Comparison of Factorization-based Filtering for Landing Navigation

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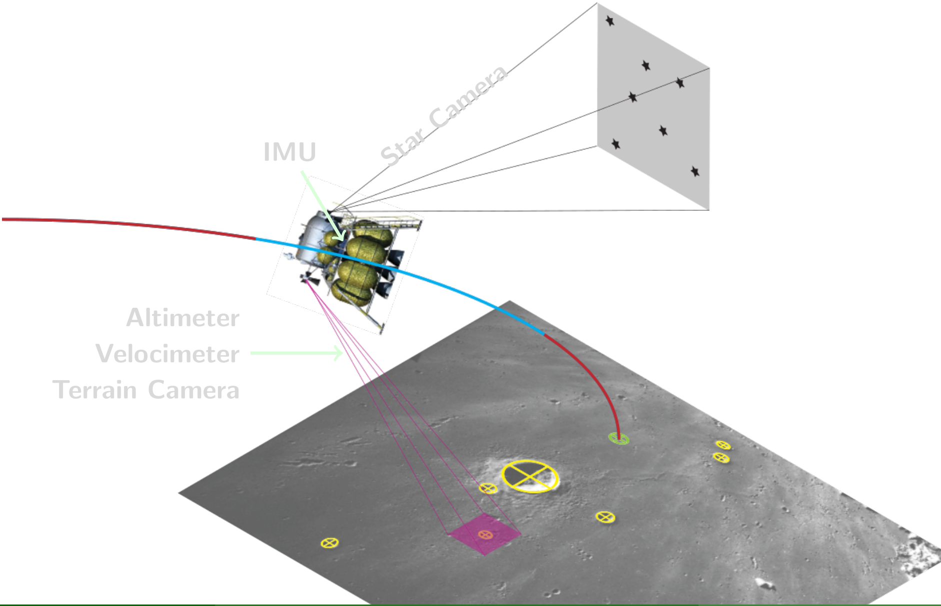
Motivation



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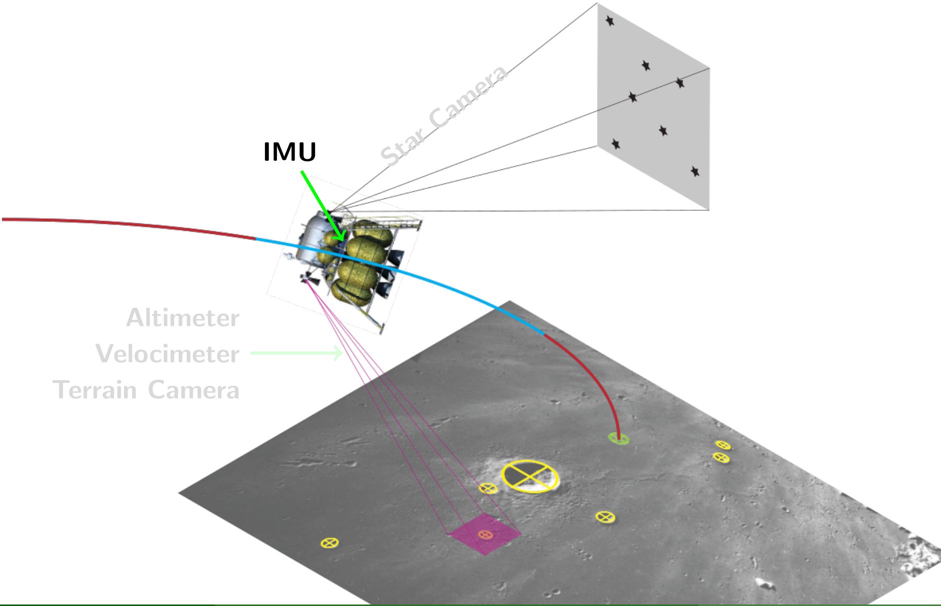

