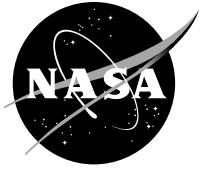# Applicability and Limitations of Reliability Allocation Methods

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Abstract

Reliability allocation process may be described as the process of assigning reliability requirements to individual components within a system to attain the specified system reliability. For large systems, the allocation process is often performed at different stages of system design. The allocation process often begins at the conceptual stage. As the system design develops, more information about components and the operating environment becomes available, different allocation methods can be considered. Reliability allocation methods are usually divided into two categories: weighting factors and optimal reliability allocation. When properly applied, these methods can produce reasonable approximations.

Reliability allocation techniques have limitations and implied assumptions that need to be understood by system engineers. Applying reliability allocation techniques without understanding their limitations and assumptions can produce unrealistic results.

This report addresses weighting factors, optimal reliability allocation techniques, and identifies the applicability and limitations of each reliability allocation technique.

1.0   Introduction

The reliability allocation process, an important element of reliability engineering, helps system designers select components and apply appropriate design strategies to meet system reliability requirements. The allocation process involves generating a reliability block diagram and specifying system reliability requirements. Reliability requirements are then assigned to subsystems and related components.

The allocation process is dynamic and is typically considered throughout the design phases (Rau, 1984). At the conceptual phase when little or nothing is known about the system, simple allocation methods such as equal allocation, can be applied to assign equal reliability to subsystems. As the system design develops and more information about components and the operating environment becomes available, different allocation methods and reliability improvement techniques may also be considered.

The allocation of system reliability involves solving the following inequality:

$$f(R_1(t), R_2(t), ..., R_n(t)) \geq R_s(t) \tag{1}$$

where f is a function that relates components reliabilities to system reliability, R_1 is the reliability at time t of the ith component, $R_s(t)$ is reliability goal at time t.

Reliability allocation methods are usually divided into two categories: weighting factors and optimal reliability allocation. The former method often involves using subjective and objective assessments to estimate the allocation factors and failure rates of subsystems. The latter method involves applying mathematical programming techniques to estimate the optimum number of units and components reliability levels that either maximizes the system reliability or minimizes the system cost, given some constraints.

This report describes weighting factors and optimal reliability allocation techniques and identifies the applicability and limitations of each reliability allocation technique. Section 2.0 describes traditional and nontraditional weighting factor allocation techniques. Section 3.0 discusses optimal reliability allocation, specifically cost minimization, and redundancy allocation methods.

2.0 Weighting Factor Methods

In general, weighting factor methods assign reliability requirements to system components using criteria that are considered important for system performance. Once the system reliability requirement is defined, the subsystems reliability are estimated as the product of subsystem's allocation factor and the system's required reliability. The subsystem allocation factors are usually evaluated numerically by the system design team. This section describes two traditional and nontraditional techniques.

2.1 AGREE Method

The AGREE allocation method was developed for electronic equipment and takes into account the complexity and the significance of each subassembly and assumes that subassemblies are in series and have exponential failure distribution.

Unit complexity is assessed based on number of modules, where a module is a single functional unit. The unit importance factor is defined as the probability that the system will fail if the unit fails. An importance value of one implies that the unit is essential for successful system operation. A value of zero means that the unit has no impact on system performance. The AGREE allocation is expressed as follows:

$$\lambda_i = \frac{n_i \left[-\log_e Rs(T) \right]}{N \cdot E_i \cdot t_i}$$

(2)

where
λ_i allocated failure rate of unit i
$Rs(T)$ system reliability
n_i number of modules in unit i
E_i importance factor of unit i
t_i number of hours unit i will be required to operate in T system hours
N total number of modules in the system

AGREE allocation applies to series configurations. The method can be used when information is available about system complexity and the importance of each subsystem to system success. Also, this method can produce inaccurate reliability estimates for redundant configuration and when the importance factor of subsystems is relatively low (Kececioglu, 1991). Hence, this technique should be used when all unit importance factors are close to one (Kapur and Lamberson, 1977).

2.2 ARINC Method

The ARINC allocation method applies to subsystems that are in series, are independent, and have constant failure rates. The method also assumes that mission time for each subsystem equals system mission time. ARINC allocation requires the availability of component failure rates, which can be estimated using predicted or field data. With this information, the weighting factor w_i and component failure rate λ_i are calculated as follows:

$$w_i = \frac{\lambda_i}{\sum_{i=1}^{n} \lambda_i} \quad i = 1, 2, \ldots, n \tag{3}$$

$$\lambda_i = w_i \cdot \lambda_s \tag{4}$$

where λ_s the system's failure rate and n represents the number of units or subsystems.

