Stochastic-Strength-Based Damage Simulation Tool for Ceramic Matrix Composites

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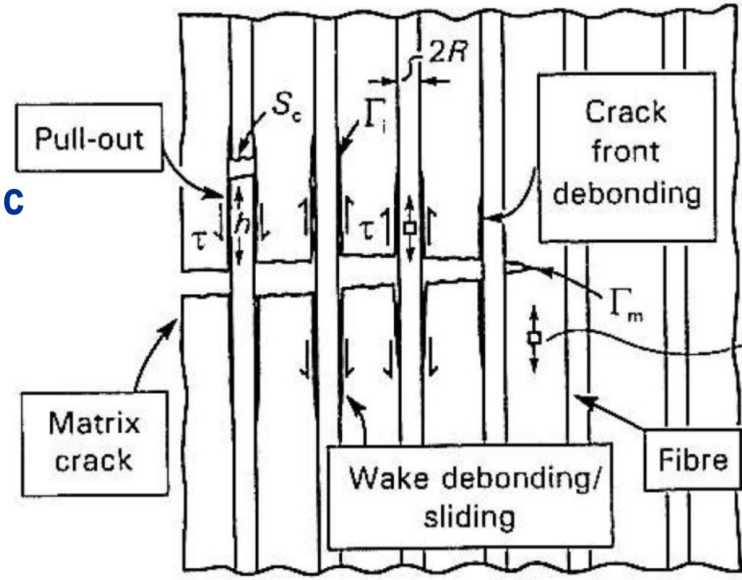
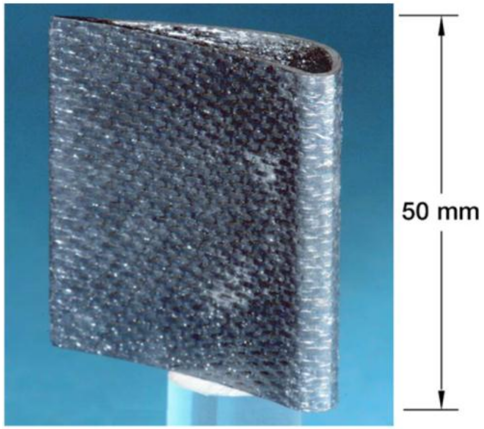
Ceramic Matrix Composites (CMCs)

- Ceramic matrix composites (CMC) are being developed for hot section of advanced turbine engines (2700° F) and other uses.
- Specific mechanisms governing response of ceramic matrix composites need to account for in analysis approach:
  - Brittle material response
  - Weak, compliant interface
  - Residual stresses present due to processing
- Robust, efficient analysis tools required to analyze deformation, failure and life of these materials.

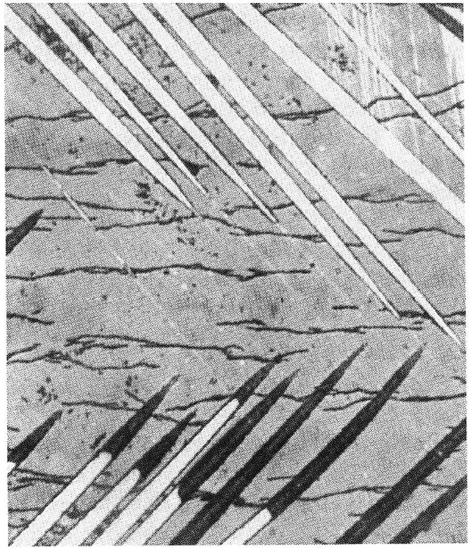
Failure mechanisms





SiC/SiC (silicon carbide) CMC stator vane

Periodic matrix cracking failure mechanism



➔ *CMCs are designed to have "graceful failure" (non-linear stress-strain response) as opposed to brittle failure*

Scope, Technical Challenge & Approach

❑ Predict the strength and service life of ceramic ***<u>& composite structures</u>***

CMC - ***C***eramic ***M***atrix ***C***omposites & ***PMC*** - ***P***olymer ***M***atrix ***C***omposites

- **Need to account for:**
  - Wide variability in the strength of individual components (probabilistic/stochastic strength)
  - How damage response changes with loading types (multiaxial loading, flexural loading, size effect)
  - How composite architecture effects strength/damage response
  - How strength degrades with time and fluctuating load
- **Approach → Combine two NASA developed codes:**

  (MAC/GMC) : composite micromechanics analysis &

  (CARES/Life) : probability of failure prediction of ceramic components

  with commercial finite element analysis **(Abaqus)**

Outline

1. Overview: Describe the MAC and CARES codes

   – **MAC/GMC:** composite ***micromechanics*** model

   – **CARES Unit Sphere:** multiaxial ***stochastic strength*** model
   (isotropy & anisotropy)

2. Applying CARES to the MAC code to simulate stochastic damage progression in a ceramic matrix composite (CMC)

   ❖ ***Cellular Automaton***: Encouraging failure of adjacent elements - mimics crack-like growth

   ❖ Visualization of element-by-element failure propagation for fiber, matrix, and interface

3. Stress-strain response of a SiC-RBSN laminate (literature circa 1990)

   ❖ Off-axis loading

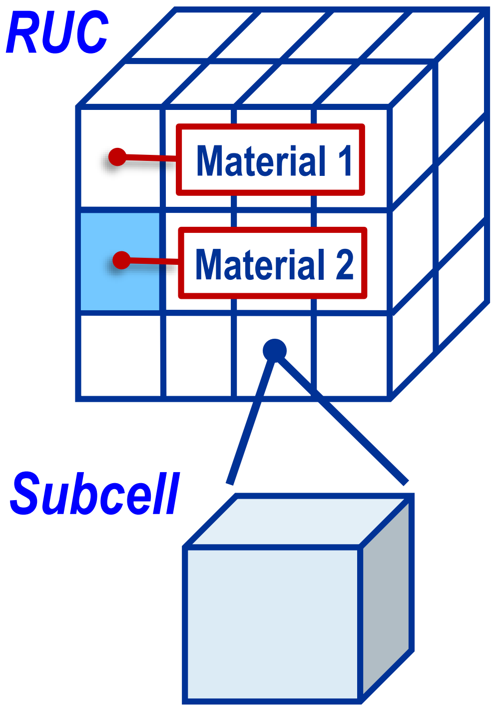
   ❖ notched specimen

4. Time-dependent slow crack growth notional example

MAC/GMC Micromechanics Analysis Code

Repeating Unit Cell (RUC) of composite material

- ❖ RUC made of material subcells
- ❖ Multiscale capability



- ***FEAMAC:*** *MAC/GMC embedded in FEA as constitutive material*

CARES/Life: Life Prediction Code For Advanced Ceramics

- Predicts the probability of failure of ceramic components under thermomechanical loading
- Combines Weibull & Weakest Link theory with concepts of linear elastic fracture mechanics (the *Batdorf Unit Sphere* model)
- CARES is a post-processor to FEA

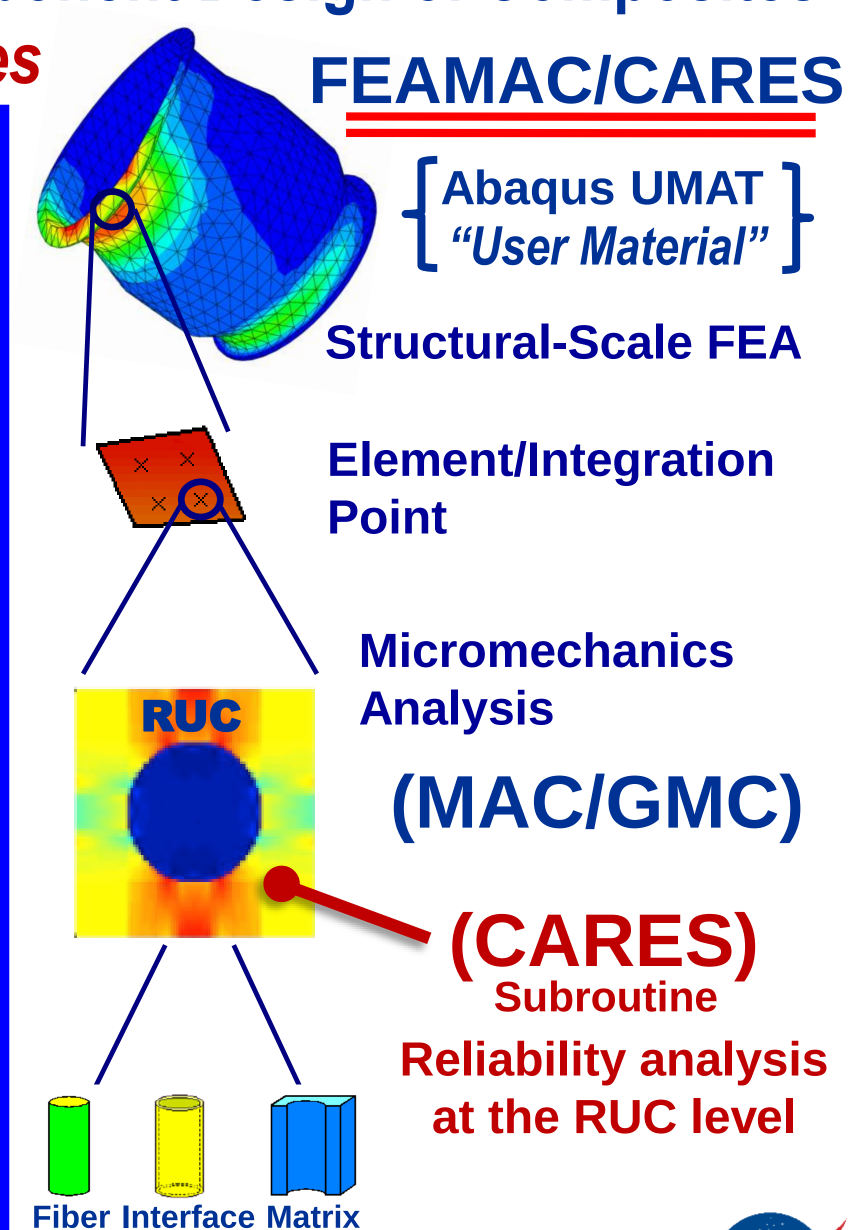
Component Reliability Analysis Capability:

- ➢ Transient loads and temperatures
- ➢ Fast-Fracture Rupture
- ➢ Time-dependent (da/dt) crack growth
- ➢ Cycle-dependent (da/dn) crack growth
- ➢ Multiaxial stress failure models *(PIA & Unit Sphere & Tsai-Wu & Tsai-Hill)*
- ➢ Proof test

Approach for Life Prediction & Component Design of Composites

➢ *Combine CARES, MAC & FEA codes*

Move CARES from the macroscopic scale of the structure to the microscale of the individual RUC material constituents



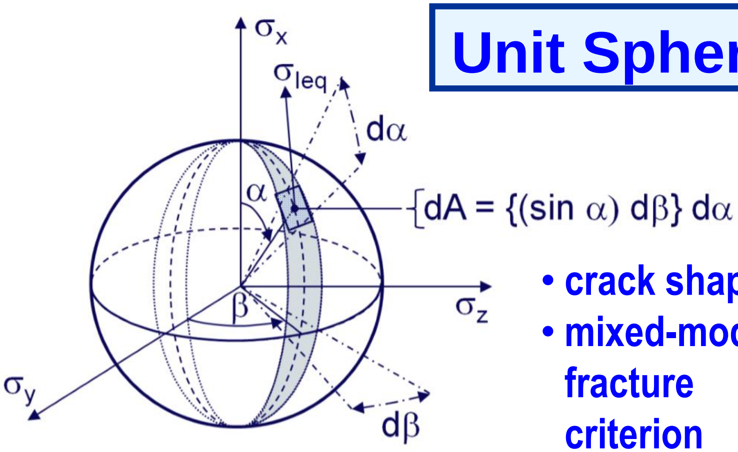
❖ FEAMAC/CARES Capability:

- Individual constituent and component level probability of failure tracked (for failure initiation)
- Individual & concurrent failure modes
- Laminate level analysis capability
- Progressive damage capability/simulation
  - ➢ Subcells killed at random failure thresholds

Debonding/crack path physics at constituent level not explicitly included

Fracture-mechanics-based failure criteria to predict probability of failure/damage of a structure over time.

Unit Sphere

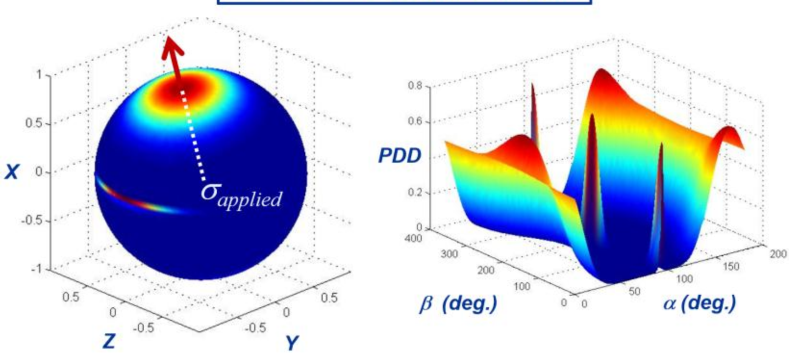


- **crack shape**
- **mixed-mode fracture criterion**

- Two models for anisotropic strength response:
  - K_{Ic} / Critical strength
  - Flaw orientation bias

(Transversely isotropic (K_{Ic}) Material)

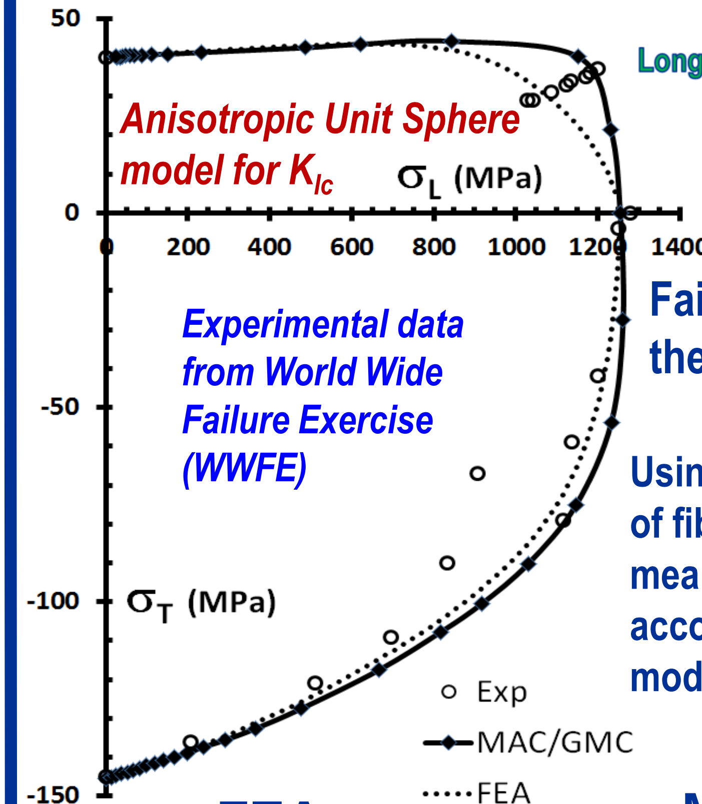
For a 15° offset uniaxial load

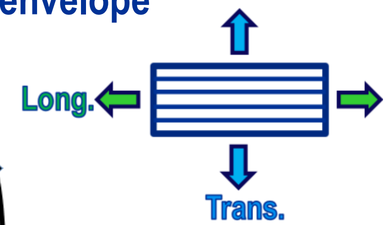


Unit Sphere Probability Density Distribution For Orientation Of Critical Flaws

Prediction of multiaxial strength:

Unidirectional E-Glass/Epoxy PMC under biaxial loading *50% P_f* (fail. prob.) envelope

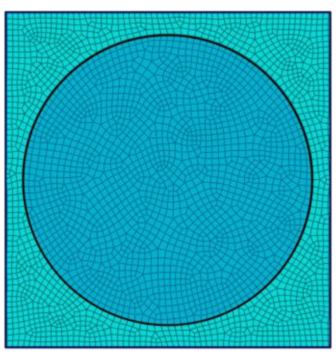




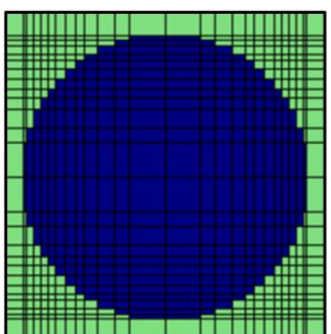
Failure criterion at the unit-cell level

Using microstresses of fiber & matrix means more accurate accounting of failure modes

FEA

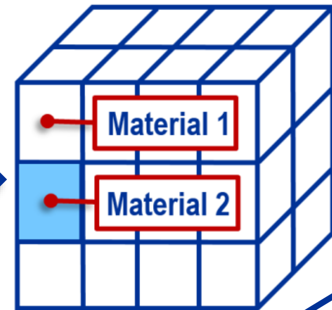


MAC/GMC



Progressive Damage Criterion

Calculate failure probability, P_f, for each material constituent of the RUC associated with an element integration point



RUC

CARES calculated $P_{f\,(CARES)}$ of RUC

Random number generated $P_{f(Random)}$ of RUC

$$P_{f\,(CARES)} \geq P_{f\,(Random)}$$

Yes

Fail *all* material constituent subcells
➢ *Kill elastic modulus*

No

Don't fail subcells

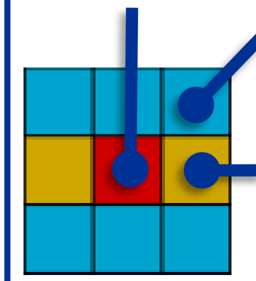
Encourages more rapid damage propagation than failing individual subcells

Random Element Failure vs: Neighbor Influenced Failure (Cellular Automaton Enhancement)

Encourage more abrupt failure and "crack-like" damage growth patterns

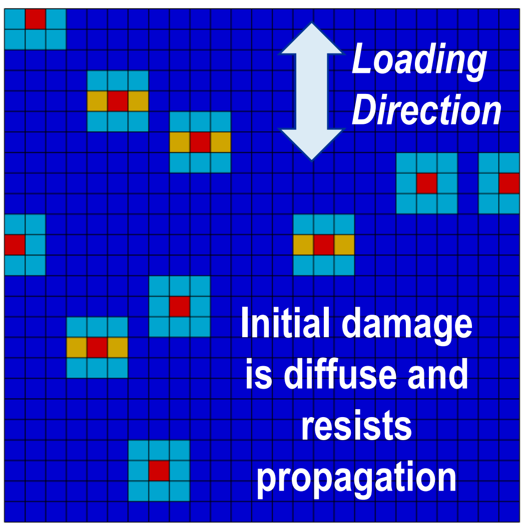
A cellular automaton is a collection of "colored" cells on a grid that evolves through discrete time steps according to a set of rules based on the states of neighboring cells

Rule: When failure of an element is encountered, the random failure threshold of the neighboring elements are adjusted to that of the failed element. *Load state determines which elements have highest probability of failure*



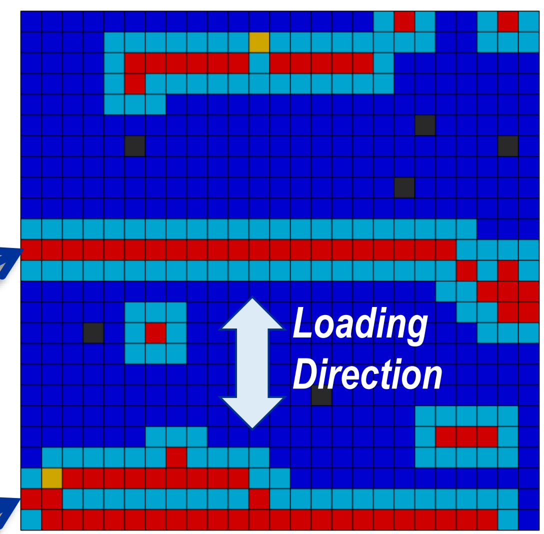
Adjacent 2 elements with highest $P_{f\,(CARES)}$ has $P_{f\,(Random)}$ adjusted

Random element failure



Example: 0^o Ply uniaxial ramp load 25x25 FEA mesh

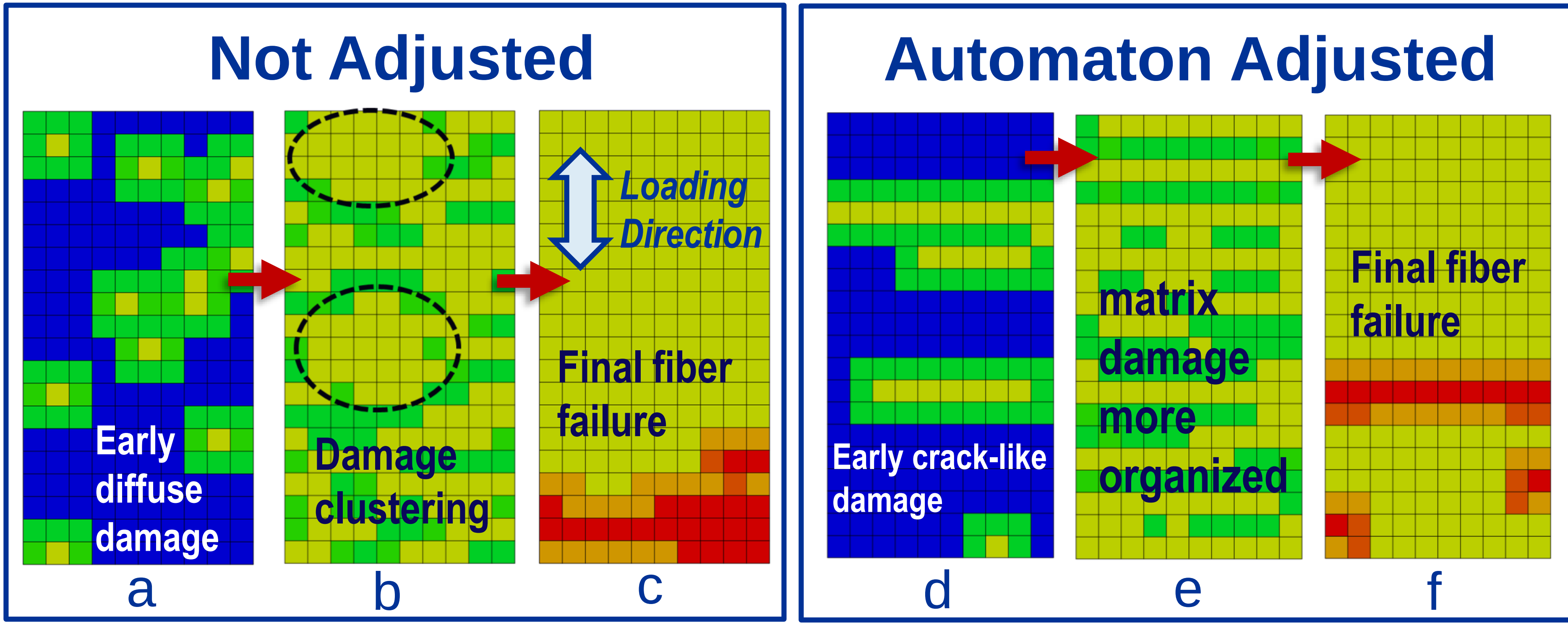
Cellular automaton



Adjusted element $P_{f\,(Random)}$ more likely to be lower than original $P_{f\,(Random)}$ and fail sooner as load increases – *enhancing damage propagation*

0^o single ply tensile specimen

Progression of damage in FE model of a unidirectional ply under longitudinal loading



- Matrix failure
- Adjacent to failed matrix
- Fiber failure
- Adjacent to failed fiber
- No failure

(a) and (d) ; early matrix damage

(b) and (e) ; progression to substantial matrix damage

(c) and (f) ; final composite failure (fiber failure)

Example: SiC/RBSN Laminated Composite in *On-Axis* & *Off-Axis* Loading

- **Tested by Bhatt & Phillips (1990)**

➢ **displays key mechanisms/features for model material**

- SCS-6 fiber/Reaction Bonded Silicon Nitride matrix composite examined in detail by NASA ➢ several papers published
- Laminated CMCs of interest to industry and less complex than woven composites
  - ➢ failure modes are not conflicted with complex fiber architecture
- [0] & [0/90] laminates display nonlinearity due to matrix failure, followed by fiber failure.
- Remaining ply orientations display sudden brittle failure.

