Stochastic-Strength-Based Damage Simulation Tool for Ceramic Matrix Composites

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Ceramic Matrix Composites (CMCs)

- Ceramic matrix composites (CMC) are being developed for hot section of advanced turbine engines (2700° F) and other uses.

- Specific mechanisms governing response of ceramic matrix composites need to account for in analysis approach:
  - Brittle material response
  - Weak, compliant interface
  - Residual stresses present due to processing

- Robust, efficient analysis tools required to analyze deformation, failure and life of these materials.

→ CMCs are designed to have “graceful failure” (non-linear stress-strain response) as opposed to brittle failure
Scope, Technical Challenge & Approach

- Predict the strength and service life of ceramic & composite structures

  CMC - Ceramic Matrix Composites &  PMC - Polymer Matrix Composites

- Need to account for:
  - Wide variability in the strength of individual components (probabilistic/stochastic strength)
  - How damage response changes with loading types (multiaxial loading, flexural loading, size effect)
  - How composite architecture effects strength/damage response
  - How strength degrades with time and fluctuating load

- Approach ➔ Combine two NASA developed codes:
  (MAC/GMC) : composite micromechanics analysis &
  (CARES/Life) : probability of failure prediction of ceramic components with commercial finite element analysis (Abaqus)
Outline

1. Overview: Describe the MAC and CARES codes
   - **MAC/GMC**: composite *micromechanics* model
   - **CARES Unit Sphere**: multiaxial *stochastic strength* model
     *(isotropy & anisotropy)*

2. Applying CARES to the MAC code to simulate stochastic damage progression in a ceramic matrix composite (CMC)
   - **Cellular Automaton**: Encouraging failure of adjacent elements - mimics crack-like growth
   - Visualization of element-by-element failure propagation for fiber, matrix, and interface

3. Stress-strain response of a SiC-RBSN laminate (literature circa 1990)
   - Off-axis loading
   - Notched specimen

4. Time-dependent slow crack growth notional example
MAC/GMC Micromechanics Analysis Code

- **FEAMAC**: MAC/GMC embedded in FEA as constitutive material
- **CARES/Life**: Life Prediction Code For Advanced Ceramics
  - Predicts the probability of failure of ceramic components under thermomechanical loading
  - Combines Weibull & Weakest Link theory with concepts of linear elastic fracture mechanics (the Batdorf Unit Sphere model)
  - CARES is a post-processor to FEA

Component Reliability Analysis Capability:
- Transient loads and temperatures
- Fast-Fracture Rupture
- Time-dependent \((da/dt)\) crack growth
- Cycle-dependent \((da/dn)\) crack growth
- Multiaxial stress failure models (PIA & Unit Sphere & Tsai-Wu & Tsai-Hill)
- Proof test

Repeating Unit Cell (RUC) of composite material
- RUC made of material subcells
- Multiscale capability

RUC

Material 1

Material 2

Subcell
Approach for Life Prediction & Component Design of Composites

- Combine CARES, MAC & FEA codes

Move CARES from the macroscopic scale of the structure to the microscale of the individual RUC material constituents

- FEAMAC/CARES Capability:
  - Individual constituent and component level probability of failure tracked (for failure initiation)
  - Individual & concurrent failure modes
  - Laminate level analysis capability
  - Progressive damage capability/simulation
  - Subcells killed at random failure thresholds

- Debonding/crack path physics at constituent level not explicitly included

FEAMAC/CARES Capability:

- Abaqus UMAT “User Material”
- Structural-Scale FEA
- Element/Integration Point
- Micromechanics Analysis

(MAC/GMC) Subroutine

Reliability analysis at the RUC level
Fracture-mechanics-based failure criteria to predict probability of failure/damage of a structure over time.

Prediction of multiaxial strength:
Unidirectional E-Glass/Epoxy PMC under biaxial loading 50% $P_f$ (fail. prob.) envelope

Unit Sphere

- crack shape
- mixed-mode fracture criterion

Two models for anisotropic strength response:
- $K_{lc}$ / Critical strength
- Flaw orientation bias

(Transversely isotropic ($K_{lc}$) Material)

For a 15$^\circ$ offset uniaxial load

Unit Sphere Probability Density Distribution For Orientation Of Critical Flaws

Experimental data from World Wide Failure Exercise (WWFE)

Anisotropic Unit Sphere model for $K_{lc}$

Failure criterion at the unit-cell level

Using microstresses of fiber & matrix means more accurate accounting of failure modes
Progressive Damage Criterion

Calculate failure probability, $P_f$, for each material constituent of the RUC associated with an element integration point.

