Incorporation of Damage and Failure Into an Orthotropic Elasto-Plastic Three-Dimensional Model with Tabulated Input Suitable for Use in Composite Impact Problems

Robert K. Goldberg and Kelly S. Carney
NASA Glenn Research Center, Cleveland OH
Paul DuBois
George Mason University, Fairfax VA
Canio Hoffarth, Bilal Khaled and Subramaniam Rajan
Arizona State University, Tempe AZ
Gunther Blankenhorn
Livermore Software Technology Corporation, Livermore CA

Abstract

A material model which incorporates several key capabilities which have been identified by the aerospace community as lacking in the composite impact models currently available in LS-DYNA® is under development. In particular, the material model, which is being implemented as MAT 213 into a tailored version of LS-DYNA being jointly developed by the FAA and NASA, incorporates both plasticity and damage within the material model, utilizes experimentally based tabulated input to define the evolution of plasticity and damage as opposed to specifying discrete input parameters (such as modulus and strength), and is able to analyze the response of composites composed with a variety of fiber architectures. The plasticity portion of the orthotropic, three-dimensional, macroscopic composite constitutive model is based on an extension of the Tsai-Wu composite failure model into a generalized yield function with a non-associative flow rule. The capability to account for the rate and temperature dependent deformation response of composites has also been incorporated into the material model. For the damage model, a strain equivalent formulation is utilized to allow for the uncoupling of the deformation and damage analyses. In the damage model, a diagonal damage tensor is defined to account for the directionally dependent variation of damage. However, in composites it has been found that loading in one direction can lead to damage in multiple coordinate directions. To account for this phenomena, the terms in the damage matrix are semi-coupled such that the damage in a particular coordinate direction is a function of the stresses and plastic strains in all of the coordinate directions. The onset of material failure, and thus element deletion, is being developed to be a function of the stresses and plastic strains in the various coordinate directions. Systematic procedures are being developed to generate the required input parameters based on the results of experimental tests.

Introduction

As composite materials are gaining increased use in aircraft components where impact resistance under high energy impact conditions is important (such as the turbine engine fan case), the need for accurate material models to simulate the deformation, damage and failure response of polymer matrix composites under impact conditions is becoming more critical. Within commercially available transient dynamic finite element codes such as LS-DYNA® [1], there are several material models currently available for application to the analysis of composites. The available models include relatively simple equations where criteria related to ratios of stresses to failure strengths are used to signify failure. More sophisticated sets of material models, based on continuum damage mechanics approaches (such as Matzenmiller et al [2]), are also available where the initiation and accumulation of damage is assumed to be the primary driver of any nonlinearity in the composite response. While these material models have been utilized with some level of success in modeling the nonlinear and impact response of polymer composites, there are some areas where the predictive capability can be improved. Most importantly, the existing models often require correlation based on structural level impact tests, which significantly limits the use of these methods as predictive tools. Furthermore, the current models generally assume that the nonlinear response of the composite can be modeled either by using a deformation based plasticity approach (such as in Sun and Chen [3]) or by a continuum damage mechanics approach (such as in Matzenmiller et al [2]). By using a plasticity based model, the nonlinear unloading and strain softening observed in actual composites [4] cannot be simulated. However, by using a continuum damage mechanics based model, the rate dependence in the material response, which is often observed in composites under high strain rate conditions [5], is difficult to incorporate in a theoretically consistent manner. Furthermore, a continuum damage mechanics approach cannot fully account for the significant nonlinearity that is observed in the shear stress-strain response [6]. Therefore, a modeling approach in which a plasticity based deformation model is combined with a damage model (specifically designed to account for the nonlinear unloading and strain softening observed after the peak stress) can provide some advantages. The input to current material models currently generally consists of point-wise properties (such as a specified failure stress or failure strain) that lead to curve fit approximations to the material stress-strain curves. This type of approach leads either to models with only a few parameters, which provide a crude approximation at best to the actual stress-strain curve, or to models with many parameters which require a large number of complex tests to characterize. An improved approach would be to use tabulated data, in which the material stress-strain curves are explicitly entered into the model in a discretized form. The discretized data, obtained from a well-defined straightforward set of experiments, would allow the complete stress-strain response of the material to be accurately defined. In addition, while many of the existing models are designed to be used with two-dimensional shell elements, to properly capture the through-thickness response of the material, which may be significant in impact applications, a fully three-dimensional formulation suitable for use with solid elements would be desirable.

