On the VHF Source Retrieval Errors Associated with Lightning Mapping Arrays (LMAs)

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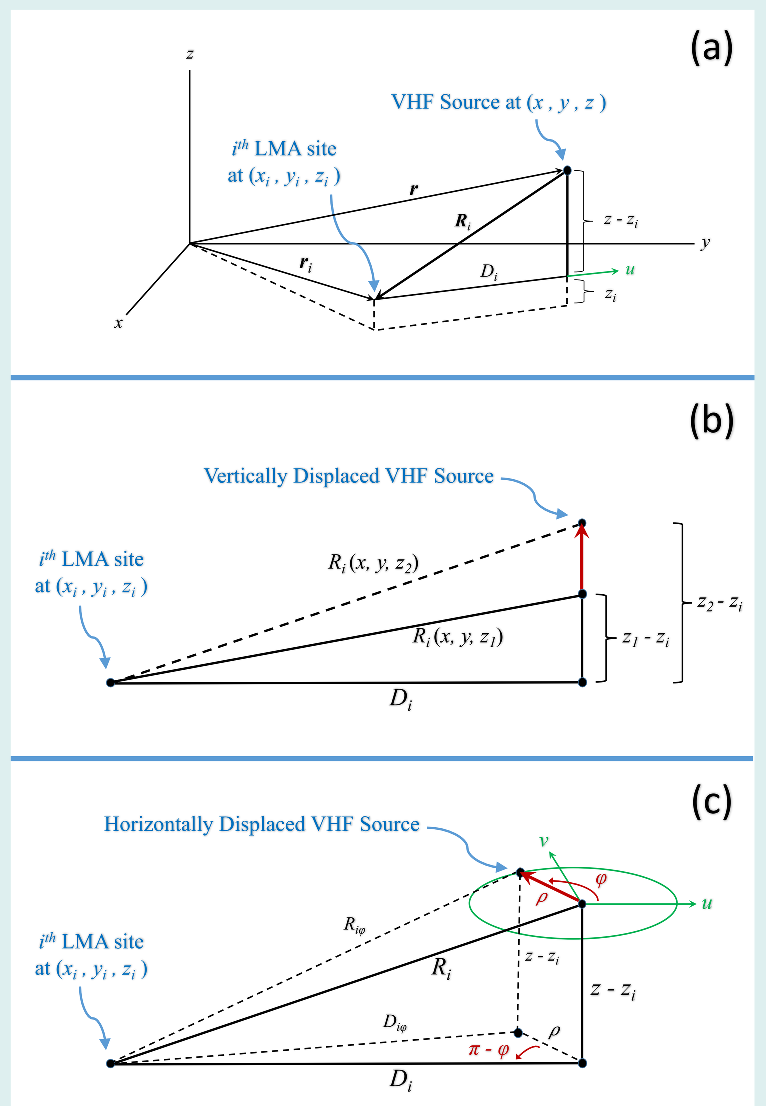
1. OVERVIEW

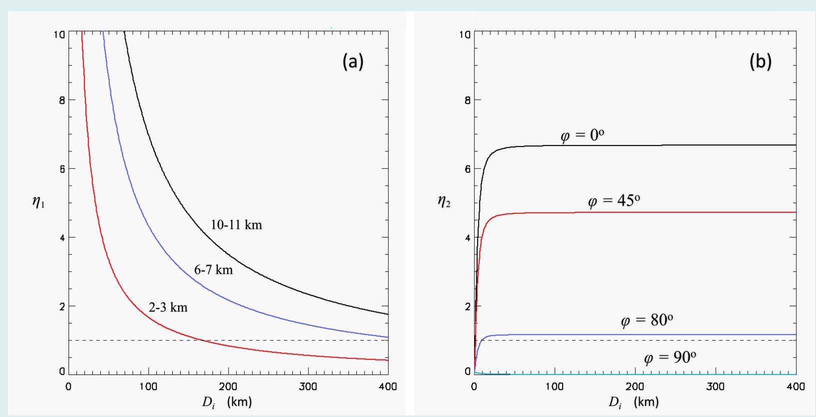
This presentation examines in detail the *standard retrieval method*: that of retrieving the (x, y, z, t) parameters of a lightning VHF point source from multiple ground-based Lightning Mapping Array (LMA) time-of-arrival (TOA) observations. The solution is found by minimizing a chi-squared function via the Levenberg-Marquardt algorithm. The associated forward problem is examined to illustrate the importance of signal-to-noise ratio (SNR). Monte Carlo simulated retrievals are used to assess the benefits of changing various LMA network properties. A *generalized retrieval method* is also introduced that, in addition to TOA data, uses LMA electric field amplitude measurements to retrieve a transient VHF dipole moment source.