Bhatt, R.T., and Phillips, R.E.: "Laminate Behavior for SiC Fiber-Reiinforced Reaction-Bonded Silicon Nitride Matrix Composites." J. of Comp. Tech. & Res. V. 12, No. 1, Spring 1990, pp. 13-23.

SiC/RBSN

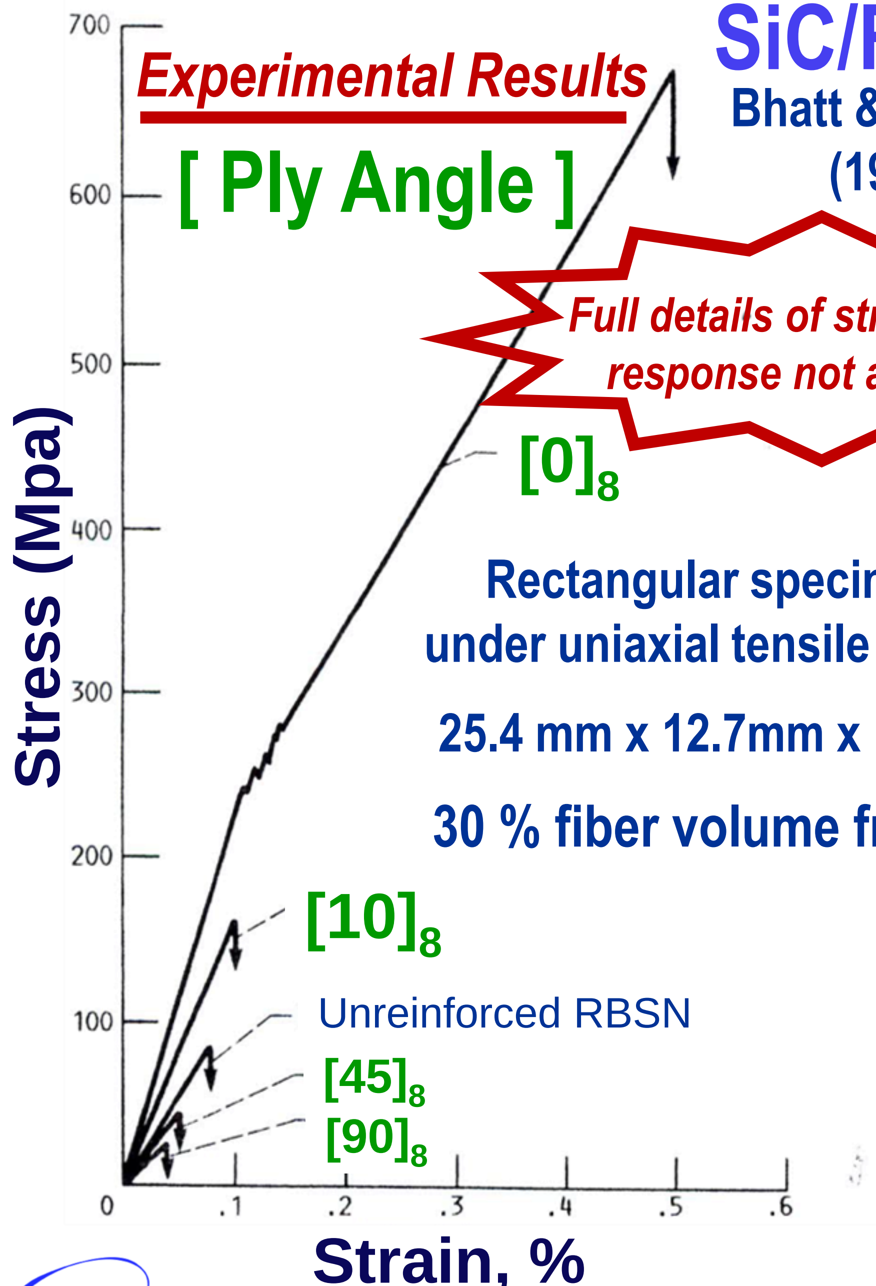
Bhatt & Phillips (1990)

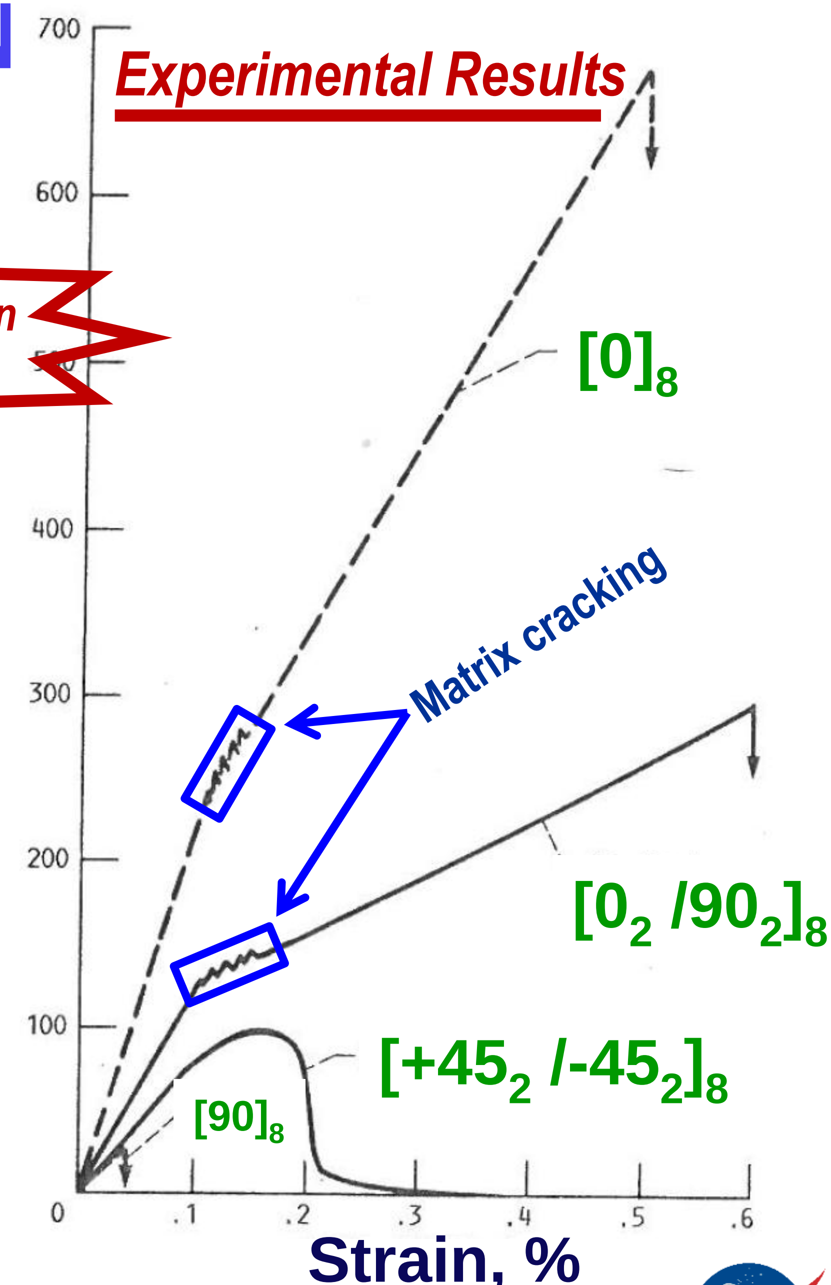
Full details of stress-strain response not available

Rectangular specimens under uniaxial tensile loading

25.4 mm x 12.7mm x 1.2 mm

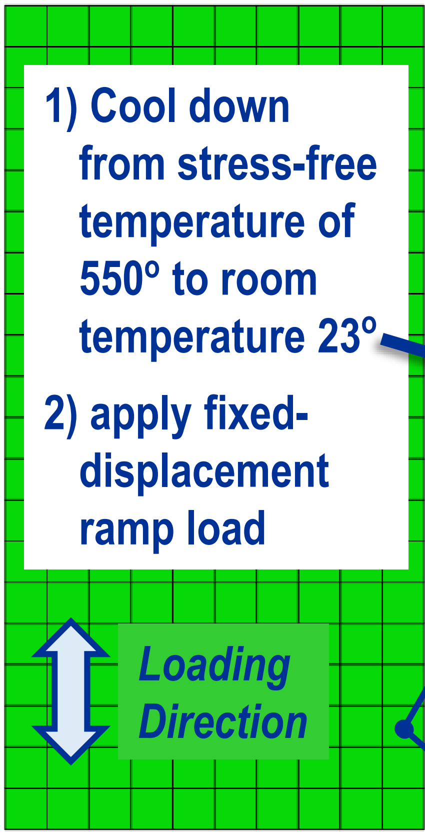
30 % fiber volume fraction





SiC/RBSN Example Procedure & Setup

Abaqus FEA
S4 Shell elements
Fixed-displacement ramp load



(10x20 mesh)

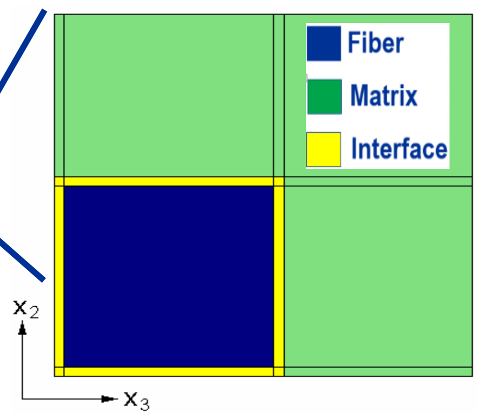
Stochastic strength analysis:

(from individual trials / simulations / realizations)

- Use ***CARES Unit Sphere*** failure criterion
  - *assume Isotropic material constituent strength – for simplicity and initial testing*

Residual stresses in constituents

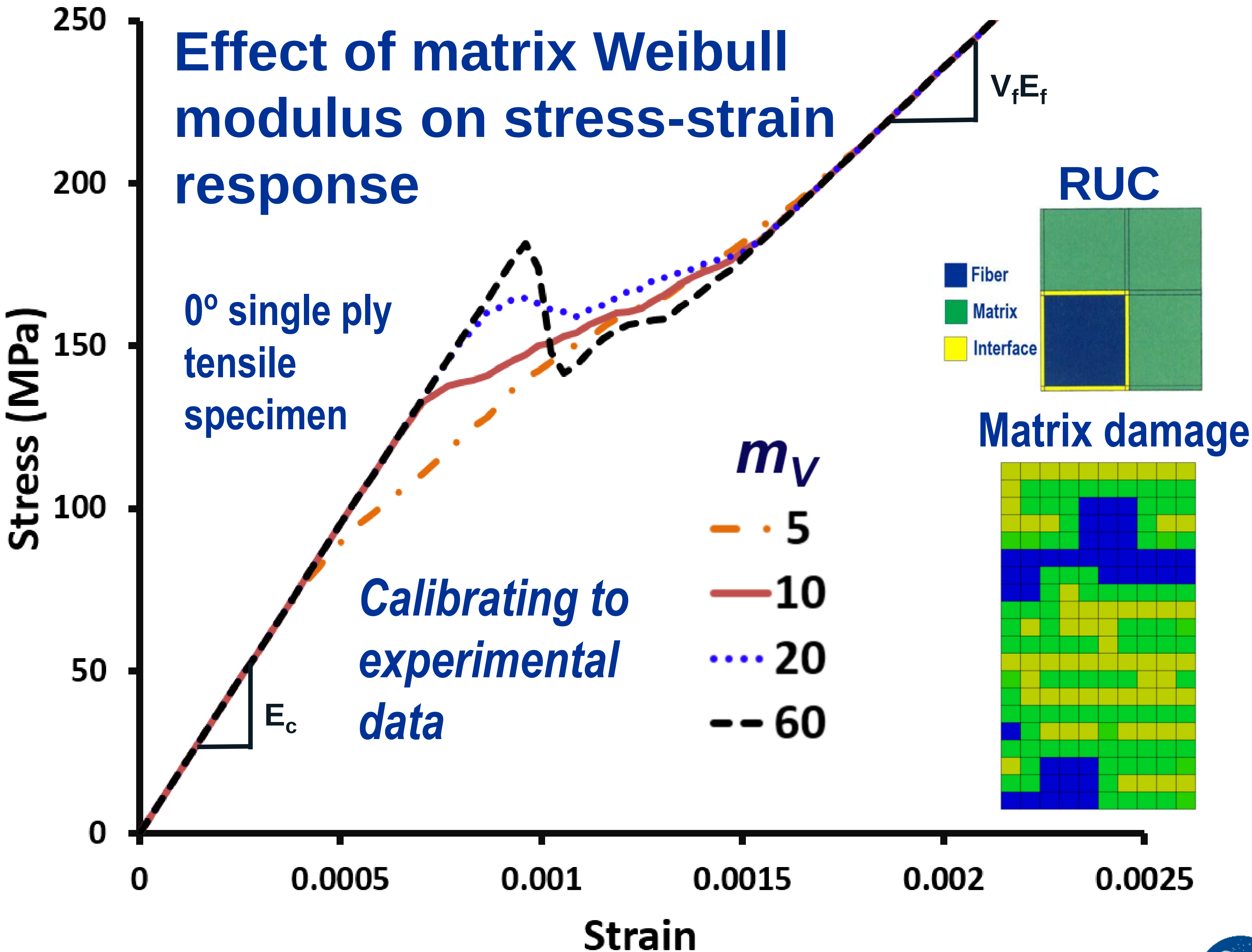
MAC/GMC RUC



- Weibull parameters correlated to experimental results for 0° tensile specimen
- Interface strength made large:
  - *Encourage matrix to fail before interface*

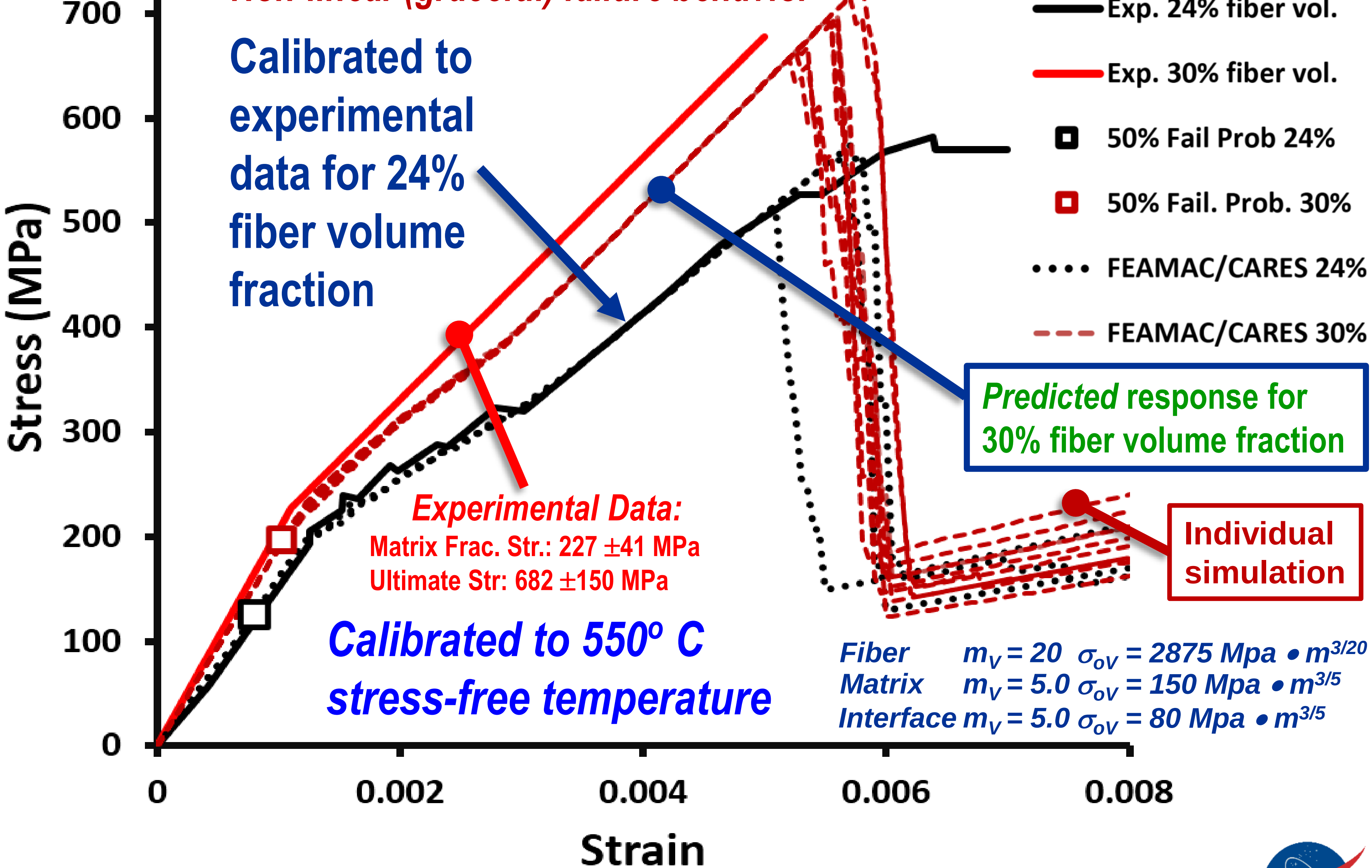
- *Interfacial failure modes and sliding resistance not considered*

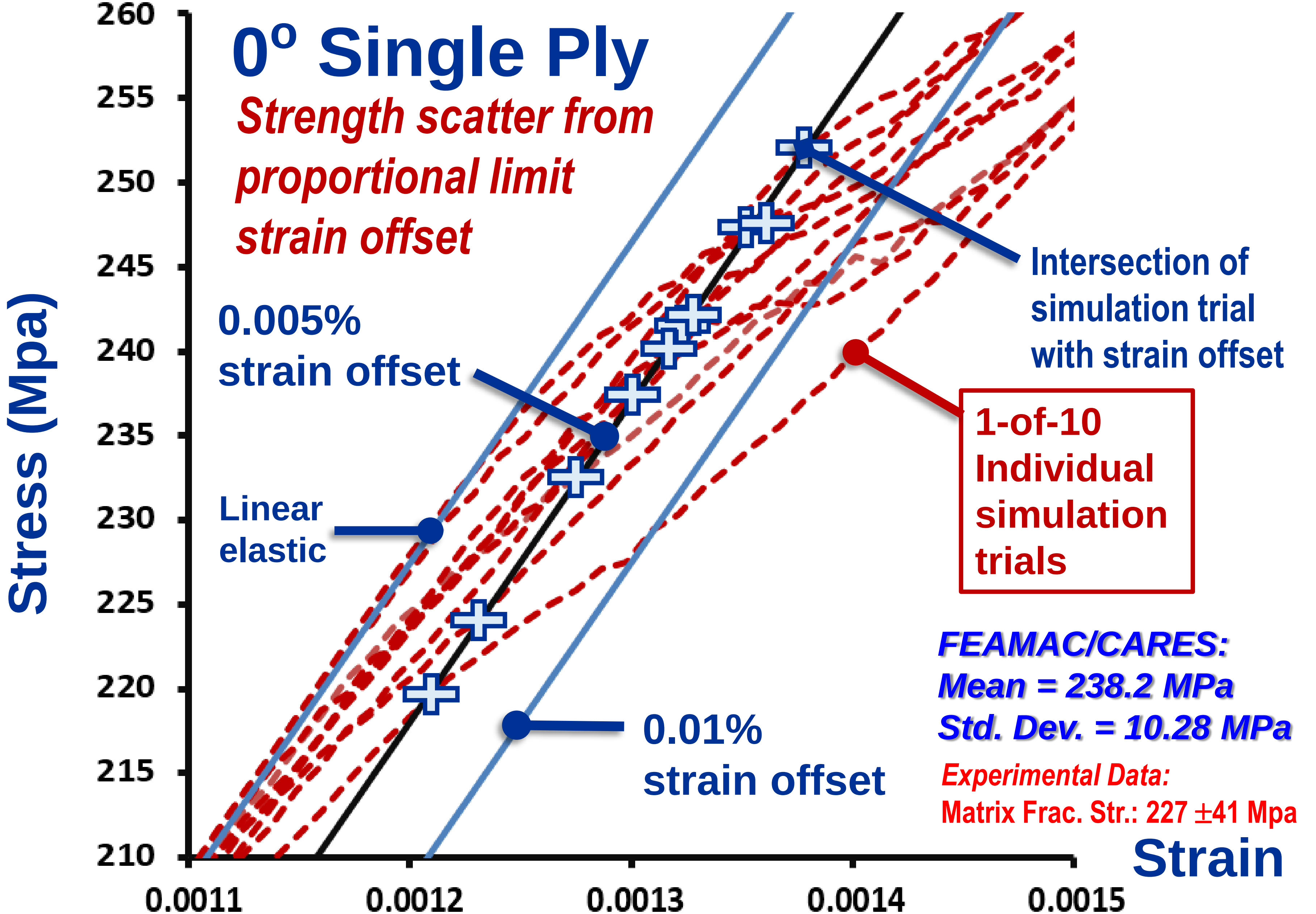
Effect of matrix Weibull modulus on stress-strain response