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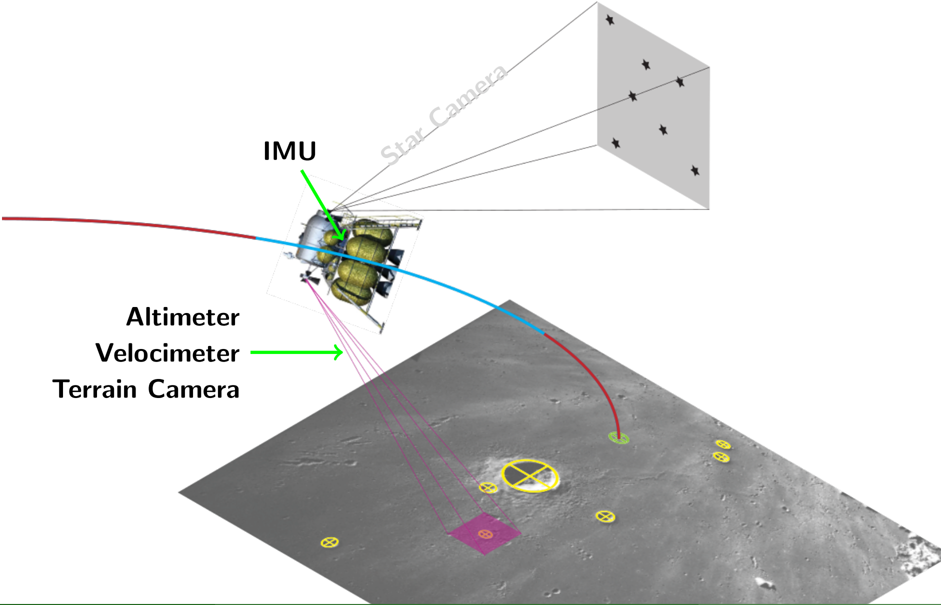

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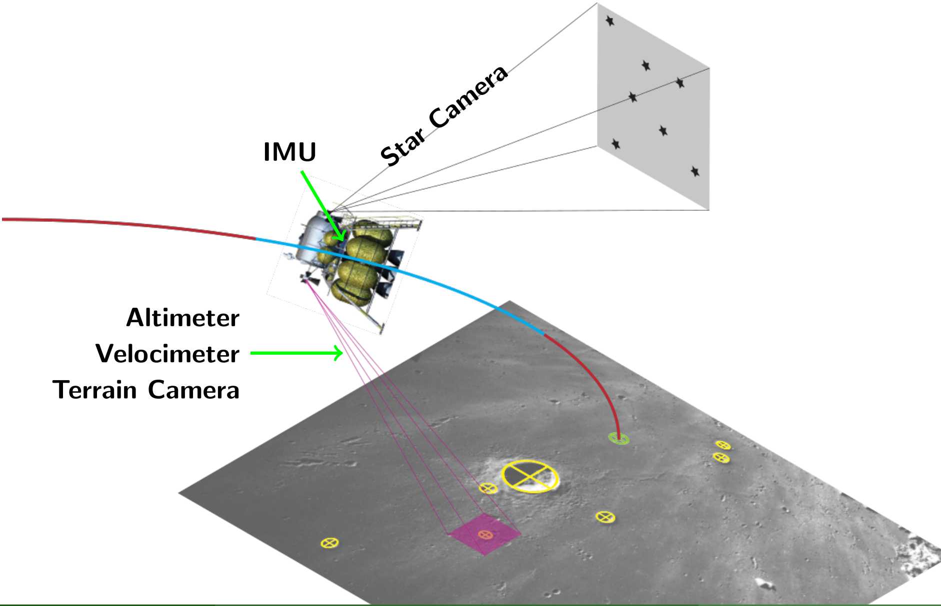

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- Blend data from sensors
  - inertial measurement unit
  - star camera
  - altimeter
  - velocimeter
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The Linear Problem

- Consider the linear state-space model

$$\boldsymbol{x}_k = \boldsymbol{F}_{k-1}\boldsymbol{x}_{k-1} + \boldsymbol{w}_{k-1}$$
$$\boldsymbol{z}_k = \boldsymbol{H}_k\boldsymbol{x}_k + \boldsymbol{v}_k$$

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- The well-known Kalman filter produces the conditional mean and covariance through a two-stage recursion:

Initial Cond.	$\boldsymbol{m}_{k-1}^+ = \boldsymbol{m}_0$
	$\boldsymbol{P}_{k-1}^+ = \boldsymbol{P}_0$
Mean Prop.	$\boldsymbol{m}_k^- = \boldsymbol{F}_{k-1}\boldsymbol{m}_{k-1}^+$
Cov. Prop.	$\boldsymbol{P}_k^- = \boldsymbol{F}_{k-1}\boldsymbol{P}_{k-1}^+\boldsymbol{F}_{k-1}^T + \boldsymbol{Q}_{k-1}$
Kalman Gain	$\boldsymbol{K}_k = \boldsymbol{P}_k^-\boldsymbol{H}_k^T[\boldsymbol{H}_k\boldsymbol{P}_k^-\boldsymbol{H}_k^T + \boldsymbol{R}_k]^{-1}$
Mean Update	$\boldsymbol{m}_k^+ = \boldsymbol{m}_k^- + \boldsymbol{K}_k(\boldsymbol{z}_k - \boldsymbol{H}_k\boldsymbol{m}_k^-)$
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Covariance Constraints

- By definition, the covariance matrix **must** be
  - symmetric
  - positive definite
- A proper filtering recursion should always maintain these properties.
- For the linear case, where the Kalman filter is theoretically exact, do these properties hold?

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Given that $\boldsymbol{P}_{k-1}^{+} = (\boldsymbol{P}_{k-1}^{+})^T$, it is clear from

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$$\begin{aligned} \boldsymbol{P}_k^+ &= \boldsymbol{P}_k^- - \boldsymbol{K}_k \boldsymbol{H}_k \boldsymbol{P}_k^- \\ &= \boldsymbol{P}_k^- - \boldsymbol{P}_k^- \boldsymbol{H}_k^T [\boldsymbol{H}_k \boldsymbol{P}_k^- \boldsymbol{H}_k^T + \boldsymbol{R}_k]^{-1} \boldsymbol{H}_k \boldsymbol{P}_k^- \end{aligned}$$

Therefore, the updated covariance matrix is algebraically symmetric.

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Symmetry: General Comment

In the worst case, brute-force symmetrization can be used:

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Given that $\boldsymbol{P}_{k-1}^{+} > \boldsymbol{0}$ and that $\boldsymbol{F}_{k-1}$ is full rank, the noise-free propagation of covariance is guaranteed to be positive definite; therefore,

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Positive Definiteness: Update

Consider the measurement update that results from

$$\boldsymbol{H}_k = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1+\delta \end{bmatrix}, \quad \boldsymbol{P}_k^- = \boldsymbol{I}_3\,, \quad \text{and} \quad \boldsymbol{R}_k = \delta^2 \boldsymbol{I}_2$$

where $\delta^2 < \epsilon_{\text{roundoff}}$ but $\delta > \epsilon_{\text{roundoff}}$.

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Positive Definiteness: Update

In this case, although $\boldsymbol{H}_k$ clearly has a rank of 2,

$$\boldsymbol{H}_k \boldsymbol{P}_k^- \boldsymbol{H}_k^T + \boldsymbol{R}_k = \begin{bmatrix} 3 & 3+\delta \\ 3+\delta & 3+2\delta \end{bmatrix}$$

with roundoff, which is a singular matrix.

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- Can be mitigated with factorization-based filtering methods.

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The Cholesky Square-Root Filter

- For the nonlinear state-space model

$$\boldsymbol{x}_k = \boldsymbol{f}(\boldsymbol{x}_{k-1}) + \boldsymbol{w}_{k-1}$$
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- The Cholesky square-root filter is given by the recursion:

Mean Prop.	$\boldsymbol{m}_k^-$	$= \boldsymbol{f}(\boldsymbol{m}_{k-1}^+)$
SRF Prop.	$\boldsymbol{S}_k^-$	$= \mathrm{qr}\{[\boldsymbol{F}_{k-1}\boldsymbol{S}_{k-1}^+ \mid \boldsymbol{T}_{k-1}]^T\}^T$
Innov. SRF	$\boldsymbol{Y}_k$	$= \mathrm{qr}\{[\boldsymbol{H}_k\boldsymbol{S}_k^- \mid \boldsymbol{L}_k]^T\}^T$
Cross Cov.	$\boldsymbol{C}_k$	$= \boldsymbol{S}_k^-\left[\boldsymbol{H}_k\boldsymbol{S}_k^-\right]^T$
Update Factors	$\boldsymbol{U}_k$	$= \boldsymbol{C}_k(\boldsymbol{Y}_k^-)^T$
Kalman Gain	$\boldsymbol{K}_k$	$= \boldsymbol{U}_k\boldsymbol{Y}_k^{-1}$
Mean Update	$\boldsymbol{m}_k^+$	$= \boldsymbol{m}_k^- + \boldsymbol{K}_k(\boldsymbol{z}_k - \boldsymbol{h}(\boldsymbol{m}_k^-))$
SRF Update	$\boldsymbol{S}_k^+$	$= \mathrm{cholupdate}\{(\boldsymbol{S}_k^-)^T, \boldsymbol{U}_k, -1\}^T$

Comments on the Methods

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  - can guarantee positive definiteness
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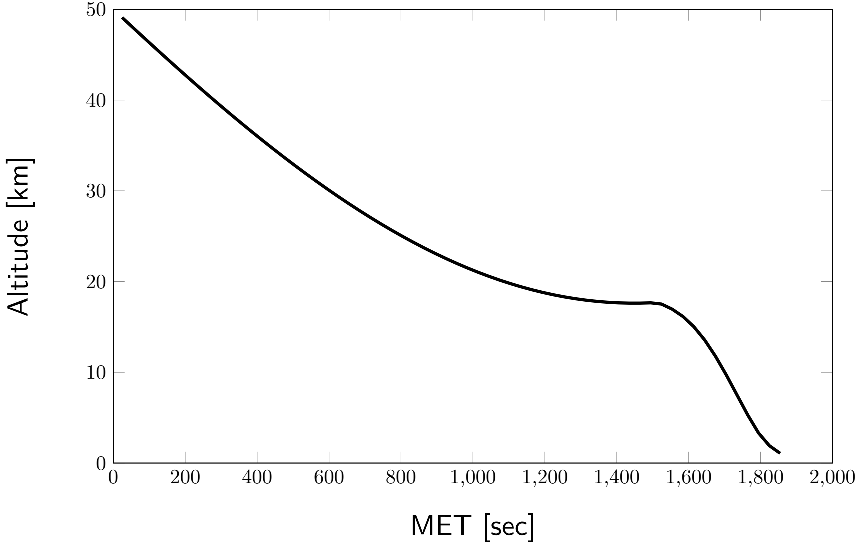
- UDU Factorization
  - guarantees symmetry of the covariance matrix
  - simple check for positive definiteness
  - does not require square root operations
  - more complicated, structurally

Trajectory


Altitude [km]
MET [sec]
0
10
20
30
40
50
0
200
400
600
800
1,000
1,200
1,400
1,600
1,800
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IMU Model

- **Inertial Measurement Unit** output is given by

$$\Delta \boldsymbol{v}_{m,k} = \Delta \boldsymbol{v}_k + \boldsymbol{b}_v + \boldsymbol{w}_{v,k}$$
$$\Delta \boldsymbol{\theta}_{m,k} = \Delta \boldsymbol{\theta}_k + \boldsymbol{b}_\theta + \boldsymbol{w}_{\theta,k}$$

where

  - $\Delta \boldsymbol{v}_k$ is the true, integrated, non-gravitational acceleration
  - $\Delta \boldsymbol{\theta}_k$ is the true, integrated angular velocity

- Sensor specifications

Accelerometer
  - Bias $(1\sigma) = 300\mu g$
  - Noise $(1\sigma) = 35\mu g/\sqrt{\mathrm{Hz}}$
  - Frequency $= 40$ Hz
  - Active: always

Gyro
  - Bias $(1\sigma) = 1^\circ/\mathrm{hr}$
  - Noise $(1\sigma) = 0.07^\circ/\sqrt{\mathrm{hr}}$
  - Frequency $= 40$ Hz
  - Active: always