To begin to address these needs, a new composite material model, designated as MAT 213, is being developed and implemented for use within LS-DYNA. The material model is meant to be a fully generalized model suitable for use with any composite architecture (unidirectional, laminated or textile). For the deformation model, the commonly used Tsai-Wu composite failure criteria [6] has been generalized and extended to a strain-hardening plasticity model with a quadratic yield function and a non-associative flow rule. For the damage model, a strain equivalent formulation has been developed, which allows the plasticity and damage calculations to be uncoupled, and thus allows the plasticity calculations to take place in the effective (undamaged) stress space. In traditional damage mechanics models such as the one developed by Matzenmiller et al [2], a load in a particular coordinate direction is assumed to result in a stiffness reduction only in the direction of the applied load. However, as will be described in more detail later in this paper, in the current model a semi-coupled formulation is developed in which a load in one direction results in a stiffness reduction in all of the coordinate directions.

In the following sections of this paper, a summary of the rate-independent deformation model is presented. Next, the strain equivalent semi-coupled damage model is discussed, along with the procedures that need to be used to properly characterize the damage model. A discussion of the approaches that are being considered to account for material failure will be discussed.

Deformation Model

A general quadratic three-dimensional orthotropic yield function based on the Tsai-Wu failure model is specified as follows, where 1, 2, and 3 refer to the principal material directions.

$$f(\sigma)=a+\begin{pmatrix}F_1 & F_2 & F_3 & 0 & 0 & 0\end{pmatrix}\begin{pmatrix}\sigma_{11}\\ \sigma_{22}\\ \sigma_{33}\\ \sigma_{12}\\ \sigma_{23}\\ \sigma_{31}\end{pmatrix} + \begin{pmatrix}\sigma_{11} & \sigma_{22} & \sigma_{33} & \sigma_{12} & \sigma_{23} & \sigma_{31}\end{pmatrix}\begin{pmatrix}F_{11} & F_{12} & F_{13} & 0 & 0 & 0\\ F_{12} & F_{22} & F_{23} & 0 & 0 & 0\\ F_{13} & F_{23} & F_{33} & 0 & 0 & 0\\ 0 & 0 & 0 & F_{44} & 0 & 0\\ 0 & 0 & 0 & 0 & F_{55} & 0\\ 0 & 0 & 0 & 0 & 0 & F_{66}\end{pmatrix}\begin{pmatrix}\sigma_{11}\\ \sigma_{22}\\ \sigma_{33}\\ \sigma_{12}\\ \sigma_{23}\\ \sigma_{31}\end{pmatrix} \tag{1}$$

In the yield function, σ_{ij} represents the stresses and F_{ij} and F_k are coefficients that vary based on the current values of the yield stresses in the various coordinate directions. By allowing the coefficients to vary, the yield surface evolution and hardening in each of the material directions can be precisely defined. The values of the normal and shear coefficients can be determined by simplifying the yield function for the case of unidirectional tensile and compressive loading in each of the coordinate directions along with shear tests in each of the shear directions. In the above equation, the stresses are the current value of the yield stresses in the normal and shear directions. To determine the values of the off-axis coefficients (which are required to capture the stress interaction effects), the results from 45° off-axis tests in the various coordinate directions can be used.

