A Space-Time Discontinuous-Galerkin Approach for Separated Flows

Scott M. Murman, Laslo T. Diosady, Anirban Garai, Marco Ceze
NASA ARC

Background

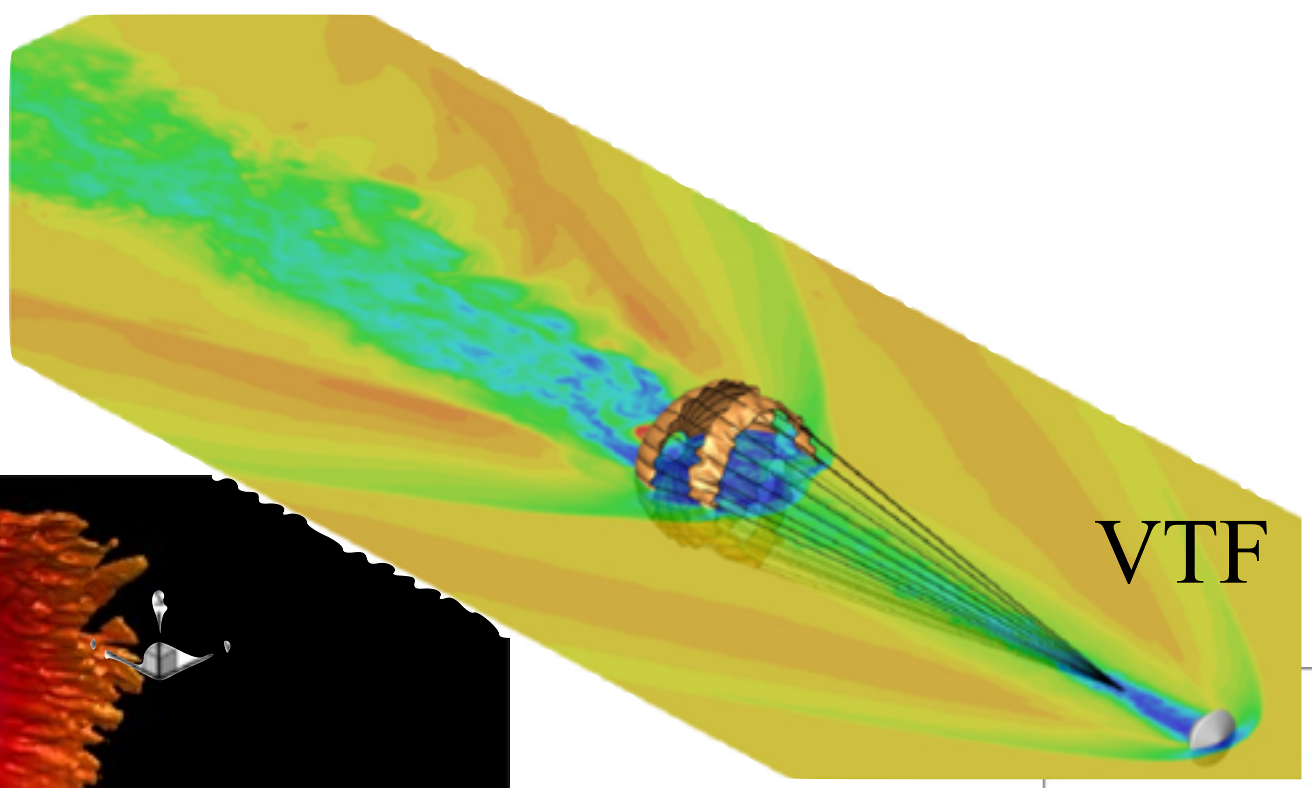
- Developing a spectral-element DG capability for separated flows over the past few years
  - Led by investment from TTT/RCA
  - *Diosady & Murman AIAA 2013-2870, 2014-2784, 2015-0294*
- Effort has grown recently w/ collaboration from other projects
  - Desired synergy between R&D and engineering
- Opportunity to share broader vision of effort & fill in technical gaps for AIAA audience

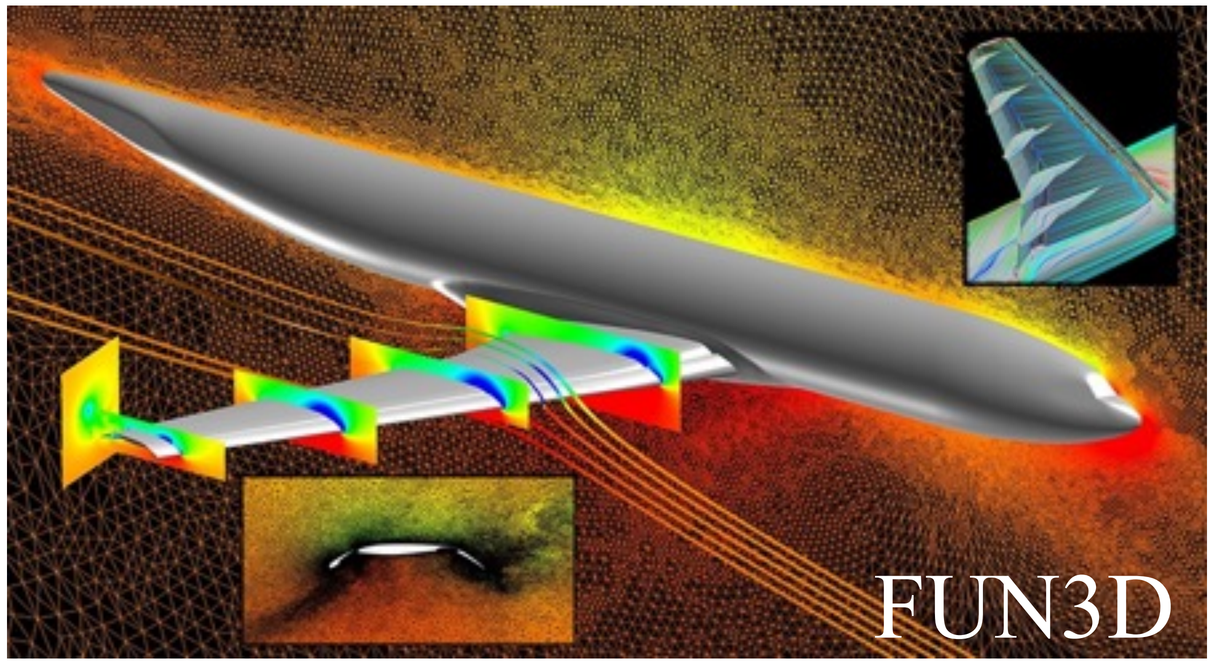
Context

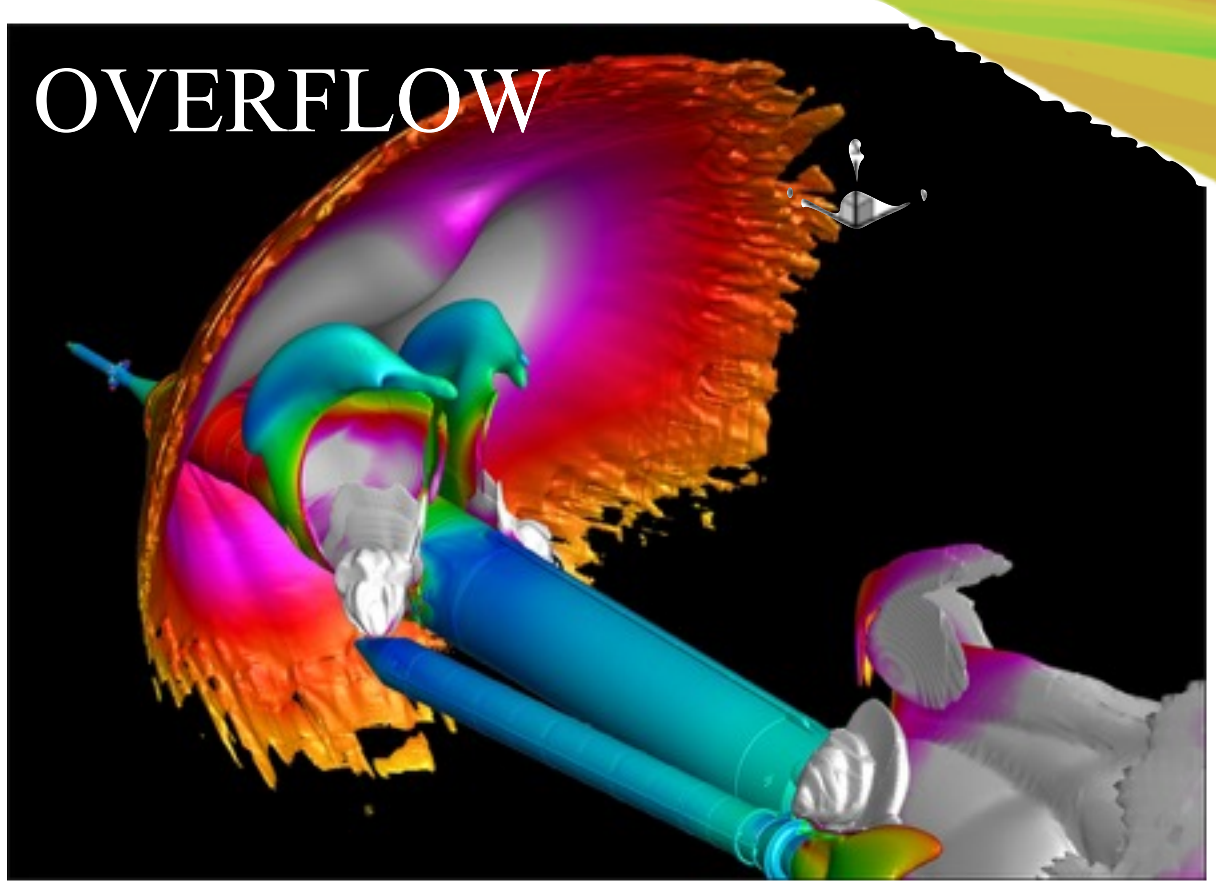
- No single optimal algorithm/method/solver
  - Pareto front of optimal choices
- Different groups prioritize differently
- Our priorities are derived from the needs of numerous projects within NASA - ARMD, HEOMD, STMD - and industrial partners
- Current NASA technology based primarily on RANS/DES
  - Works well for many engineering tasks
- Supplement existing capability w/ scale-resolving methods