2. FORWARD PROBLEM: SNR ANALYSES

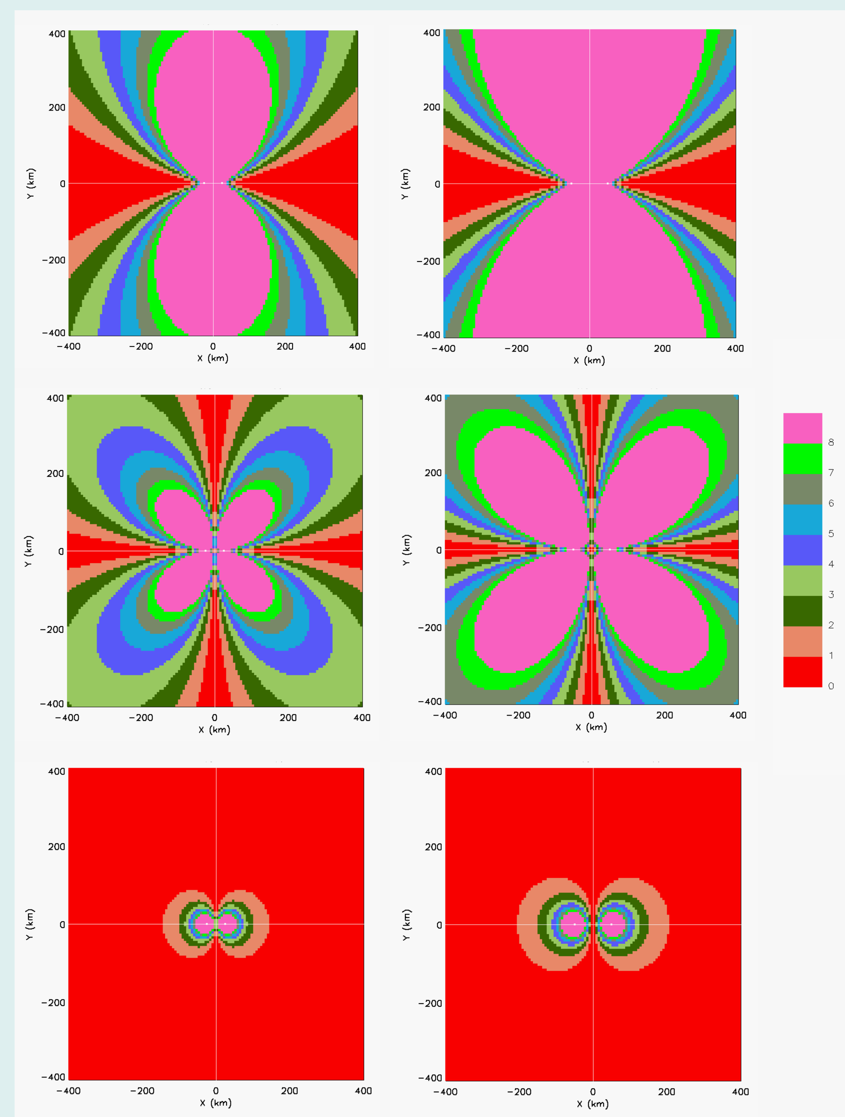
Performing the forward problem illustrates how well the measurements (TOA, or difference in TOA) track changes in the VHF point source. The sensitivity of 1 sensor (or 2 sensor) systems to various source displacements is examined.



Basic geometry (top), vertical displacement (middle), horizontal displacement (bottom)..