- CARES calculated $P_f\text{(CARES)}$ of RUC
- Random number generated $P_f\text{(Random)}$ of RUC

If $P_f\text{(CARES)} \geq P_f\text{(Random)}$:
- Encourages more rapid damage propagation than failing individual subcells
  - Fail all material constituent subcells
    - Kill elastic modulus
- Yes

If $P_f\text{(CARES)} < P_f\text{(Random)}$:
- Don’t fail subcells
- No
Random Element Failure vs: Neighbor Influenced Failure  
(Cellular Automaton Enhancement) 

Encourage more abrupt failure and “crack-like” damage growth patterns 

A cellular automaton is a collection of "colored" cells on a grid that evolves through discrete time steps according to a set of rules based on the states of neighboring cells. 

Rule: When failure of an element is encountered, the random failure threshold of the neighboring elements are adjusted to that of the failed element. \textit{Load state determines which elements have highest probability of failure} 

Failed element 

Adjacent element 

Adjacent 2 elements with highest \( P_f \) (CARES) has \( P_f \) (Random) adjusted 

Random element failure 

Example: 0° Ply uniaxial ramp load 25x25 FEA mesh 

Cellular automaton 

Adjusted element \( P_f \) (Random) more likely to be lower than original \( P_f \) (Random) and fail sooner as load increases – enhancing damage propagation
0° single ply tensile specimen
Progression of damage in FE model of a unidirectional ply under longitudinal loading

(a) and (d); early matrix damage
(b) and (e); progression to substantial matrix damage
(c) and (f); final composite failure (fiber failure)

Matrix failure
Adjacent to failed matrix
Fiber failure
Adjacent to failed fiber
No failure
Example: SiC/RBSN Laminated Composite in *On-Axis* & *Off-Axis* Loading

- Tested by Bhatt & Phillips (1990)

- Displays key mechanisms/features for model material

  - SCS-6 fiber/Reaction Bonded Silicon Nitride matrix composite examined in detail by NASA ➔ several papers published
  
  - Laminated CMCs of interest to industry and less complex than woven composites ➔ failure modes are not conflicted with complex fiber architecture

  - [0] & [0/90] laminates display nonlinearity due to matrix failure, followed by fiber failure.

  - Remaining ply orientations display sudden brittle failure.

SiC/RBSN
Bhatt & Phillips
(1990)

Experimental Results

[ Ply Angle ]

Full details of stress-strain response not available

Rectangular specimens under uniaxial tensile loading
25.4 mm x 12.7 mm x 1.2 mm
30 % fiber volume fraction

Stress (Mpa)
Strain, %

Unreinforced RBSN
[45]₈
[90]₈

[0]₈

[10]₈

[0₂/90₂]₈

[+45₂/-45₂]₈

[0]₈

[90]₈

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SiC/RBSN Example Procedure & Setup

Abaqus FEA
S4 Shell elements
Fixed-displacement ramp load

Stochastic strength analysis:
(from individual trials / simulations / realizations)

- Use CARES Unit Sphere failure criterion
  - assume isotropic material constituent strength
    – for simplicity and initial testing

  1) Cool down from stress-free temperature of 550° to room temperature 23°
  2) apply fixed-displacement ramp load

Residual stresses in constituents

MAC/GMC RUC

- Weibull parameters correlated to experimental results for 0° tensile specimen
- Interface strength made large:
  - Encourage matrix to fail before interface

- Interfacial failure modes and sliding resistance not considered
Effect of matrix Weibull modulus on stress-strain response

0° single ply tensile specimen

Calibrating to experimental data

Matrix damage

Stress (MPa)