A non-associative flow rule is used to compute the evolution of the components of plastic strain. The plastic potential for the flow rule is shown below

$$h=\sqrt{H_{11}\sigma_{11}^2+H_{22}\sigma_{22}^2+H_{33}\sigma_{33}^2+2H_{12}\sigma_{11}\sigma_{22}+2H_{23}\sigma_{22}\sigma_{33}+2H_{31}\sigma_{33}\sigma_{11}+H_{44}\sigma_{12}^2+H_{55}\sigma_{23}^2+H_{66}\sigma_{31}^2} \tag{2}$$

where σ_{ij} are the current values of the stresses and H_{ij} are independent coefficients, which are assumed to remain constant. The values of the coefficients are computed based on average plastic Poisson's ratios [7]. The plastic potential function in Equation (2) is used in a flow law to compute the components of the plastic strain rate, where the usual normality hypothesis from classical plasticity [8] is assumed to apply and the variable $\dot{\lambda}$ is a scalar plastic multiplier.

$$\dot{\boldsymbol{\varepsilon}}^p = \dot{\lambda}\frac{\partial h}{\partial \boldsymbol{\sigma}} \tag{3}$$

By utilizing the principal of the equivalence of plastic work [8], the plastic potential function can be found to be equal to the equivalent stress and the scalar plastic multiplier can be found to be equal to the effective plastic strain [7].

To compute the current value of the yield stresses needed for the yield function, the common practice in plasticity constitutive equations is to use analytical functions to define the evolution of the stresses as a function of the components of plastic strain (or the effective plastic strain). Alternatively, in the developed model tabulated stress-strain curves are used to track the yield stress evolution. The user is required to input twelve experimentally obtained stress versus plastic strain curves in a tabulated, discretized form. Specifically, the required curves include uniaxial tension and compression curves in each of the normal directions (1,2,3), shear stress-strain curves in each of the shear directions (1-2, 2-3 and 3-1), and 45 degree off-axis tension curves in each of the 1-2, 2-3 and 3-1 planes. The 45 degree curves are required in order to properly capture the stress interaction effects. By utilizing tabulated stress-strain curves to track the evolution of the deformation response, the experimental stress-strain response of the material can be captured to a much higher degree of accuracy. Currently, only static test data is considered. The methodology is being adjusted to account for rate and temperature effects. To account for rate and temperature effects, a series of curves are generated for a variety of strain rates and temperatures. The table look up feature in LS-DYNA is then used to track the evolution of the yield stresses for varying temperatures and rates. To track the evolution of the deformation response along each of the stress-strain curves, the effective plastic strain is chosen to be the tracking parameter. Using a numerical procedure based on the radial return method [8] in combination with an iterative approach, the effective plastic strain is computed for each time/load step and the modified yield stresses are computed based on the effective plastic strain.

Several important issues have been identified which needed to be accounted for in the numerical implementation. One potential difficulty in computing the yield function coefficients based on the various input stress-strain curves is that due to numerical approximation or experimental variability, or just the nature of the actual composite response, the set of coefficients computed for the yield function shown in Equation (1) may result in a non-convex yield surface. For a valid plasticity based analysis, a convex yield surface is required [8]. In order to ensure a convex yield surface, the baseline stress-strain curves may need to be modified in order to yield appropriate coefficients of the yield function. In line with the approximation commonly used with the Tsai-Wu failure criteria upon which the yield function is based, the F_{12} coefficient of the yield function can be modified based on the following expression, which is compatible with a von Mises type of yield function [6]

$$F_{12} = -\frac{1}{2}\sqrt{F_{11}F_{22}} \tag{4}$$

In this expression, for each value of the effective plastic strain in the generated stress versus effective plastic strain curves the F_{11} and F_{22} coefficients are computed based on the appropriate input curves, and a modified value of the F_{12} coefficient is computed for the selected value of the effective plastic strain. Similar modifications can be applied to the F_{13} and F_{23} coefficients in order to ensure a convex yield surface.