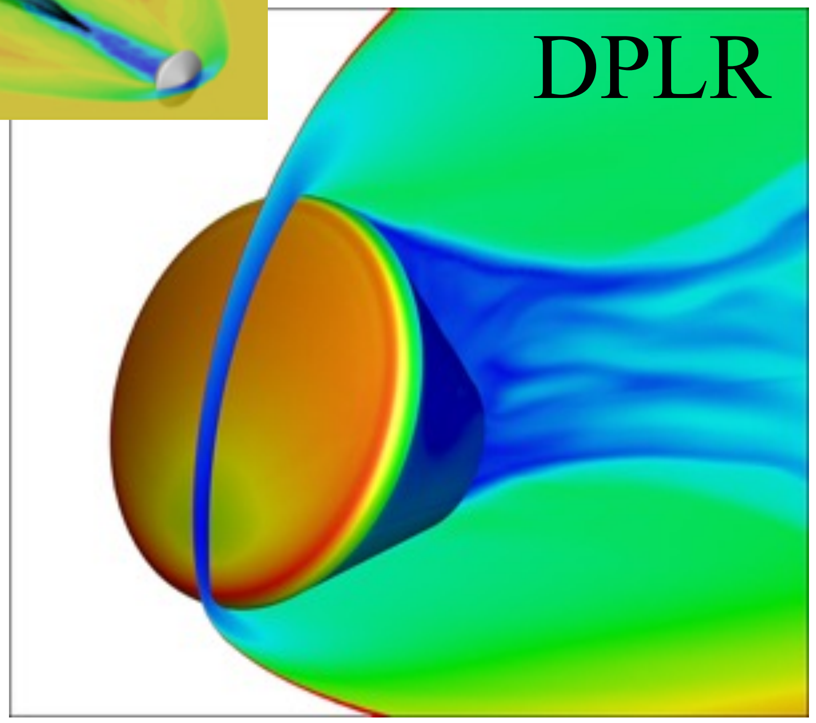
Target Applications

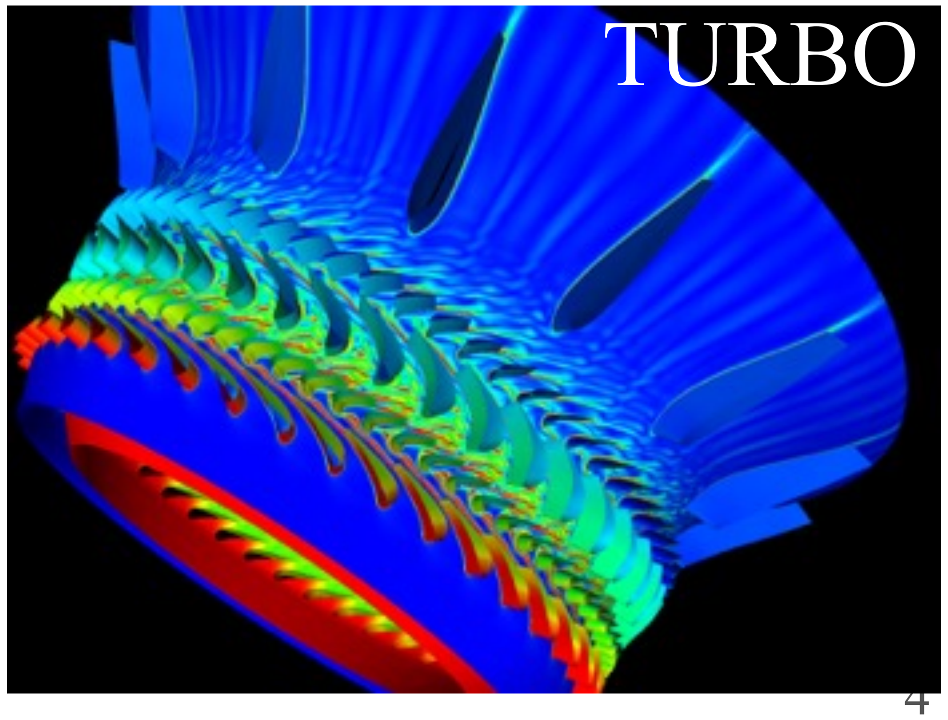
- Complex geometry - unstructured mesh
- Complex physics - scale-resolving methods
- High-Re, combustion, chemistry - fully implicit methods
- Computational intensive - high-order, adaptive methods
- Multi-disciplinary, multi-physics - robust, extensible methods









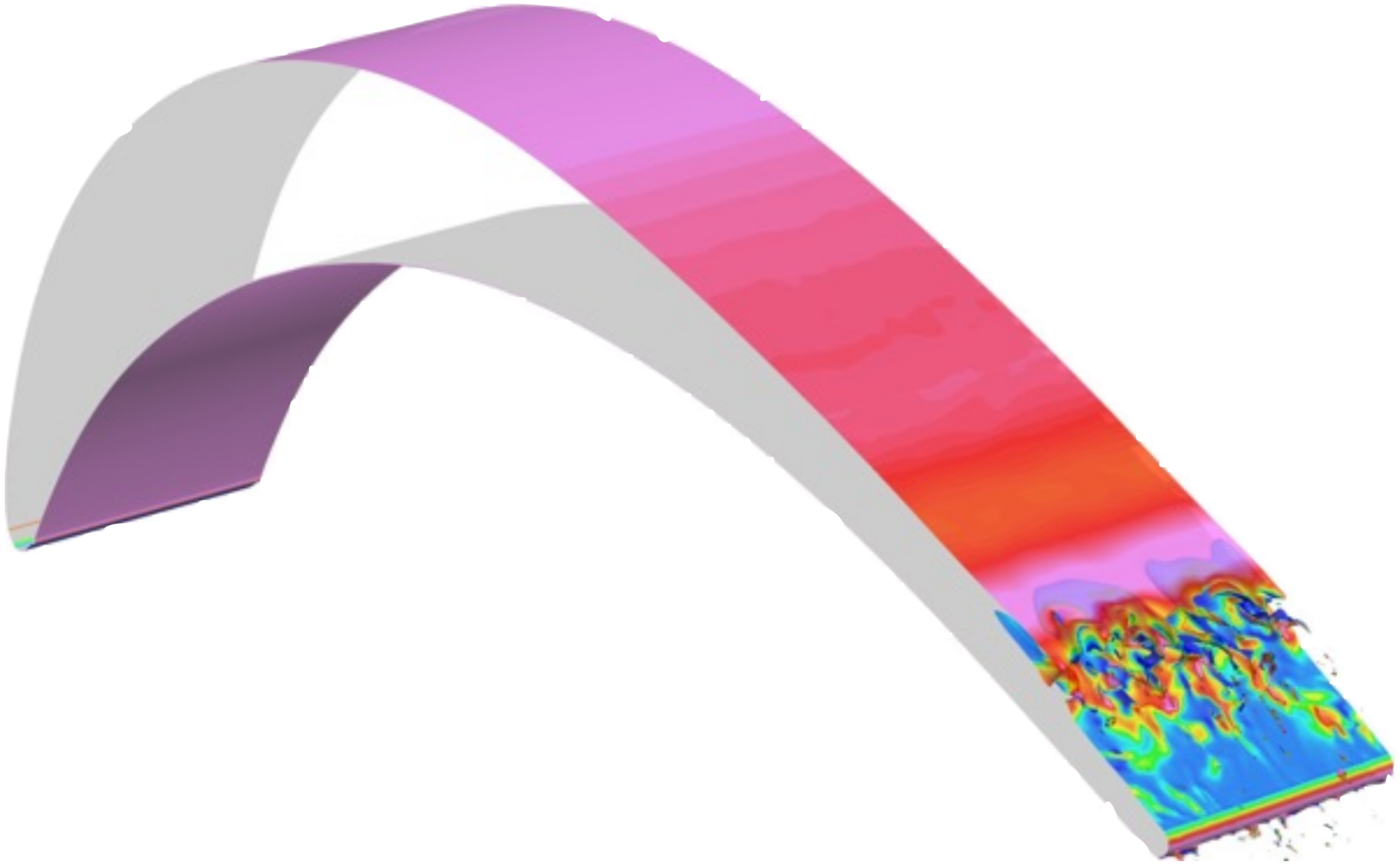


Approach

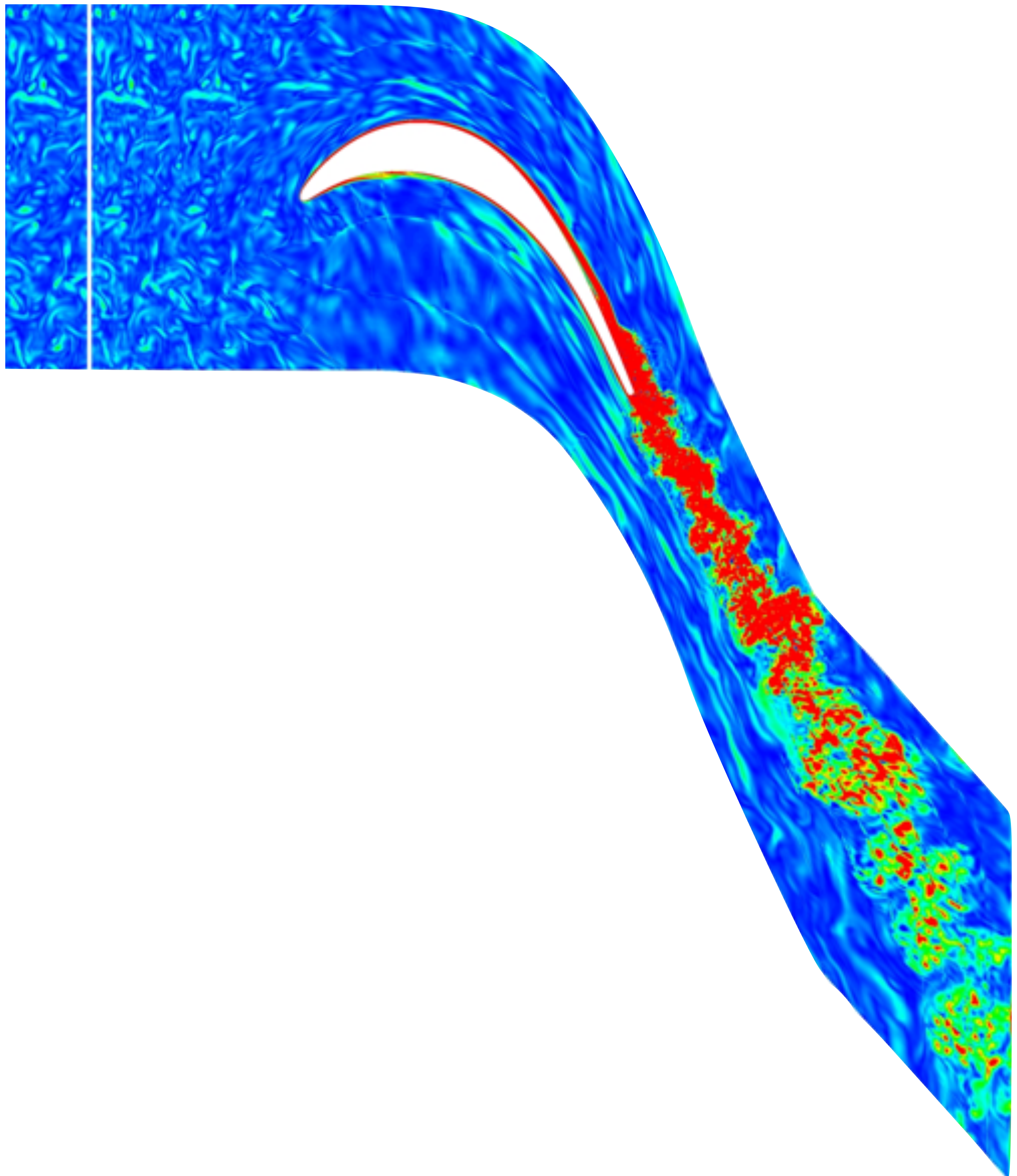
- DG spectral-element formulation
  - Unstructured arbitrary order
  - Variational Multiscale Method (VMM) for scale-resolving
- Fully implicit space-time
  - Entropy-stable, consistent all-speed scheme
  - *h*-*p* adaptation in space and time
- Galerkin formulation
  - Demonstrated success for relevant applications

Approach

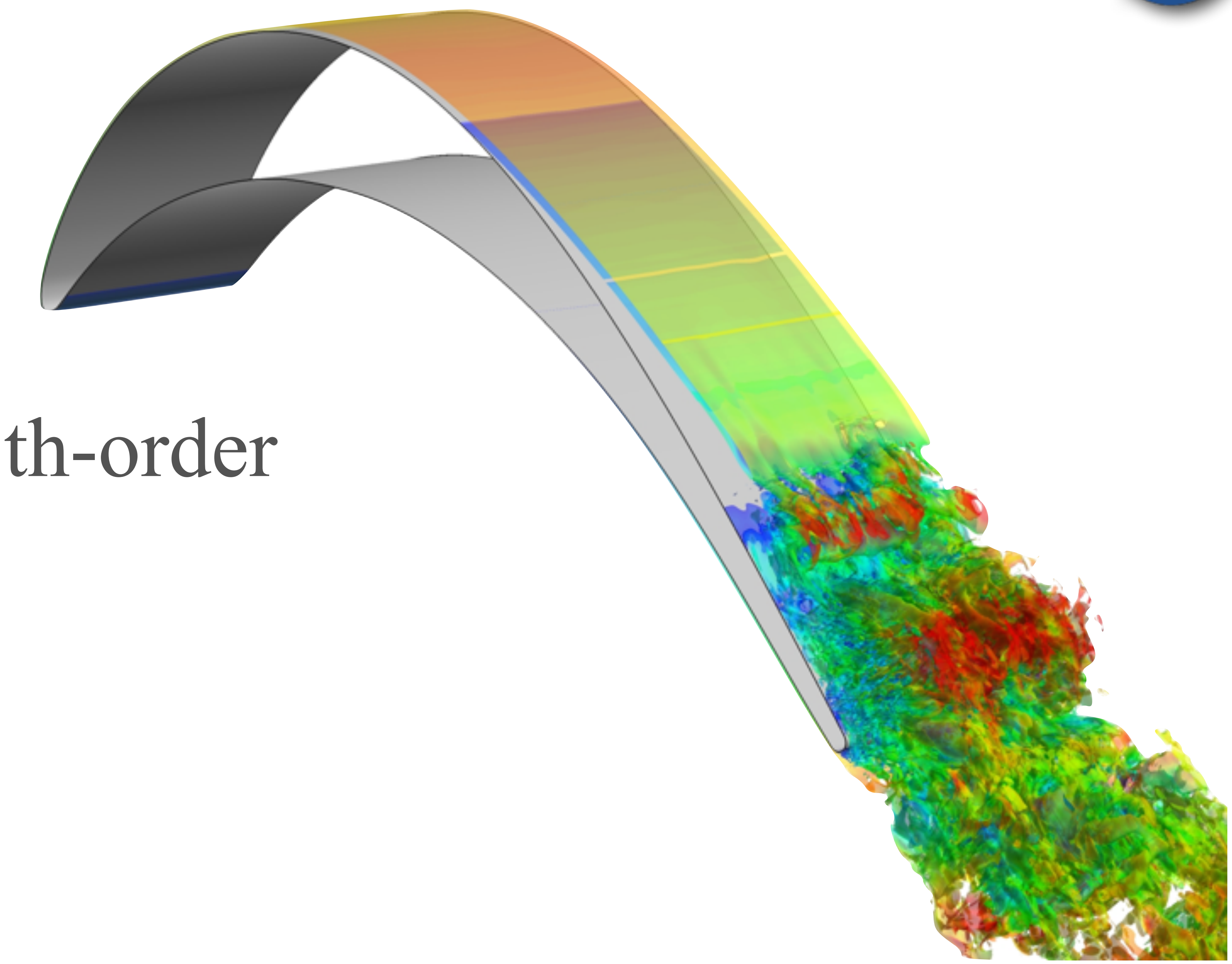
- Three main thrusts
  - New algorithms and methods
  - Optimized for next-gen exascale hardware
  - Novel physical models
- Informally known as the eddy solver
- Currently lower TRL than production methods