Strain

$E_c$

$V_fE_f$

$m_V$

5

10

20

60

RUC

Matrix damage

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0° Single Ply $[0]_8$

Non-linear (graceful) failure behavior

Calibrated to experimental data for 24% fiber volume fraction

Calibrated to 550°C stress-free temperature

Experimental Data:
Matrix Frac. Str.: 227 ± 41 MPa
Ultimate Str: 682 ± 150 MPa

Predicted response for 30% fiber volume fraction

Individual simulation

Fiber $m_v = 20 \ \sigma_{ov} = 2875$ Mpa $m^{3/20}$
Matrix $m_v = 5.0 \ \sigma_{ov} = 150$ Mpa $m^{3/5}$
Interface $m_v = 5.0 \ \sigma_{ov} = 80$ Mpa $m^{3/5}$
**0° Single Ply**

*Strength scatter from proportional limit strain offset*

**Stress (Mpa)**

- 0.005% strain offset
- 0.01% strain offset

**Linear elastic**

Intersection of simulation trial with strain offset

**1-of-10 Individual simulation trials**

**FEAMAC/CARES:**
- Mean = 238.2 MPa
- Std. Dev. = 10.28 MPa

**Experimental Data:**
- Matrix Frac. Str.: 227 ± 41 Mpa

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*PLS is defined as the stress at 0.005% strain offset:*

Prediction for Ten Trials for 90° Fiber Orientation
Assuming matrix and interface are isotropic strength materials

FEAMAC/CARES analysis was for a single ply to speed computation

\( [90]_8 \)

Experimental Data:
Frac. Str.: 27 \( \pm 3 \) MPa

Brittle behavior

Note: very few specimens were tested which means the range of uncertainty (the confidence bounds) for \( m \) is large!
Predictions for Ten Trials for 10° and 45° Fiber Orientations

Assuming matrix and interface are isotropic strength materials

FEAMAC/CARES analysis was for a single ply to speed computation

**10° Fiber Orientation**

![Stress-Strain Curve for 10° Fiber Orientation]

**45° Fiber Orientation**

![Stress-Strain Curve for 45° Fiber Orientation]

**Experimental Data:**

- [10]₈: Frac. Str.: 162 MPa
- [45]₈: Frac. Str.: 43 MPa

CARES calculated 50% matrix failure probability prior to any damage initiation
Prediction for Ten Trials for $[+45_2/-45_2]_S$ Fiber Orientation
Assuming matrix and interface are isotropic strength materials

FEAMAC/CARES analysis was for four plys (+45/-45/-45/+45) to speed computation

$[+45/-45]_S$

Matrix cracks approx. normal to loading direction

Note: very few specimens were tested which means the range of uncertainty (the confidence bounds) for $m$ is large!

Experimental Data:
Matrix Frac. Str.: 75 ±10 MPa; $m \approx 8.8$
Ultimate Str: 88 ±16 MPa; $m \approx 6.3$

Neither graceful or brittle behavior

1-of-10 Individual simulation trials
For [+45/-45]_s Fiber Orientation; 10 trials at final (matrix) failure; deformed plots

FEAMAC/CARES analysis was for four plies (+45/-45/-45/+45) to speed computation
Prediction for Ten Trials for $[0/90]_s$ Fiber Orientation

Assuming matrix and interface are isotropic strength materials

FEAMAC/CARES analysis was for four plys (0/90/90/0) to speed computation $[0/90]_s$

**Experimental Data:**
Matrix Frac. Str.: $127 \pm 26$ MPa
Ultimate Str: $294 \pm 87$ MPa

Note: very few specimens were tested which means the range of uncertainty (the confidence bounds) for $m$ is large!

CARES calculated 50% matrix failure probability prior to any damage initiation

Non-linear (graceful) failure behavior

1-of-10 Individual simulation trials
0° Double-Notched Tensile Specimen

Failure mode showed axial splitting of matrix

FEAMAC/CARES analysis was for a single ply to speed computation

Experiment

[0]₈

Axial splitting

Fiber Direction

Loading Direction

LONGITUDINAL CRACKS
**Time-dependent Failure Example: Static Loading**

(Matrix Damage Accumulation From Slow Crack Growth)

**Strain response for applied static tensile load over time**

**Applied Static Load (MPa)**

- 170
- 180
- 190
- 200

Damage increases with time and load

**Effect of N**

**Slow Crack Growth**

**Power Law:**

\[
\frac{da}{dt} = AK^{N_{\text{eq}}}
\]