Altimeter Model

- **Spherical Altitude** measurement is given by

$$z_k = (\|\boldsymbol{r}^i_{\text{alt},k}\| - r_{\text{sph}}) + b_{\text{alt}} + v_{\text{alt},k}$$

where

$$\boldsymbol{r}^i_{\text{alt},k} = \boldsymbol{r}^i_{\text{imu},k} + \boldsymbol{T}^i_{c,k}\boldsymbol{r}^c_{\text{alt/imu}}$$

- Sensor specifications
  - Bias $(1\sigma) = 0.5 \text{ m}$
  - Noise $(1\sigma) = [500,\ 5] \text{ m}$
  - Frequency $= 10 \text{ Hz}$
  - Active: $h \leq 15 \text{ km}$

Star Camera Model

- **Quaternion Star Camera** measurement is given by

$$\bar{\boldsymbol{z}}_k = \bar{\boldsymbol{q}}_{\text{err},k} \otimes \bar{\boldsymbol{q}}_c^{\text{sc}} \otimes \bar{\boldsymbol{q}}_{i,k}^{c}$$

where

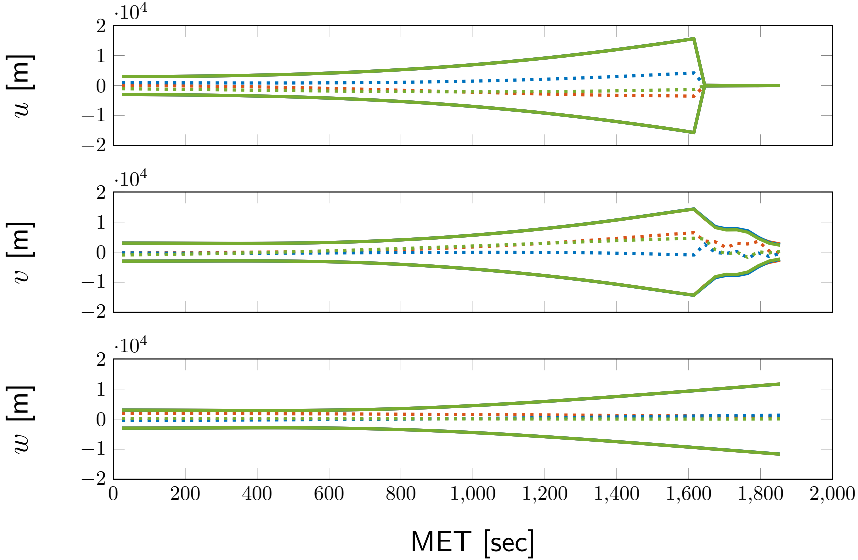
$$\bar{\boldsymbol{q}}_{\text{err},k} = \begin{bmatrix} \sin\left(\frac{1}{2}\|\boldsymbol{\theta}_{\text{err},k}\|\right)\frac{\boldsymbol{\theta}_{\text{err},k}}{\|\boldsymbol{\theta}_{\text{err},k}\|} \\ \cos\left(\frac{1}{2}\|\boldsymbol{\theta}_{\text{err},k}\|\right) \end{bmatrix} \quad \text{and} \quad \boldsymbol{\theta}_{\text{err},k} = \boldsymbol{b}_{sc} + \boldsymbol{v}_{sc,k}$$

- Sensor Specifications
  - Bias $(1\sigma) = 10''$
  - Noise $(1\sigma) = 30''$
  - Frequency $= 1$ Hz
  - Active: when not thrusting