Turbomachinery Benchmarks

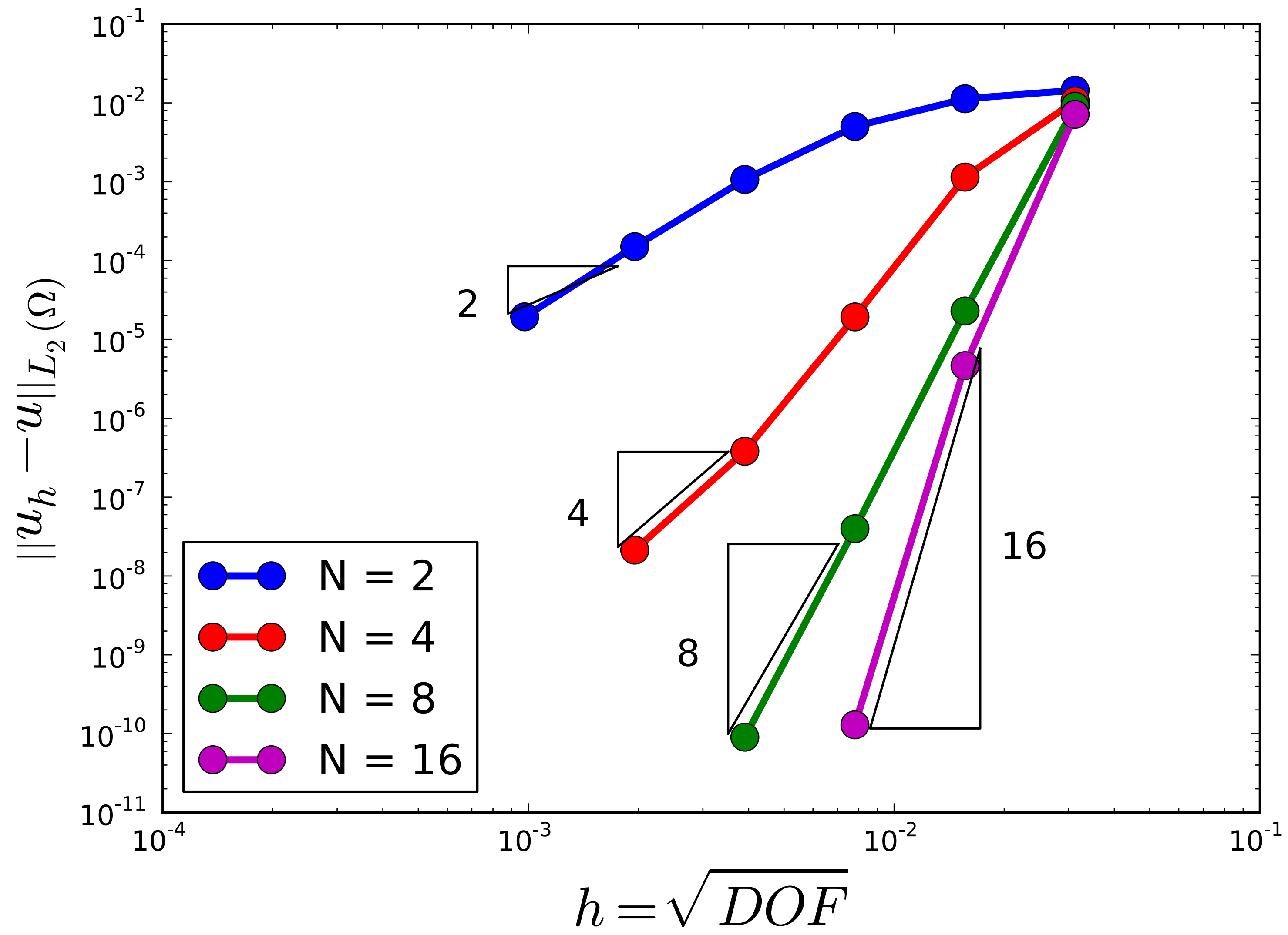


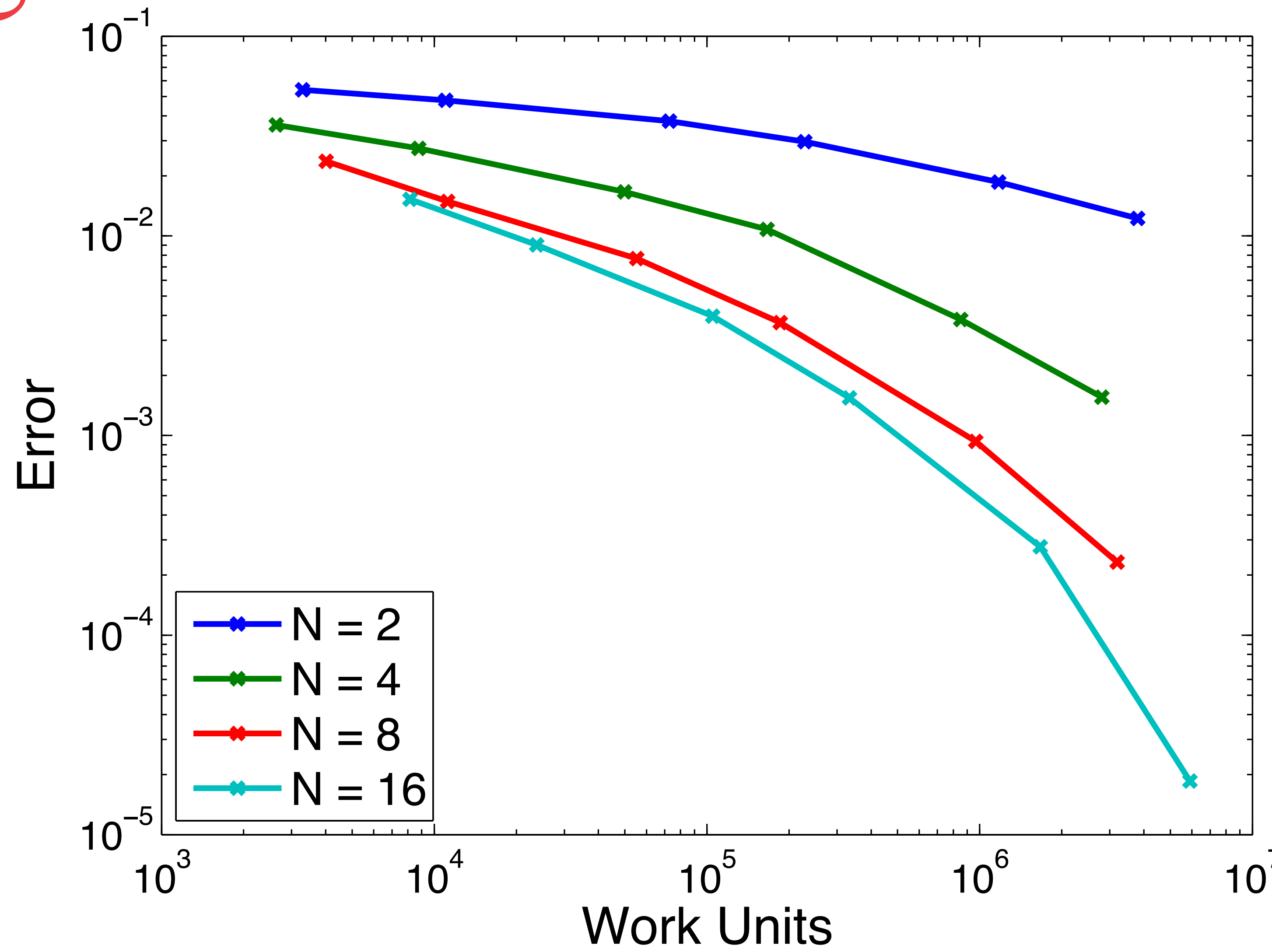
8th-order



- Developed PML approach for non-reflective BC
  - *Garai et al. AIAA 2016-1338*
- Developed physics-based approach for freestream turbulence
- *Garai et al., ASME GT2015-42773, GT2016-56700*

What is high-order?