**Weibull Parameters**

- \(m = 7\) (Weibull slope)
- \(\sigma_0 = 106\) Mpa \(\cdot\) mm\(^{3/7}\)

**Fatigue Parameters**

- \(N = 20\) (fatigue slope)
- \(B = 1.0E9\) MPa\(^2\) \(\cdot\) sec

**Service life prediction**

Longitudinal stress applied to a \(0^\circ\) SiC/RBSN ply

10 time increments per time magnitude

**Note:** Parameter “B” is related to parameter “A”

**CARES 50%**

- 170
- 180
- 190

**P\text{f} for matrix failure**

- 170
- 180
- 190
Conclusions

• Progressive damage simulation of composite structures incorporating probabilistic material strength models is possible with the FEAMAC/CARES code

• The Unit Sphere multiaxial model was used to predict the strength response of a SiC-RBSN composite for various fiber orientations under uniaxial tension

• Reasonable correlation to matrix cracking strength experimental data was achieved assuming the matrix was an isotropic material with $m \approx 5$, and assuming residual stresses from thermal processing were present

• Brittle behavior vs: non-brittle failure (*graceful failure*) demonstrated

• Localized damage modes at stress concentration features shown

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Extra Material
Abstract:
Reported here is a coupling of two NASA developed codes: CARES (Ceramics Analysis and Reliability Evaluation of Structures) with the MAC/GMC (Micromechanics Analysis Code/ Generalized Method of Cells) composite material analysis code. The resulting code is called FEAMAC/CARES and is constructed as an Abaqus finite element analysis UMAT (user defined material). Here we describe the FEAMAC/CARES code and an example problem (taken from the open literature) of a laminated CMC in off-axis loading is shown. FEAMAC/CARES performs stochastic-strength-based damage simulation response of a CMC under multiaxial loading using elastic stiffness reduction of the failed elements.
Constituent properties of SiC/RBSN with anisotropic thermal expansion coefficients

<table>
<thead>
<tr>
<th>Constituent</th>
<th>Modulus, GPa</th>
<th>Poisson ratio</th>
<th>Longitudinal coefficient of thermal expansion, $\alpha_L$ (m/m/°C)</th>
<th>Transverse coefficient of thermal expansion, $\alpha_T$ (m/m/°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fiber</td>
<td>390</td>
<td>0.17</td>
<td>4.1×10^{-6}</td>
<td>1.84×10^{-6}</td>
</tr>
<tr>
<td>Matrix</td>
<td>110</td>
<td>0.22</td>
<td>2.2×10^{-6}</td>
<td>2.2×10^{-6}</td>
</tr>
<tr>
<td>Interface</td>
<td>1.8</td>
<td>0.22</td>
<td>2.0×10^{-6}</td>
<td>2.0×10^{-6}</td>
</tr>
</tbody>
</table>

**Assumed Weibull Parameters:**

- **Fiber**
  - $m_V = 20$
  - $\sigma_{oV} = 2875$ Mpa $\cdot m^{3/20}$
- **Matrix**
  - $m_V = 5.0$
  - $\sigma_{oV} = 150$ Mpa $\cdot m^{3/5}$
- **Interface**
  - $m_V = 5.0$
  - $\sigma_{oV} = 80$ Mpa $\cdot m^{3/5}$
$0^\circ$ Double-Notched vs: Central-Hole Tensile Specimen

Loading Direction

Early matrix damage

Matrix damage progression
**Power Law:** - Slow Crack Growth (SCG)

\[
\frac{da}{dt} = AK_{eq}^N
\]

**Combined Power Law & Walker Law:** SCG and Cyclic Fatigue

\[
\frac{da}{dt} = A_1 g K_{eq}^N + A_2 f_c (1 - R)^Q K_{eq}^N
\]
Modeling individual time steps in the life prediction methodology enables simulating transient events such as turbine start-up/shut-down or atmospheric re-entry. A computationally efficient methodology has been developed that can extrapolate the reliability calculation for an arbitrary number of $Z$ cycles – where each cycle is described by $k$ number of time steps. This conceivably allows the coupling of other effects such as stiffness degradation and oxidation effects on the individual time steps and this can be accounted for interactively within the transient finite element and micromechanics analysis.
Transient Life Prediction Theory - Power Law SCG