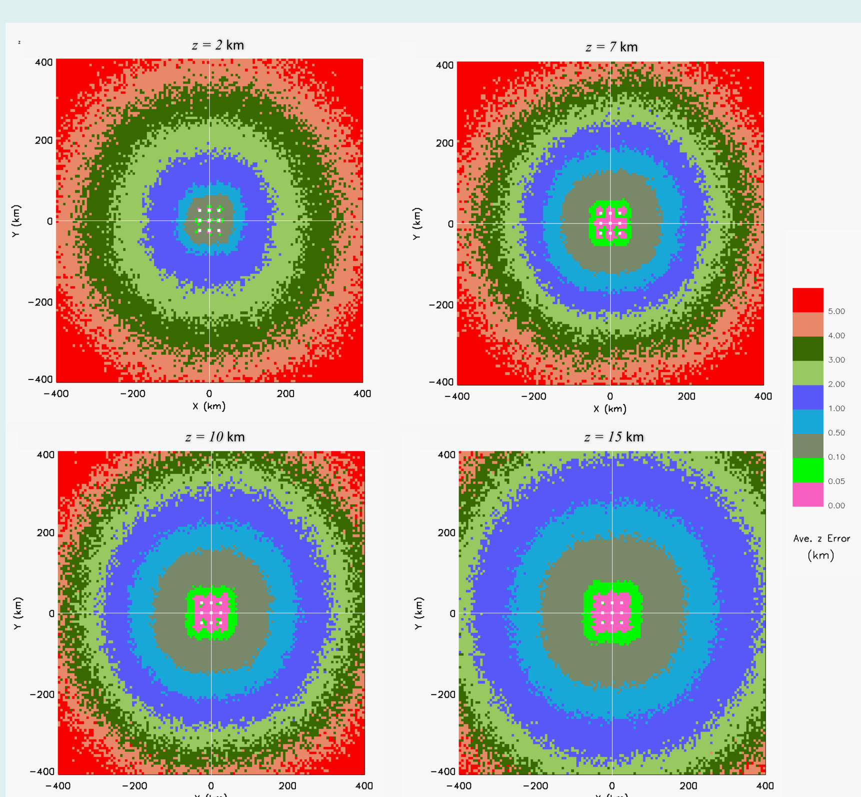


The **SNR associated with a single sensor** when a source a horizontal distance D_i away is displaced vertically (left plot), and horizontally (right plot).



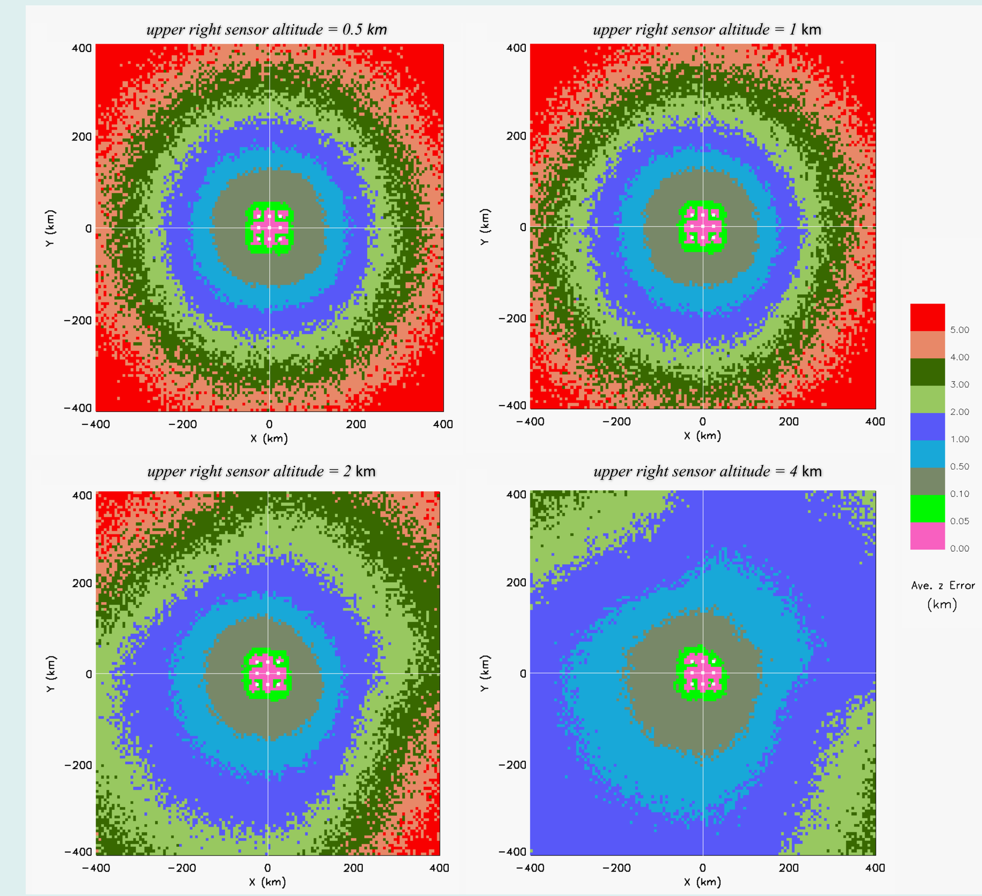
The **SNR of a two sensor system** when a source is displaced in x-direction (top row), y-direction (middle row), and vertically (bottom row). Distance between sensors is: 50 km (left column), 100 km (right column).