0º Single Ply [0]$_8$

Non-linear (graceful) failure behavior

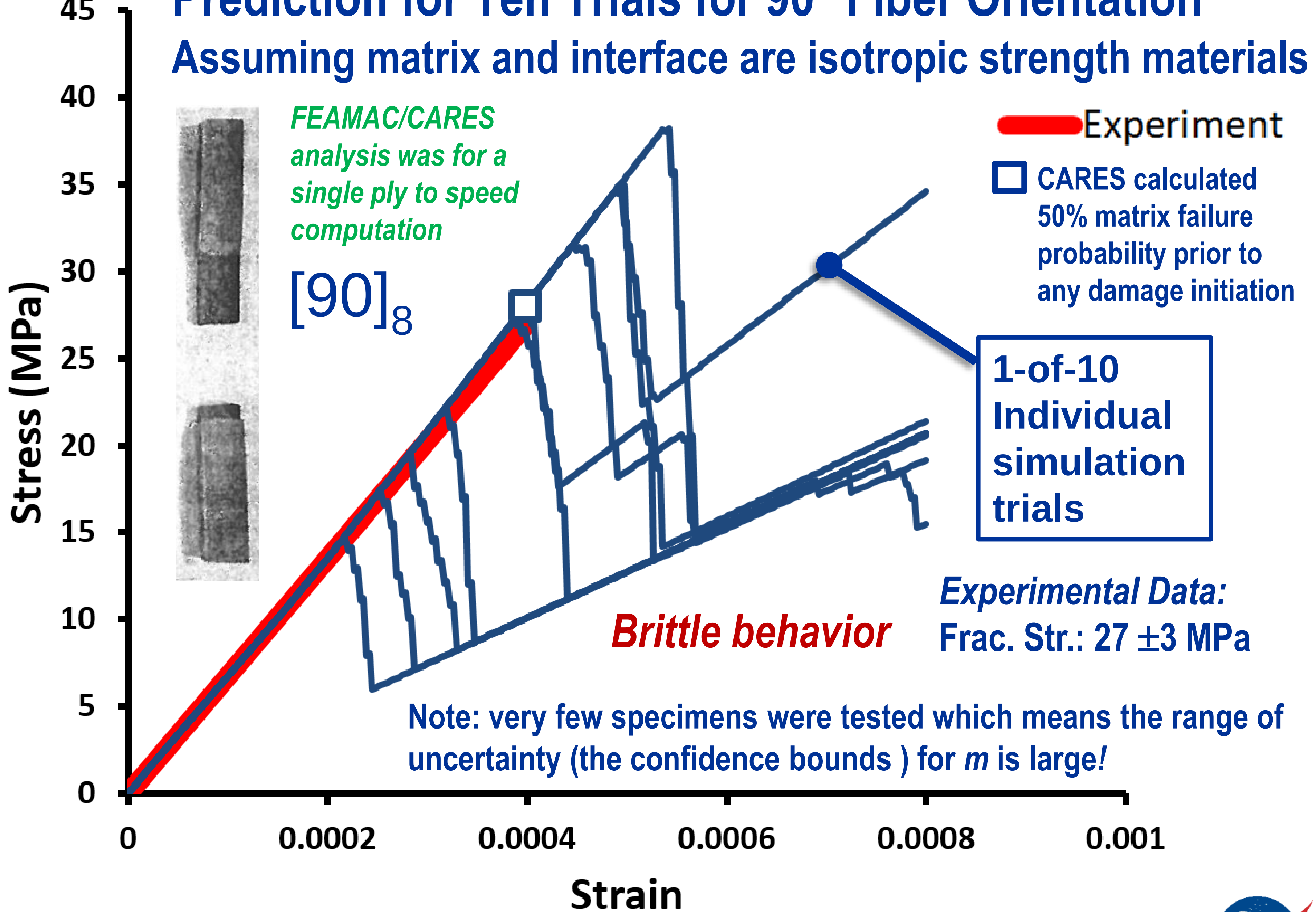




PLS is defined as the stress at 0.005% strain offset:

Kalluri, S; Calomino, A; and Brewer, D., " Computation of Variability in the Average Thermal and Mechanical Properties of a Melt-Infiltrated SiC/SiC Composite", High Temperature Ceramic Matrix Composites 5, M. Singh, R.J. Kearns, E. Lara-Curzio, R. Naslain, Eds, 2004, pp. 279-284

Prediction for Ten Trials for 90° Fiber Orientation
Assuming matrix and interface are isotropic strength materials

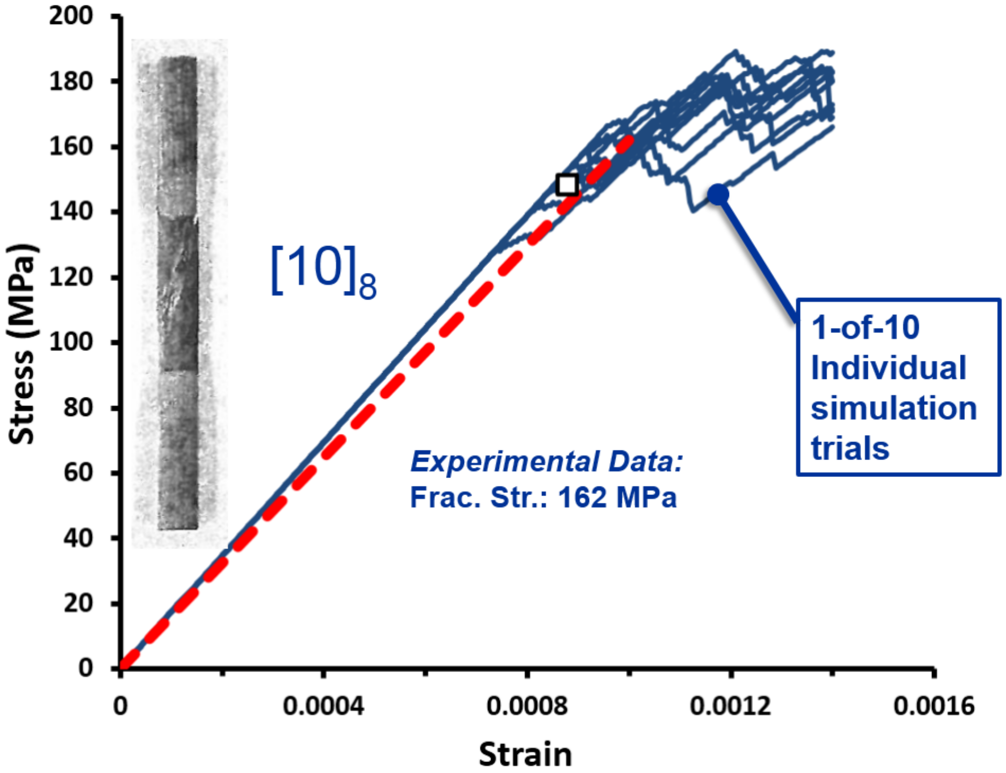


Predictions for Ten Trials for 10° and 45° Fiber Orientations

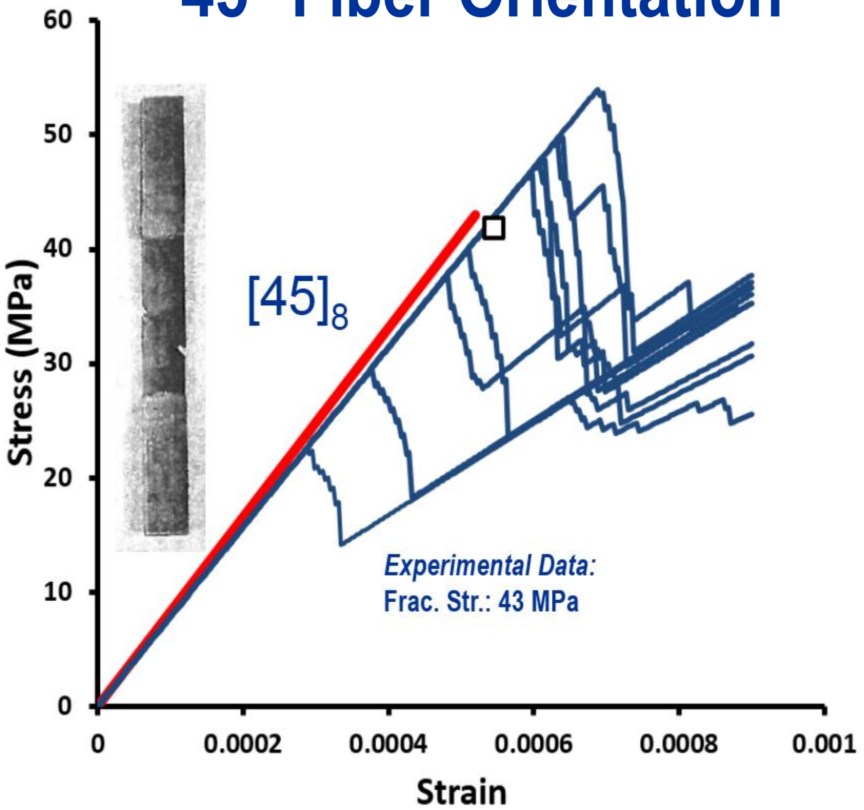
Assuming matrix and interface are isotropic strength materials

FEAMAC/CARES analysis was for a single ply to speed computation

10° Fiber Orientation



45° Fiber Orientation

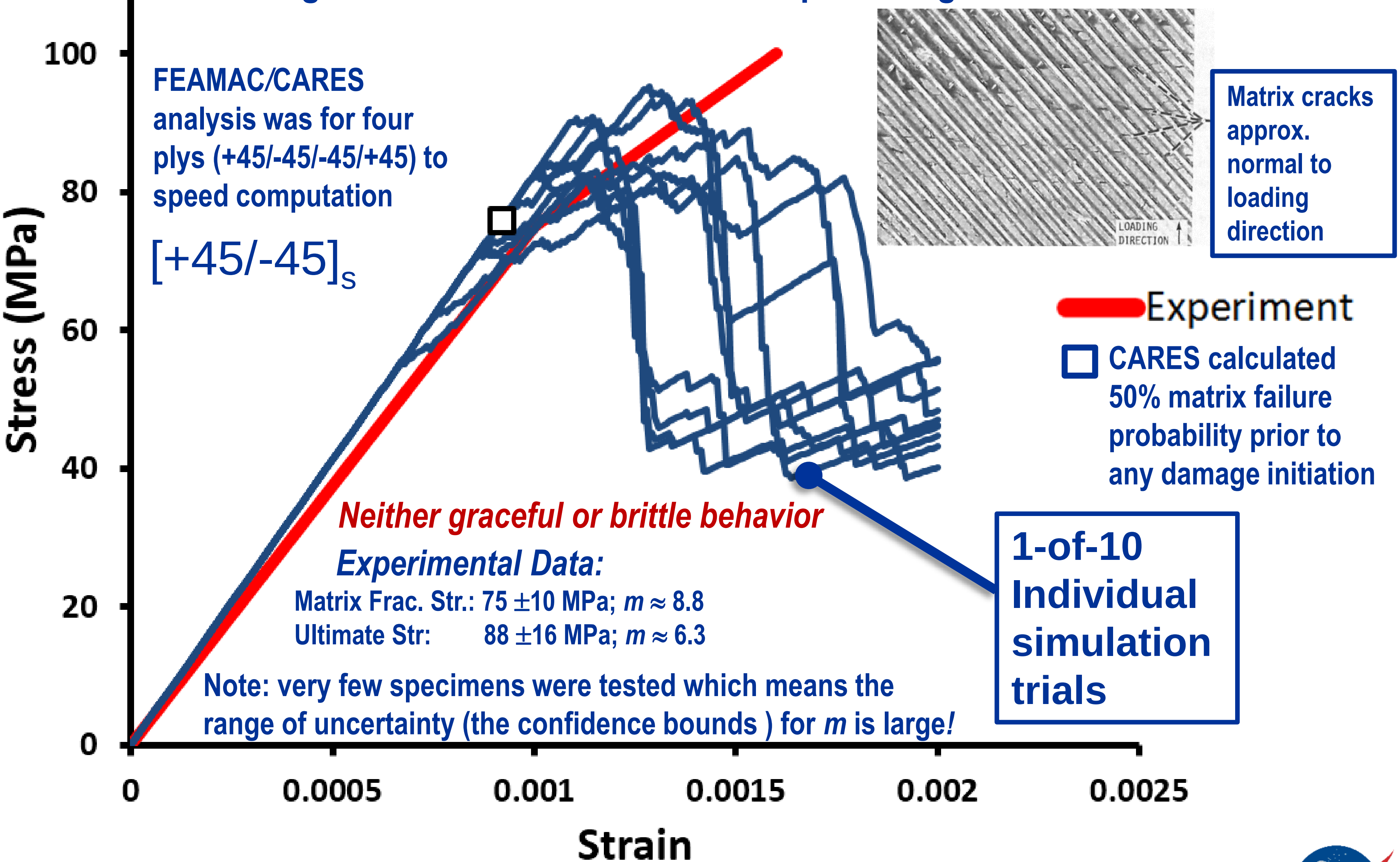


Experiment

CARES calculated 50% matrix failure probability prior to any damage initiation

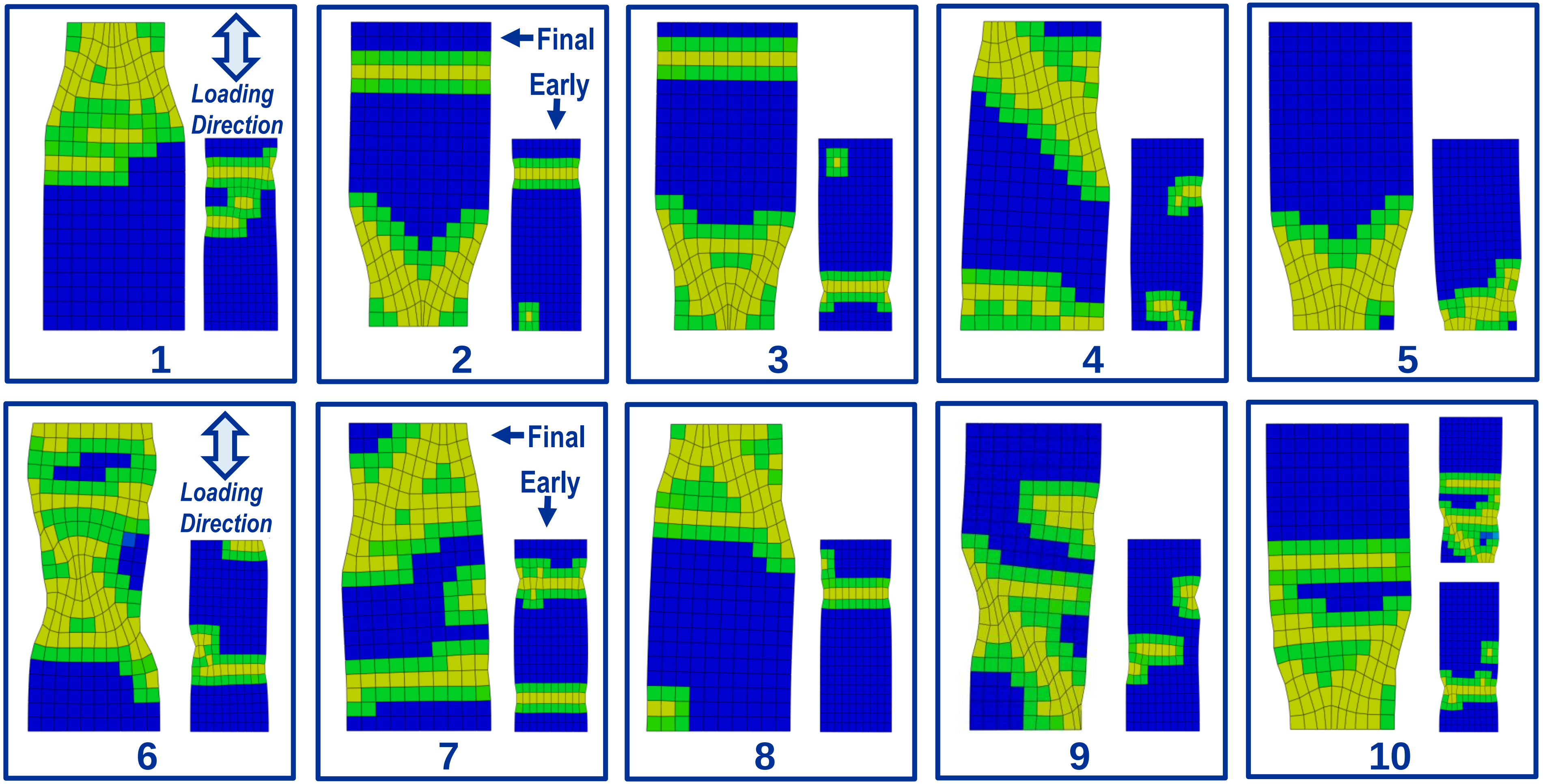
Prediction for Ten Trials for $[+45_2/-45_2]_s$ Fiber Orientation

Assuming matrix and interface are isotropic strength materials



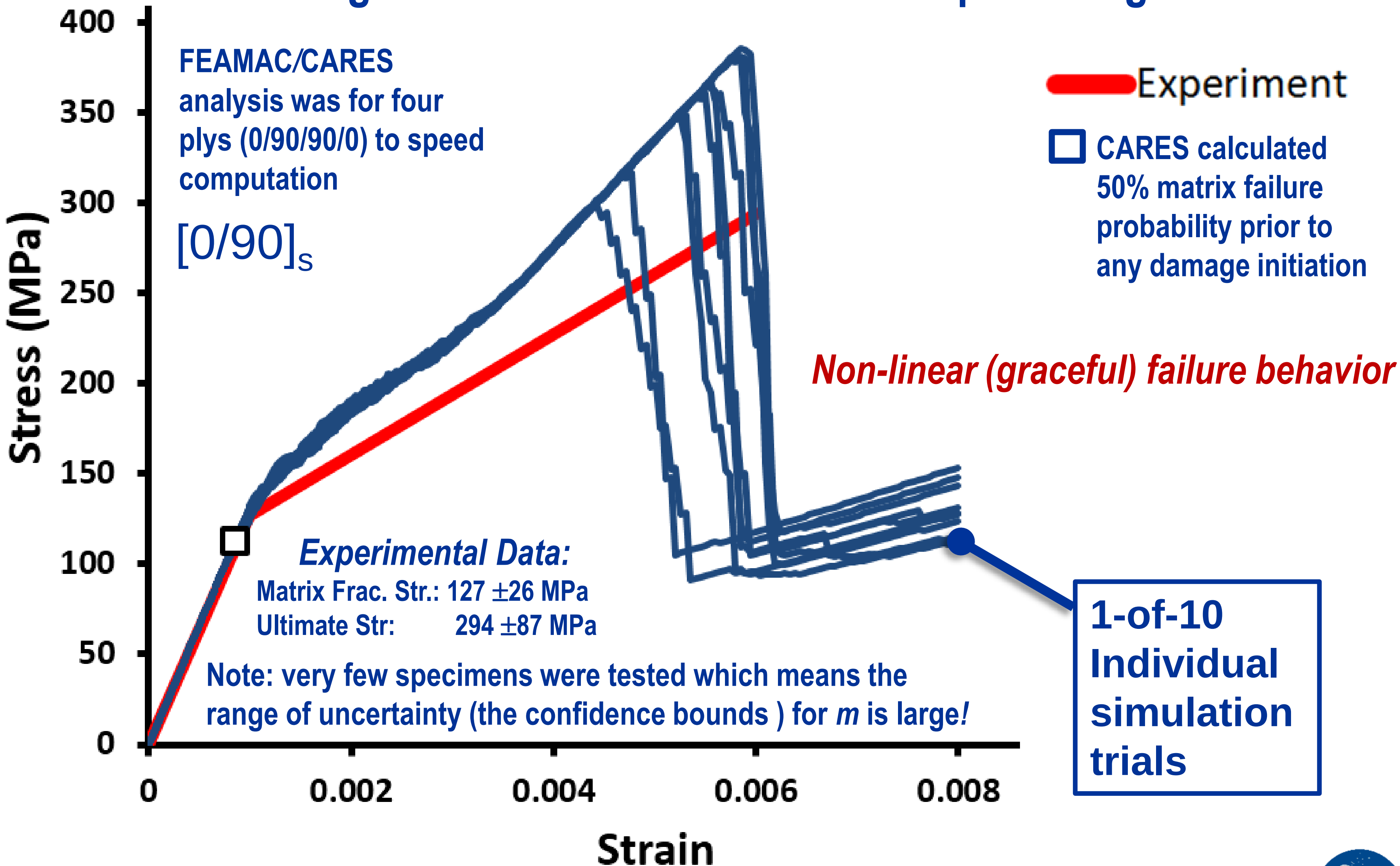
For [+45/-45]$_s$ Fiber Orientation; 10 trials at final (matrix) failure; deformed plots

FEAMAC/CARES analysis was for four plys (+45/-45/-45/+45) to speed computation



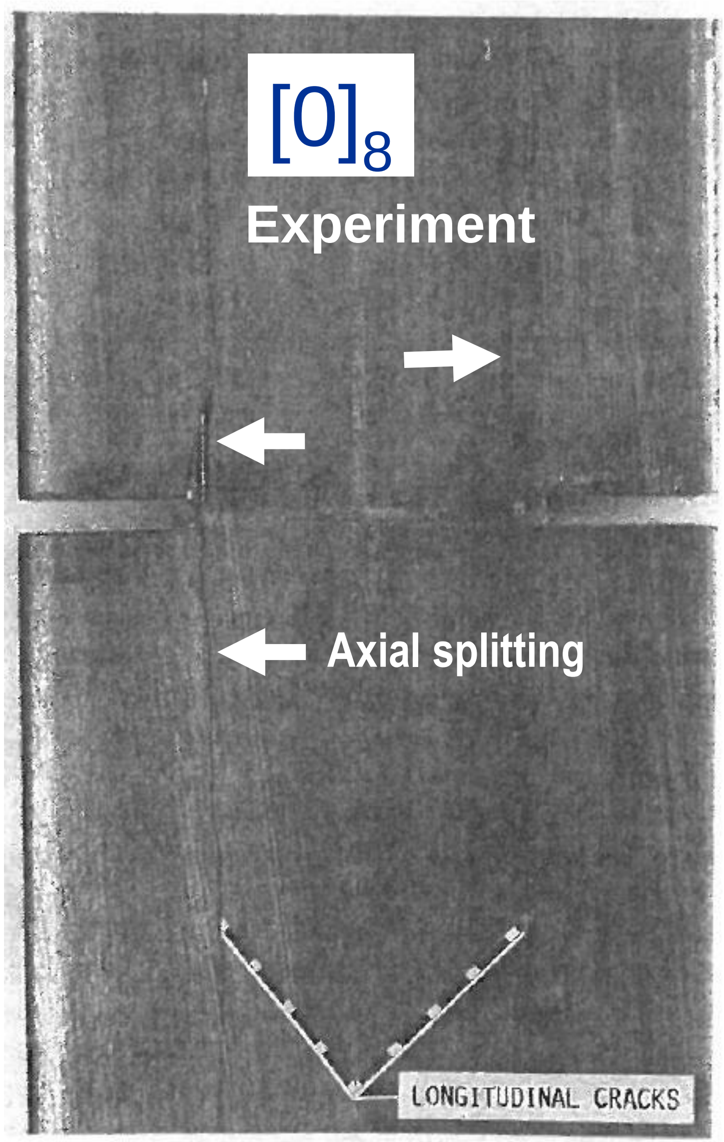
Prediction for Ten Trials for $[0/90]_S$ Fiber Orientation