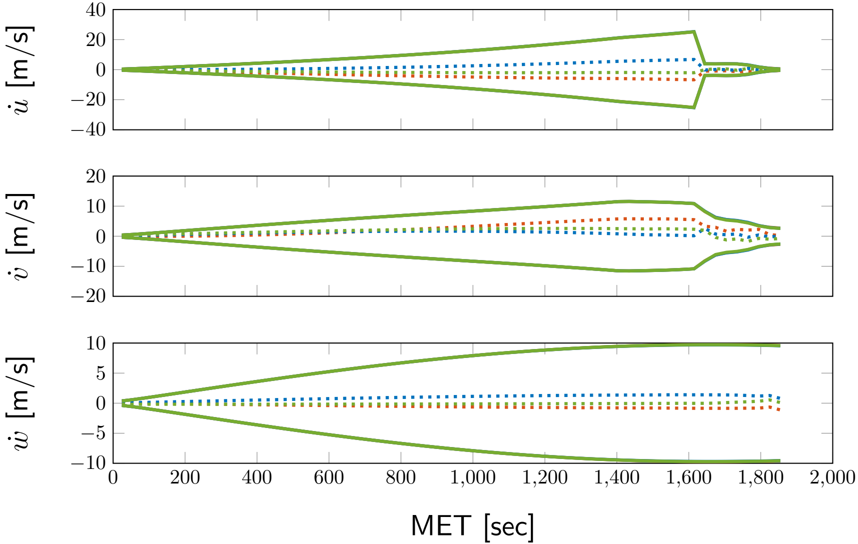
Monte Carlo Comparison

- Assess statistical consistency
  - 1000 Monte Carlo trials
  - Resample initial states and noises
  - Compute sample covariance
  - Compare to single run performance
  - Look at full covariance, UDU factorized, and Cholesky factorized filters