3. BASELINE MONTE CARLO SIMULATION

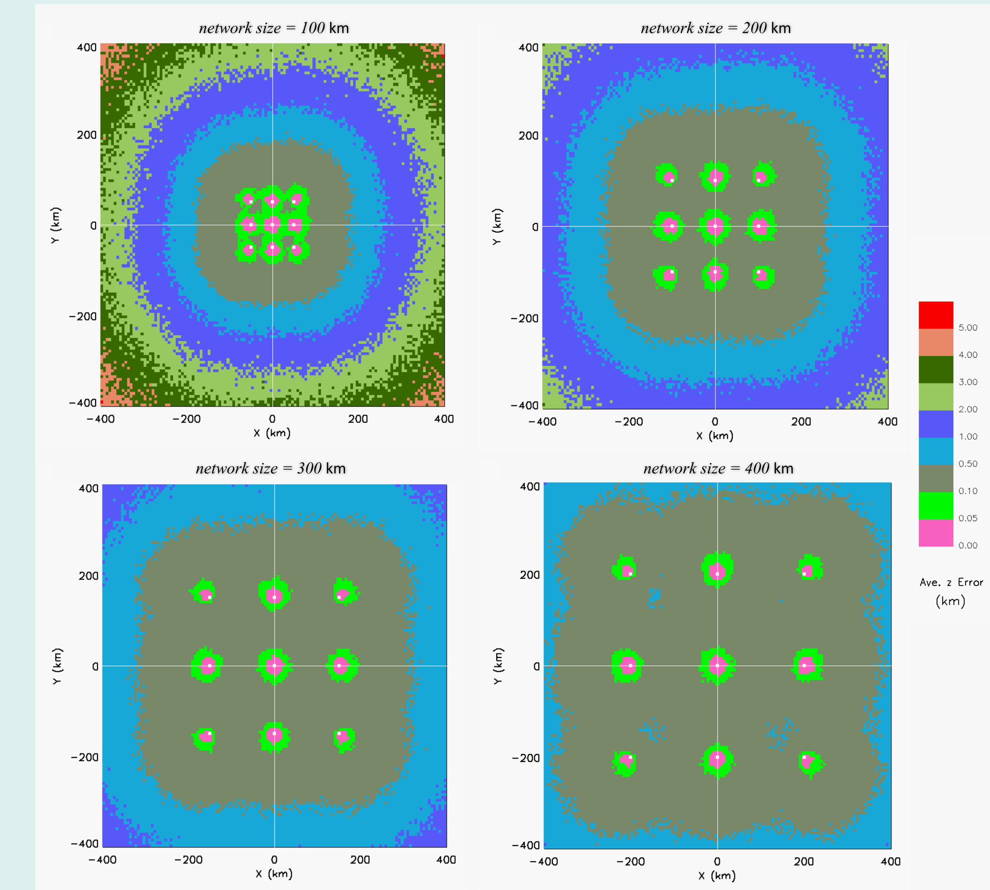


The baseline run showing the **mean altitude retrieval error** as a function of source altitude (given at the top of each plot) and horizontal source location relative to the fixed (3x3) Cartesian LMA network.