In the numerical algorithm, the standard elastic constitutive equation is used to compute the new stresses for a particular iteration (i+1) within a particular time step (n+1), where the flow law (Equation (3)) is applied to compute the components of the increments of plastic strain as shown below.

$$\boldsymbol{\sigma}_{n+1}^{i+1} = \boldsymbol{\sigma}_n + \mathbf{C} : \left[\Delta\boldsymbol{\varepsilon} - \Delta\lambda_{n+1}^{i+1} \left.\frac{\partial h}{\partial \boldsymbol{\sigma}}\right|_{n+1}^{i} \right] \quad (5)$$

where $\mathbf{C}$ is the standard elastic stiffness matrix, $\Delta\boldsymbol{\varepsilon}$ is the increment in total strains, and the remaining terms are as defined previously. In Equation (5), the direction of the plastic correction vector (the derivative of the plastic potential function with respect to stress) has been selected to be a dynamic value based on the stresses and the plastic potential function computed in the previous iteration. The common practice in the radial return algorithm is to use a constant vector based on the derivative of the plastic potential function based on the trial elastic stresses [8]. However, it has been found that the yield surface can rotate as the plastic strain evolves based on the anisotropic nature of the yield function. One example of where the yield surface can rotate occurs when a unidirectional carbon fiber composite is analyzed. In this case, the yield stress in the longitudinal (fiber) direction effectively remains constant (due to the relatively rigid behavior of the carbon fibers) and the yield stresses in the transverse directions can vary significantly (due to the more ductile response of the polymer matrix). As the transverse yield stress changes while the longitudinal yield stress remains constant, the yield surface will rotate. This rotation of the yield surface will result in the vector that the stresses need to take to “return” to the yield surface changing direction as the plastic strain evolves.

Damage Model

The deformation portion of the material model provides the majority of the capability of the model to simulate the nonlinear stress-strain response of the composite. However, in order to capture the nonlinear unloading and local softening of the stress-strain response often observed in composites [4], a complementary damage law is required. In the damage law formulation, strain equivalence is assumed, in which for every time step the total, elastic and plastic strains in the actual and effective stress spaces are the same [4]. The utilization of strain equivalence permits the plasticity and damage calculations to be uncoupled, as all of the plasticity computations can take place in the effective stress space.

The first step in the development of the damage model is to relate the actual stresses to a set of effective stresses by use of a damage tensor $\mathbf{M}$

$$\boldsymbol{\sigma} = \mathbf{M}\boldsymbol{\sigma}_{eff} \quad (6)$$

The effective stress rate tensor can be related to the total and plastic strain rate tensors by use of the standard elasto-plastic constitutive equation

$$\dot{\boldsymbol{\sigma}}_{eff} = \mathbf{C}\left(\dot{\boldsymbol{\varepsilon}} - \dot{\boldsymbol{\varepsilon}}_p\right) \tag{7}$$

where **C** is the standard elastic stiffness matrix and the actual total and plastic strain rate tensors are used due to the strain equivalence assumption.

An algorithm to carry out the uncoupled plasticity/damage analysis is summarized below. In the algorithm, the superscript "n" represents values computed in the previous time step, and the superscript "n+1" indicates values to be computed in the current time step. In the first step of the algorithm, the actual stresses are converted into effective stresses using the damage tensor **M**. In the second step, the plasticity calculations are carried out in the effective stress space to compute the current value of the plastic strain rate, and the effective stress values are updated. Next, in step 3 the damage tensor is modified based on the computed plastic strain rate. Finally, in step 4 the modified damage tensor is used to compute the updated values of the actual stresses based on the updated effective stresses. The algorithm is summarized symbolically below, where Δt is the time step.