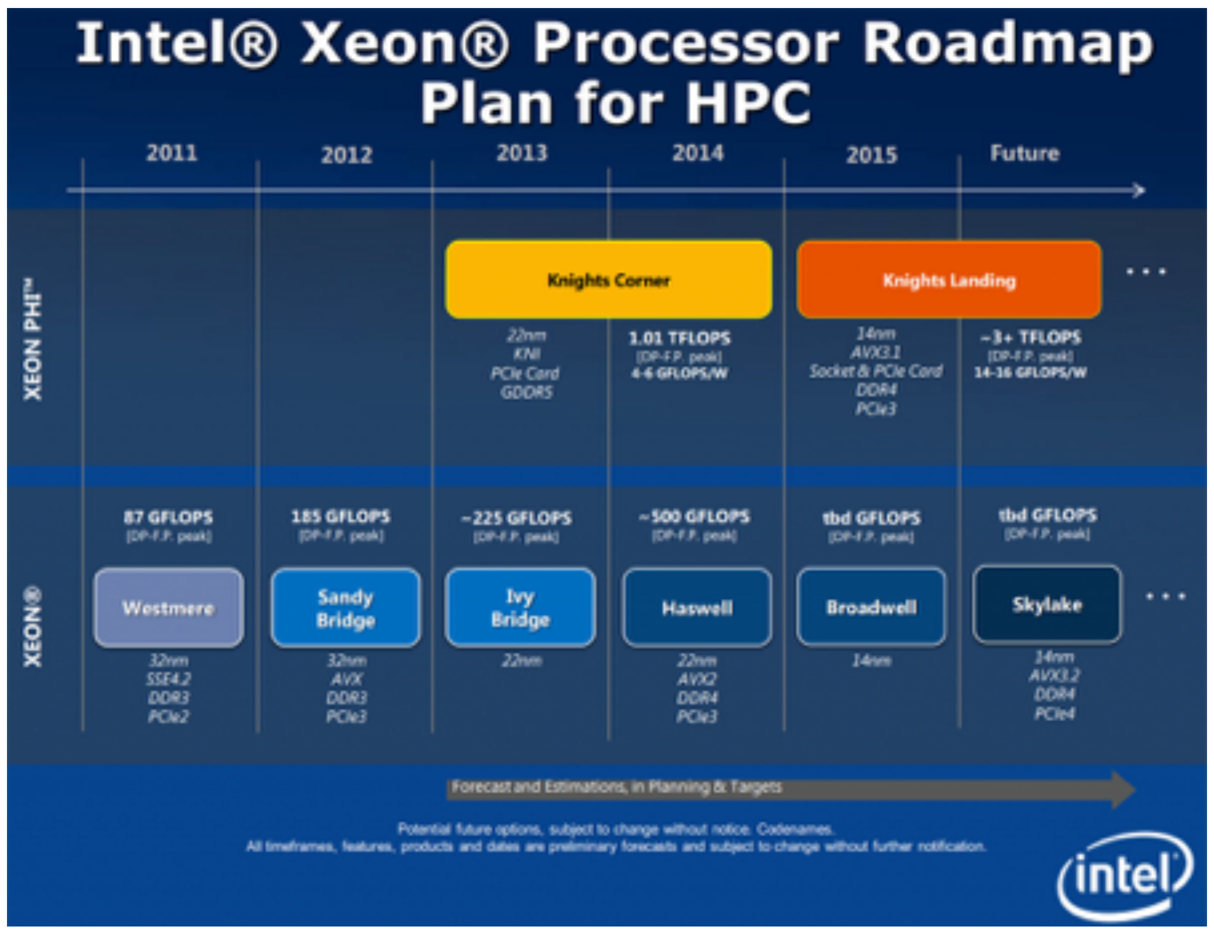


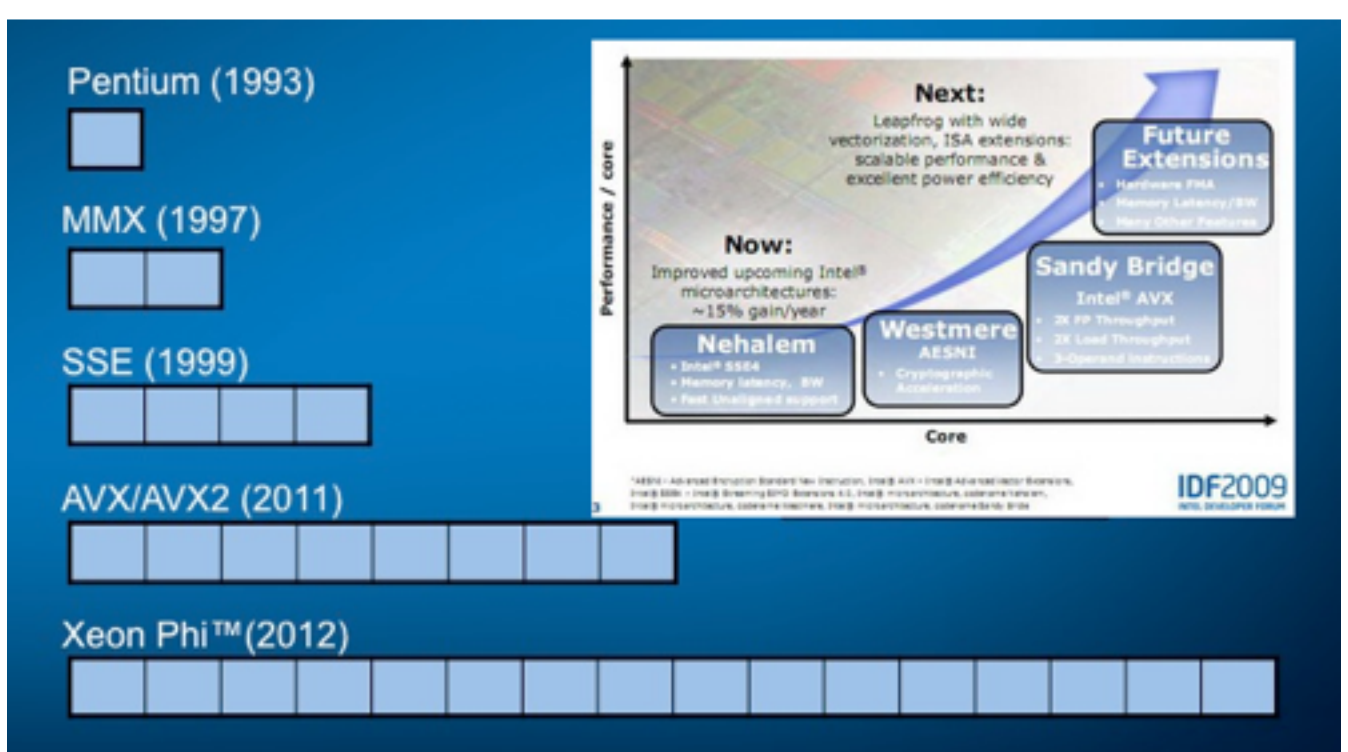


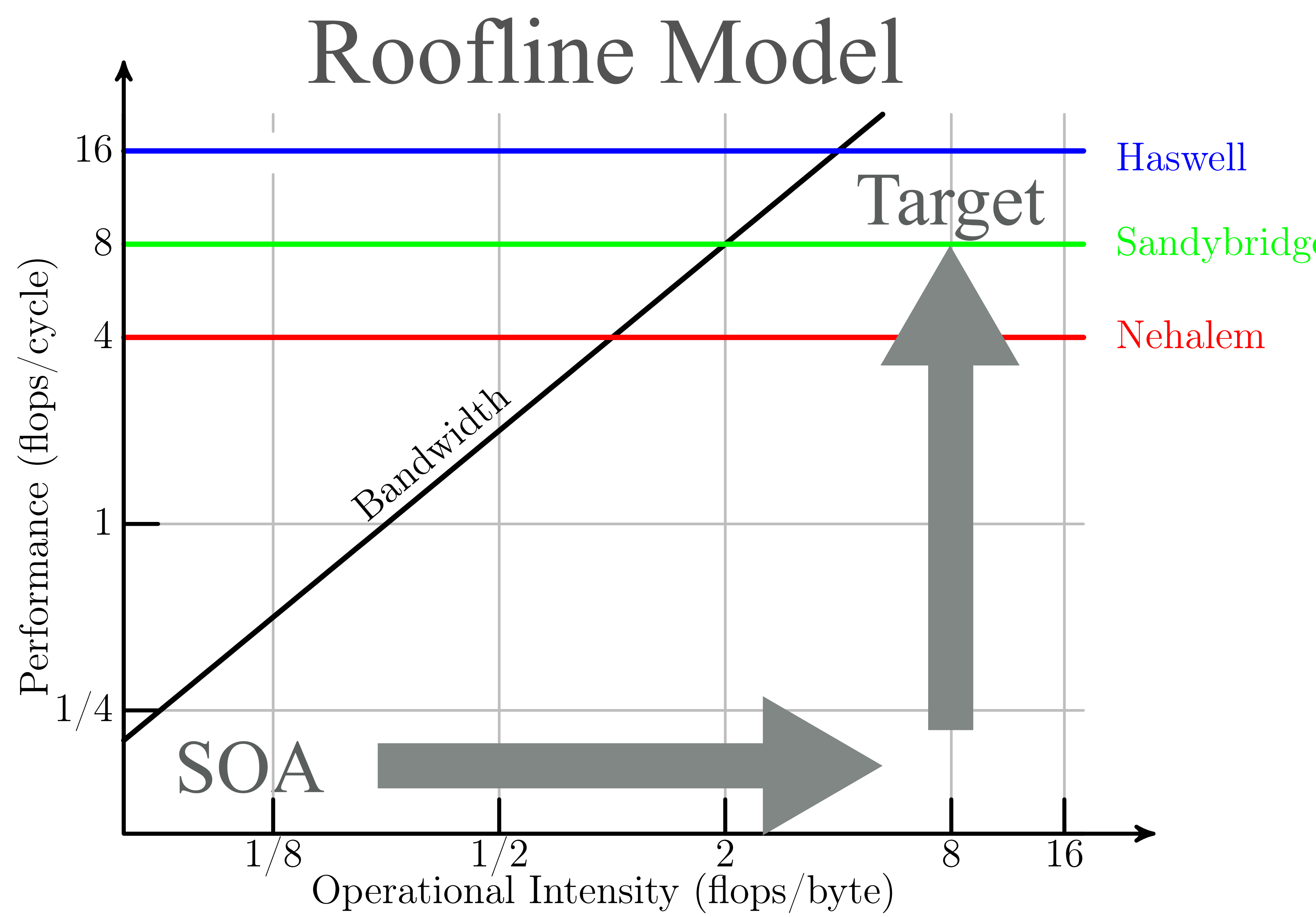
- We want to approach spectral limit in space and time
- Leads to efficiency gains and improved physical models
- Better match for current/future hardware
  - Less data movement, more flops for the same level of accuracy

Hardware-optimized Kernels

- Current algorithms achieve < 5% of machine peak
- Spectral elements a good match for current & future hardware