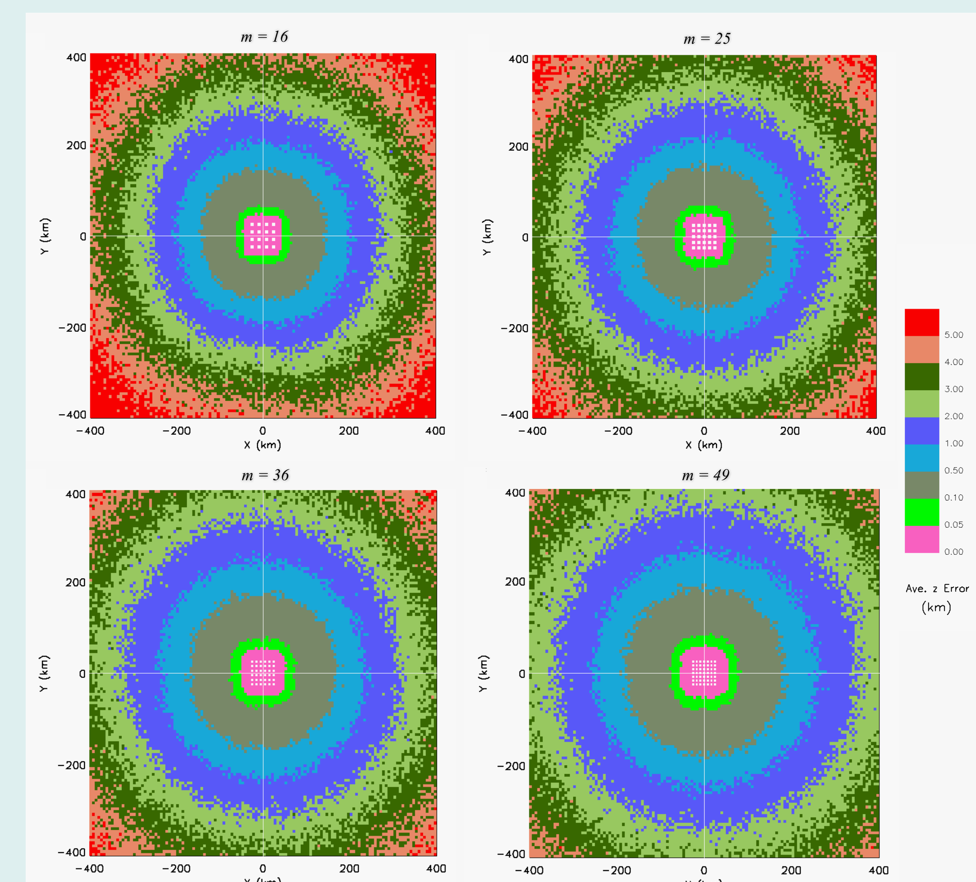
4. MORE SIMULATIONS: EFFECT OF ALTERING CERTAIN LMA NETWORK PARAMETERS



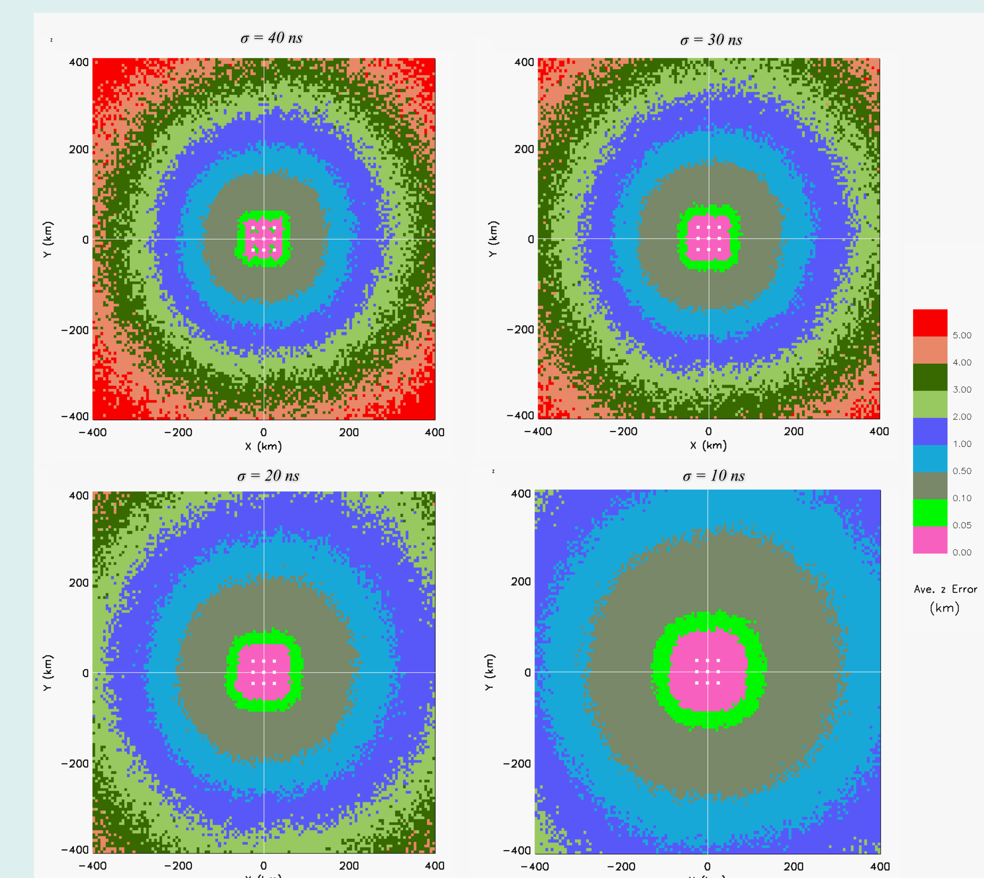
Elevating the north east sensor.