$$\begin{aligned}
&1.)\ \boldsymbol{\sigma}_{eff}^{n} = \left(\mathbf{M}^{-1}\right)^{n}\boldsymbol{\sigma}^{n}\\
&2.)\ \boldsymbol{\sigma}_{eff}^{n+1} = \boldsymbol{\sigma}_{eff}^{n} + \mathbf{C}\left(\dot{\boldsymbol{\varepsilon}} - \dot{\boldsymbol{\varepsilon}}_p\right)\Delta t\\
&3.)\ \mathbf{M}^{n+1} = \mathbf{M}^{n} + \Delta\mathbf{M}\left[\dot{\boldsymbol{\varepsilon}}_p\right]\\
&4.)\ \boldsymbol{\sigma}^{n+1} = \mathbf{M}^{n+1}\boldsymbol{\sigma}_{eff}^{n+1}
\end{aligned} \tag{8}$$

As specified in Equation (6), the effective and actual stresses are related through a damage tensor. Given the usual assumption that the actual stress tensor and the effective stress tensor are symmetric, Equation (6) can be rewritten in the following form, where the damage tensor **M** is assumed to have a maximum of 36 independent components:

$$\begin{pmatrix}\sigma_{11}\\ \sigma_{22}\\ \sigma_{33}\\ \sigma_{12}\\ \sigma_{23}\\ \sigma_{31}\end{pmatrix} = \left[\mathbf{M}\right]\begin{pmatrix}\sigma_{11}^{eff}\\ \sigma_{22}^{eff}\\ \sigma_{33}^{eff}\\ \sigma_{12}^{eff}\\ \sigma_{23}^{eff}\\ \sigma_{31}^{eff}\end{pmatrix} \tag{9}$$

In many damage mechanics models for composites, for example the models discussed in [2] and [4], the damage tensor is assumed to be diagonal or manipulated to be a diagonal tensor. The implication of a diagonal damage tensor is that loading the composite in a particular coordinate direction only leads to a stiffness reduction in the direction of the load due to the formation of matrix cracks perpendicular to the direction of the load. However, several recent unpublished experimental studies conducted at NASA Glenn Research Center have shown that in actual composites, particularly those with complex fiber architectures, a load in one coordinate direction can lead to stiffness reductions in multiple coordinate directions.

One approach to incorporating the coupling of damage modes would be to use a non-diagonal damage tensor. However, while this formulation would allow for directional coupling, it would have the side effect of a unidirectional load in the actual stress space resulting in a multiaxial load in the effective undamaged space. For the strain equivalent combined plasticity damage formulation envisioned for this model, this would be an undesirable side effect as the plasticity calculations could be adversely affected due to the introduction of nonphysical stresses.

To avoid the undesired stress coupling, a diagonal damage tensor is required. However, to account for the damage interaction in at least a semi-coupled sense, each term in the diagonal damage matrix should be a function of the plastic strains in each of the normal and shear coordinate directions. The plastic strains are used since in the current model they track the current state of load and deformation in the material.

These results suggest that the relation between the actual stress and the effective stress should be based on a multiplicative combination of the damage terms as opposed to an additive combination of the damage terms. For example, for the case of plane stress, the relation between the actual and effective stresses could be expressed as follows:

$$
\begin{aligned}
\sigma_{11} &= \left(1-d_{11}^{11}\right)\left(1-d_{22}^{11}\right)\left(1-d_{12}^{11}\right)\sigma_{11}^{eff} \\
\sigma_{22} &= \left(1-d_{11}^{22}\right)\left(1-d_{22}^{22}\right)\left(1-d_{12}^{22}\right)\sigma_{22}^{eff} \\
\sigma_{12} &= \left(1-d_{11}^{12}\right)\left(1-d_{22}^{12}\right)\left(1-d_{12}^{12}\right)\sigma_{12}^{eff}
\end{aligned}
\tag{10}
$$

where for each of the damage terms d_{ij}^{kl} the subscript indicates the direction of the load which initiates the particular increment of damage and the superscript indicates the direction in which the damage takes place. Note that for the full three-dimensional case the stress in a particular coordinate direction is a function of the damage due to loading in all of the coordinate directions (1, 2, 3, 12, 31 and 23). By using a polynomial to describe the damage, the coupled terms represent the reduction to the degree of damage resulting from the fact that in a multiaxial loading case the area reductions are combined.