As stated earlier, weighting factors are estimated based on prediction or observed data, hence application of ARINC allocation relies on the availability of a bill of materials or field data from similar equipment. The method also applies to series configurations only. This method should provide reasonable reliability allocation estimates when there is a clear understanding of the system requirements.

2.3   Jeong and Koh Method

This method was primary developed to allocate reliability requirements to a two-level switching system, as shown in Figure 1. The method involves the allocation of reliability to each subsystem (first level) and then the allocation of reliability to the lowest level components (PCBAs). The reliability allocation for the lowest level accounts for unit's repair time. The first level consists of subsystems (blocks) connected in series. This method assumes that each subsystem fails independently of each other. The second level consists of PCBAs units connected in parallel-series configuration. Redundant units are maintainable with a constant mean time to repair, μ.

In this method, subsystem allocation factors are calculated as a function of numerical ratings of system complexity, system importance, and impact on system failure. The three ratings for each subsystem are multiplied together to give an overall rating for the subsystem (Eq. (5)). The subsystem ratings (WF_i) are then normalized so that their sum is 1. The allocated failure rate for each subsystem is the required system failure rate (λ_s) multiplied by the weighting factor associated with each subsystem (Eq. (6)). The weighting factor (WF_i) and failure rate (λ_i) of block i are estimated as follows

$$WF_i = CF_i \cdot IF_i \cdot SF_i \tag{5}$$

$$\lambda_i = \left(\frac{WF_i}{\sum_{i=1}^{m} WF_i} \lambda_s, \quad i = 1, 2, \ldots, m \right) \tag{6}$$

where CF_i is the complexity factor of block i, IF_i is the importance factor of block i, and SF_i is the failure scale factor of block i. Description and calculation of CF_i, IF_i, and SF_i factors are included in Appendix A.
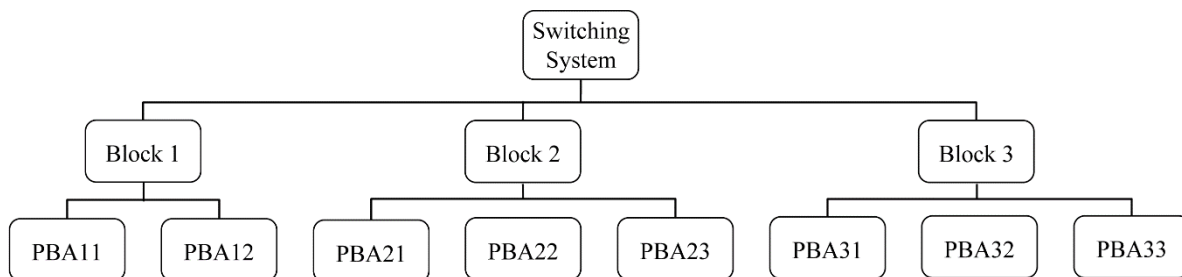

Figure 1.—Typical two-level switching system hierarchy.

Once the failure rate of first level units (λ_i) is estimated, the failure rate of second level (λ_{ij}) units is established using a two-step procedure. First, the weighting factor (WF_{ij}) and failure rate (λ_{ij}) of PBA_{ij} are estimated as follows:

$$WF_{ij} = CF_{ij} \cdot IF_{ij} \cdot SF_{ij} \tag{7}$$

$$\lambda_{ij} = \left(\frac{WF_{ij}}{\sum_{j=1}^{nj} WF_{ij}} \lambda_i, \quad j = 1,2,\ldots,n \right) \tag{8}$$

The second step involves obtaining the adjusted failure rate of the redundant configuration. Since redundant units are repairable, the redundant configuration's failure rate (λ_{ij}) is adjusted to take into account the maintainability factor. Jeong and Koh (1995) derived a mathematical model to estimate the adjusted failure rate (λ^a_{ij}) of redundant configurations. The model applies only when $\mu \gg \lambda^a_{ij}$. The derivation of the adjusted failure rate is included in Appendix B. The adjusted failure rate of redundant configurations with a failure rate λ_{ij} and mean time to repair μ are estimated as follows:

$$\lambda^a_{ij} = \sqrt{\frac{\mu \cdot \lambda_{ij}}{2}} \tag{9}$$

From my perspective, the allocation criteria and approach used to allocate reliability requirements at different levels is simple and takes into consideration the system complexity, functionality, criticality, and maintainability, which are important reliability factors. This method can be adapted to allocate component reliability to multiple levels of the system design. In addition, this method requires prior knowledge about the configuration, functionality of system components, so the method is appropriate for advanced stages of the system design.