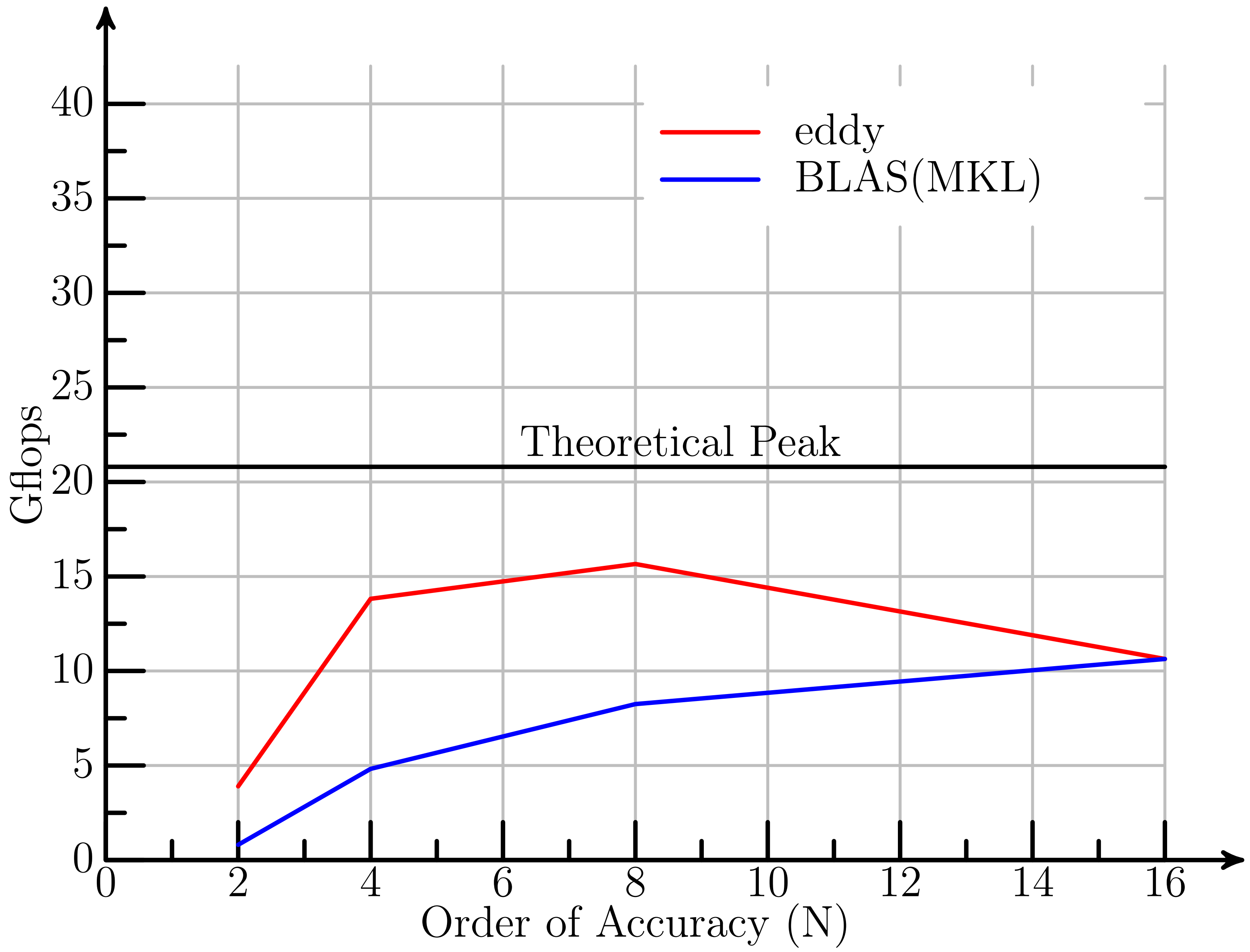




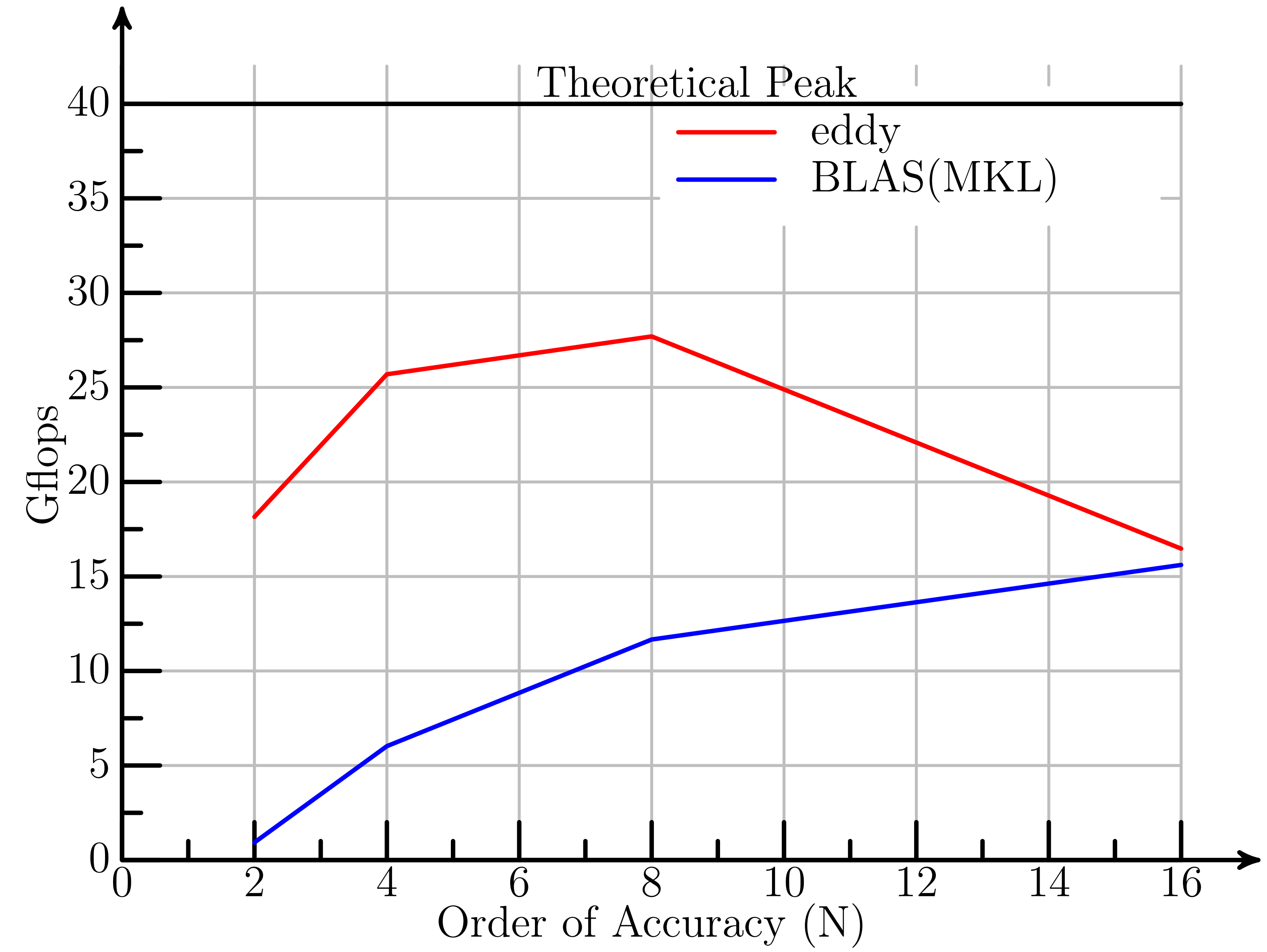
Hardware-optimized Kernels

- Tensor-product sum-factorization linear algebra kernels
- Benchmark represents ~20% of code

Sandybridge (2.6GHz)

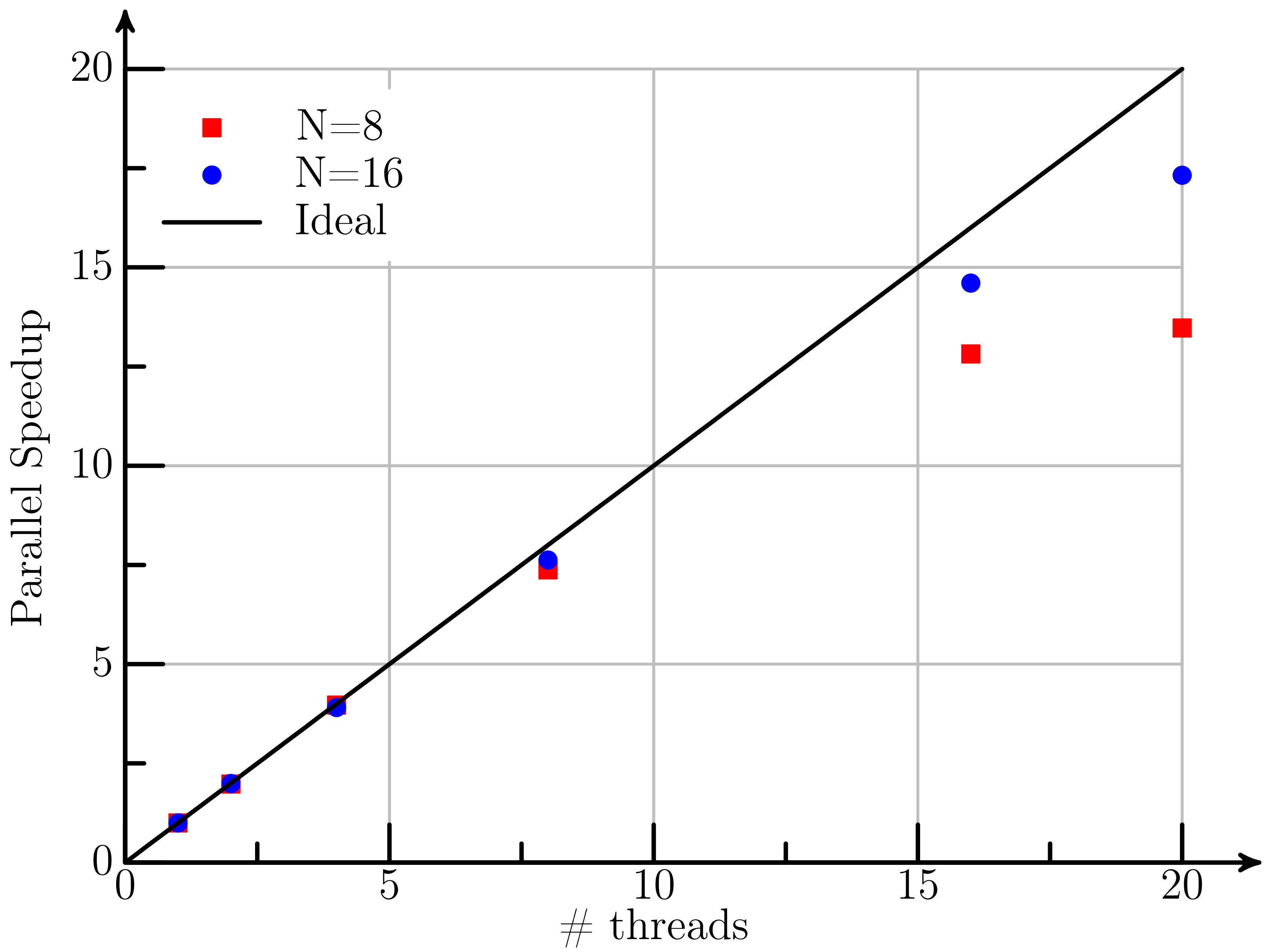


Haswell (2.5GHz)



Time-parallel

- Exploit multiple levels of parallelism
  - Parallel in space across nodes (MPI)
  - Parallel in time within node (OpenMP)
  - Parallel within loops on chip (SIMD vectorization)

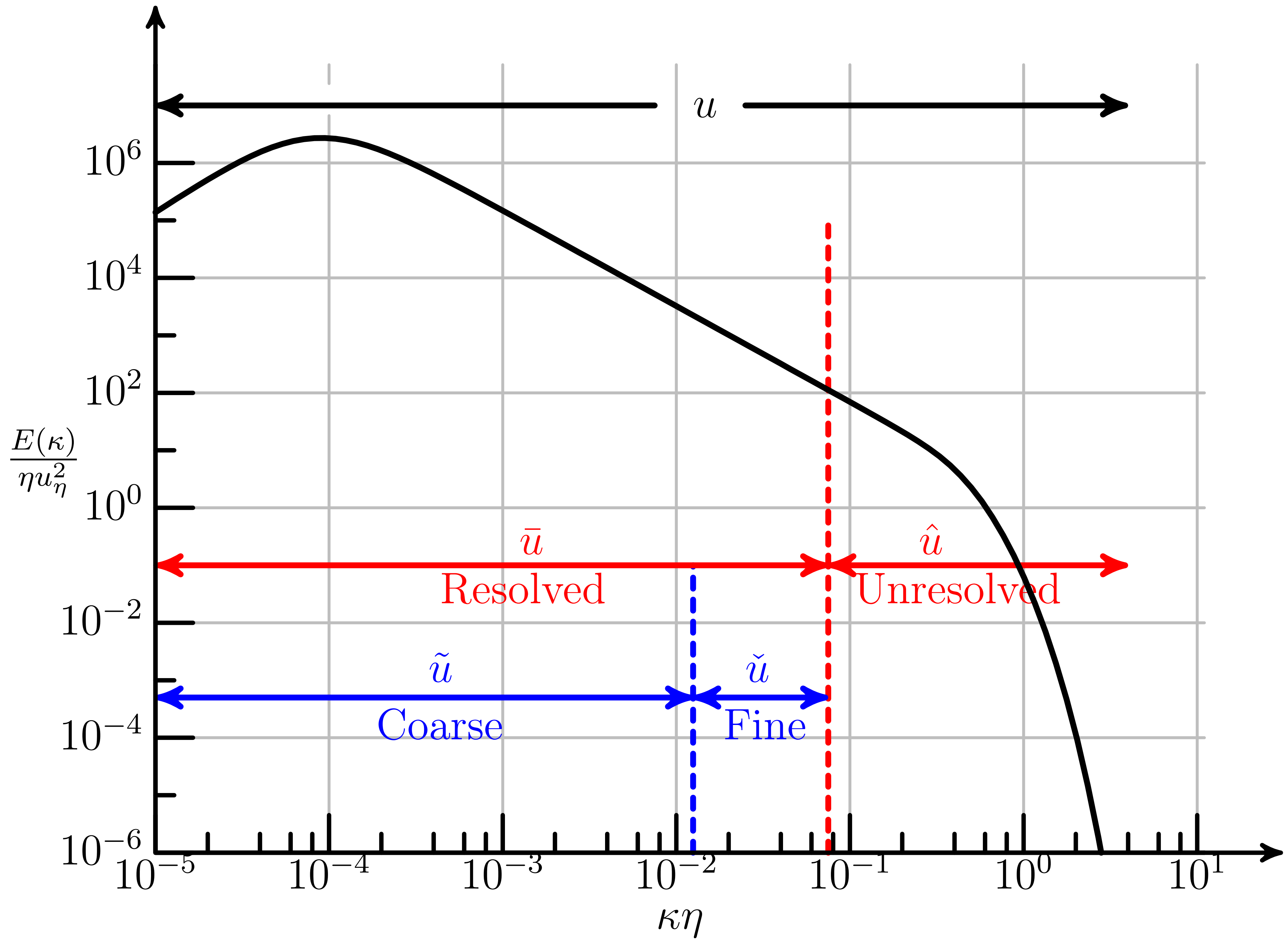


- 500 Gflops per Haswell node for 8th-order

Scale-resolving Models

- Improved numerics changes how we *do* CFD
  - Efficiency, automation, error estimates
- Consistent predictive models would change how we *use* CFD
  - Certification through simulation
- Need to prioritize new modeling approaches
  - Tighter coupling of numerics and modeling
- Current work is not a DG solver development it is a framework for examining scale-resolving models and methods