Reliability formula for $k$ discrete time steps over $Z$ cycles:

$$P_{SV}(t_k) = \exp\{-\frac{\sum_{i=1}^{n} V_i}{4\pi} \int_{\Omega} \left[ \cdots \left[ \frac{\sigma_{Ieq,k,T_{\text{max}}}}{\sigma_{0BV,k}} \right]^{N_{V,k}} \right. \right.$$

$$\left. + \frac{\sigma_{N_{V,k}, Z\Delta t_k}}{N_{V,k}^{2-2}} \frac{m_{V,k}(N_{V,k}-2)}{m_{V,k} N_{V,k}} \right]_{k \in \mathbb{N}} + \frac{\sigma_{N_{V,j}, Z\Delta t_j}}{N_{V,j}^{2-2}} \frac{m_{V,j}(N_{V,j}-2)}{m_{V,j} N_{V,j}} \right]_{j \in \mathbb{N}} + \cdots$$

Individually, each time step:

- Each time step can have different loading, Weibull, and fatigue parameters.
- Compatibility of failure probability is maintained between the individual time steps.
Unit Sphere Multiaxial (Batdorf) Model:
Puts linear elastic fracture mechanics into Weibull weakest-link theory

- **Incremental failure probability is the product of two probabilities**:

\[ \Delta P_f = P_1 \cdot P_2 \]

- **Component failure probability**:

\[ P_f = 1 - \exp \left\{ - \int_V \int_0^{\sigma_c} P_1(\sigma_c) P_2(\sigma_c) \, d\sigma_c \, dV \right\} \]

- **Mixed-Mode Fracture Criteria**:
  - Normal stress (shear-insensitive cracks)
  - Maximum tensile stress
  - Total coplanar strain energy release rate
  - Noncoplanar (Shetty)

- **Flaw Shapes**:
  - Griffith crack
  - Penny-shaped crack

\[ P_2 \text{ involves integration of an equivalent stress } \sigma_e, \]
where \( \sigma_e \geq \sigma_c \), over the surface of a unit radius sphere (all possible flaw orientations) divided by the total surface area of the unit radius sphere.

\[ \sigma_e \text{ is a function of an assumed crack shape and multiaxial fracture criterion} \]
Anisotropic Unit Sphere model defined in a material coordinate system reference frame

All possible vectors normal to the flaw plane of the fiber or tow or unidirectional lamina

Width of belt defines extent of misalignment

Similar to Puck’s composite failure criterion except in a probabilistic framework

When represented on a unit radius sphere of all possible flaw plane orientations the flaw plane normals define an equatorial belt distribution
**Multiaxial Performance:** biaxial response predicted from a MAC/GMC RUC for combined longitudinal (L) and transverse (T) loading on a unidirectional PMC vs: FEA. 50% probability of failure envelope.

**PMCs from WWFE**

Unit Sphere parameters adjusted so GMC results matched FEA results for uniaxial tension and compression. Intermediate points are predictions.

Differences in RUC stress fields for a transverse strain:

- **GMC RUC**
- **HF - GMC RUC**
- **FEA of RUC**
0° single ply tensile specimen (Load parallel to fiber axis)

Mesh effect & time step sensitivity

CARES calculated 50% matrix failure probability prior to any damage initiation

Trade off: Mesh sensitivity vs: localization of damage
Residual matrix stresses after cool-down from temperature

- Effect of anisotropic fiber-thermal-expansion-coefficient, $\alpha_f$ on RUC

"The fiber is orthotropic, with different coefficients of thermal expansion along and perpendicular to the fiber axis."