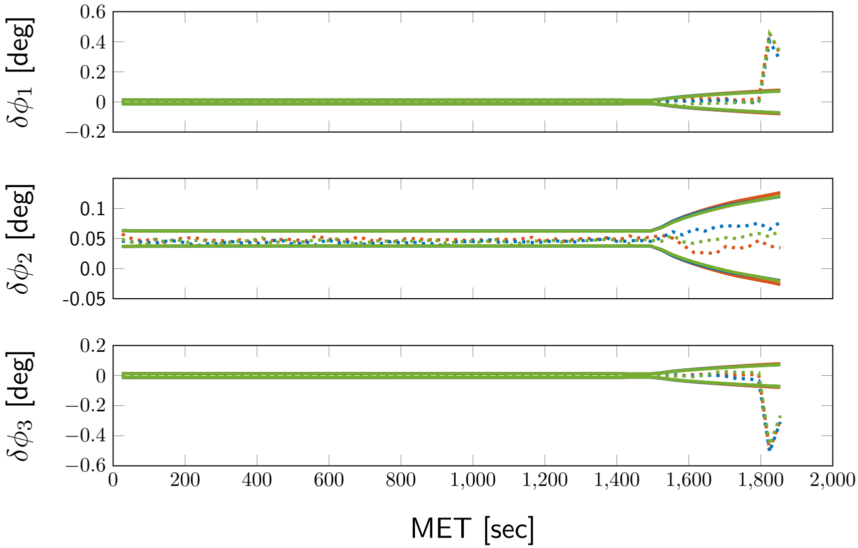
Monte Carlo: Position



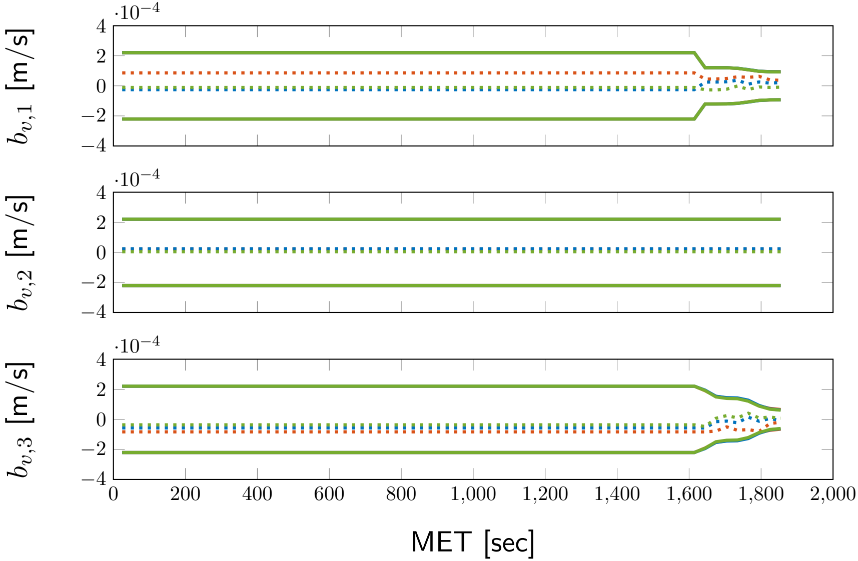
Monte Carlo: Velocity



Monte Carlo: Attitude



Monte Carlo: Accel. Bias



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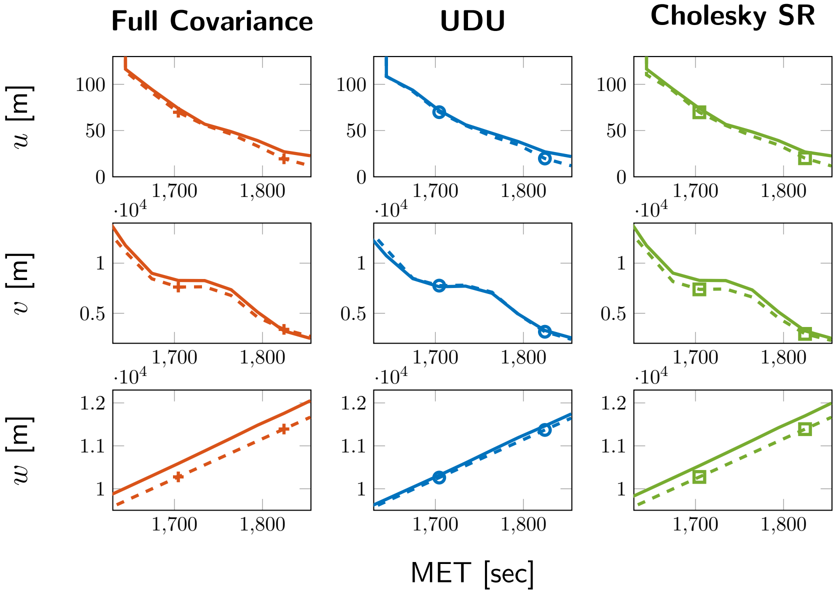
- Observations
  - Some full covariance trials failed
  - All UDU and Cholesky factorized trials successful
  - Translational uncertainty growth before altimeter turns on
  - Rotational uncertainty growth after star camera turns off
    - errors caused by sampling

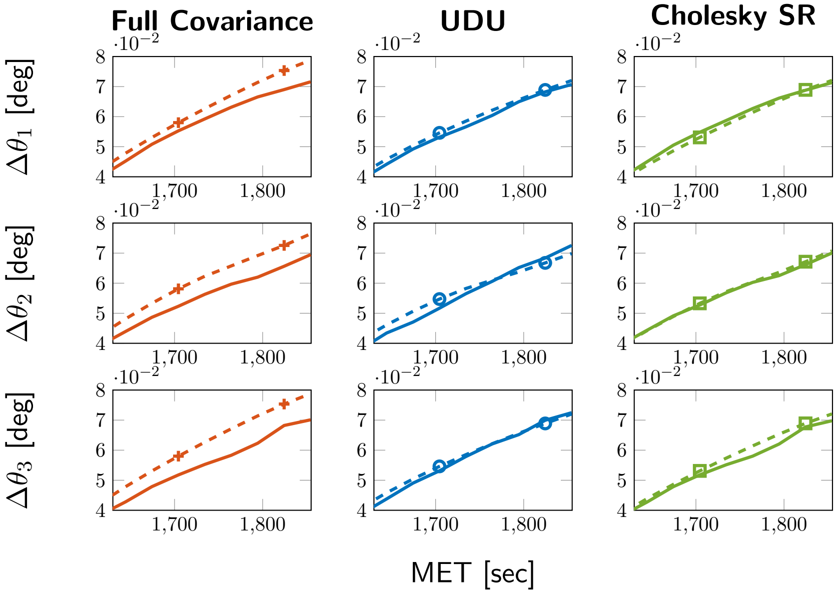
Terminal Descent Analysis

- More in-depth analysis during terminal descent
  - Same simulation, same configuration
  - Enhanced view in terminal descent

Grid Comparison: Position



Grid Comparison: Attitude



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- Which filter should you use?
  - vector vs. scalar processing of data
  - computational resources available

Acknowledgments

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Questions?