Variational Multiscale Method



- Explicit separation of scales (*Hughes et al., 1998, Collis, 2001*)
- Filtering is variational projection operator
- Assume unresolved scales only interact w/ finest resolved scales
- Extended VMM to dynamic procedure, varying coefficient in space & time

Dynamic Modeling

- Dynamic (parameter-free) models are a necessity for complex flows
  - Automatically adjust to physics, numerics
  - $C_S = 0.18$ for HIT, $C_S = 0.065$ for shear flow - 10x change in eddy viscosity
- Successful approaches have been built upon strong physical understanding
  - Scale similarity, homogeneity, local isotropy, near-wall asymptotics
- New approaches need to leverage these lessons learned

Dynamic VMM Model

$$\tau\left(u,\bar{w}\right) \simeq -2\left(\left(C_1\Delta\right)^2 \|\check{S}_{i,j}\|\check{S}_{i,j}, \check{w}_{i,j}\right)$$

- Variational Leonard stresses
  - Requires high-order (N ≥ 4)

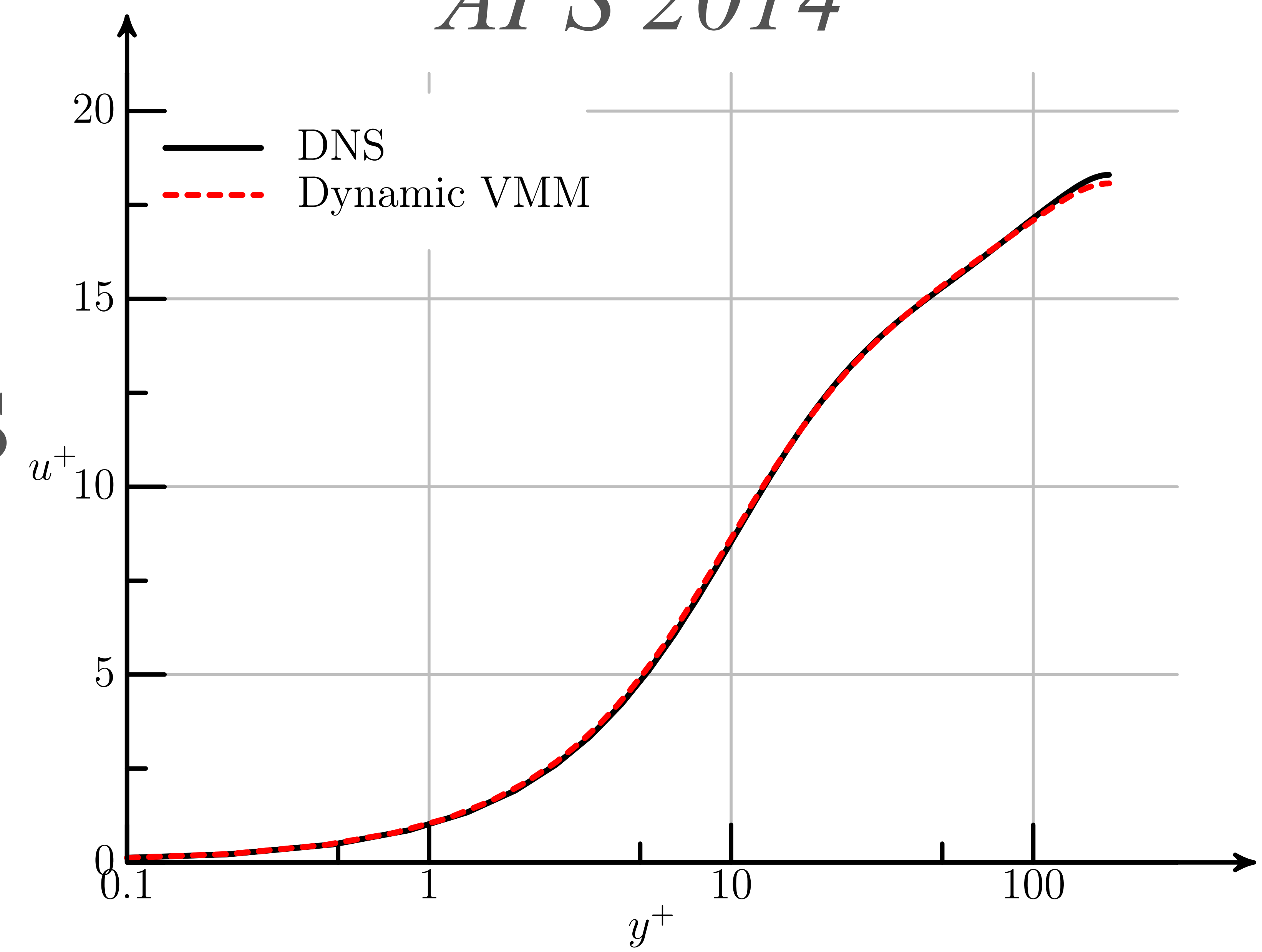
$$\left(\bar{u}_i^h\bar{u}_j^h - \bar{u}_i^H\bar{u}_j^H, \bar{w}_{i,j}^H\right) = -2\left(\left(C_1\Delta\right)^2 \|\check{S}_{i,j}^h\|\check{S}_{i,j}^h, \check{w}_{i,j}^H\right) + 2\left(\left(C_1\Delta\right)^2 \|\check{S}_{i,j}^H\|\check{S}_{i,j}^H, \check{w}_{i,j}^H\right)$$

- Using state as test function gives variational analogue to Germano procedure
- Can also provide analogue to Lilly’s least-square
- Entropy-stable compressible formulation in full paper

Channel Flow

APS 2014

- Previous work demonstrates dynamic model converges to DNS w/ sufficient resolution (consistency)

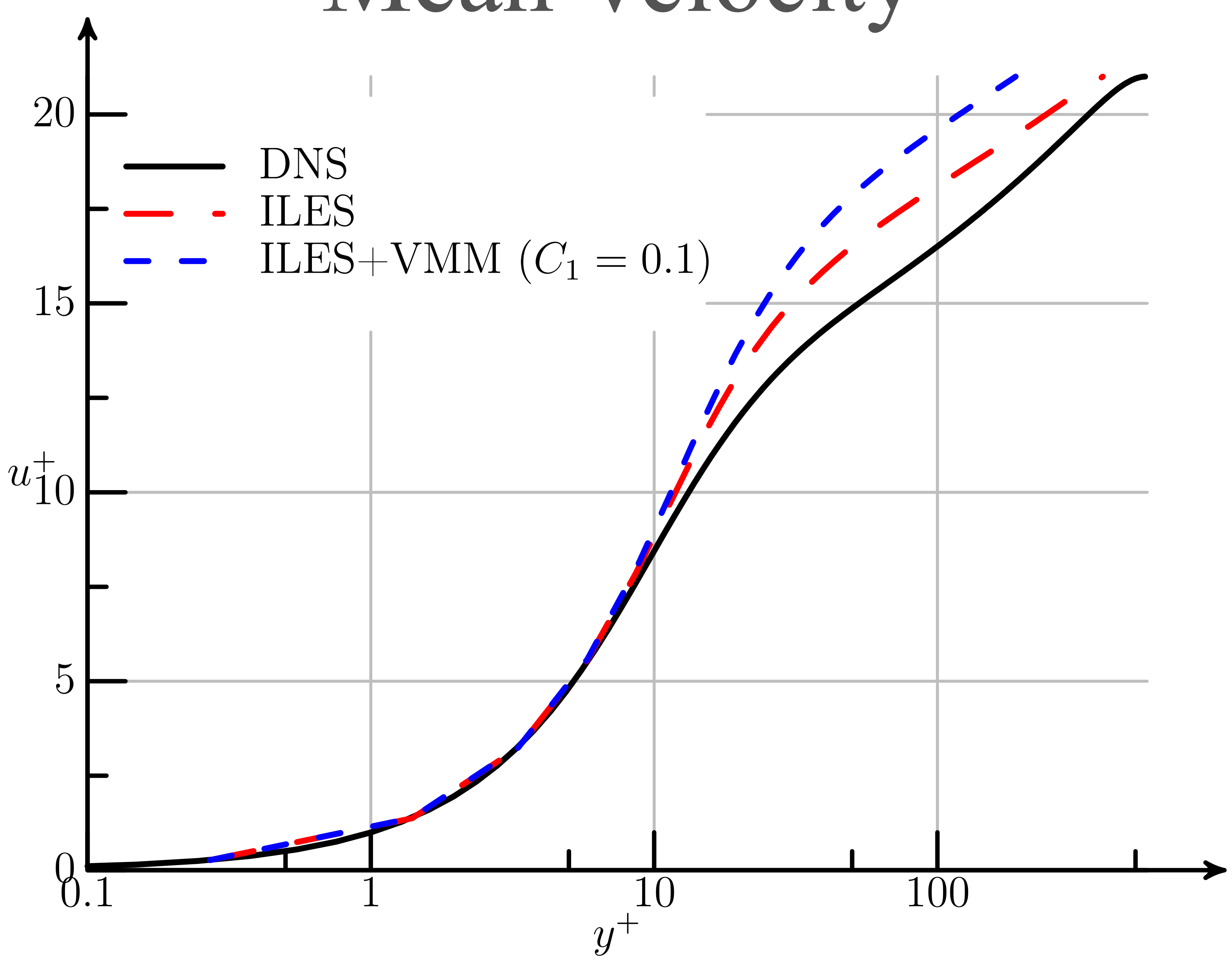


- Practical simulations never have sufficient resolution
- Examine behavior on realistic coarse mesh
  - $Re_\tau = 544$
  - 4th-order in time, 8th-order in space
  - $\Delta t^+ = 1$, $\Delta x^+ = 100$, $\Delta y^+ = 1$, $\Delta z^+ = 50$