There are two primary items needed for model characterization and input for the damage portion of the material model. First, the values of the various damage parameter terms d_{ij}^{kl} need to be defined in a tabulated manner as a function of the effective plastic strain. Similar to the deformation model, the values of the damage parameters are defined in a tabulated, discretized form in order to reflect the actual material behavior in the most accurate manner possible. The values are tabulated as a function of the effective plastic strain in order to provide a unified framework to simultaneously track the evolution of multiple damage parameters under multiaxial loading conditions. As mentioned above, since in the context of the current model the plastic strains are used to represent the nonlinear deformation of the material, using the effective plastic strain as an equivalent parameter to track the damage parameter evolution should be reasonable. Note that for the case of uniaxial loading the effective plastic strain equals the uniaxial plastic strain, which maintains consistency with the formulation described above. In addition to characterizing the damage parameters, the various input stress-strain curves need to be converted into plots of effective (undamaged) stress versus effective plastic strain in order to carry out the calculations required by the deformation (plasticity) model. As an example of how this process could be carried out, assume that a material is loaded unidirectionally in the 1 direction. At multiple points, once the actual stress-strain curve has become nonlinear, the total strain (ε_{11}), actual stress (σ_{11}), and average local, damaged (reduced) modulus E_{11}^{d11} in the 1 direction can be measured. Assuming that the original, undamaged modulus E_{11} is known, since the damage in the 1 direction is assumed to be only due to the load in the 1 direction (due to the uniaxial load), the damage parameters and effective stress in the 1 direction can be computed at a particular point along the stress-strain curve as follows:

$$
\begin{aligned}
&1-d_{11}^{11}=\frac{E_{11}^{d11}}{E_{11}}\\
&M_{11}=1-d_{11}^{11}\\
&\sigma_{11}^{eff}=\frac{\sigma_{11}}{M_{11}}\\
&\varepsilon_{11}^{p}=\varepsilon_{11}-\frac{\sigma_{11}^{eff}}{E_{xx}}
\end{aligned}
\tag{11}
$$

These values need to be determined at multiple points, representing different values of plastic strain, in order to fully characterize the evolution of damage as the plastic strain increases.

With this information, an effective stress versus plastic strain $\left(\varepsilon_{11}^{p}\right)$ plot can be generated. From this plot, the effective plastic strain corresponding to the plastic strain in the 1 direction at any particular point can be determined by using the equations shown below, which are based on applying the principal of the equivalence of plastic work [8] in combination with Equation (2), simplifying the expressions for the case of unidirectional loading in the 1 direction [7]:

$$
\begin{aligned}
&h=\sqrt{H_{11}\left(\sigma_{11}^{eff}\right)^{2}}\\
&\varepsilon_{e}^{p}=\int\frac{\sigma_{11}^{eff}\,d\varepsilon_{11}^{p}}{h}
\end{aligned}
\tag{12}
$$

where ε_{e}^{p} is the effective plastic strain and $d\varepsilon_{11}^{p}$ is the increment of plastic strain in the 1 direction. From this data, plots of the effective stress in the 1 direction versus the effective plastic strain as well as plots of the damage parameter d_{11}^{11} as a function of the effective plastic strain can be generated. By measuring the damaged modulus in the other coordinate directions at each of the measured values of plastic strain in the 1 direction, the value of the damage parameters $d_{11}^{22}, d_{11}^{12}, d_{11}^{33}$, etc. can be determined as a function of the plastic strain in the 1 direction, and thus as a function of the effective plastic strain. To determine the remaining required damage terms, this process would need to be repeated by the loading the material in the other coordinate directions.