Assuming matrix and interface are isotropic strength materials



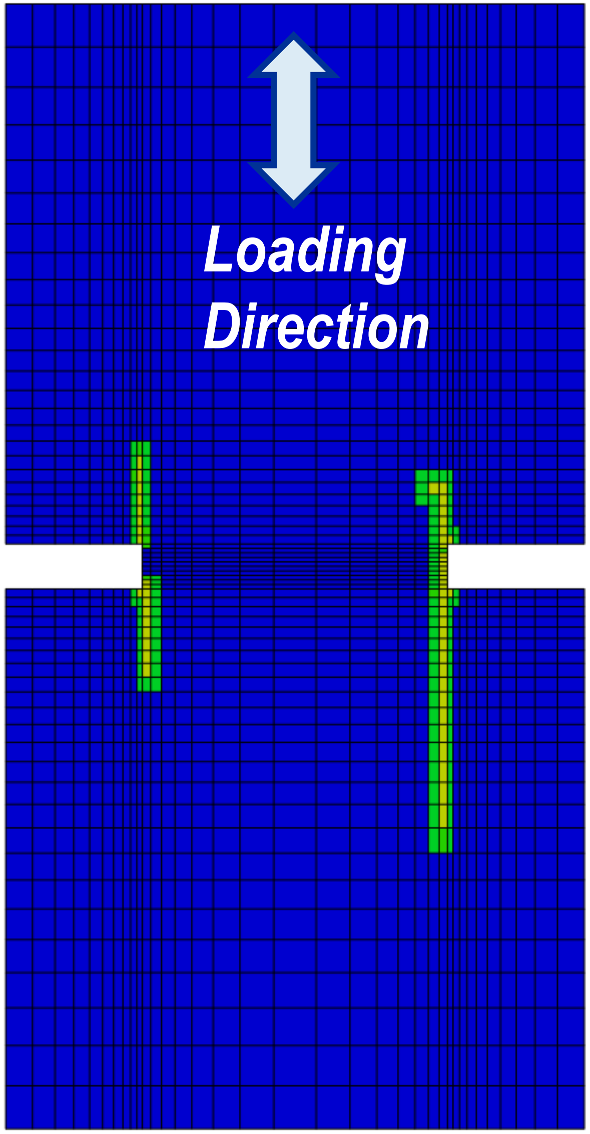
0° Double-Notched Tensile Specimen

Failure mode showed axial splitting of matrix



Fiber Direction

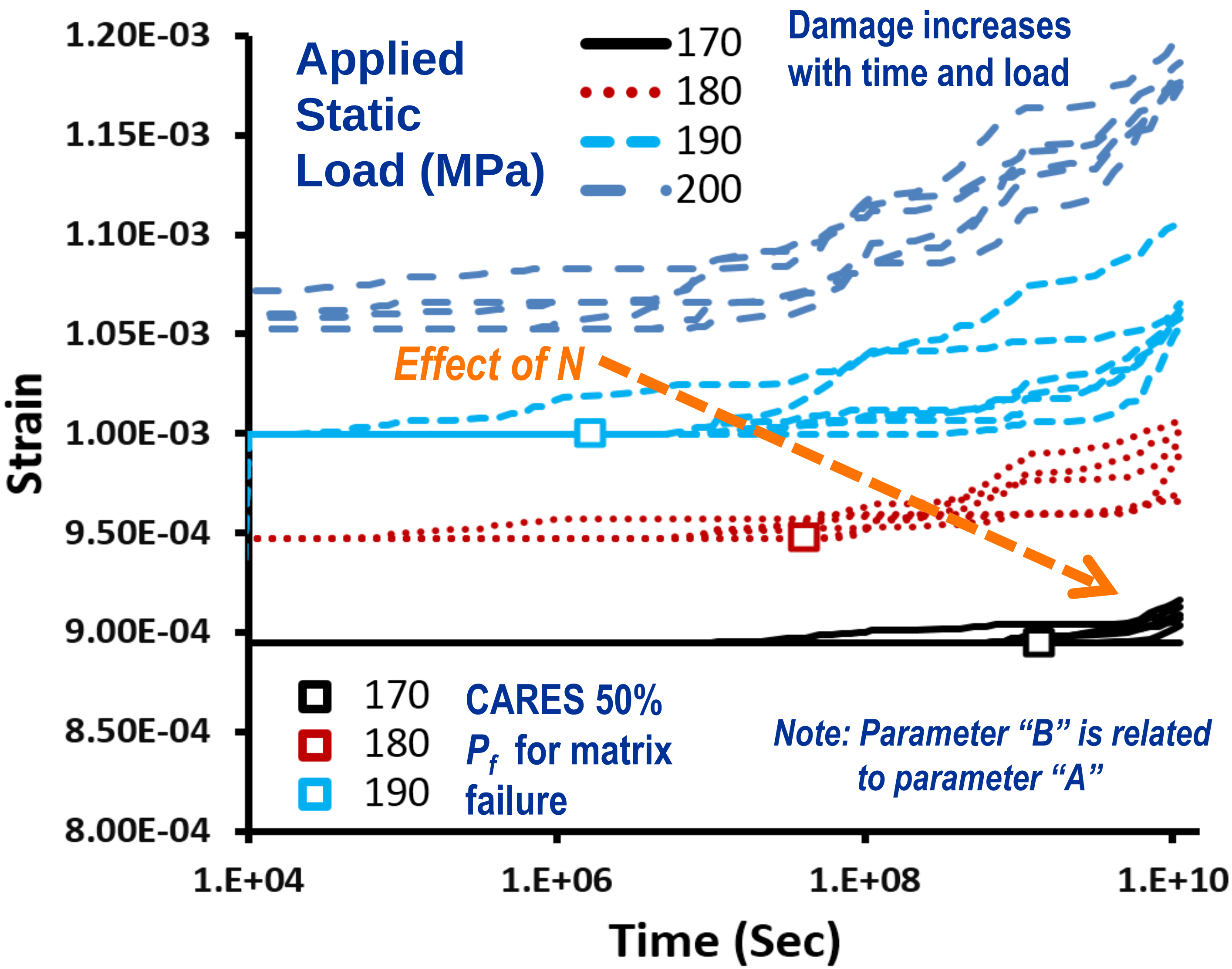
FEAMAC/CARES analysis was for a single ply to speed computation



Time-dependent Failure Example: Static Loading

(Matrix Damage Accumulation From Slow Crack Growth)

Strain response for applied static tensile load over time



Service life prediction

Longitudinal stress applied to a 0° SiC/RBSN ply

10 time increments per time magnitude

Slow Crack Growth Power Law:

$$\frac{da}{dt} = AK_{Ieq}^{N}$$

Weibull Parameters

m = 7 **(Weibull slope)**

σ_o = 106 Mpa • mm$^{3/7}$

Fatigue Parameters

N = 20 **(fatigue slope)**

B = 1.0E9 MPa^2• sec

Conclusions

- Progressive damage simulation of composite structures incorporating probabilistic material strength models is possible with the FEAMAC/CARES code
- The Unit Sphere multiaxial model was used to predict the strength response of a SiC-RBSN composite for various fiber orientations under uniaxial tension
- Reasonable correlation to matrix cracking strength experimental data was achieved assuming the matrix was an isotropic material with $\boldsymbol{m} \approx 5$, and assuming residual stresses from thermal processing were present
- Brittle behavior vs: non-brittle failure *(graceful failure)* demonstrated
- Localized damage modes at stress concentration features shown

Acknowledgement

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Extra Material

Abstract:

Reported here is a coupling of two NASA developed codes: CARES (Ceramics Analysis and Reliability Evaluation of Structures) with the MAC/GMC (Micromechanics Analysis Code/ Generalized Method of Cells) composite material analysis code. The resulting code is called FEAMAC/CARES and is constructed as an Abaqus finite element analysis UMAT (user defined material). Here we describe the FEAMAC/CARES code and an example problem (taken from the open literature) of a laminated CMC in off-axis loading is shown. FEAMAC/CARES performs stochastic-strength-based damage simulation response of a CMC under multiaxial loading using elastic stiffness reduction of the failed elements.

Constituent properties of SiC/RBSN with anisotropic thermal expansion coefficients

Constituent	Modulus, GPa	Poisson ratio	Longitudinal coefficient of thermal expansion, α_L (m/m/°C)	Transverse coefficient of thermal expansion, α_T (m/m/°C)
Fiber	390	0.17	4.1×10^{-6}	1.84×10^{-6}
Matrix	110	0.22	2.2×10^{-6}	2.2×10^{-6}
Interface	1.8	0.22	2.0×10^{-6}	2.0×10^{-6}

Assumed Weibull Parameters:

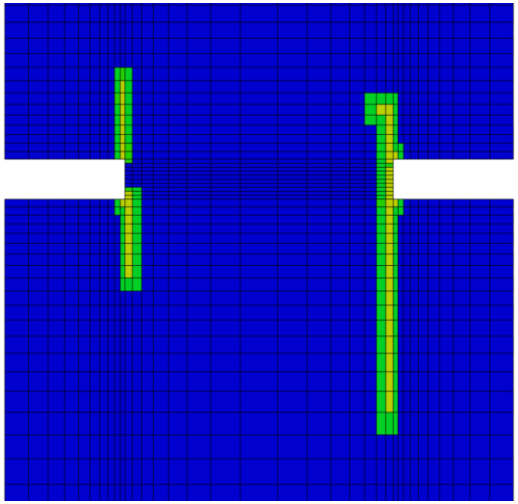
Fiber $m_V = 20$ $\sigma_{oV} = 2875$ Mpa • $m^{3/20}$

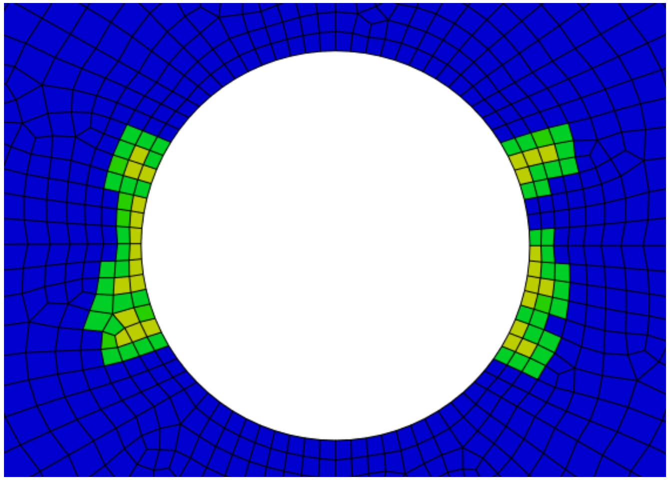
Matrix $m_V = 5.0$ $\sigma_{oV} = 150$ Mpa • $m^{3/5}$

Interface $m_V = 5.0$ $\sigma_{oV} = 80$ Mpa • $m^{3/5}$

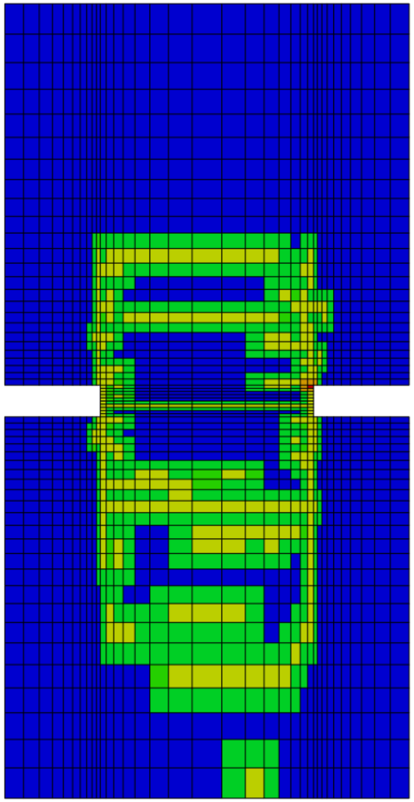
0° Double-Notched vs: Central-Hole Tensile Specimen

Loading Direction

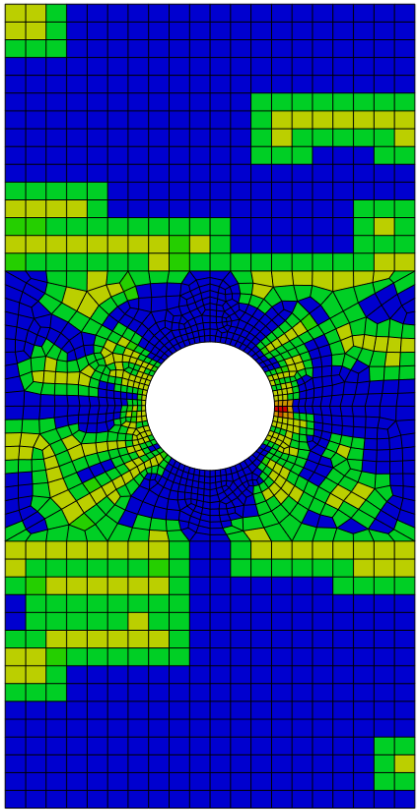




Early matrix damage



Matrix damage progression



Time-Dependent Life Prediction Theory - Slow Crack Growth and Cyclic Fatigue Crack Growth Laws

Power Law: - Slow Crack Growth (SCG)

$$\frac{da}{dt} = AK_{Ieq}^{N}$$

Combined Power Law & Walker Law: SCG and Cyclic Fatigue

$$\frac{da}{dt} = A_1\, g\, K_{Ieq}^{N} + A_2\, f_c (1-R)^{Q}\, K_{Ieq}^{N}$$

Time-Dependent Life Prediction Theory - Slow Crack Growth and Cyclic Fatigue Crack Growth Laws with discrete time steps

Modeling individual time steps in the life prediction methodology enables simulating transient events such as turbine start-up/shut-down or atmospheric re-entry. A computationally efficient methodology has been developed that can extrapolate the reliability calculation for an arbitrary number of Z cycles – where each cycle is described by k number of time steps. This conceivably allows the coupling of other effects such as stiffiness degradation and oxidation effects on the individual time steps and this can be accounted for interactively within the transient finite element and micromechanics analysis.

Transient Life Prediction Theory - Power Law SCG

Reliability formula for *k* discrete time steps over Z cycles:

$$P_{SV}(t_k) = \exp\{-\sum_{i=1}^{n} \frac{V_i}{4\pi}[\int_{\Omega}[\ \ldots[\ \ [\ (\frac{\sigma_{Ieq,k,T\max}}{\sigma_{0BVk}})^{N_{V,k}-2} +$$

$$\frac{\sigma_{Ieq,k}^{N_{V,k}}\, Z\Delta t_k}{\sigma_{0BV,k}^{N_{V,k}-2}\, B_{V,k}}]_k^{\frac{m_{V,k}(N_{V,j}-2)}{m_{V,j}(N_{V,k}-2)}} + \frac{\sigma_{Ieq,j}^{N_{V,j}}\, Z\Delta t_j}{\sigma_{0BV,j}^{N_{V,j}-2}\, B_{V,j}}]_j^{\frac{m_{V,j}\ (N_{V,i}-2)}{m_{V,i}(N_{V,j}-2)}} + \ldots.$$

$$\ldots + \frac{\sigma_{Ieq,2}^{N_{V,2}}\, Z\Delta t_2}{\sigma_{0BV,2}^{N_{V,2}-2}\, B_{V,2}}]_2^{\frac{m_{V,2}\ (N_{V,1}-2)}{m_{V,1}(N_{V,2}-2)}} + \frac{\sigma_{Ieq,1}^{N_{V,1}}\, Z\Delta t_1}{\sigma_{0BV,1}^{N_{V1}-2}\, B_{V,1}}]_1^{\frac{m_{V,1}}{N_{V,1}-2}}\, d\Omega]_i\}$$

Individual time step: Each time step can have different loading, Weibull, and fatigue parameters. Compatibility of failure probability is maintained between the individual time steps

Unit Sphere Multiaxial (Batdorf) Model:

Puts linear elastic fracture mechanics into Weibull weakest-link theory

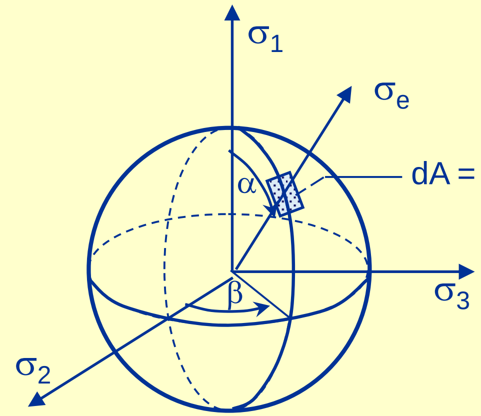
- **Incremental failure probability is the product of two probabilities:**

$$\Delta P_f = P_1 \cdot P_2$$

$\boldsymbol{P_1}$ = Probability of the existence of a crack having a critical strength between σ_c and $\sigma_c + \Delta\sigma_c$ in the incremental volume $\Delta \boldsymbol{V}$

$\boldsymbol{P_2}$ = Probability a crack having a critical strength of σ_c will be oriented in a direction such that it will fail under the applied multiaxial stress state

$\boldsymbol{P_2}$ involves Integration of an equivalent stress σ_e, where $\sigma_e \geq \sigma_c$, over the surface of a unit radius sphere (all possible flaw orientations) divided by the total surface area of the unit radius sphere



σ_e is a function of an assumed crack shape and multiaxial fracture criterion

- **Component failure probability:**

$$P_f = 1 - \exp\left\{ -\int_V \left[\int_0^{\sigma_e} P_1(\sigma_c)\, P_2(\sigma_c)\, \mathrm{d}\sigma_c \right] \mathrm{d}V \right\}$$

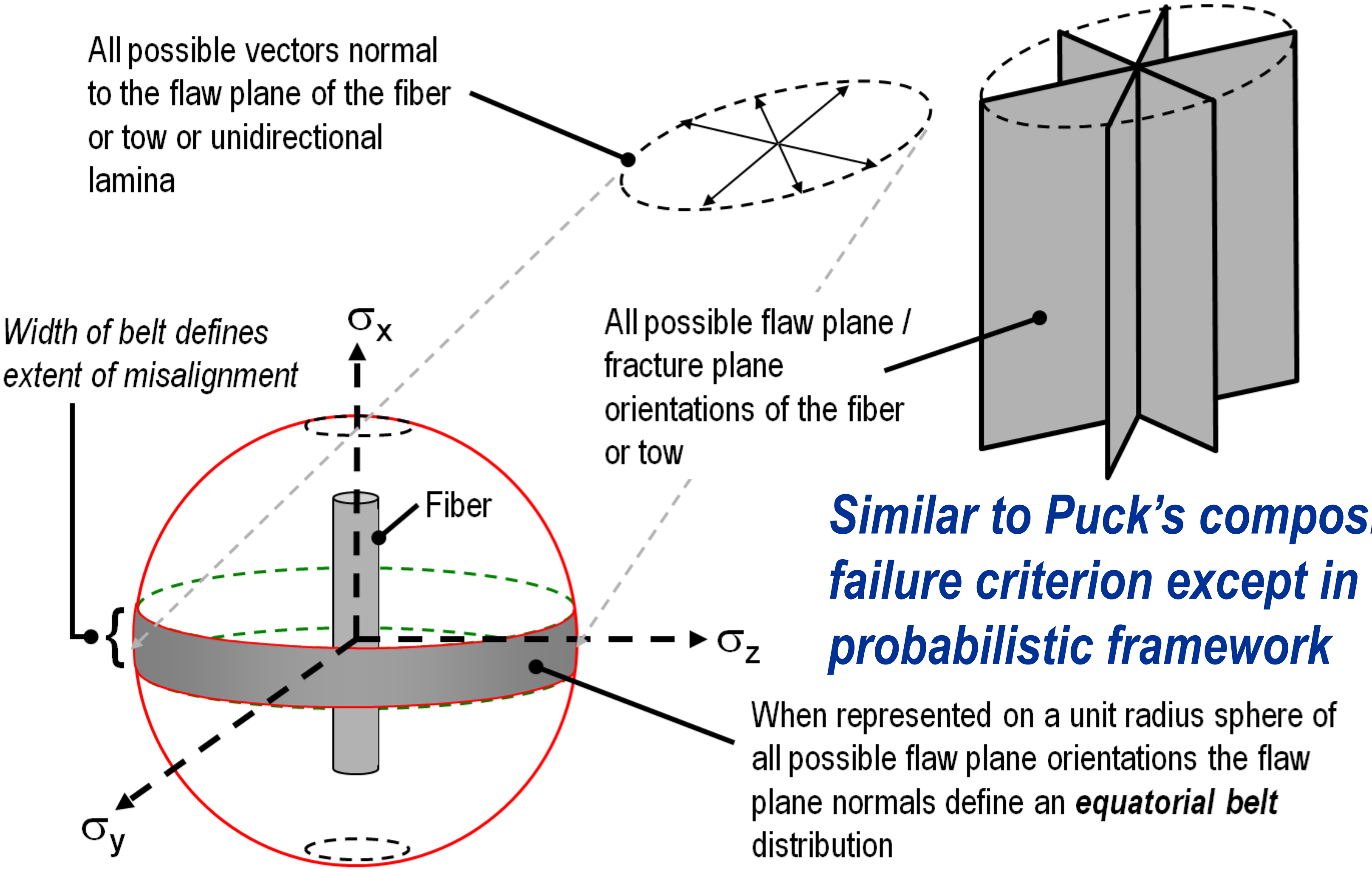
Mixed-Mode Fracture Criteria:

- Normal stress (shear-insensitive cracks)
- Maximum tensile stress
- Total coplanar strain energy release rate
- Noncoplanar (Shetty)

Flaw Shapes:

- Griffith crack
- Penny-shaped crack

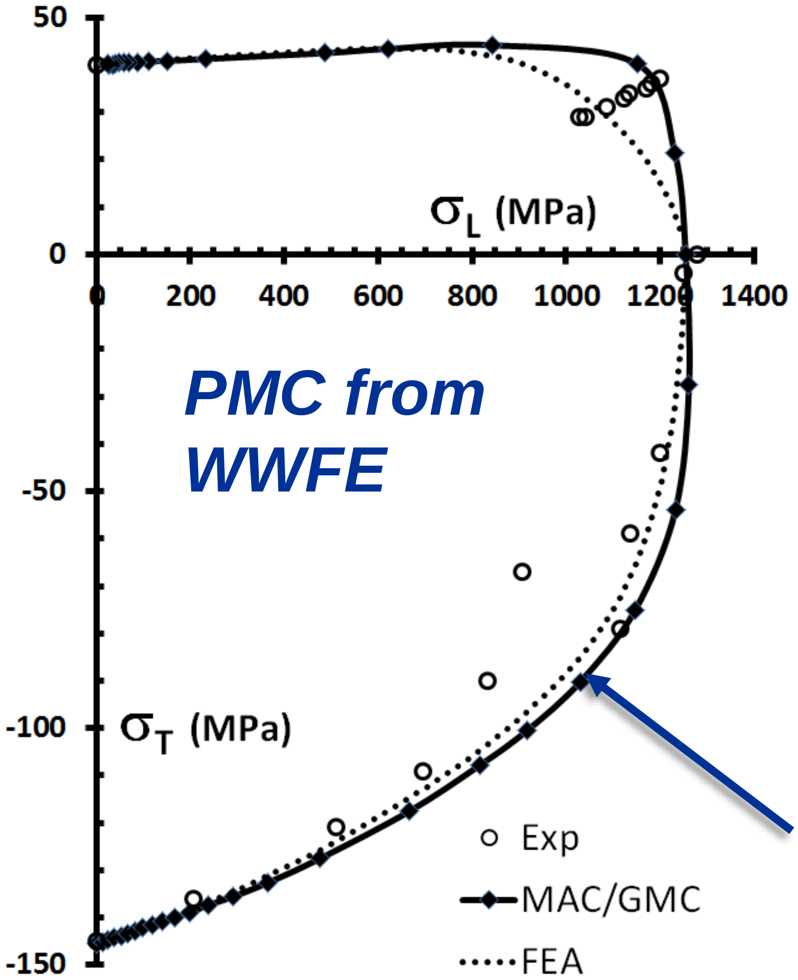
Anisotropic Unit Sphere model defined in a material coordinate system reference frame



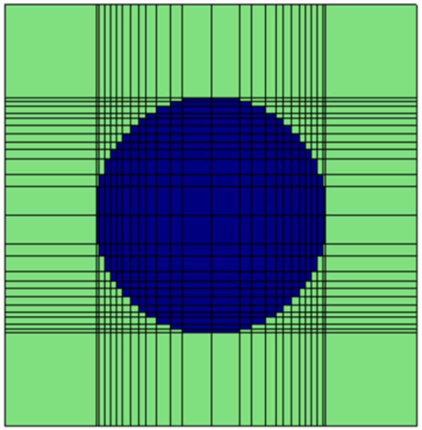
Similar to Puck's composite failure criterion except in a probabilistic framework

Multiaxial Performance: biaxial response predicted from a MAC/GMC RUC for combined longitudinal (L) and transverse (T) loading on a unidirectional PMC vs: FEA.