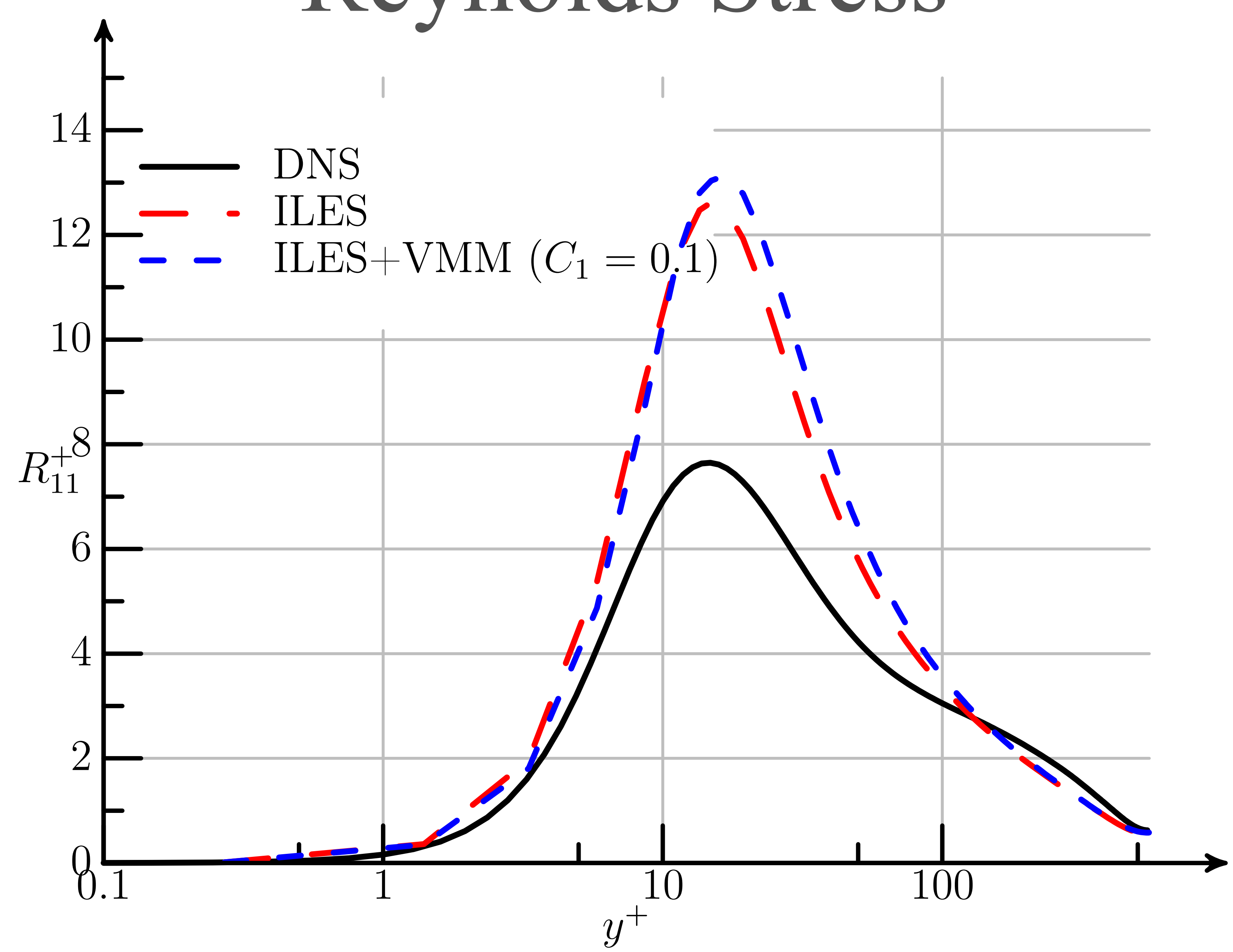
Channel Flow

- VMM w/ fixed coefficient degrades performance

Mean Velocity

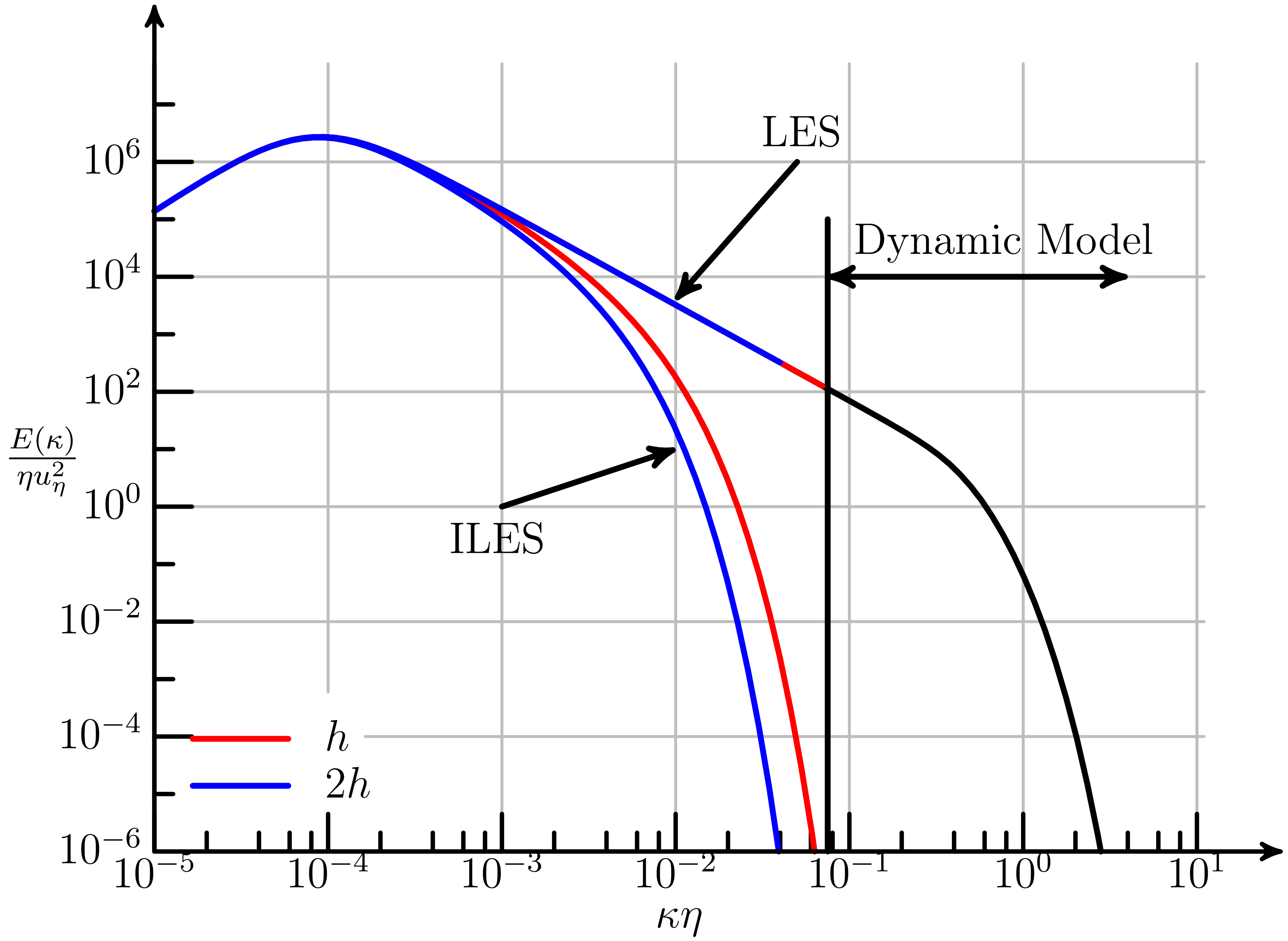


Reynolds Stress



Idealized Behavior

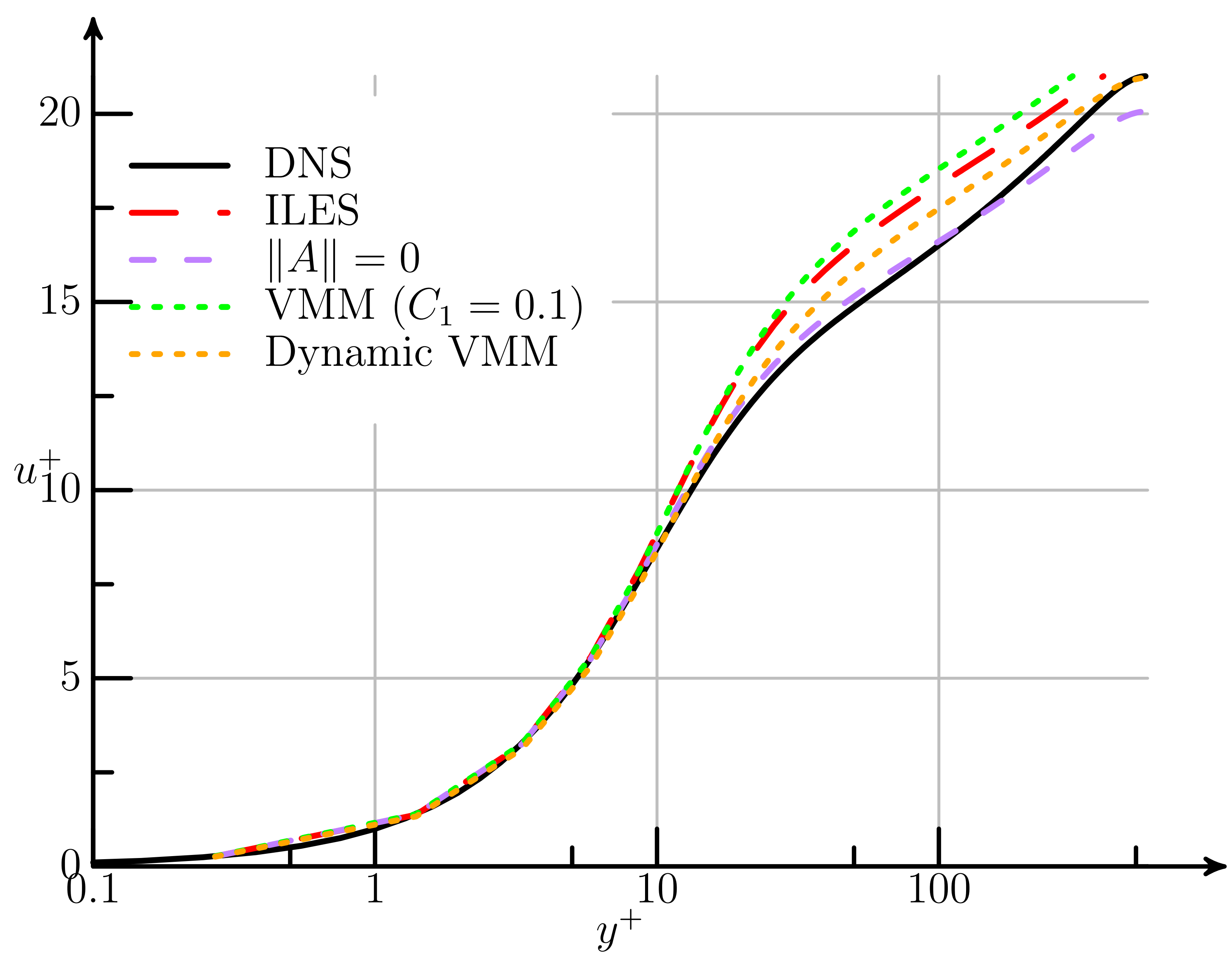
- ILES always resolves lower Re
- Dynamic approach resolves inertial range uses model for dissipation scales
- Requires non-dissipative scheme (*e.g.* skew symmetry)
- Entropy-stable schemes inherently dissipative
- Completely remove numerical dissipation as first test



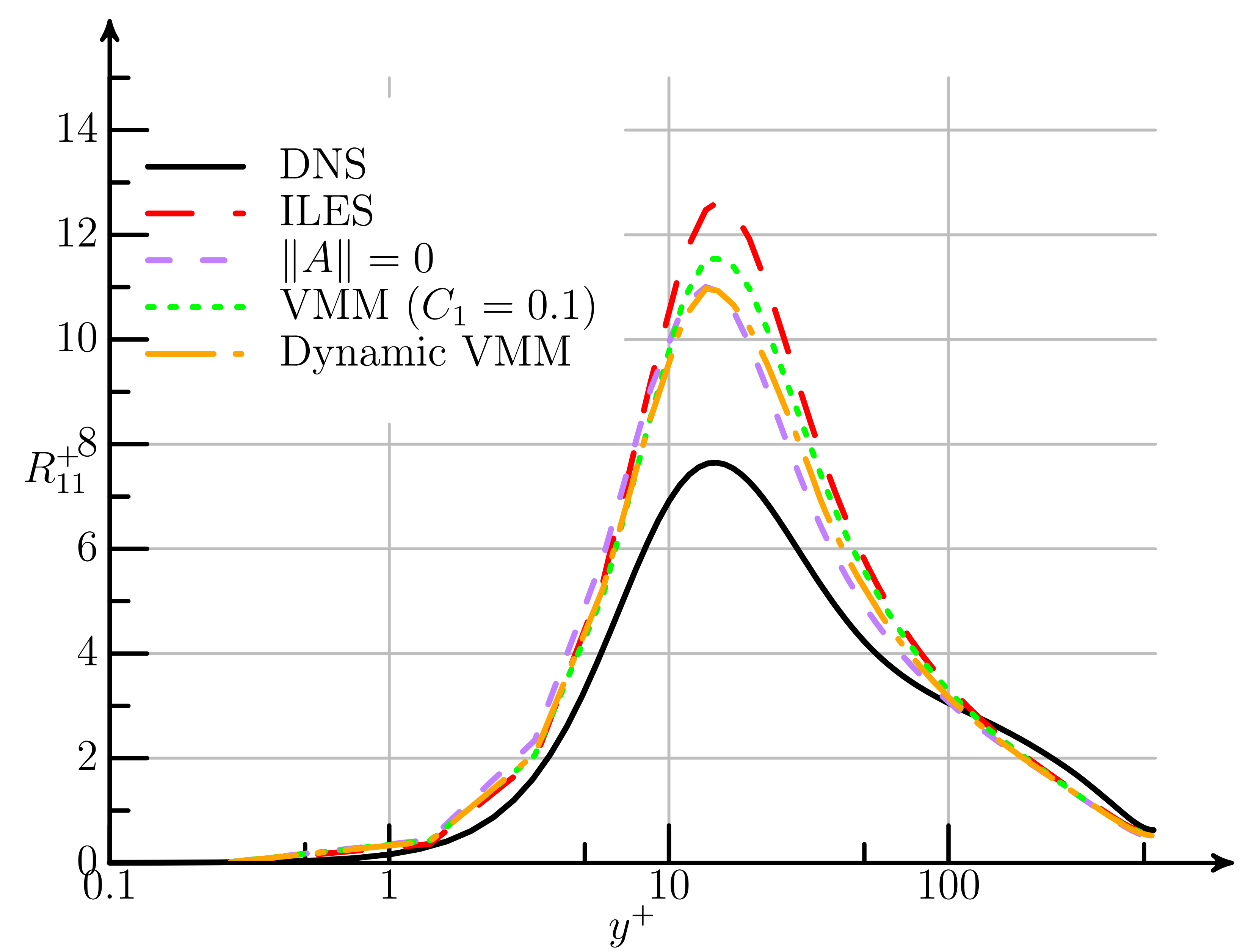
Channel Flow

- Dynamic procedure least sensitive to current mesh resolution
  - Examine trends w/ changing resolution through higher Re