2.4   Wang and Others Method

Wang et al. (2001) developed a method to allocate reliability requirements to a CNC system composed of units connected in a series configuration. They defined seven factors that are considered important for CNC lathe system performance: frequency of failure, criticality of failure, maintainability, complexity, manufacturing technology, working condition, and cost. In essence the method consists of estimating the weighting factors using objective and subjective assessment, adjusting each factor using important weighting factors and calculating the failure rate of each subsystem.

Estimating the weighting factors involves calculating the relative ratio for failure rate and the average relative ratio for failure rate. The estimation of the relative ratio factor for the frequency of failure, criticality of failure, maintainability, and complexity require historical failure data and criticality analysis data. The mathematical expression for calculating these factors is included in Appendix C.

Once the relative ratio is estimated, the average relative ratio for failure rate is calculated as follows:

$$\gamma_{ki} = \frac{1}{2} \sum_{j=1}^{n} \beta^k_{ij}, \quad k = 1,2,\ldots,m; \quad i = 1,2,\ldots,n \tag{10}$$

where
γ_{ki} average relative ratio for failure rate of kth criterion and i^{th} subsystem
n number of subsystems
m number of allocation factors
β^k_{ij} relative ratio for ratio rate allocation between the ith subsystem and the jth subsystem

The failure rate allocation factors are calculated as follows:

$$\alpha_i = \sum_{k=1}^{m} \gamma_{ki} \cdot w_k, \quad i = 1, 2, \ldots, n \tag{11}$$

where
a_i comprehensive allocation factor for the ith subsystem
w_k weight or the importance of the kth criterion
γ_{ki} average relative ratio for failure rate of kth criterion and i^{th} subsystem
n number of subsystems
m number of allocation criteria

Once the failure rate allocation factor k_i is calculated, the failure rate for the ith subsystem is estimated as follows:

$$\lambda_i = \frac{a_i}{\sum\limits_{i=1}^{n} a_i} \lambda_s \tag{12}$$

where
λ_i failure rate of ith subsystem
a_i failure rate allocation factor of the ith subsystem
λ_s system's failure rate
n number of subsystems

This method applies only to series configurations and requires the availability of equipment historical data in order to reduce subjectivity and produce credible and reasonable allocation estimates. Although the Wang and others method was developed for CNC lathe systems, it can also be adapted for other systems by adjusting the weighting factors. As stated earlier, this method utilizes objective and subjective assessments to estimate allocation factors and requires calculation of importance weight factors which involves expert judgment.

3.0 Optimal Reliability Allocation Methods

In general, optimal reliability allocation is concerned with the allocation of individual component reliability to meet some desired level of system reliability or cost, subject to a set of constraints such as cost, weight, and volume. Optimal reliability allocation methods involve applying mathematical programming techniques to obtain the best possible combination of components reliability that maximizes system reliability or minimizes system cost. A reliability allocation problem may be formulated as maximization of system reliability, subject to cost restrictions, or minimization of system cost, subject to

attaining some specified level of system reliability. Reliability maximization methods are applied to obtain the optimal number of redundant components of a system configuration, given a set of constraints. The cost minimization method is often applied to allocate reliability requirements to parallel-series systems while minimizing system cost.

3.1 Cost Minimization Problem Formulation

The cost minimization formulation process usually begins by establishing the system configuration and the desired level of reliability. Once system reliability requirements are established, the problem objective and constraints are described by mathematical expressions. The formulation process consists of obtaining the system reliability function (reliability model) in terms of its components' reliability and cost reliability function. This process was established to achieve a minimum total system cost that satisfies the system reliability requirement. Although reliability cost functions can be established using empirical data, such data is often difficult to obtain. The exponentially increasing functions are used to represent the relationship between cost and reliability of components (Majety, Dawande, and Rajgopal, 1999). A general formulation of a cost minimization reliability allocation problem of a series configuration shown in Figure 2 can be represented as follows:

Minimize

$$C_s = \sum_{i=1}^{n} C_i(R_i) \tag{13}$$

Subject to

$$R_s \geq R_r \tag{14}$$

$$R_{i,\min} \leq R_i \leq R_{i,\max}, \quad i = 1, 2, \ldots, N \tag{15}$$

where
C_s the total cost of the system
C_i the cost of ith stage or subsystem
R_r the system reliability requirement
R_s the system reliability goal
R_i the reliability of the ith stage or subsystem
$R_{i,\max}$ maximum achievable reliability of the ith component
$R_{i,\min}$ minimum achievable reliability of the ith component
N number of stages or subsystems