50% probability of failure envelope.



GMC RUC Used

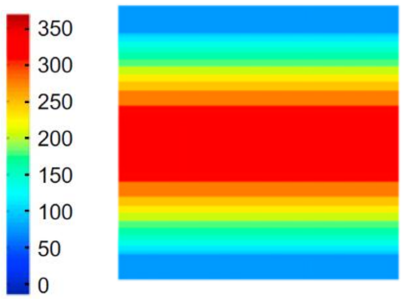


Unit Sphere parameters adjusted so GMC results matched FEA results for uniaxial tension and compression.

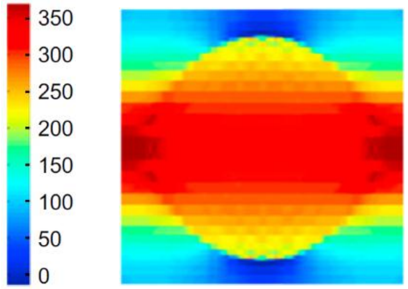
Intermediate points are predictions

Differences in RUC stress fields for a transverse strain:

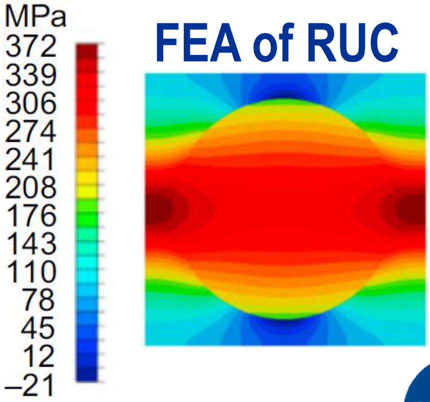
GMC RUC



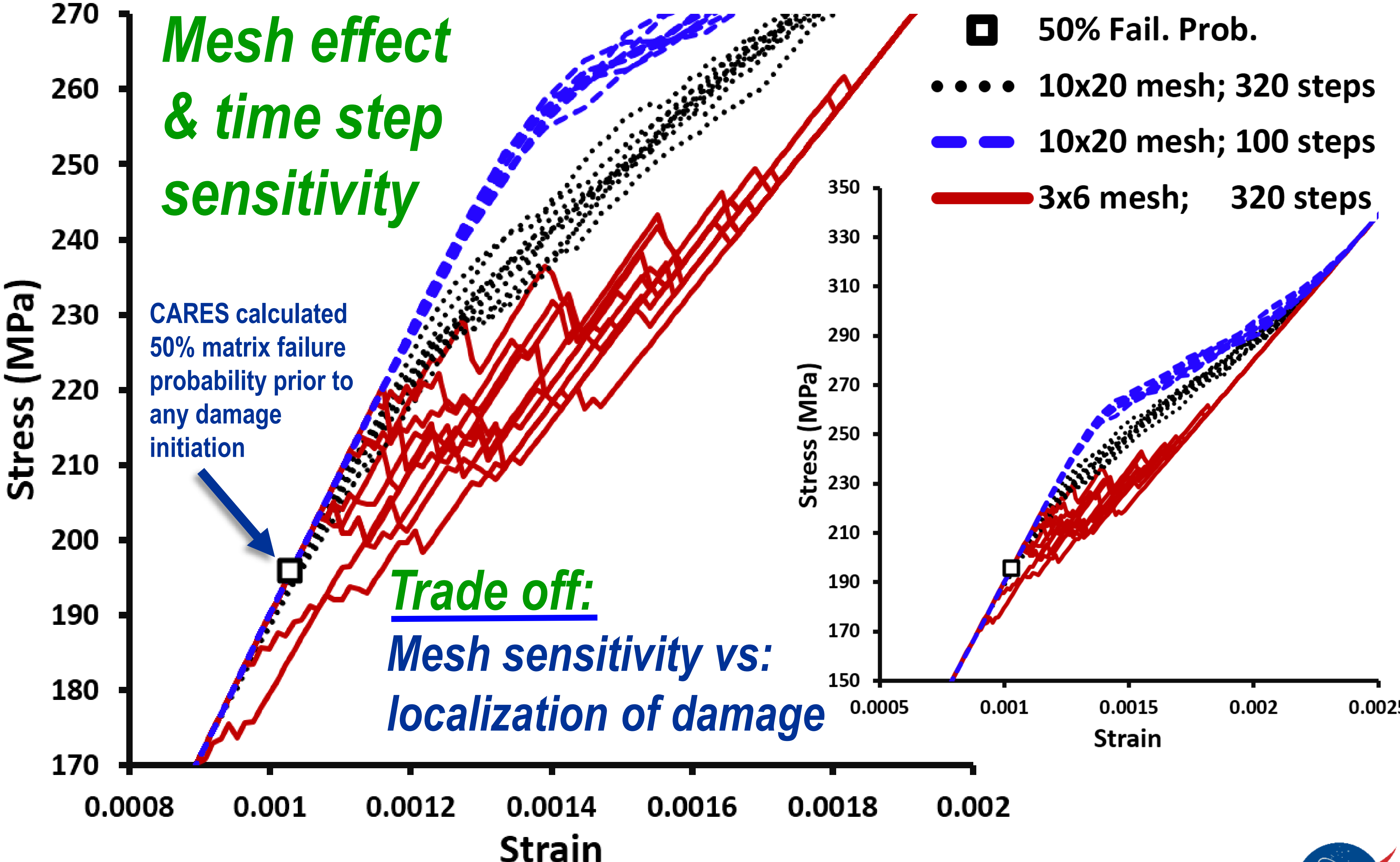
HF - GMC RUC



FEA of RUC

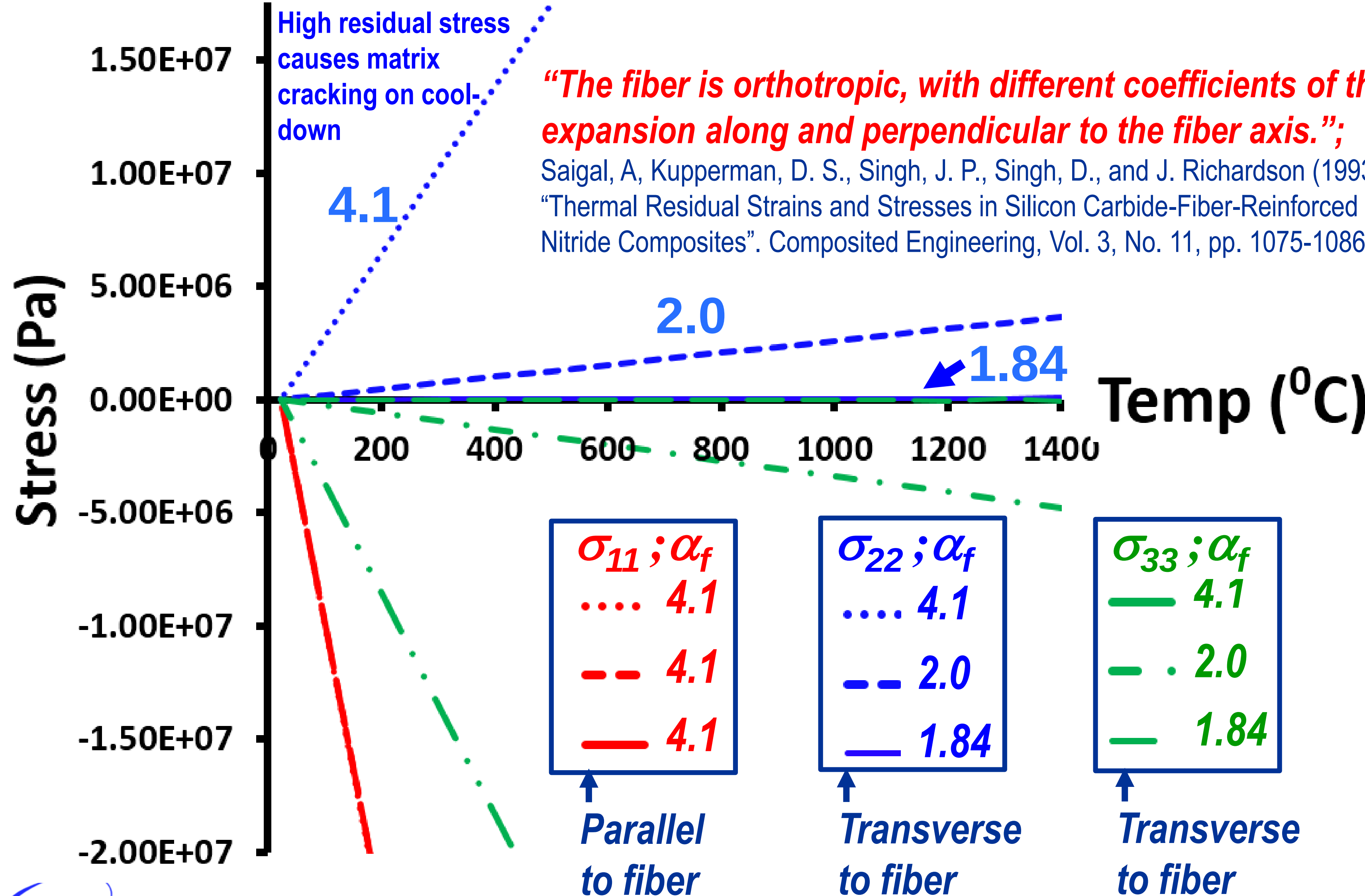


0º single ply tensile specimen *(Load parallel to fiber axis)*



Residual matrix stresses after cool-down from temperature

- Effect of anisotropic fiber-thermal-expansion-coefficient, α_f on RUC



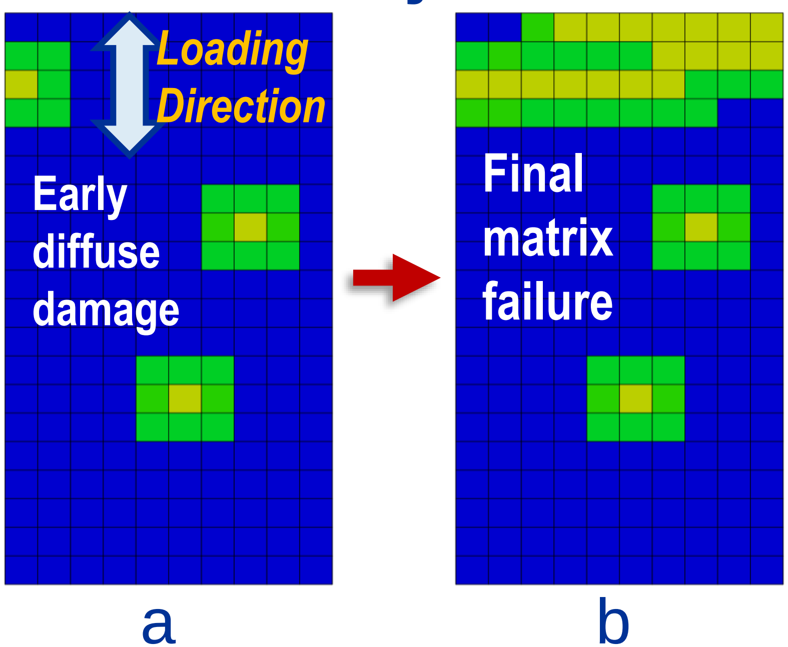
"The fiber is orthotropic, with different coefficients of thermal expansion along and perpendicular to the fiber axis.";

Saigal, A, Kupperman, D. S., Singh, J. P., Singh, D., and J. Richardson (1993): "Thermal Residual Strains and Stresses in Silicon Carbide-Fiber-Reinforced Silicon Nitride Composites". Composited Engineering, Vol. 3, No. 11, pp. 1075-1086.

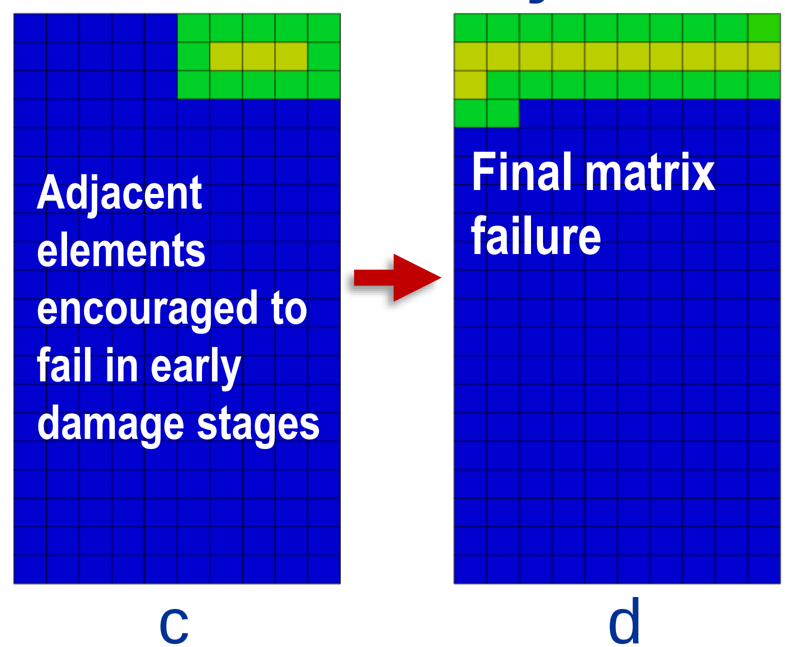
90º single ply tensile specimen

Progression of damage in FE model of a unidirectional ply under *transverse* loading

Not Adjusted



Automaton Adjusted

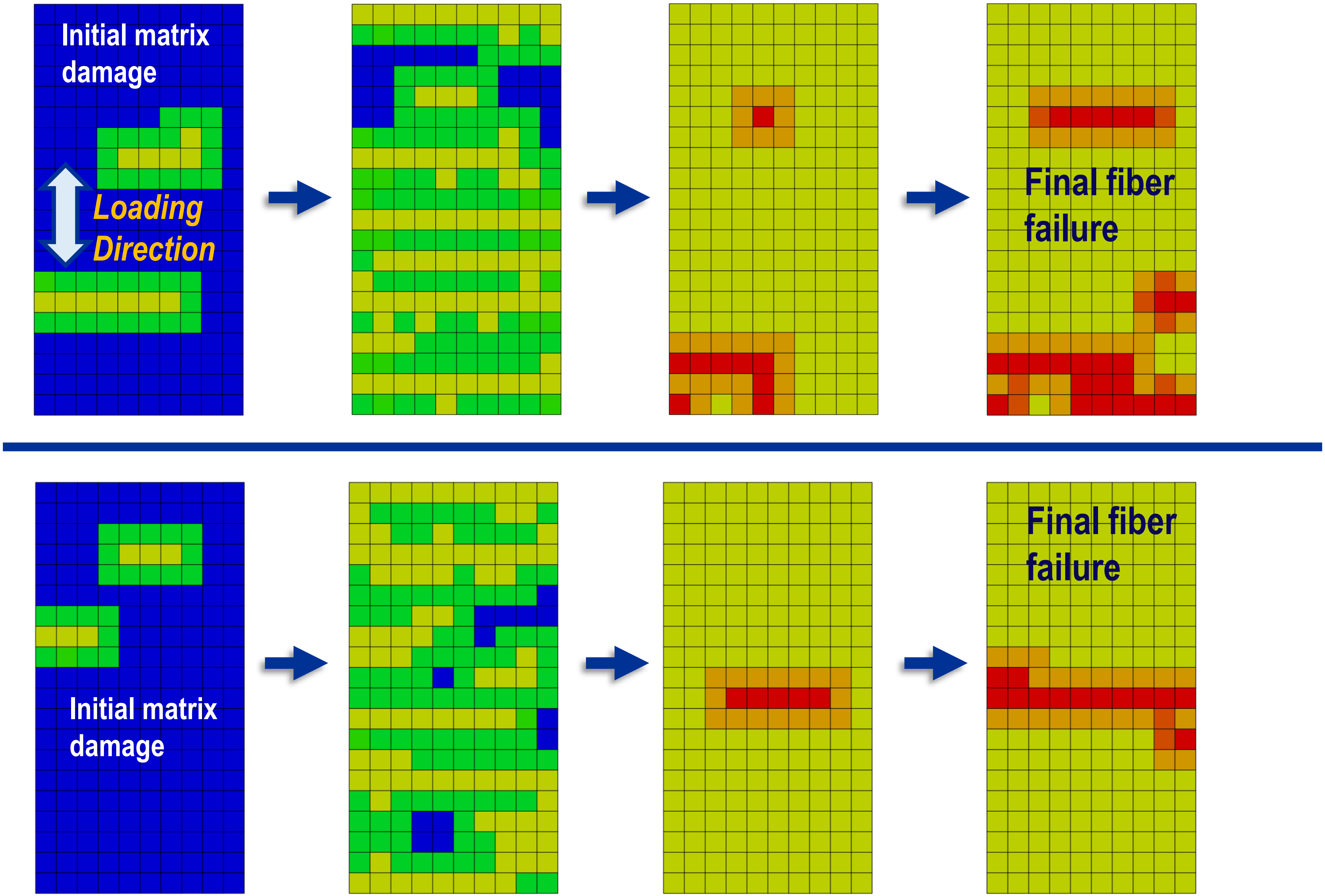


- Matrix failure
- Adjacent to failed matrix
- No failure

(a) and (c) ; early matrix damage

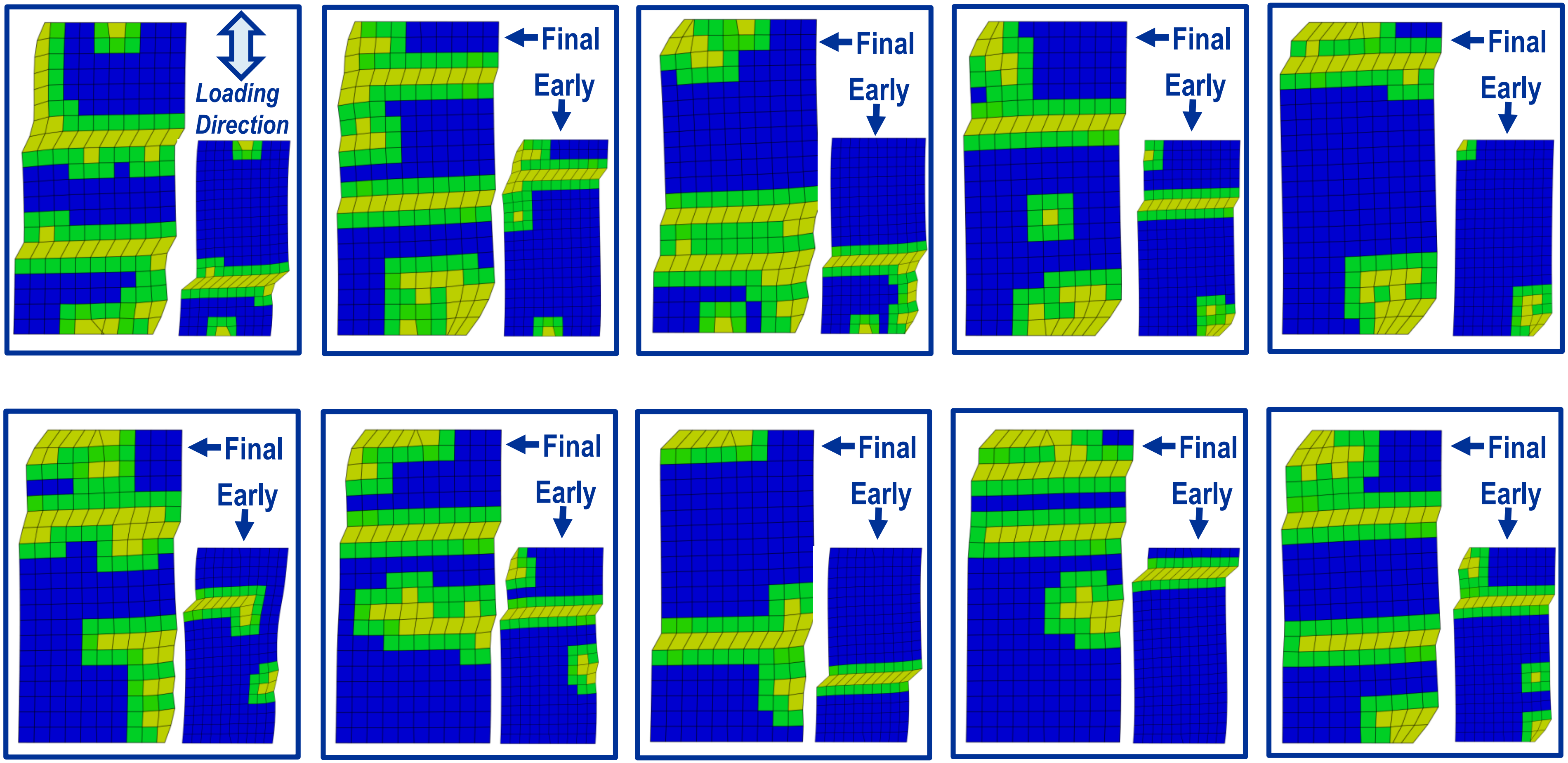
(b) and (d) ; final composite failure (matrix failure)

Damage progression of 0º tensile specimen - two trials (undeformed plot)

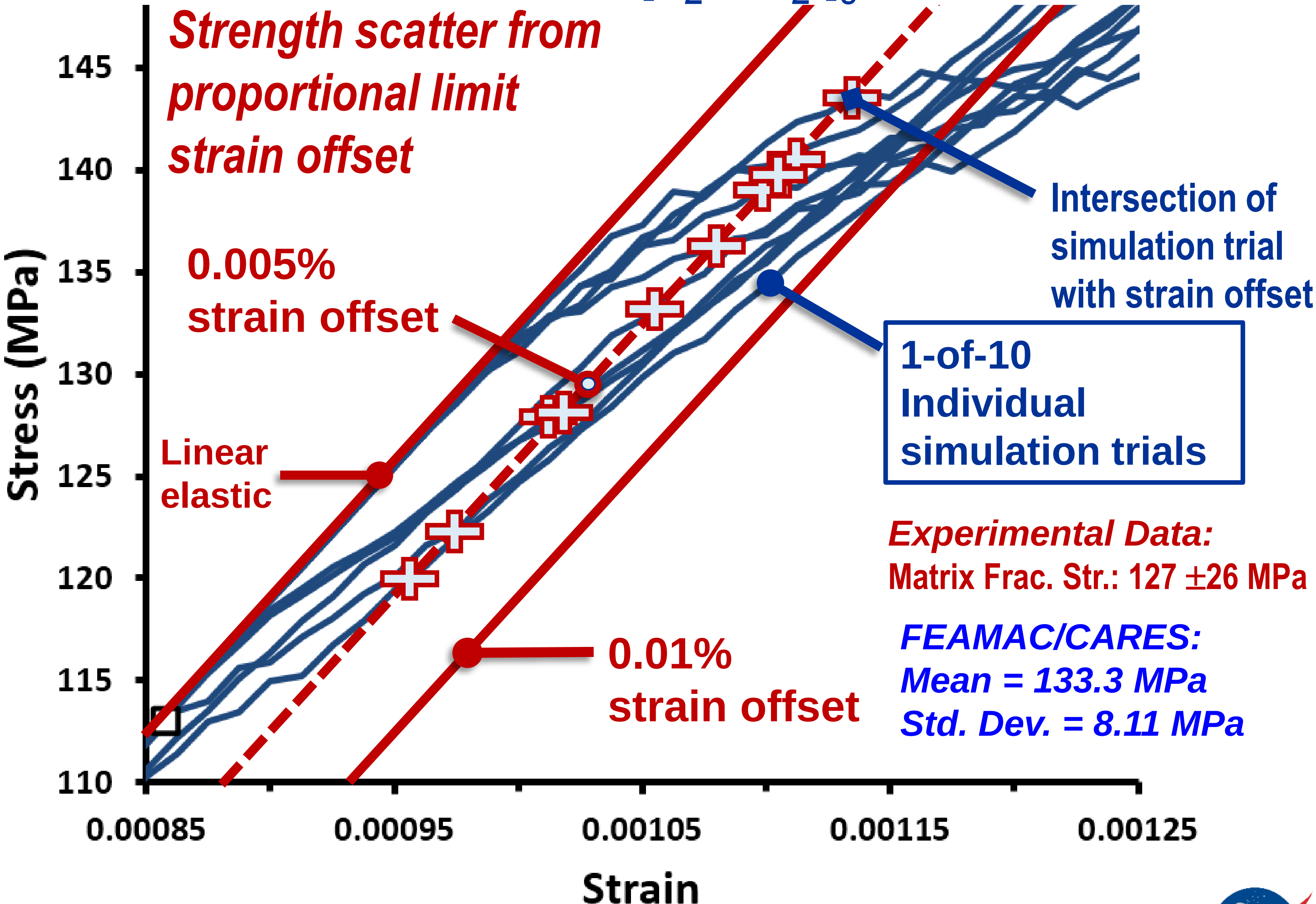


10^0 off-axis tensile specimen; 10 trials at final (matrix) failure; deformed plots

- Edges are allowed to freely deform (warp) on cool-down
- *After cool-down;* bottom edge fixed in loading direction when displacement load applied
- After cool-down; single node along top edge (middle) fixed in direction perpendicular to displacement direct.



