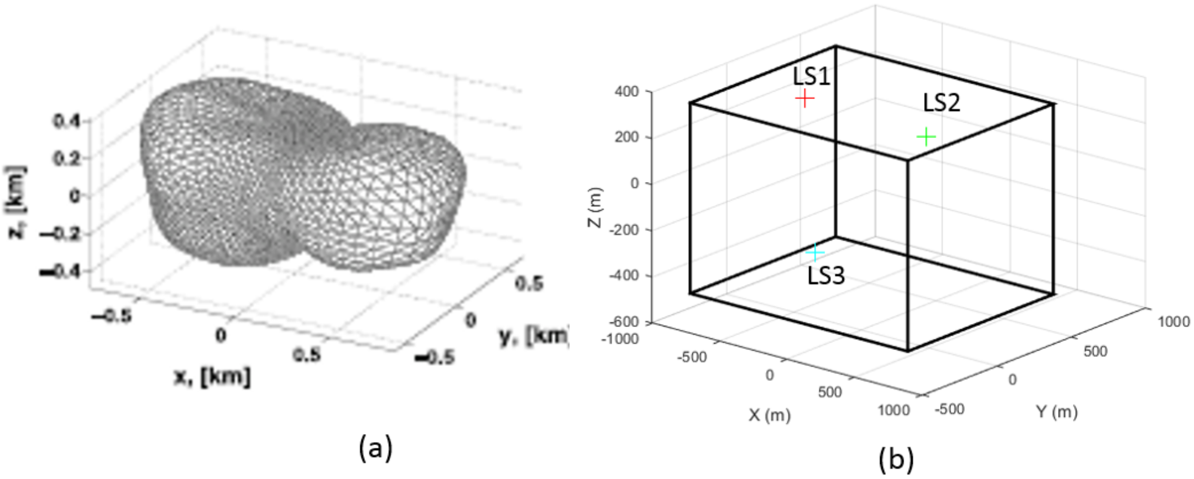

Figure 1. a) Shape of the asteroid Castalia courtesy of Takahashi and Scheeres.[7] b) Landing sites under investigation on Castalia with the box bounding the asteroid.

References

[1] Acikmese, B. and Ploen, S., "Convex Programming Approach to Powered Descent Guidance for Mars Landing," *Journal of Guidance, Control, and Dynamics*, Vol. 30, No. 5, September-October 2007, pp. 1353–1366.

[2] Acikmese, B., Blackmore, L., and Scharf, D., "Enhancements on the Convex Programming Based Powered Descent Guidance Algorithm for Mars Landing," *AIAA/AAS Astrodynamics Specialist Conference*, August 2008.

[3] Lu, P. and Liu, X., "Autonomous Trajectory Planning for Rendezvous and Proximity Operations by Conic Optimization," *Journal of Guidance, Control, and Dynamics*, Vol. 36, No. 2, 2013, pp. 375–389.

[4] Liu, X. and Lu, P., "Solving Nonconvex Optimal Control Problems by Convex Optimization," *Journal of Guidance, Control, and Dynamics*, Vol. 37, No. 3, 2014, pp. 750–765.

[5] Pinson, R. and Lu, P., "Rapid Generation of Optimal Asteroid Powered Descent Trajectories via Convex Optimization," *2015 AAS/AIAA Astrodynamics Specialist Conference*, 2015.

[6] Vallado, D., *Fundamentals of Astrodynamics and Applications*, Microcism Press, California, 2013.