Once the problem model is formulated, the solution is obtained by applying programming solution techniques such as heuristic, nonlinear, dynamic, or mixed integer programming. Most optimal reliability allocation methods require extensive calculations and solutions are obtained using computer codes.
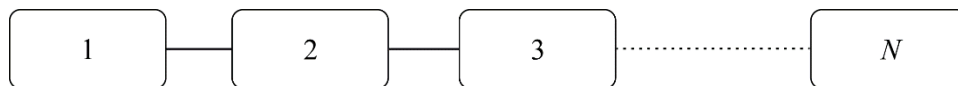

Figure 2.—An *N*-stage series system.

3.2 Reliability Optimization Methods—Redundancy Allocation

Literature on reliability allocation methods contains an abundance of design for reliability allocation methods. Most of these methods were developed to solve redundant optimization problems for simple and complex system configurations. Some methods have been successfully applied to solve reliability allocation problems for diverse applications. For example, Yang et al., (1999) applied a generic method to a reliability allocation problem in a nuclear power plant. Goel et al. (2003) used mixed integer nonlinear programming to optimize a chemical plant system availability. Wattanapongsakorn and Levitan (2001) used a simulated annealing method to optimize reliability for a distributed system consisting of hardware, software, and network components.

The effectiveness and computational efficiency of any method depends to a large extent on the complexity of the system, number of variables, and number of constraints. Tillman, Hwang, and Kuo (1977) noted that only a few methods have proven to be effective when applied to large-scale nonlinear programming problems. Nakagawa and Miyazaki (1981) stated that it is impractical to use dynamic programming to solve problems with more than three constraints because the computation time is excessive. In another study, Nakagawa and Nakashima (1977) demonstrated that the Misra and Ljubojevic (1973) method produced unsatisfactory results because the number of variables increases when applied to multistage series systems.

The next subsections consider three reliability optimization methods that have produced satisfactory results.

3.2.1 Sharma and Venkateswaran Method

The Sharma and Venkateswaran (1971) method used by Tillman, Hwang, and Kuo (1977) is a simple heuristic method that can be applied to multistage system problems with any number of constraints. Basically, the method consists of adding redundancy to the stage that has the lowest reliability. The process begins by assigning a redundant unit to the stage with the lowest reliability. If any constraint is violated, the most recent redundant component is removed. The resulting number is the optimum allocation for that stage. At this point this stage is removed from consideration. If all stages have been removed from consideration, the current number of redundant allocated components is the optimum configuration of the system. Otherwise, the process repeats.

Kuo, Hwang, and Tillman (1978) demonstrated that this method is effective in solving multistage series system problems with nonlinear constraints. They applied this method to a five-stage system configuration with three nonlinear constraints. The problem was formulated as a minimization problem, subject to two constraints which were defined as exponential functions and found an optimal solution.

Kuo, Hwang, and Tillman also applied the Sharma and Venkateswaran method to a bridge configuration and a multistage series configuration with linear constraints. In both cases, the method yielded suboptimal solutions. Nakagawa and Miyazaki (1981) applied the method to a parallel-series reliability problem with one linear constraint and noted that this method produced inaccurate solutions as the number of stages increases.

3.2.2 Misra and Ljubojevic Method

Misra and Ljubojevic developed a method for solving optimization problems with multiple constraints and described the computational procedure in detail. In essence, the method breaks up the r-constraints problem into individual r-problems, each having one constraint. A desirability factor, defined as the ratio of the percentage increase in the system reliability to the percentage increase of the corresponding cost, is introduced to determine the stage where a redundancy should be added.

Kuo, Hwang, and Tillman applied this approach to find the optimum redundancy of a four-stage series system, subject to two linear constraints. The problem was formulated to maximize system reliability, subject to weight and cost constraints. They concluded that the Misra and Ljubojevic method can be effectively applied to solve reliability allocation problems involving linear constraints. They also observed an increase in computational time as the number of constraints increased.

4.0   Summary and Conclusions

Reliability allocation methods assign reliability requirements to system components taking into consideration those factors that are important for system performance. Reliability allocation methods are typically divided into two categories: weighting factors and optimal reliability allocation factors. Weighting factor methods are flexible and are usually estimated using objective and subjective assessments. Objective assessments require availability of historical data and mathematical model definitions. Subjective assessments rely on expert judgment. A reasonable reliability approximation can be obtained when important system design information such as bill of material and operating conditions are available. Weighting factor methods can only be applied for series configurations and when it is reasonable to assume that components are statistically independent and failure rates are constant.