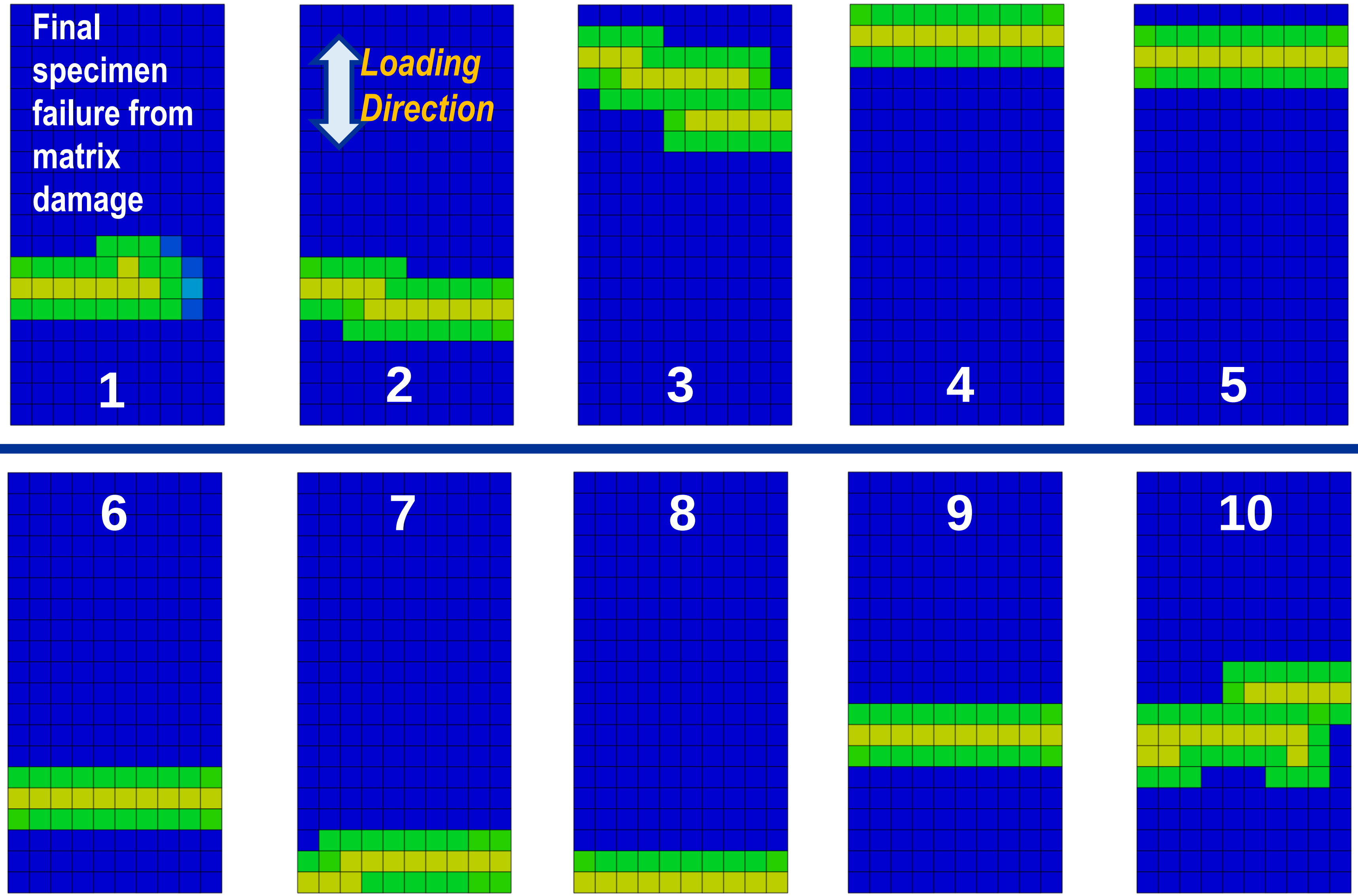
Prediction for Ten Trials for $[0_2/90_2]_s$ Fiber Orientation

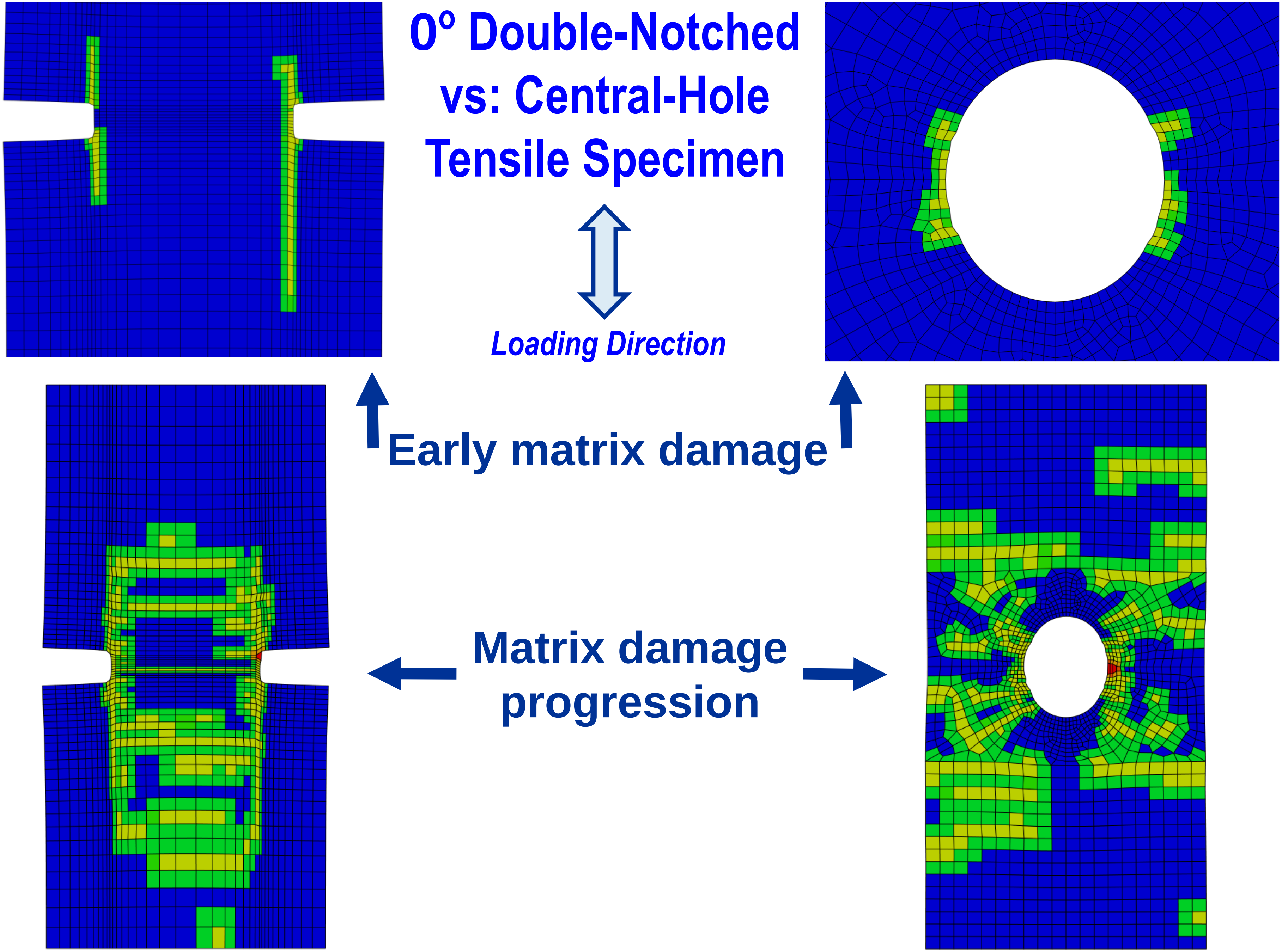


Experimental Data:
Matrix Frac. Str.: 127 ±26 MPa

FEAMAC/CARES:
Mean = 133.3 MPa
Std. Dev. = 8.11 MPa

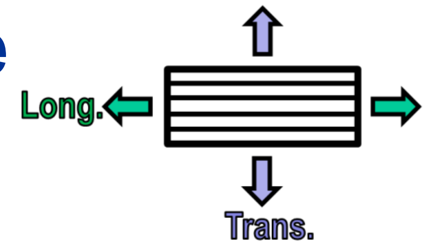
90º Tensile specimen *at final failure for 10 trials* – Undeformed plots

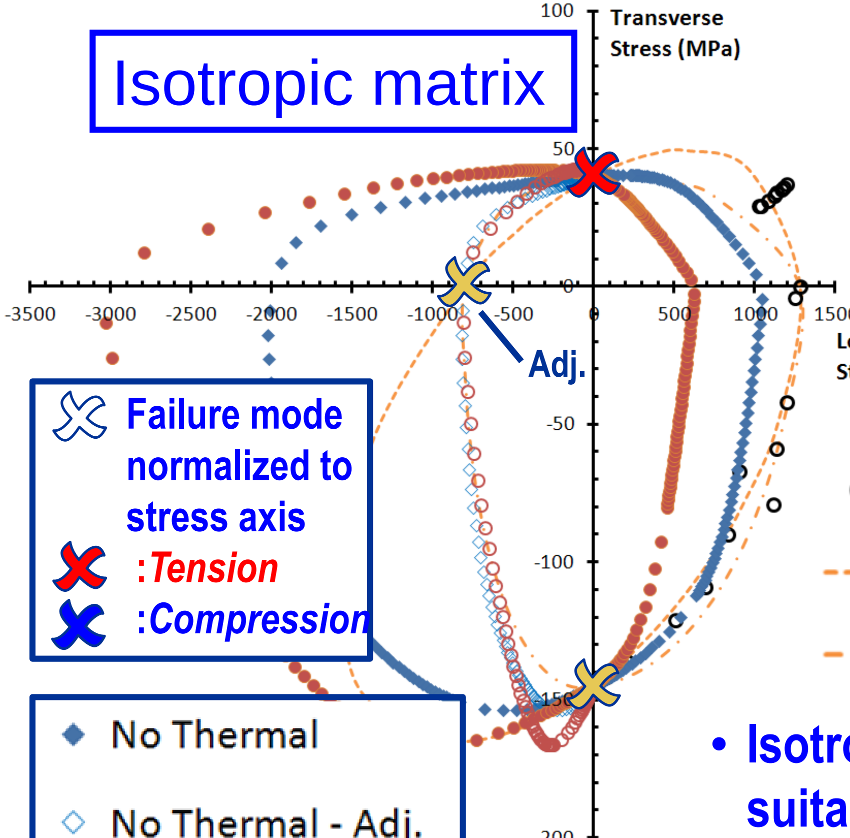


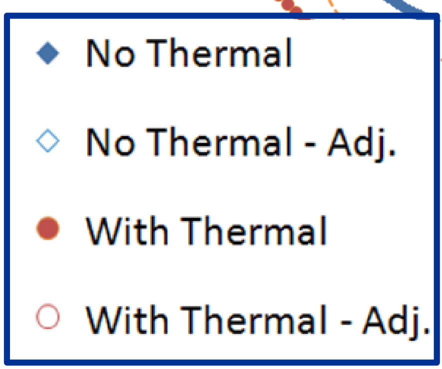

0° Double-Notched
vs: Central-Hole
Tensile Specimen
Loading Direction
Early matrix damage
Matrix damage
progression

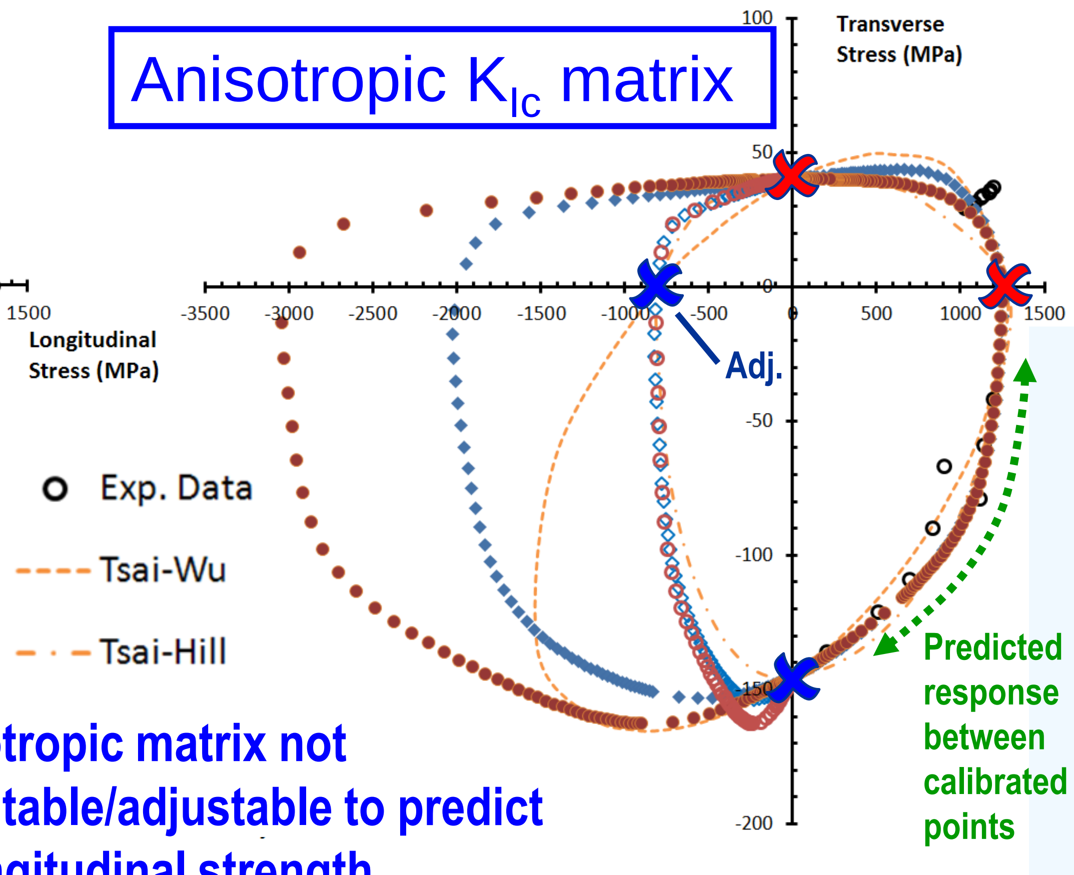
Biaxial failure envelope at 50% P_f for a composite unit cell for all sampled points and failure modes

(with and without thermal residual stresses)









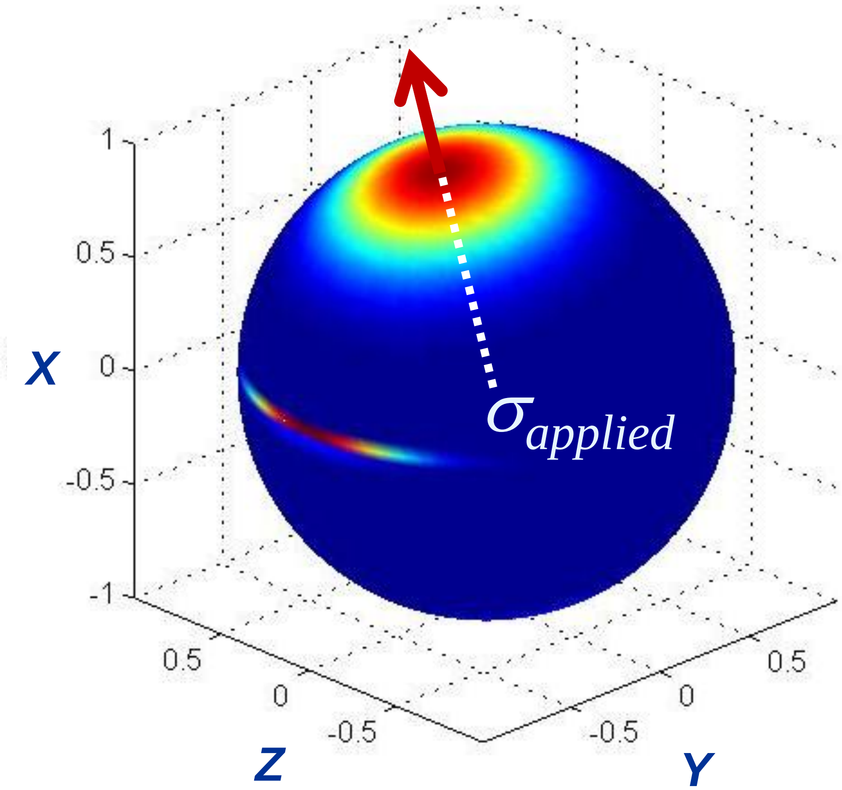
- **Isotropic matrix not suitable/adjustable to predict longitudinal strength**
- **Anisotropy in unit sphere Compares Tsai-Wu & Tsai-Hill**
- **Validates approach taken**

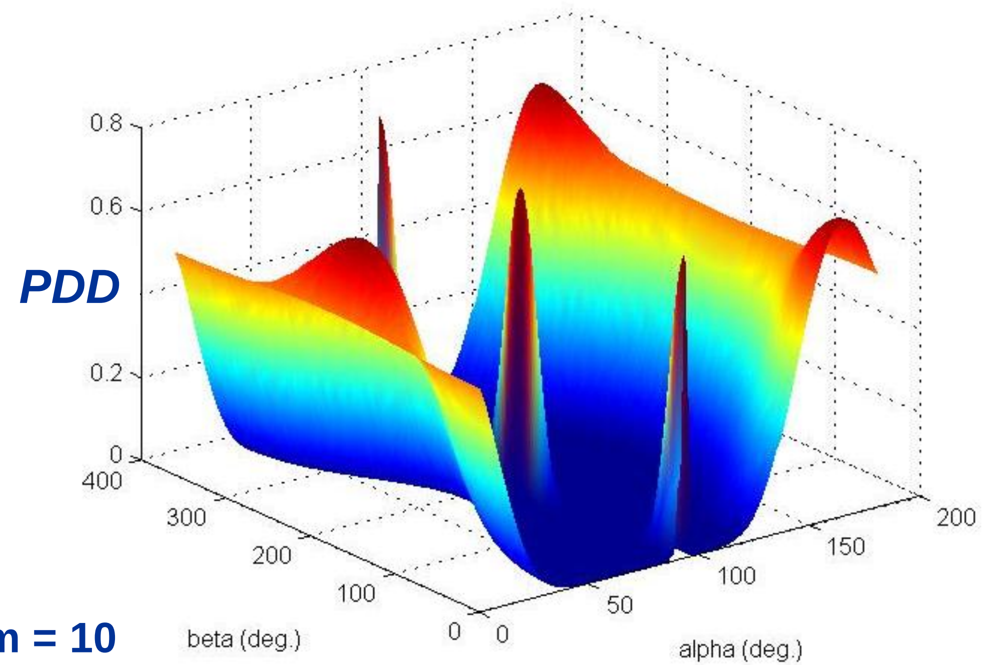
Tsai/Wu/Hill curves come from the applied stresses on the composite

Probability density distribution for orientation of critical flaws

(Transversely isotropic (K_{Ic}) Material)

For a 15^o offset uniaxial load





m = 10

P_f = 0.6321

- Critical crack initiation angle could help with determination of anisotropic elastic damage constants

CARES: Life Prediction & Component Design Tools For Advanced Ceramics

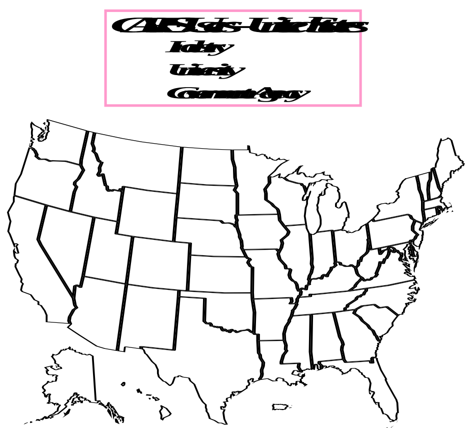
Software *(**C**eramics **A**nalysis and **R**eliability **E**valuation of **S**tructures)*

Predicts the probability of failure of ceramic components under load

- Silicon nitride, silicon carbide, alumina
- Ultra high temperature ceramics
- MEMS materials – silicon; SiC
- Glass

Utilized worldwide for life prediction of brittle material components.