90° single ply tensile specimen

Progression of damage in FE model of a unidirectional ply under *transverse* loading

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**Not Adjusted**

(a) Early diffuse damage

(b) Final matrix failure

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**Automaton Adjusted**

(c) Adjacent elements encouraged to fail in early damage stages

(d) Final matrix failure

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Matrix failure

Adjacent to failed matrix

No failure

(a) and (c) ; early matrix damage

(b) and (d) ; final composite failure (matrix failure)
Damage progression of 0° tensile specimen - two trials (undeformed plot)
10° off-axis tensile specimen; 10 trials at final (matrix) failure; deformed plots

- Edges are allowed to freely deform (warp) on cool-down
- *After cool-down:* bottom edge fixed in loading direction when displacement load applied
- *After cool-down:* single node along top edge (middle) fixed in direction perpendicular to displacement direct.
Prediction for Ten Trials for $[0_2/90_2]_s$ Fiber Orientation

Strength scatter from proportional limit strain offset

0.005% strain offset

0.01% strain offset

Linear elastic

Intersection of simulation trial with strain offset

1-of-10 Individual simulation trials

Experimental Data:
Matrix Frac. Str.: 127 ± 26 MPa

FEAMAC/CARES:
Mean = 133.3 MPa
Std. Dev. = 8.11 MPa
90° Tensile specimen *at final failure for 10 trials* – Undeformed plots

Final specimen failure from matrix damage

Loading Direction

1. 1
2. 2
3. 3
4. 4
5. 5
6. 6
7. 7
8. 8
9. 9
10. 10

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$0^\circ$ Double-Notched vs: Central-Hole Tensile Specimen

Matrix damage progression

Loading Direction

Early matrix damage

Matrix damage progression
Biaxial failure envelope at 50% $P_f$ for a composite unit cell for all sampled points and failure modes
(with and without thermal residual stresses)

- Isotropic matrix not suitable/adjustable to predict longitudinal strength
- Anisotropy in unit sphere
- Compares Tsai-Wu & Tsai-Hill
- Validates approach taken

Failure mode normalized to stress axis:
- Tension
- Compression

Isotropic matrix
Anisotropic $K_{lc}$ matrix

Predicted response between calibrated points
Tsai/Wu/Hill curves come from the applied stresses on the composite
Probability density distribution for orientation of critical flaws

(Transversely isotropic ($K_{lc}$) Material)

For a $15^\circ$ offset uniaxial load

\[ P_f = 0.6321 \]

- Critical crack initiation angle could help with determination of anisotropic elastic damage constants
CARES: Life Prediction & Component Design Tools For Advanced Ceramics

Software (Ceramics Analysis and Reliability Evaluation of Structures)

Predicts the probability of failure of ceramic components under load

- Silicon nitride, silicon carbide, alumina
- Ultra high temperature ceramics
- MEMS materials – silicon; SiC
- Glass

Utilized worldwide for life prediction of brittle material components.

- Aerospace
- Automotive
- Electronic
- Energy
- Glass
- Medical
- Power

- NASA Software of the Year Award
- R&D 100 Award
- Federal Laboratory Consortium Technology Transfer Award
- American Ceramic Society Corporate Technical Achievement Award
- Enterprise Development, Inc. Innovation Award
- NASA Steven Szabo Engineering Excellence Award

Applications

MEMS
- Microturbine (a)
- Microrocket (b)
- Pressure sensor (c)

Biomedical
- Hip joint (d)
- Dental Bridge (e)
- MEMS implants (f)

Aerospace
- Turbine blade (g)
- Rocket Nozzle (h)
- Mars Aeroshell (i)

Fuel Cell (SOFC)
- Power generation (j)
- Propulsion (k)
Macromechanics links the size scales & provides the composite response based on the composite constituent materials.

**FEAMAC:** MAC/GMC embedded in FEA as constitutive material

**GMC (1990s)**
- 1st order displacement field in subcells
- Stresses and strains piecewise constant
- Number of linear algebraic equations function of number of subcells
- Local inelasticity/damage
- No shear coupling
- No "subcell mesh" sensitivity

**HFGMC (2000s)**
- 2nd order displacement field in subcells
- Elastic stresses and strains piecewise linear
- Number of linear algebraic equations is rather large
- Local inelasticity/damage
- Has shear coupling
- Has "subcell mesh" sensitivity

We currently only use GMC in FEAMAC/CARES

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CARES: Ceramics Analysis and Reliability Evaluation of Structures

Life Prediction & Component Design Code For Advanced Ceramics

• Developed to predict the probability of failure of ceramic components under complex thermomechanical loading