Mean Velocity

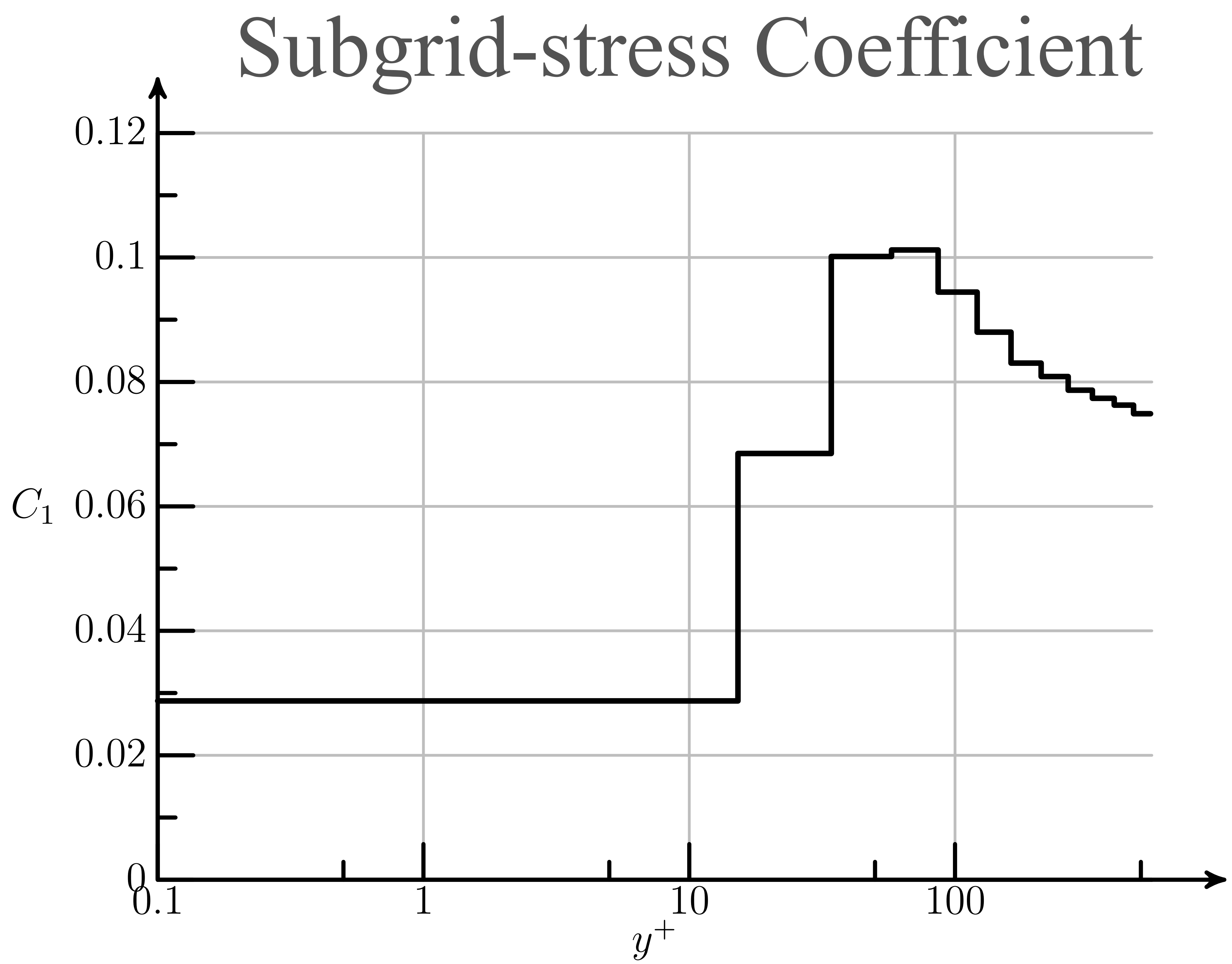


Reynolds Stress



Dynamic VMM Model

- Expected value in log layer
- Approach zero towards wall
- Decays towards centerline

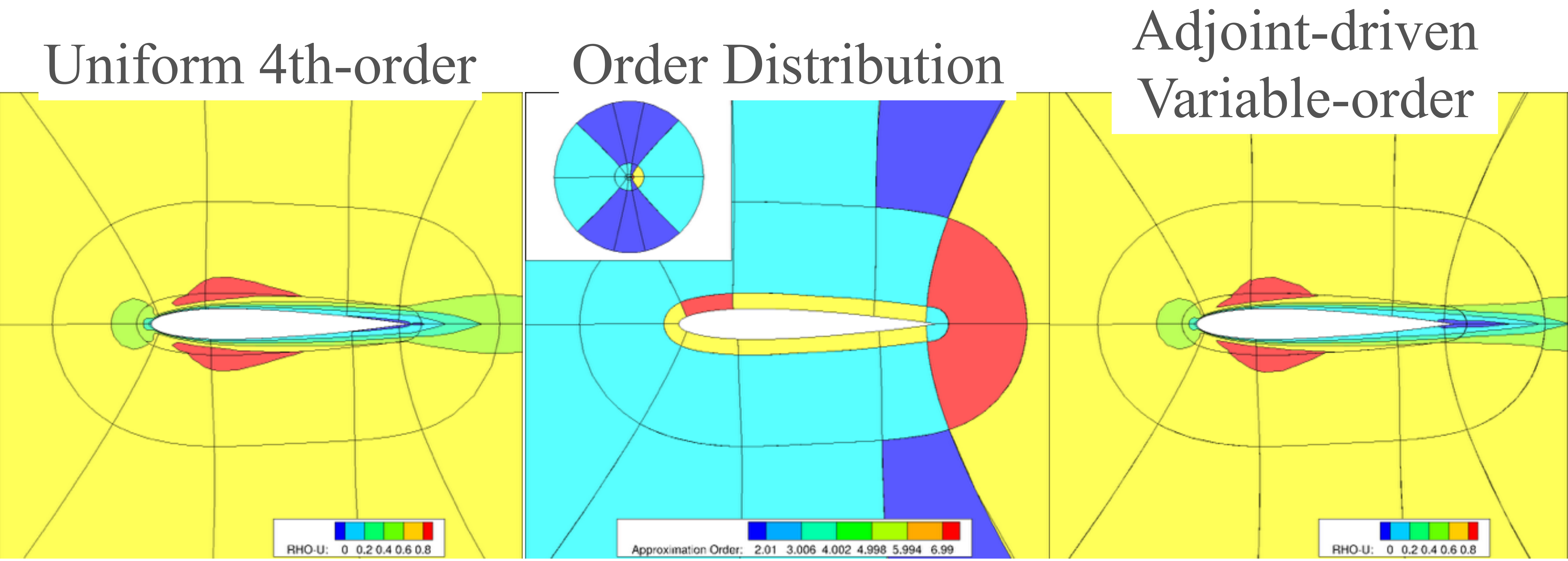


Summary

- Working prototype to experiment w/ scale-resolving methods for complex flows
- Existence proof that spectral-elements can take advantage of modern hardware
- Initial experiments w/ VMM encouraging
- Current work is extending to relevant flight geometry and conditions
  - Wall-modeled LES/VMM
  - Complex geometry (*AIAA 2015-0294*)
  - Relative motion/FSI capability

Backup

Space-time Adjoint



- Variable-order produces lower error at same cost
  - *Ceze et al. AIAA 2016-0833*
- Currently extending to space-time *h-p* adaptation and error estimates

Current Status

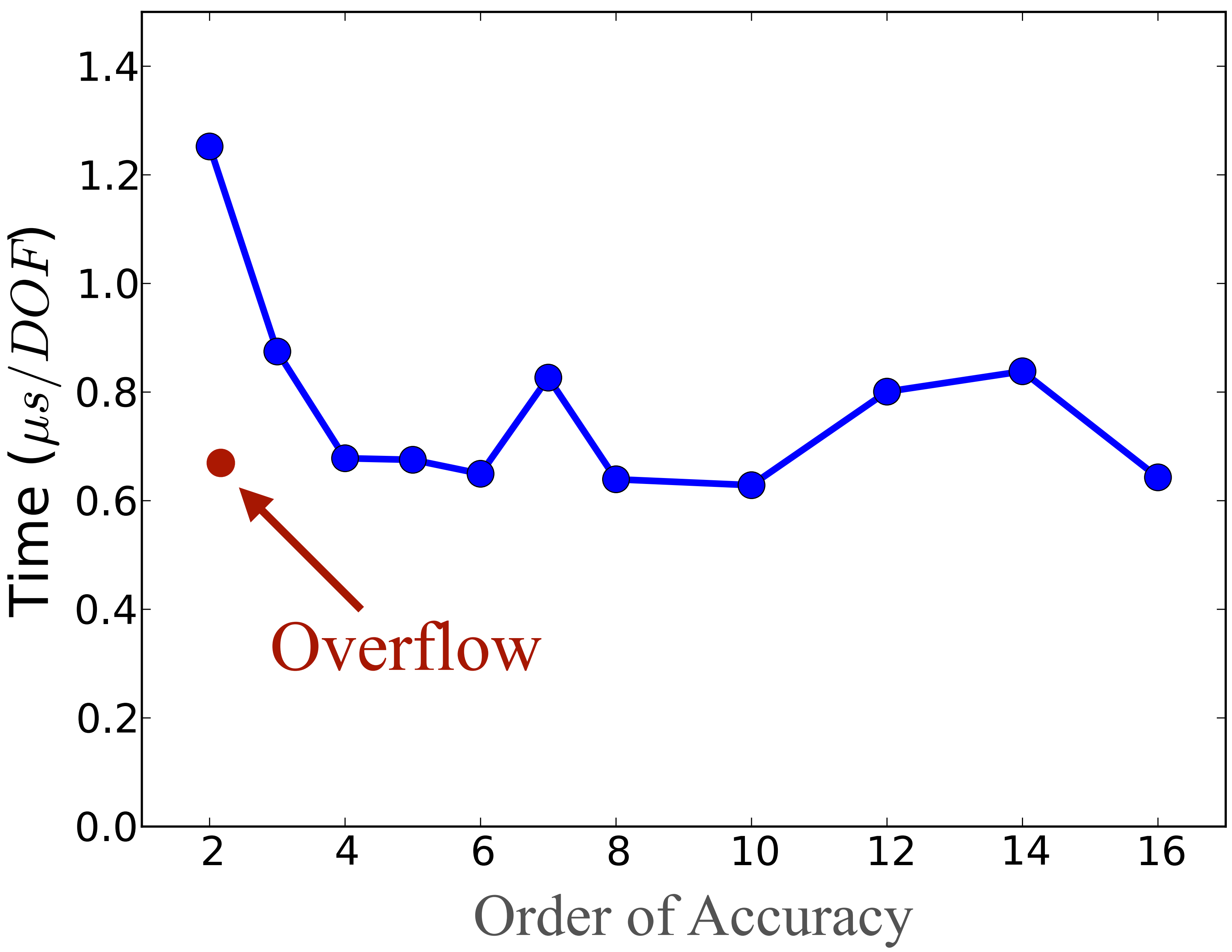
- Laslo Diosady - moving body, shock capturing
- Anirban Garai - turbomachinery, LES - AIAA 2016-XXXX
- Marco Ceze - adjoint, mesh adaptation - AIAA 2016-XXXX
- Corentin Carton de Wiart - wall modeling, hybrid-RANS

Hardware-optimized Kernels

- Optimization counterbalances increase in cost for high order

AIAA 2013-2870

Residual



AIAA 2016-XXXX

Jacobian