- Aerospace
- Automotive
- Electronic
- Energy
- Glass
- Medical
- Power



- NASA Software of the Year Award
- R&D 100 Award
- Federal Laboratory Consortium Technology Transfer Award
- American Ceramic Society Corporate Technical Achievement Award
- Enterprise Development, Inc. Innovation Award
- NASA Steven Szabo Engineering Excellence Award

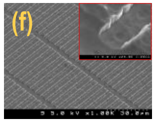
Applications

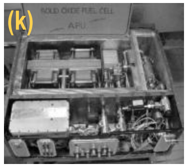
MEMS
- Microturbine (a)
- Microrocket (b)
- Pressure sensor (c)

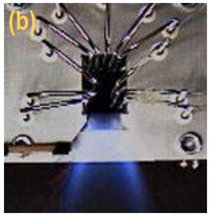
Biomedical
- Hip joint (d)
- Dental Bridge (e)
- MEMS implants (f)

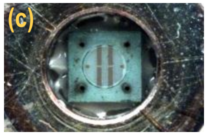
Aerospace
- Turbine blade (g)
- Rocket Nozzle (h)
- Mars Aeroshell (i)

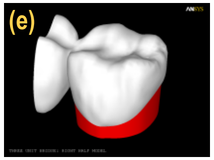
Fuel Cell (SOFC)
- Power generation (j)
- Propulsion (k)











MAC/GMC Methodology: Generalized Method of Cells (*GMC*) & High-Fidelity Generalized Method of Cells (*HFGMC*)

❖ ***Micromechanics links the size scales &** provides the composite response based on the composite constituent materials*

▪ ***FEAMAC:** MAC/GMC embedded in FEA as constitutive material*

GMC (1990s)

- 1st order displacement field in subcells
- Stresses and strains piecewise constant
- Number of linear algebraic equations function of number of subcells
- Local inelasticity/damage
- *No shear coupling*
- No "*subcell mesh*" sensitivity

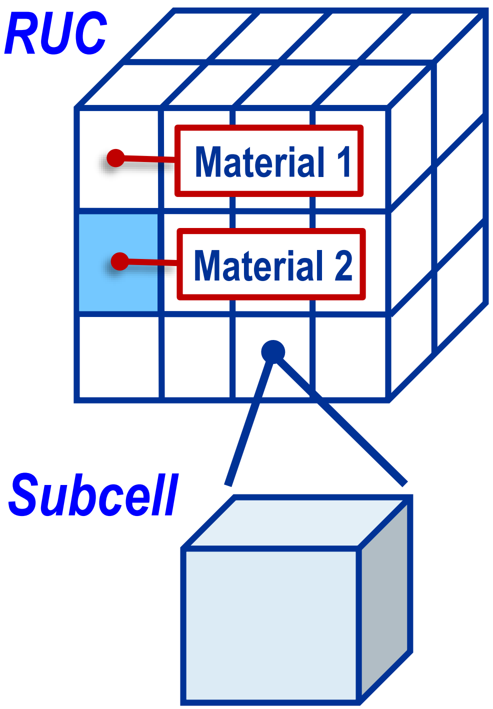
HFGMC (2000s)

- 2nd order displacement field in subcells
- Elastic stresses and strains piecewise linear
- Number of linear algebraic equations is rather large
- Local inelasticity/damage
- *Has shear coupling*
- Has "*subcell mesh*" sensitivity

Repeating Unit Cell (RUC) of composite material

❖ RUC made subcells

❖ Multiscale capability



We currently only use GMC in FEAMAC/CARES

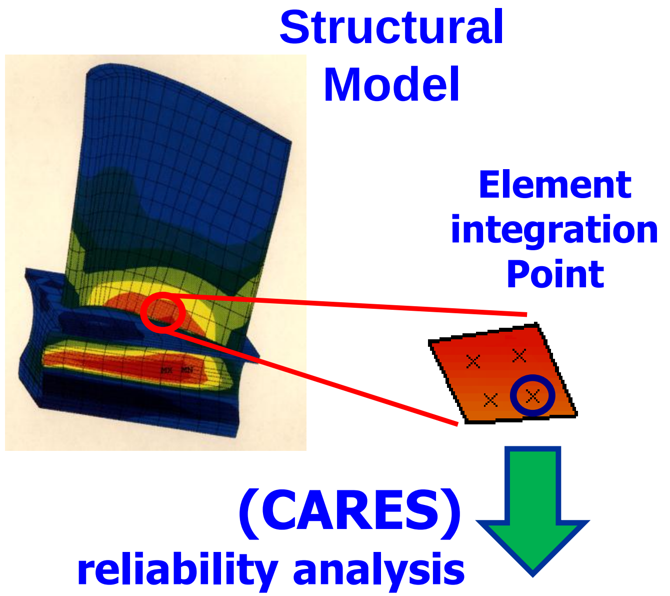
Aboudi, J.; Arnold, S.M.; and Bednarcyk, B.A. (2013) Micromechanics of Composite Materials: A Generalized Multiscale Analysis Approach, Elsevier, Oxford, UK.

Aboudi, J; Pindera, M.J.; and Arnold, S.M. (2003): Higher-Order Theory for Periodic Multiphase Materials With Inelastic Phases. Int. J. Plast., vol. 19, pp. 805–847.

CARES: *Ceramics Analysis and Reliability Evaluation of Structures*

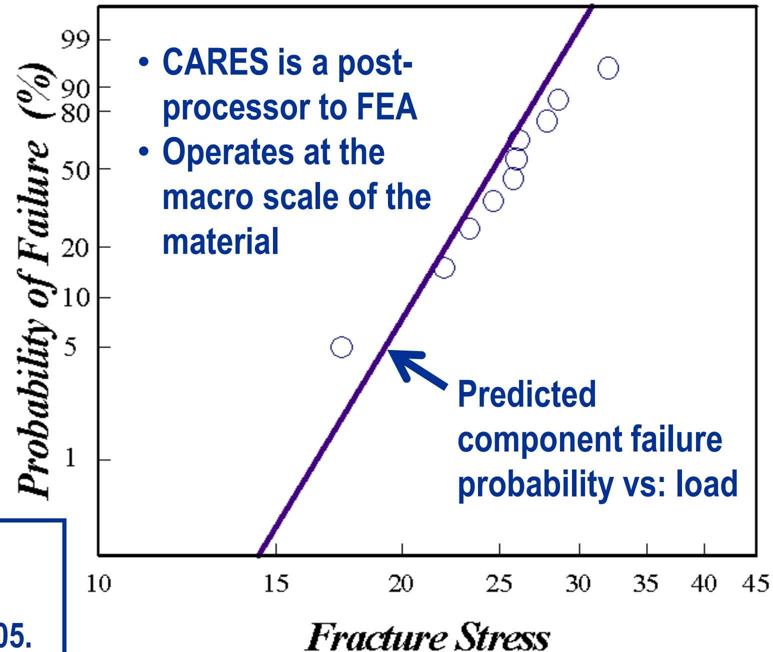
Life Prediction & Component Design Code For Advanced Ceramics

- Developed to predict the probability of failure of ceramic components under complex thermomechanical loading
- Combines Weibull & Weakest Link theory with concepts of linear elastic fracture mechanics (the ***Batdorf Unit Sphere*** model)

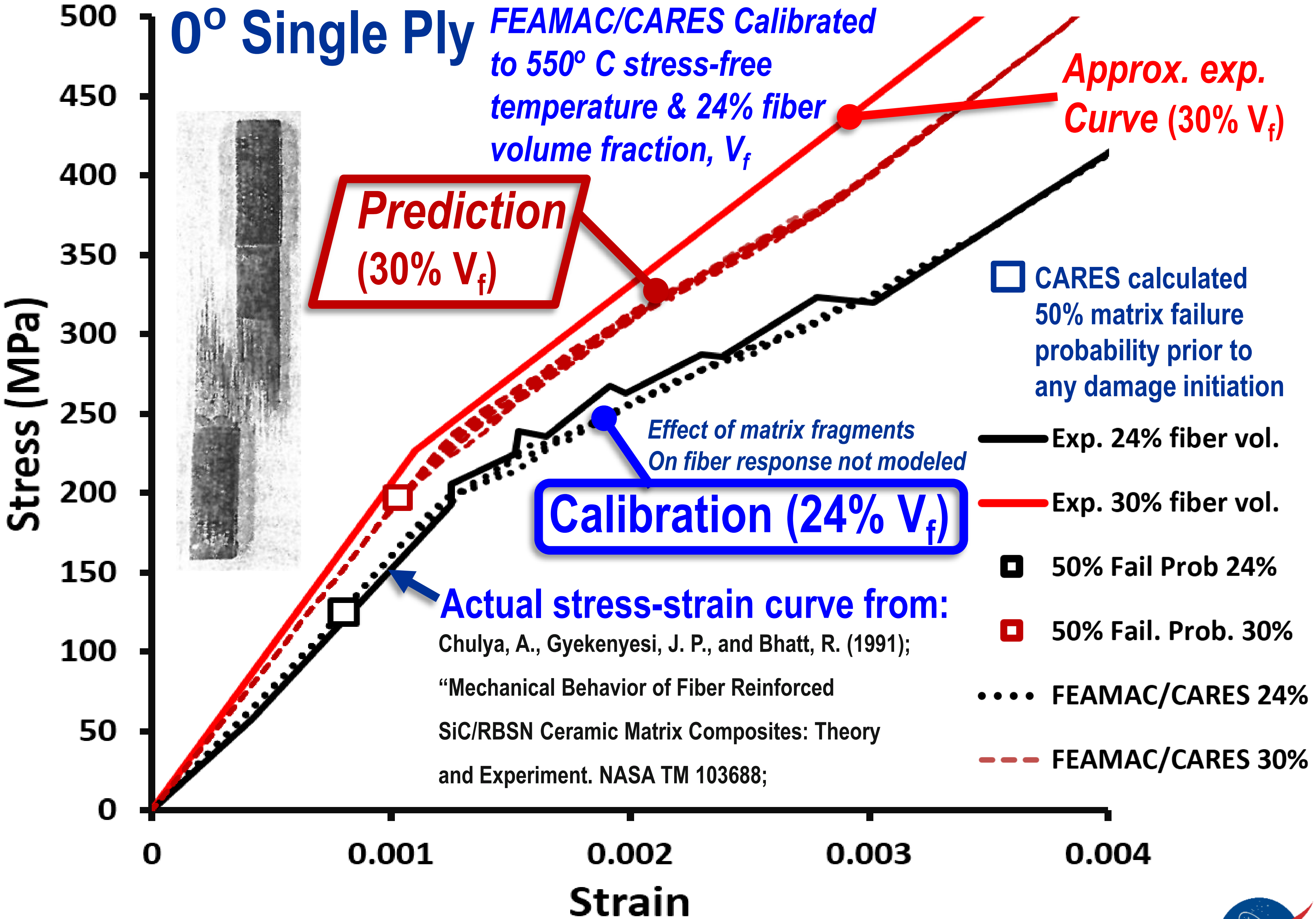


Component Reliability Analysis Capability:

- Transient loads and temperatures
- Fast-Fracture Rupture
- Time-dependent (da/dt) crack growth
- Cycle-dependent (da/dn) crack growth
- Multiaxial stress failure models
  *(PIA & **Unit Sphere** & Tsai-Wu & Tsai-Hill)*
- Proof test

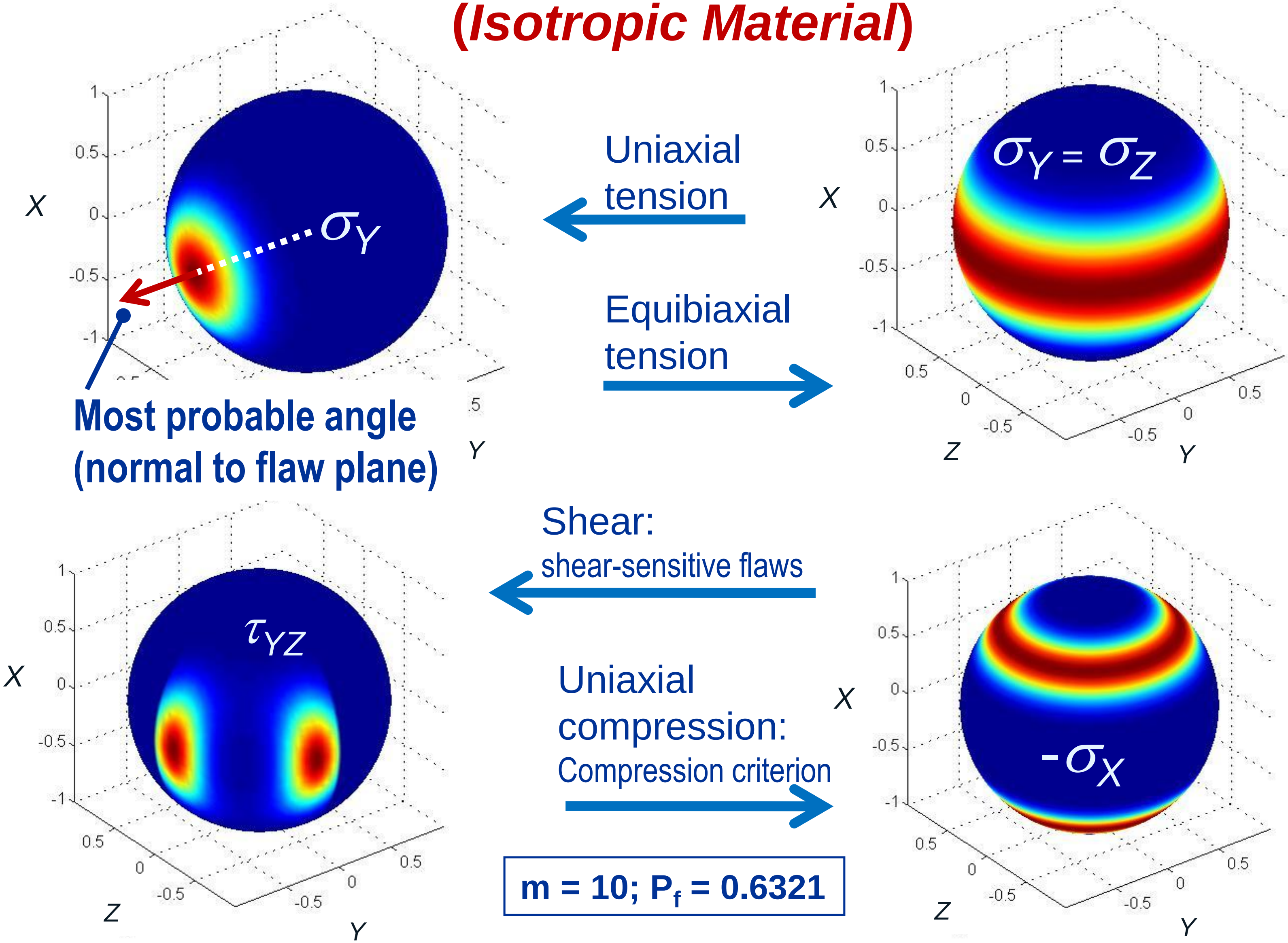


Nemeth, Jadaan, Gyekenyesi.: "Lifetime Reliability Prediction of Ceramic Structures Under Transient Thermomechanical Loads." NASA/TP-2005-212505, 2005.


0º Single Ply
FEAMAC/CARES Calibrated
to 550º C stress-free
temperature & 24% fiber
volume fraction, Vf
Approx. exp.
Curve (30% Vf)
Prediction
(30% Vf)
CARES calculated
50% matrix failure
probability prior to
any damage initiation
Effect of matrix fragments
On fiber response not modeled
Calibration (24% Vf)
Exp. 24% fiber vol.
Exp. 30% fiber vol.
50% Fail Prob 24%
50% Fail. Prob. 30%
FEAMAC/CARES 24%
FEAMAC/CARES 30%
Actual stress-strain curve from:
Chulya, A., Gyekenyesi, J. P., and Bhatt, R. (1991);
"Mechanical Behavior of Fiber Reinforced
SiC/RBSN Ceramic Matrix Composites: Theory
and Experiment. NASA TM 103688;
Stress (MPa)
500
450
400
350
300
250
200
150
100
50
0
0
0.001
0.002
0.003
0.004
Strain

Probability Density Distribution For Orientation Of Critical Flaws

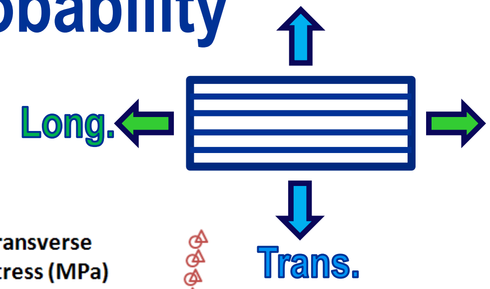
(*Isotropic Material*)

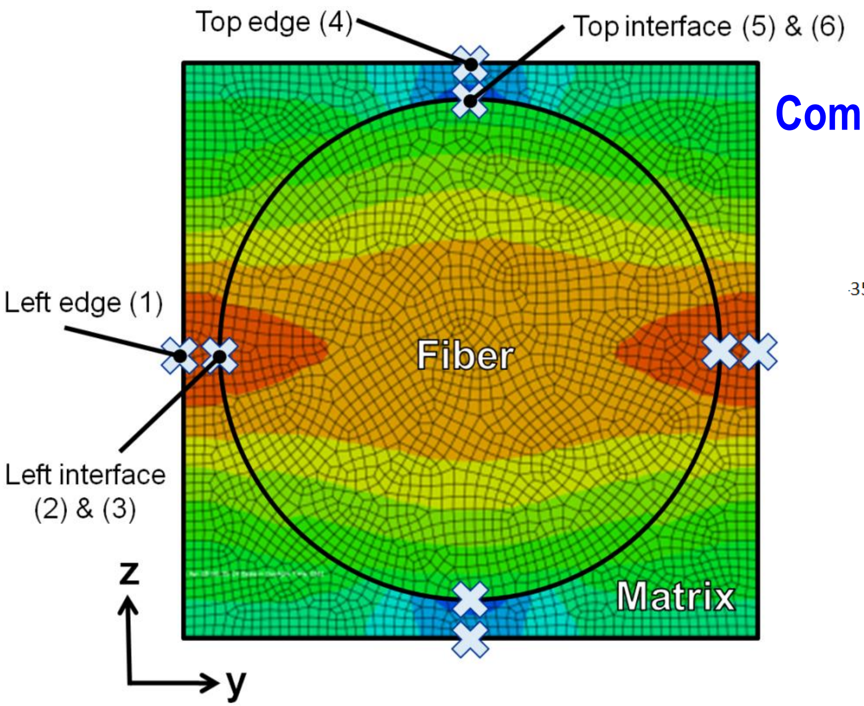


❖ **Will provide direction of damage initiation in MAC**

Nemeth, N. N.: "Probability Density Distribution of the Orientation of Strength-Controlling Flaws From Multiaxial Loading Using the Unit-Sphere Stochastic Strength Model for Anisotropy." International Journal of Fracture, Vol. 185, Issue 1-2, pp. 97-114, January, 2014

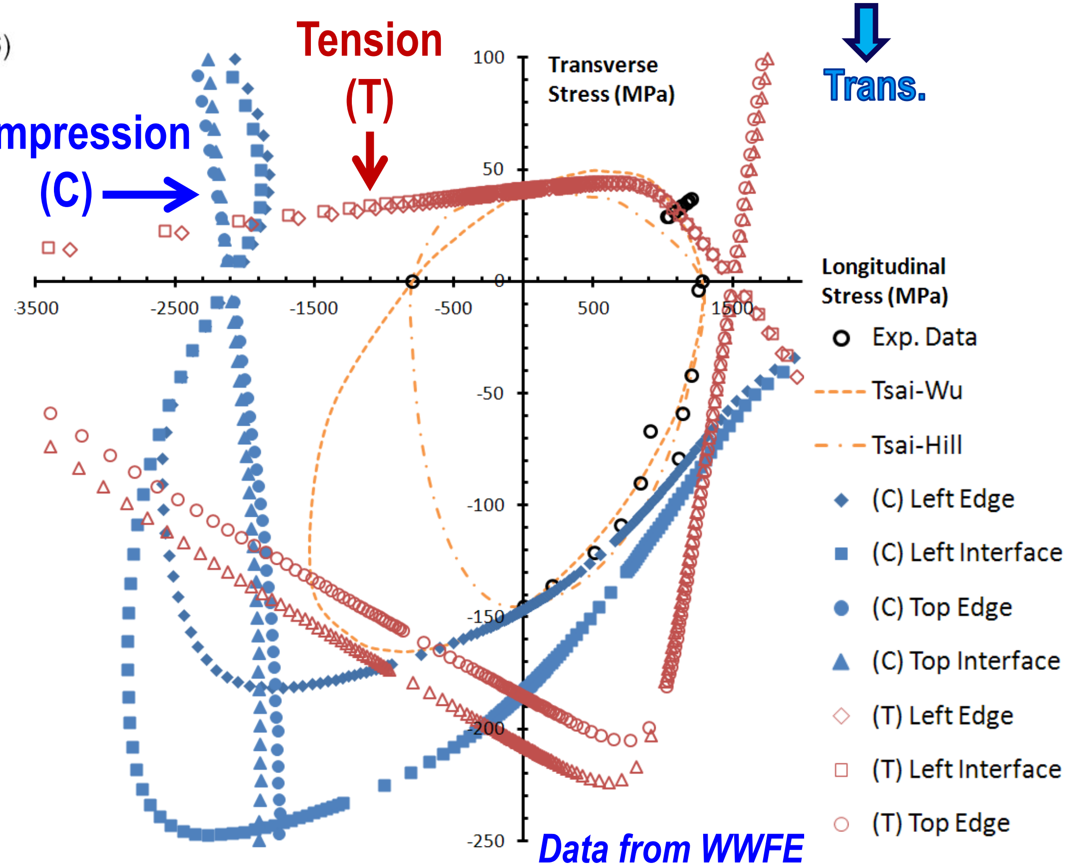
Example of a biaxial failure envelope at 50% probability of failure (P_f) for a PMC unit cell





FE model of a fiber-in-matrix unit cell

(Sampled points indicated with ✖)



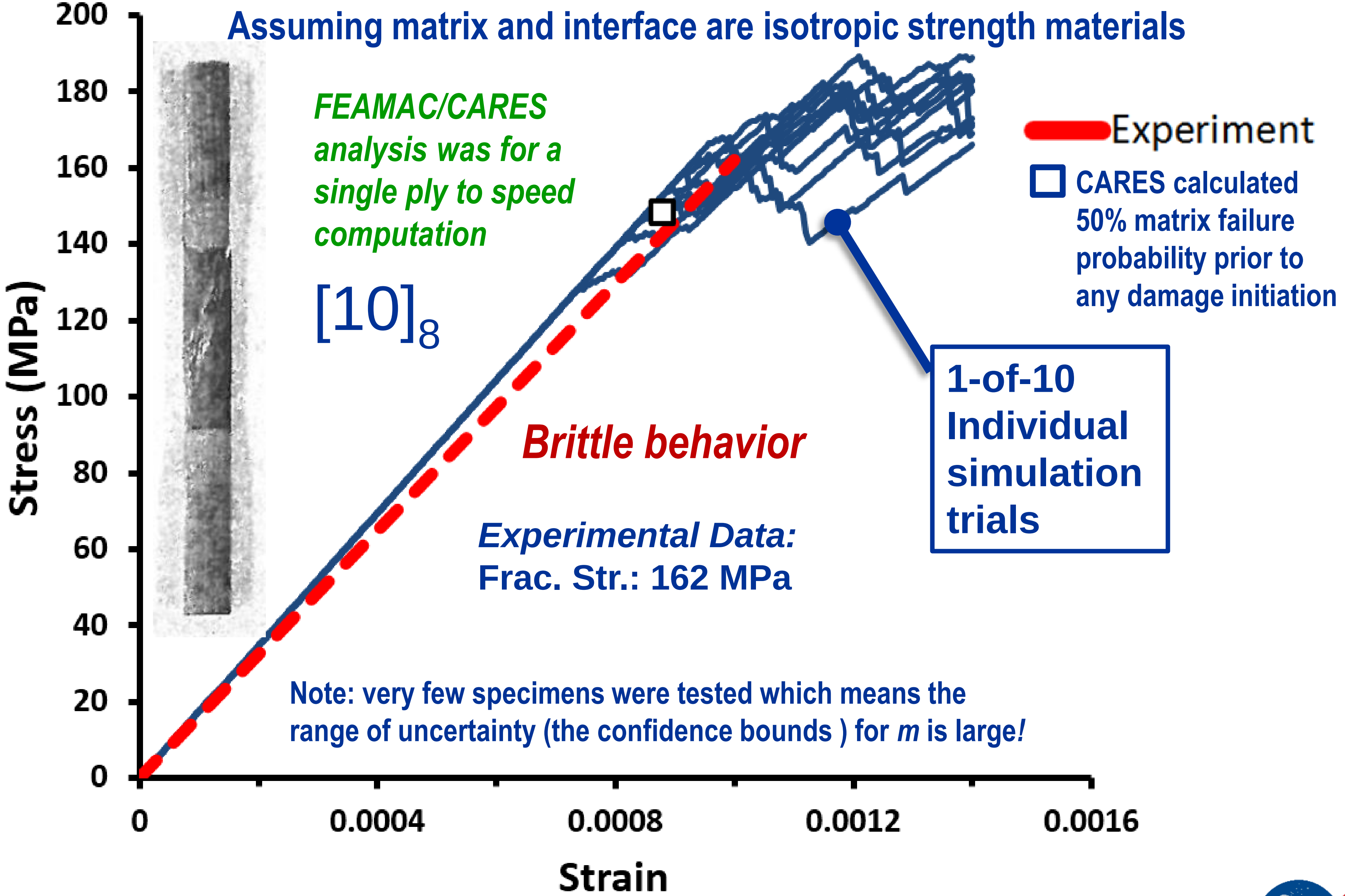
Tracking individual failure modes at sampled points for the matrix

(What mode is critical, where, and when)

Nemeth, N. N.: "Unit-Sphere Multiaxial Stochastic-Strength Model Applied to a Composite Material." Journal of Composite Materials Vol. 48(27), pp. 3395-3424, November 2014.