• Combines Weibull & Weakest Link theory with concepts of linear elastic fracture mechanics (the Batdorf Unit Sphere model)

Component Reliability Analysis Capability:

- Transient loads and temperatures
- Fast-Fracture Rupture
- Time-dependent (da/dt) crack growth
- Cycle-dependent (da/dn) crack growth
- Multiaxial stress failure models (PIA & Unit Sphere & Tsai-Wu & Tsai-Hill)
- Proof test


Structural Model

Element integration Point

(CARES) reliability analysis

- CARES is a post-processor to FEA
- Operates at the macro scale of the material

Predicted component failure probability vs: load

Fracture Stress
0° Single Ply

FEAMAC/CARES Calibrated to 550°C stress-free temperature & 24% fiber volume fraction, \( V_f \)

Approx. exp. Curve (30% \( V_f \))

Prediction (30% \( V_f \))

Calibration (24% \( V_f \))

Actual stress-strain curve from:

Effect of matrix fragments on fiber response not modeled

CARES calculated 50% matrix failure probability prior to any damage initiation
Probability Density Distribution For Orientation Of Critical Flaws

(Isotropic Material)

Uniaxial tension

Equibiaxial tension

Shear: shear-sensitive flaws

Uniaxial compression:
Compression criterion

Most probable angle (normal to flaw plane)

Will provide direction of damage initiation in MAC

Example of a biaxial failure envelope at 50% probability of failure ($P_f$) for a PMC unit cell

FE model of a fiber-in-matrix unit cell
(Sampled points indicated with ⬤)

Tracking individual failure modes at sampled points for the matrix
(What mode is critical, where, and when)

Prediction for Ten Trials for 10° Fiber Orientation
Assuming matrix and interface are isotropic strength materials

FEAMAC/CARES analysis was for a single ply to speed computation

[10]₈

Brittle behavior

Experimental Data:
Frac. Str.: 162 MPa

Note: very few specimens were tested which means the range of uncertainty (the confidence bounds) for \( m \) is large!

CARES calculated 50% matrix failure probability prior to any damage initiation

1-of-10 Individual simulation trials
Prediction for Ten Trials for 45° Fiber Orientation
Assuming matrix and interface are isotropic strength materials

FEAMAC/CARES analysis was for a single ply to speed computation

[45]_8

Brittle behavior

Experimental Data:
Frac. Str.: 43 MPa

Note: very few specimens were tested which means the range of uncertainty (the confidence bounds) for m is large!

1-of-10 Individual simulation trials
Failure envelope at 50% $P_f$ of the unit cell combining all sampled points and failure modes *(with and without thermal residual stresses)*

Anisotropic unit sphere model compared with Tsai-Wu & Tsai-Hill

Experimental data from World Wide Failure Exercise (WWFE)

Tsai/Wu/Hill curves are computed based on macro stresses

**Unit Sphere Stochastic-Strength Multiaxial Failure Criterion model**

- Two models for transverse isotropy

  - **Flaw / Fracture-Plane Orientation Anisotropy**

    - [Diagram: Polar Cap (L) and Equatorial Belt (T)]

    - *Failure probability* ➔ *surface area of a unit radius sphere (all possible flaw orientations)*, where equivalent mode I stress ($\sigma_{\text{eq}}$) exceeds critical mode I strength ($\sigma_{\text{lc}}$), divided by the total surface area of the unit radius sphere

- **Strength Orientation Anisotropy**

  - $\sigma_{\text{lc}}$ or $K_{\text{lc}}$ varies with orientation

  - [Diagram: Unit Sphere with crack shape and mixed-mode fracture criterion]

Unit Sphere Probability Density Distribution For Orientation Of Critical Flaws

(Isotropic Material)

Most probable angle (normal to flaw plane)

Uniaxial tension

Equibiaxial tension

Shear: shear-sensitive flaws

Uniaxial compression: Compression criterion

(Transversely isotropic ($K_{iso}$) Material)

For a 15° offset uniaxial load

Biaxial Failure Envelope At 50% Probability Of Failure ($P_f$) For PMC Unit Cell

Experimental data from World Wide Failure Exercise (WWFE)

FEA

MAC/GMC