To convert the 45º off-axis stress-strain curves into plots of the effective (undamaged) stress versus effective plastic strain, the total and plastic strain (permanent strain after unload) in the structural axis x direction needs to be measured at multiple points along the stress-strain curve. Given the undamaged modulus E_{xx}, and utilizing the strain equivalence hypothesis, the effective stress in the structural axis system x direction can be computed as follows:

$$\sigma_{xx}^{eff} = E_{xx}\left(\varepsilon_{xx} - \varepsilon_{xx}^{p}\right) \tag{13}$$

Given the effective stress in the structural axis system, the effective stresses in the material axis system can be generated by use of stress transformation equations. Using the material axis system stresses, the plastic potential function and effective plastic strain corresponding to each value of plastic strain can be determined using the standard stress transformation equations for the case of 45º off-axis loading and the principal of the equivalence of plastic work in combination with Equation (2) as shown below [7]:

$$\begin{aligned}
\sigma_{11}^{eff} &= 0.5\sigma_{xx}^{eff} \\
\sigma_{22}^{eff} &= 0.5\sigma_{xx}^{eff} \\
\sigma_{12}^{eff} &= -0.5\sigma_{xx}^{eff} \\
h &= 0.5\sigma_{xx}^{eff}\sqrt{H_{11} + H_{22} + 2H_{12} + H_{44}} \\
\varepsilon_{e}^{p} &= \int \frac{\sigma_{xx}^{eff}\, d\varepsilon_{xx}^{p}}{h}
\end{aligned} \tag{14}$$

The methods to predict element failure are still under development. The failure model will be based on values of the effective plastic strain, the computed damage parameters and stress invariants such as those defined in the yield function.

Failure Model

In a failure model based on tabulated input that has been developed for metals, MAT 224, the effective failure strain of the material is defined to be a function of the triaxiality, which is the ratio of the hydrostatic stress to the von Mises stress, and for the case of solid elements, the Lode parameter, which is a ratio of the J_3 deviatoric stress invariant and the cube of the von Mises stress [1]. An advantage of this approach is that the variation in the material failure response due to varying load conditions and directions is accounted for while still being based on stress and strain invariants, which maintains a level of consistency. However, this model was primarily developed for the case of isotropic materials. Developing a similar tabulated based model for composites adds a significant level of complexity due to the significant anisotropy in the material response and failure behavior. Classic composite failure criteria such as the Tsai-Wu criteria [6] account for the variation in the material failure response due to the material anisotropy and load direction with the various anisotropic constants in the failure model. Hashin [9] and Mayes and Hansen [10] argue that failure criteria can be expressed in terms of stress invariants. Similarly, Feng [11] derived a failure model in terms of strain invariants. The above listed authors also described how the Tsai-Wu failure criteria [6] can be rewritten in terms of the various stress and strain invariants that were established. The stress invariants described in the above studies were developed assuming the condition of transverse isotropy, appropriate for a unidirectional composite. The above models also developed separate criteria based on fiber driven failure and matrix driven failure. The failure model for MAT 213 will involve the determination of stress invariants appropriate for the case of general orthotropic materials, not just transversely isotropic materials. The model will permit the use of previously established failure models as special cases. The model will also allow for a general, macroscopic failure criteria, as well as specialized failure criteria based on constituent (fiber and matrix) based failure modes.

Conclusions

A generalized composite model suitable for use in polymer composite impact simulations has been developed. The model utilizes a plasticity based deformation model based on generalizing the Tsai-Wu failure criteria. A strain equivalent damage model has also been developed in which loading the material in a particular coordinate direction can lead to damage in multiple coordinate directions. Procedures have also been developed to appropriately characterize the damage model. Ongoing efforts will include refining the methods to model failure and element removal. An extensive set of verification and validation studies will be undertaken in order to fully exercise the developed model.

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