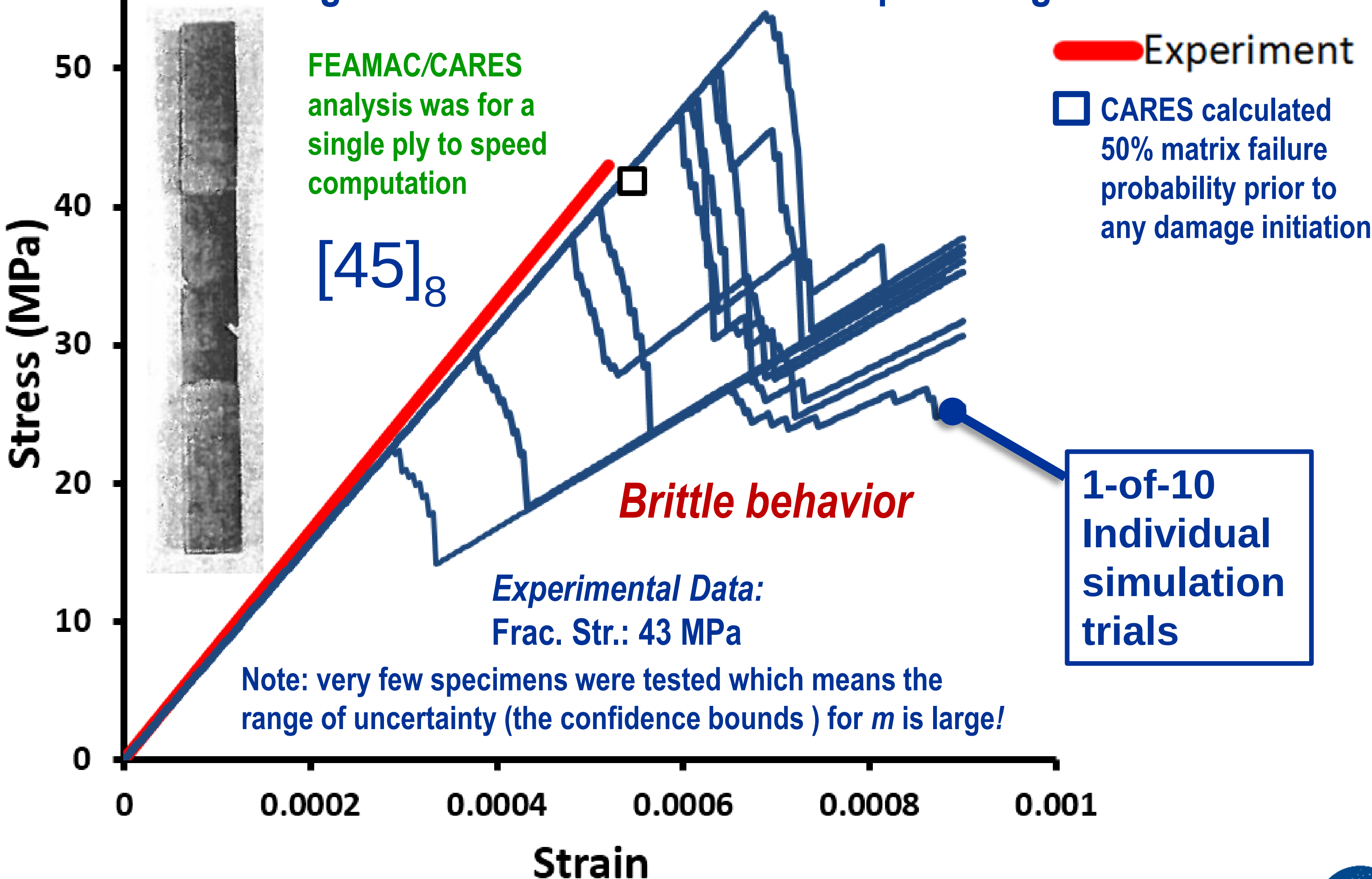
Prediction for Ten Trials for 10° Fiber Orientation

Assuming matrix and interface are isotropic strength materials



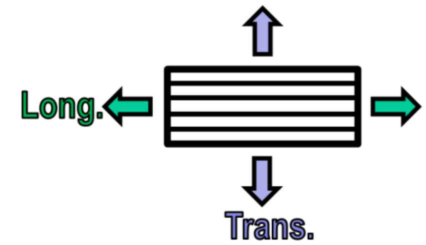
Prediction for Ten Trials for 45° Fiber Orientation

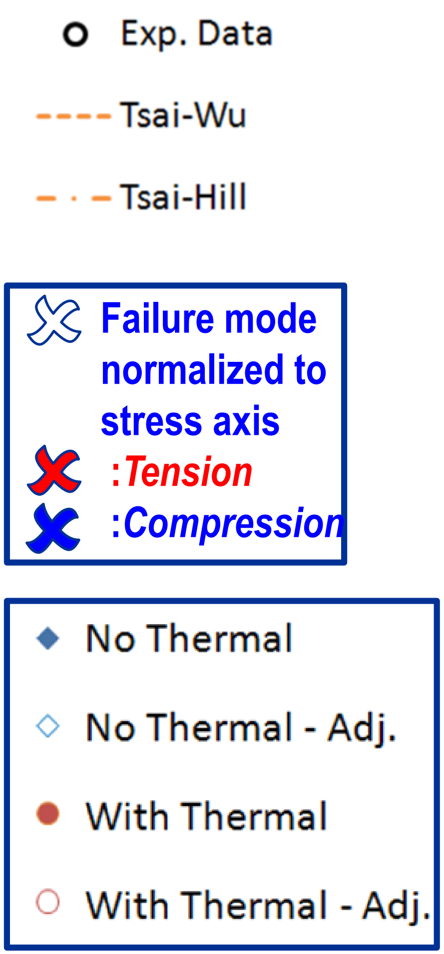
Assuming matrix and interface are isotropic strength materials

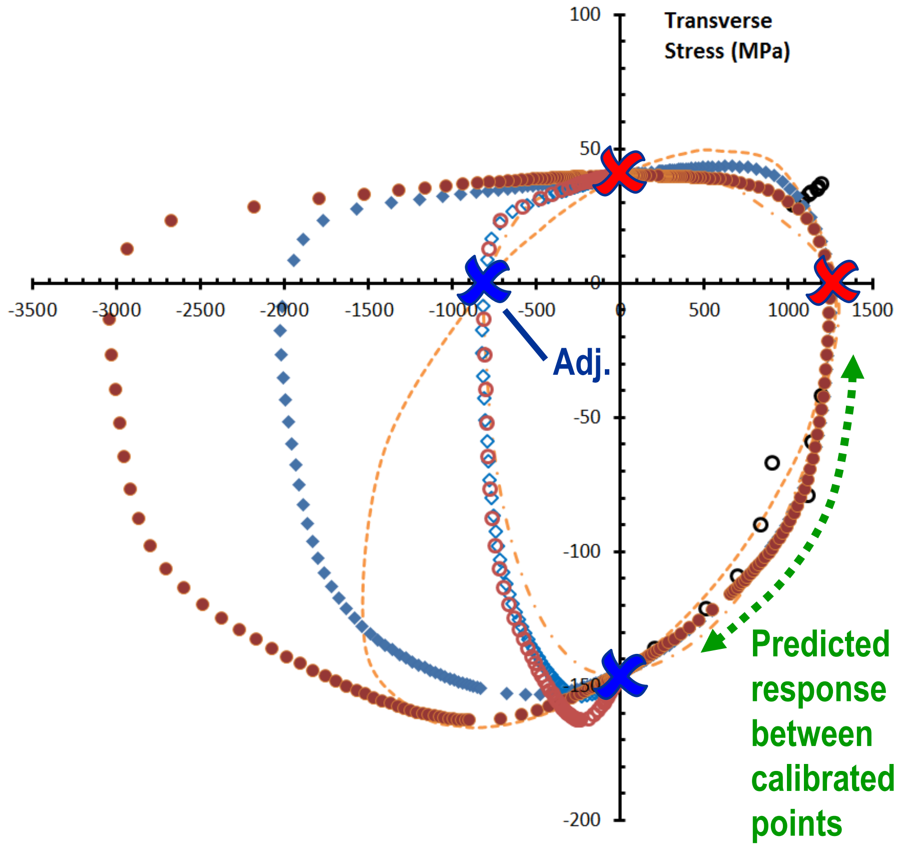


Failure envelope at 50% P_f of the unit cell combining all sampled points and failure modes

(with and without thermal residual stresses)







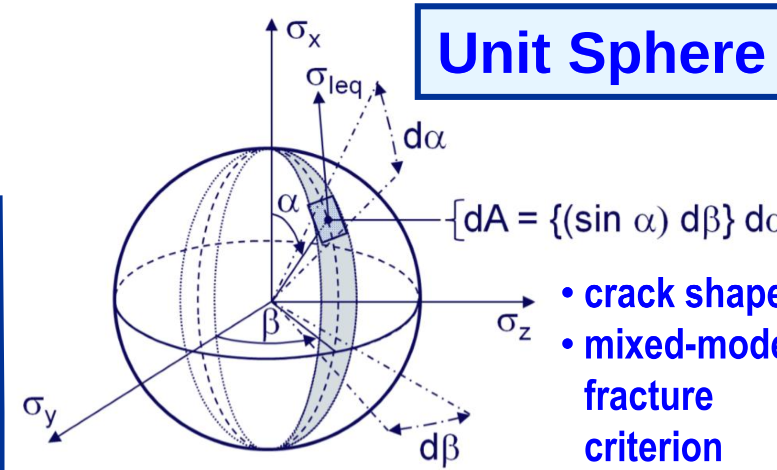
Anisotropic unit sphere model compared with Tsai-Wu & Tsai-Hill

Experimental data from World Wide Failure Exercise (WWFE)

Tsai/Wu/Hill curves are computed based on macro stresses

Nemeth, N. N.: "Unit-Sphere Multiaxial Stochastic-Strength Model Applied to a Composite Material." Journal of Composite Materials Vol. 48(27), pp. 3395-3424, November 2014.

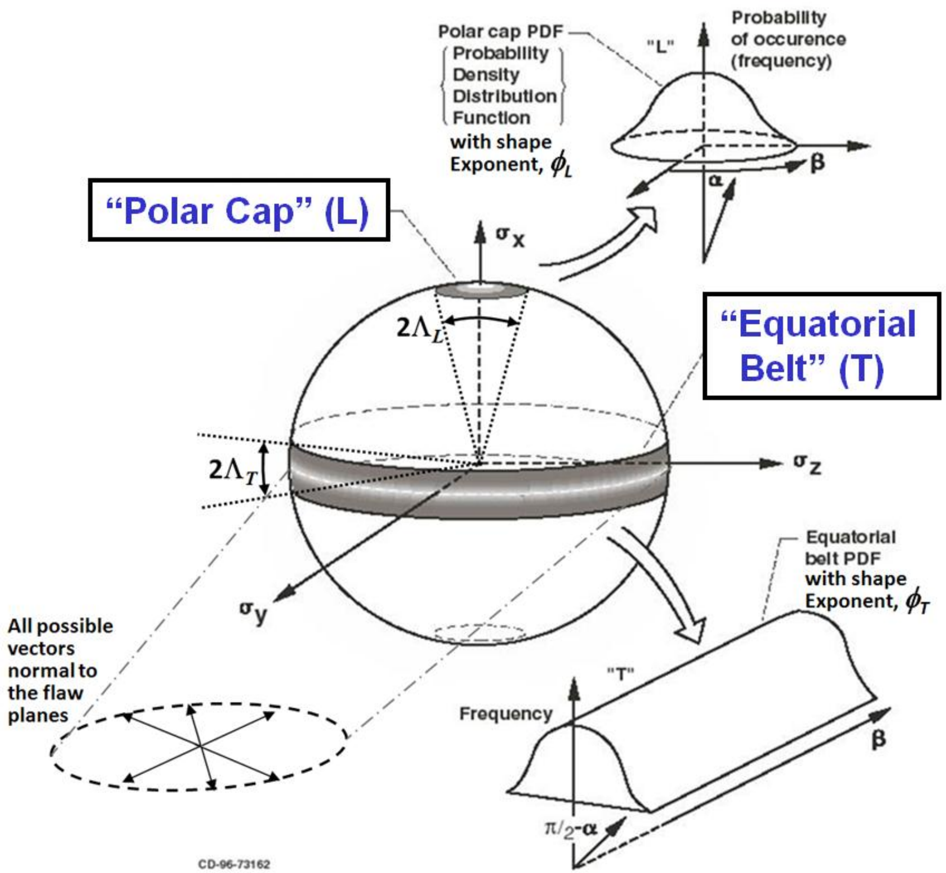
Unit Sphere Stochastic-Strength Multiaxial Failure Criterion model



- crack shape
- mixed-mode fracture criterion

➤ Two models for transverse isotropy

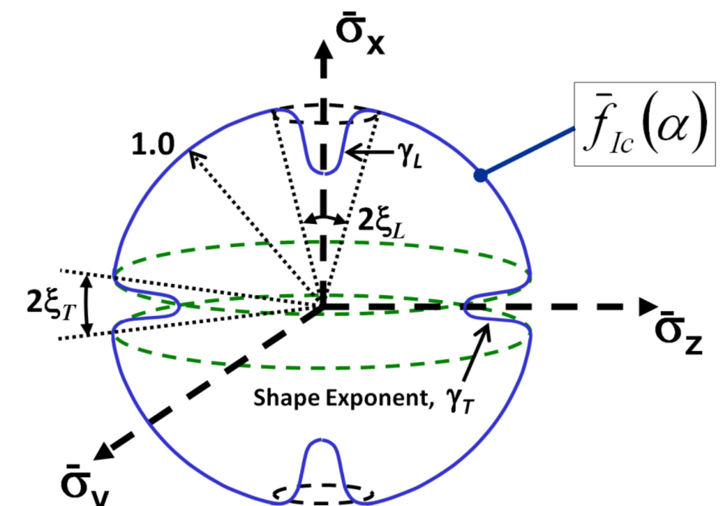
- **Flaw / Fracture-Plane Orientation Anisotropy**



Failure probability ➔ surface area of a unit radius sphere (all possible flaw orientations), where equivalent mode I stress (σ_{Ieq}) exceeds critical mode I strength (σ_{Ic}), divided by the total surface area of the unit radius sphere

- **Strength Orientation Anisotropy**

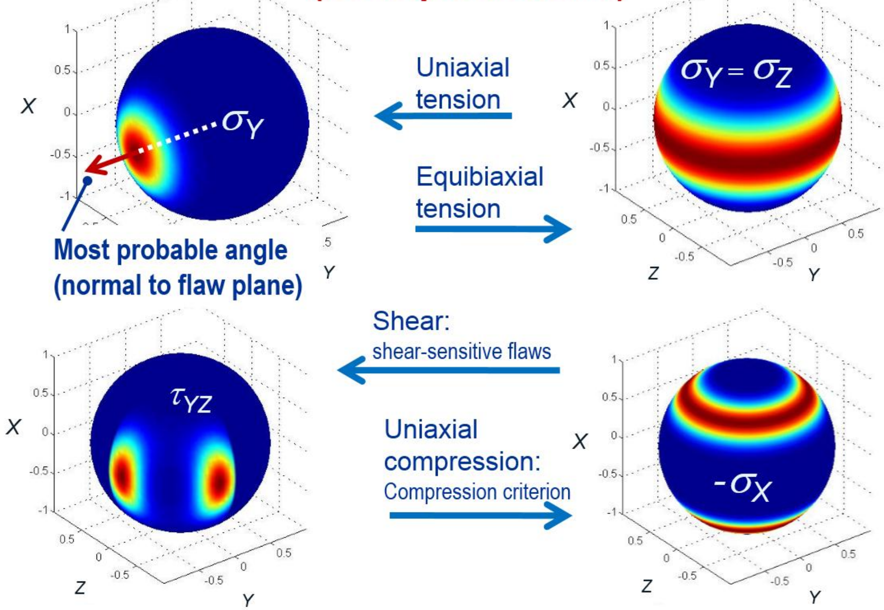
σ_{Ic} or K_{Ic} varies with orientation



Nemeth, N. N.: "Unit-Sphere Multiaxial Stochastic-Strength Model Applied to a Composite Material." Journal of Composite Materials Vol. 48(27), pp. 3395-3424, November 2014.

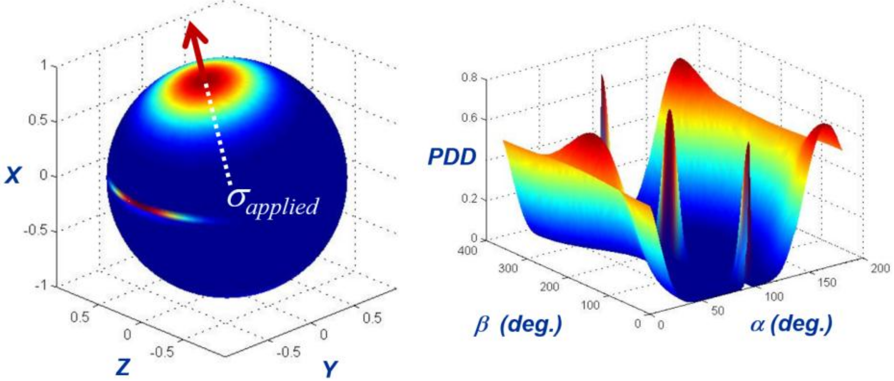
Unit Sphere Probability Density Distribution For Orientation Of Critical Flaws

(Isotropic Material)

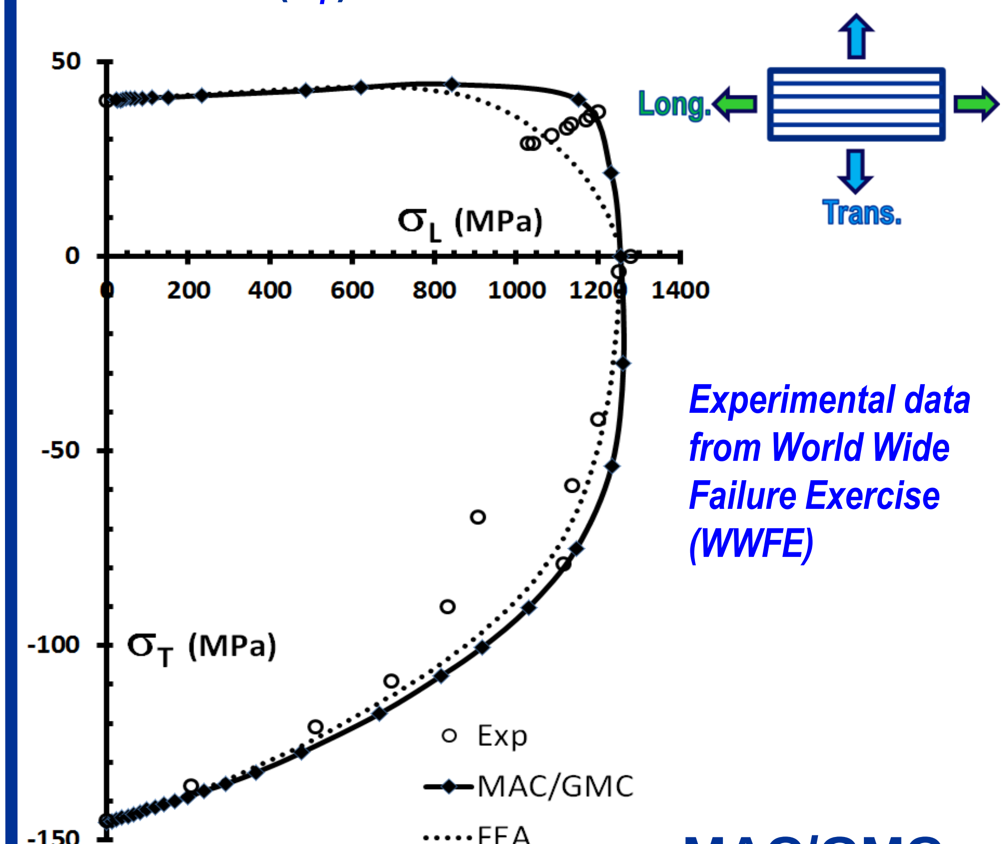


(Transversely isotropic (K_{Ic}) Material)

For a 15° offset uniaxial load

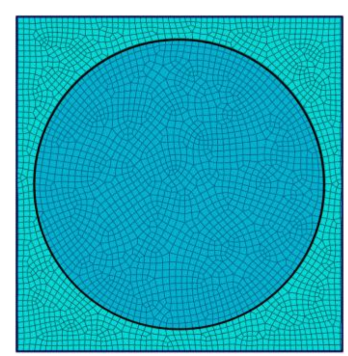


Biaxial Failure Envelope At 50% Probability Of Failure (P_f) For PMC Unit Cell



Experimental data from World Wide Failure Exercise (WWFE)

FEA



MAC/GMC