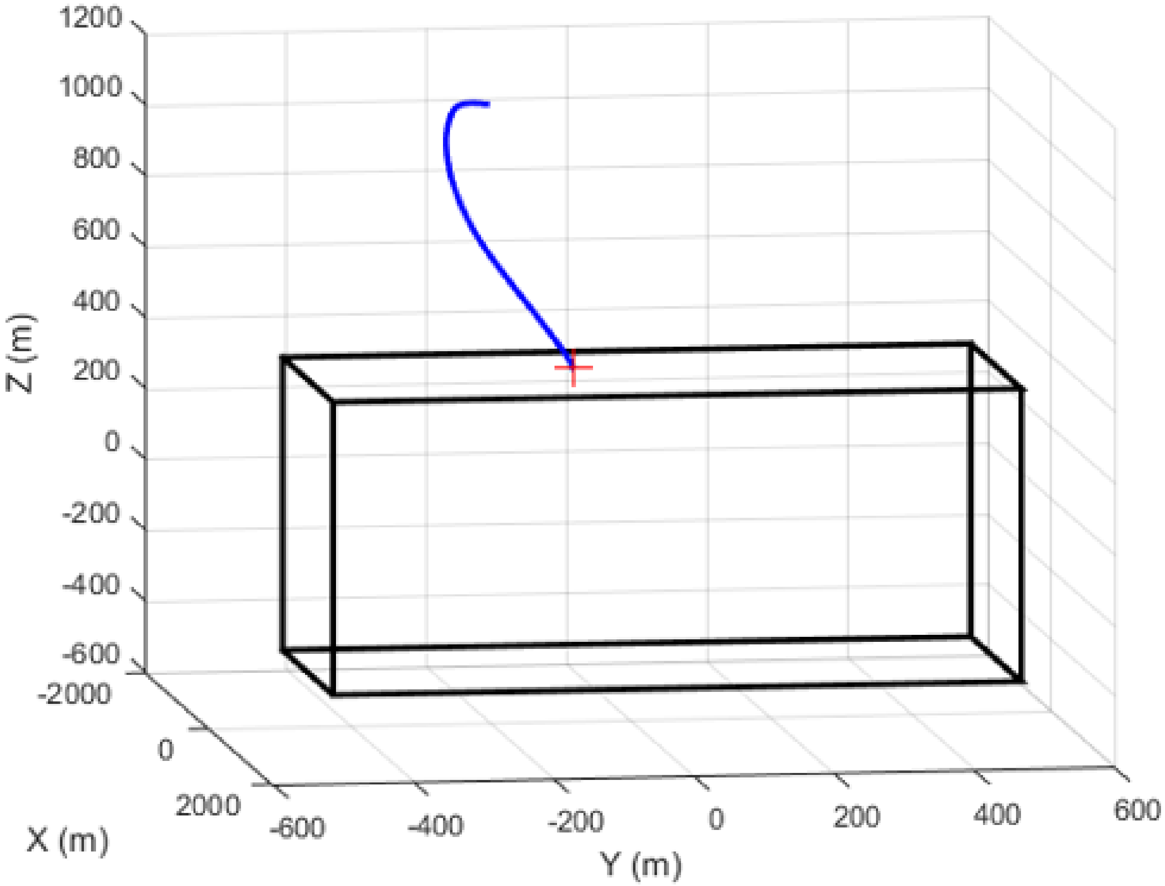

Figure 2. Designed fuel optimal trajectory landing at LS1 with a 400 second flight time.

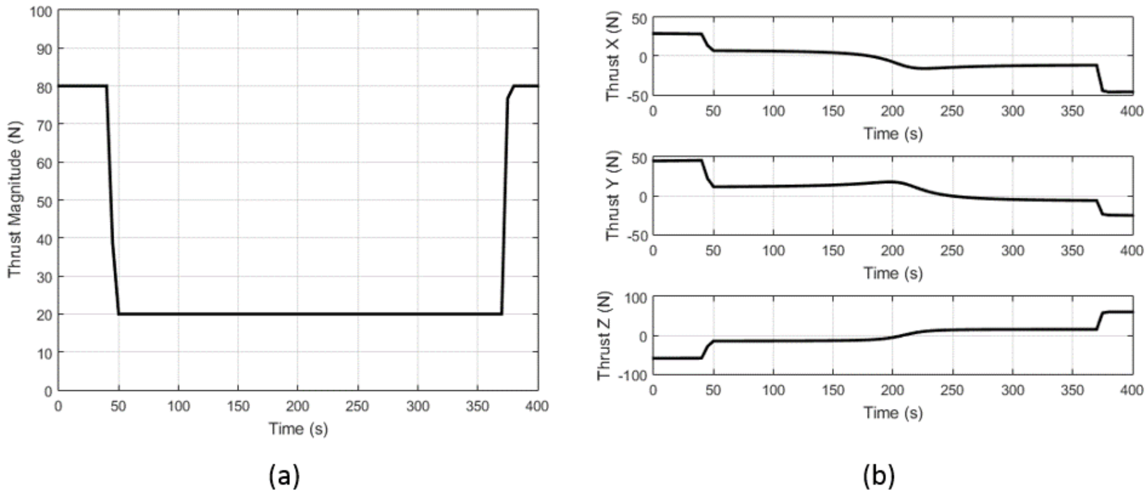

Figure 3. Thrust magnitude (a) and components (b) corresponding to the fuel optimal trajectory landing at LS1 with a 400 second flight time.

[7] Takahashi, Y. and Scheeres, D. J., “Small body surface gravity fields via spherical harmonic expansions,” *Celestial Mechanics and Dynamical Astronomy*, Vol. 119, No. 2, June 2014, pp. 169–206.

[8] Grant, M. and Boyd, S., “CVX: Matlab software for disciplined convex programming, version 2.0,” http://cvxr.com/cvx, April 2011.

[9] Brent, R. P., *Algorithms for Minimization Without Derivatives*, chap. 4-5, Prentice-Hall, NJ, 1973.

[10] Boyd, S. and Vandenberghe, L., *Convex Optimization*, Cambridge University Press, 2004.

[11] Scheeres, D. J., Ostro, S. J., Hudson, R. S., and Werner, R. A., “Orbits Close to Asteroid 4769 Castalia,” *Icarus*, Vol. 121, 1996, pp. 67–87.