Expanding the horizontal extent of the network.



Increasing the number of measurements.



Improving the measurement accuracy.

4. GENERALIZED RETRIEVAL METHOD

The vertical field E_z from He et al (2000) due to a transient dipole source is generalized so that it expresses the field at the i^{th} LMA sensor. The amplitude measurement a_i and the associated model μ_i are identified, and the generalized chi-squared is minimized to obtain a solution [note: β_i is a function of the spatial variables $(\mathbf{r}, \mathbf{r}_i)$ and the orientation angles of the dipole source; τ_i is the TOA observation at i^{th} sensor]:

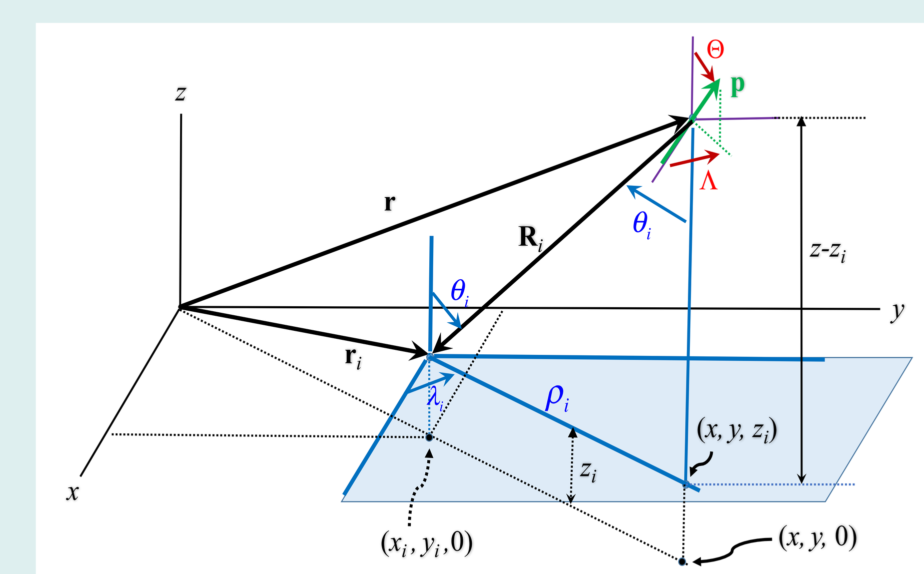
$$E_{zi}(t') = \frac{1}{2\pi\varepsilon_o c^2 R_i}\left\{\left(\beta_i - \tfrac{2}{3}\cos\Theta\right)[\ddot{p}] + \frac{3c\beta_i}{R_i}[\dot{p}] + \frac{3c^2\beta_i}{R_i^2}[p]\right\} \quad ,$$

$$\beta_i = \sin\theta_i\cos\theta_i\sin\Theta\cos(\lambda_i - \Lambda) - \sin^2\theta_i\cos\Theta + \tfrac{2}{3}\cos\Theta \ .$$

$$a_i \equiv 2\pi\varepsilon_o c^2 E_{zi}(t_i)$$

$$\mu_i = \mu_i(\mathbf{r}, \Lambda, \Theta, \mathbf{w}) = \frac{1}{R_i}\left\{w_1\left(\beta_i - \tfrac{2}{3}\cos\Theta\right) + w_2\frac{3c\beta_i}{R_i} + w_3\frac{3c^2\beta_i}{R_i^2}\right\} \ ; \quad \mathbf{w} \equiv ([\ddot{p}], [\dot{p}], [p]) \ .$$

$$\chi^2(\mathbf{r}, t, \Lambda, \Theta, \mathbf{w}) = \sum_{i=1}^{m}\frac{(\tau_i - \{t + R_i/c\})^2}{\sigma_i^2} + \sum_{i=1}^{m}\frac{(a_i - \mu_i)^2}{\tilde{\sigma}_i^2} \quad .$$



5. REFERENCES

He, S., M. Popov, and V. Romanov, 2000: Explicit full identification of a transient dipole source in the atmosphere from measurement of the electromagnetic fields at several points at ground level, *Radio Sci.*, **35**, 107-117.

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