Optimal reliability allocation methods require applying mathematical programming methods to obtain the optimum system allocation. The cost minimization problem formulation requires defining components reliability cost models, system reliability model, and upper and lower bounds of component reliability.

Optimal reliability allocation techniques can be applied to series, parallel, and complex configurations such as bridge and series parallel. However, as the number of system components and constraints increase, the solutions become more complex, requiring application of advanced programming methods. Problems are often solved using computer codes.

Appendix A.—Jeong and Koh Reliability Factors

The complexity factor is defined as the proportion of number of active components of a given block to the total number of active components. The complexity factor of block i is estimated as follows:

$$CF_i = \frac{N_i}{\sum\limits_{i=1}^{m} N_i}, \quad i = 1,2,\ldots,m \tag{16}$$

where CF, is the complexity factor of block i, N is the number of active components of unit i, and m is the total number of blocks.

The importance factor is defined as the essentiality of the block to perform its intended function. It is a function of the number (N_i) of system functions performed by block i.

This factor is calculated as follows

$$IF_i = \frac{1 - \dfrac{NF_i}{\sum NF_i}}{m-1}, \quad i = 1,2,\ldots,m \tag{17}$$

The failure scale factor is quantified in terms of the affected subscriber lines due to its failure. When the affected subscriber lines due to the failure of block i is K_i, its failure scale factor, SF_i, is calculated as follows

$$SF_i = \frac{1 - \dfrac{K_i}{\sum K_i}}{m-1}, \quad i = 1,2,\ldots,m \tag{18}$$

Appendix B.—Derivation of Adjusted Failure Rate

Jeong and Koh derived a mathematical expression for estimating the adjusted failure rate of the parallel PBA block. Basically, the model considers that the reliability of a redundant configuration can be approximated using the following expression

$$R(t) \cong e^{-\frac{2\lambda^a_{ij}{}^2}{\mu}t}$$

(19)

where λ^a_{ij} is the allocated failure rate for the overall PBA_{ij} and μ is the repair rate.

Jeong and Koh noted that the reliability model is a good approximation when $\mu \gg \lambda^a_{ij}$. Using the above reliability expression, the mean time between failure (MTBF) of the redundant configuration can be calculated as follows

$$MTBF = \int_0^\infty R(t)\,dt = \frac{\mu}{2(\lambda^a_{ij})^2} = \frac{1}{\lambda_{ij}}$$

(20)

From the above relationship the following adjusted failure rate expression is obtained

$$\lambda^a_{ij} = \sqrt{\frac{\mu \cdot \lambda_{ij}}{2}}$$

(21)

Appendix C.—Relative Ratio of the Failure Rate Calculation

The frequency of failure criterion, $\beta^{(1)}$ is calculated by

$$\beta^{(1)} = \frac{n_i}{n_i + n_j} \tag{22}$$

where n_i and n_j are the observed relative frequency of the ith subsystem U_i, and U_j, respectively. The relative failure frequency of the ith subsystem is defined as the ratio between the number of failures of the ith subsystem and the number of failures of all subsystems.

For the criticality of failure criterion, $\beta^{(2)}$ is calculated as follows

$$\beta^{(2)} = \frac{CR_j}{CR_i + CR_j} \tag{23}$$

where CR_i and CR_j represent the criticality of subsystem i and subsystem j, respectively. To calculate CR_i, for subsystem i, the failure modes of each essential component of the system are considered. The failure criticality of the jth failure mode of the kth component in a system is a product of three terms: (1) the probability of failure of this component in a specified interval, (2) the probability that the failure of a component is caused by a specific failure mode, and (3) the probability that the failure mode will cause severe damage to the system.

The maintainability criterion, $\beta^{(3)}$ is calculated as follows

$$\beta^{(3)} = \frac{T_{Dj} + T_{Hj}}{T_{Dj} + T_{Di} + T_{Hi} + T_{Hj}} \tag{24}$$

where T_{Dj} and T_{Di} are the mean down time of the ith subsystem U_i subsystem and the jth subsystem U_j, respectively, and T_{Hj} and T_{Hi} are the mean repair cost of the ith subsystem and U_i subsystem and the jth subsystem U_j, respectively.

For the complexity factor, $\beta^{(4)}$ is calculated as follows

$$\beta^{(4)} = \frac{n_i}{n_i + n_j} \tag{25}$$

where the n_i and n_j are the approximate number of essential parts in subsystem i and subsystem j, respectively.

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