Facility Measurement Uncertainty Analysis at NASA GRC

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Importance of MUA

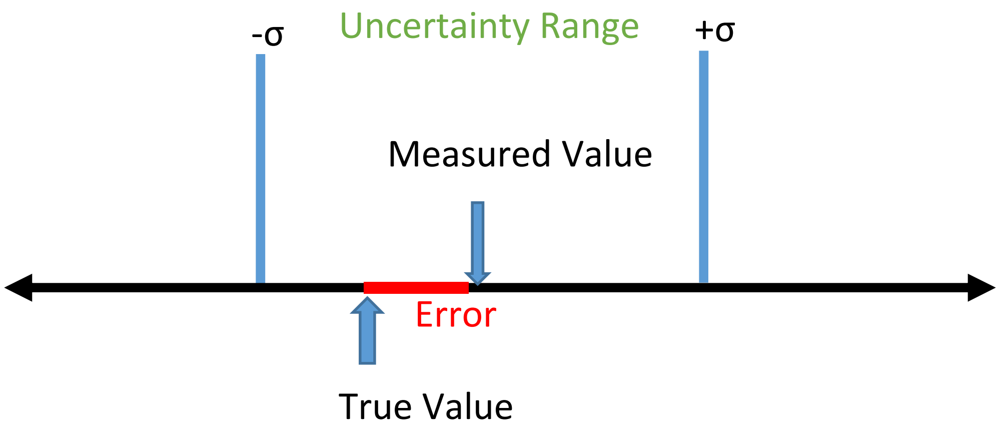
Understanding not just value, but also the process to obtain the value provides a greater understanding of the data acquired in the facilities.

Qualitative questions:	Quantitative Answers:
How good is the data?	+/- error limits on critical instruments *and* calculated values of interest
What are the facility's strengths and weaknesses?	Characterization of critical facility instruments and parameters
What instrumentation is best to measure…?	Quantification of instrumentation chain accuracy
What methods are best to measure…?	Determine percent contributions of uncertainty sources for clear understanding of where improvements should be made

Error Vs. Uncertainty

- Error of a measurement: the difference between the measured value and the unknown true value.

- Uncertainty of a measurement: an estimate of the range within which the actual value could fall, and the probability that it falls within that range[1].



[1] H. Coleman, W. Steele and H. Coleman, *Experimentation, validation, and uncertainty analysis for engineers*. Hoboken, N.J.: John Wiley & Sons, 2009.

Accuracy vs. Precision

- Accuracy:
  the ability to hit a specified point
- Precision:
  the ability to hit a consistent point.
- The two situations are not exclusive, you can have highly precise data which is not accurate and vise versa[2].

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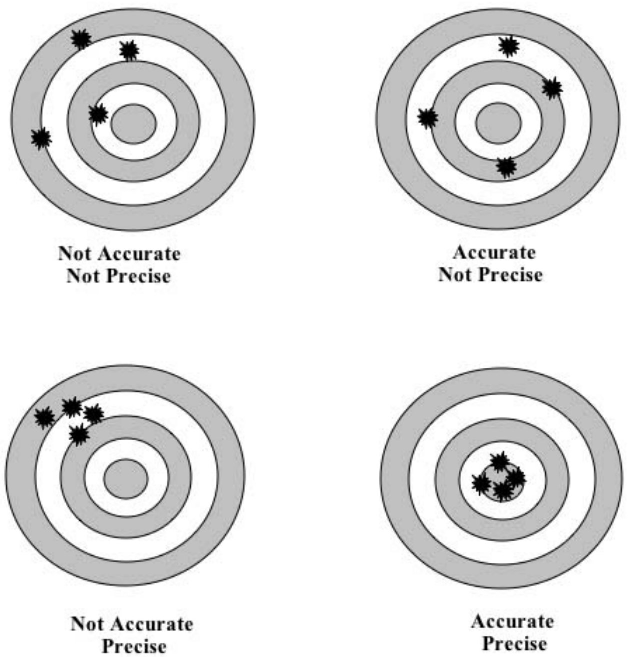
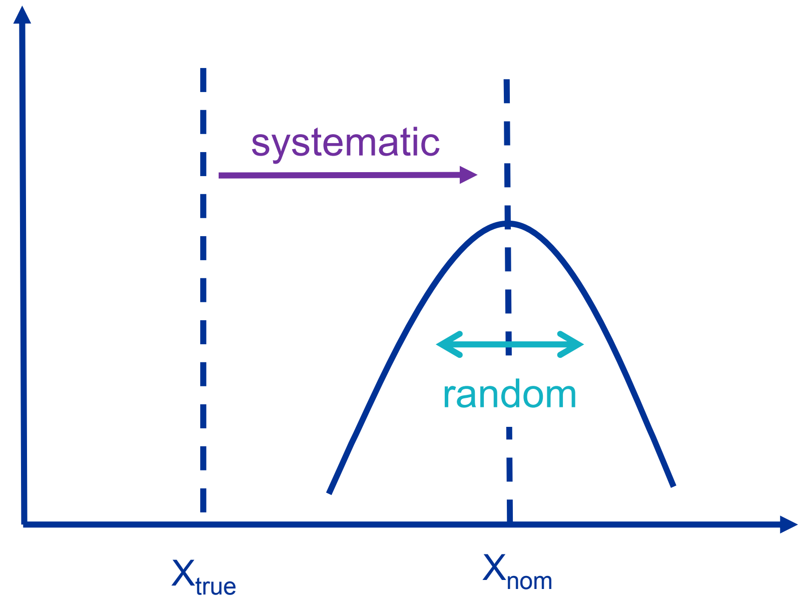


[2] Joint Committee for Guides in Metrology, 'Evaluation of Measurement Data — Guide to the Expression of Uncertainty in Measurement', JCGM/WG 1, 2008.

Uncertainty Type Classification

- **Type A**: evaluate by statistical analysis of observations
- **Type B**: evaluate by other means (based on calibration certificates, past experience, etc.)
- **Random**: the scatter of the results (repeatability, precision, scatter)
- **Systematic**: standard offset (bias, accuracy)



Customers looking to compare test results with CFD results are more concerned with systematic uncertainty effects.

Customers testing for the effect of model changes will be more concerned about random uncertainty effects.

Approaches to Uncertainty:

Statistical Process Control

- A quality control method which uses statistical techniques for regulation, characterization, and optimization of a process[3].
- Includes facility characterization and check standards
- Important for maintaining quality over time

Ground-up Analysis

- Analyze available data and spec. sheets to determine elemental uncertainties, then propagate through equations to values of interest.
- Powerful tool for determining both over-all and itemized uncertainty.
- Easy to implement “what if…?” scenario simulations for cost-benefit analysis for potential improvements

[3]J. Devore, *Probability and statistics for engineering and the sciences*. Monterey, Calif.: Brooks/Cole Pub. Co., 1982.

Approaches to Uncertainty, continued:

Statistical Process Control

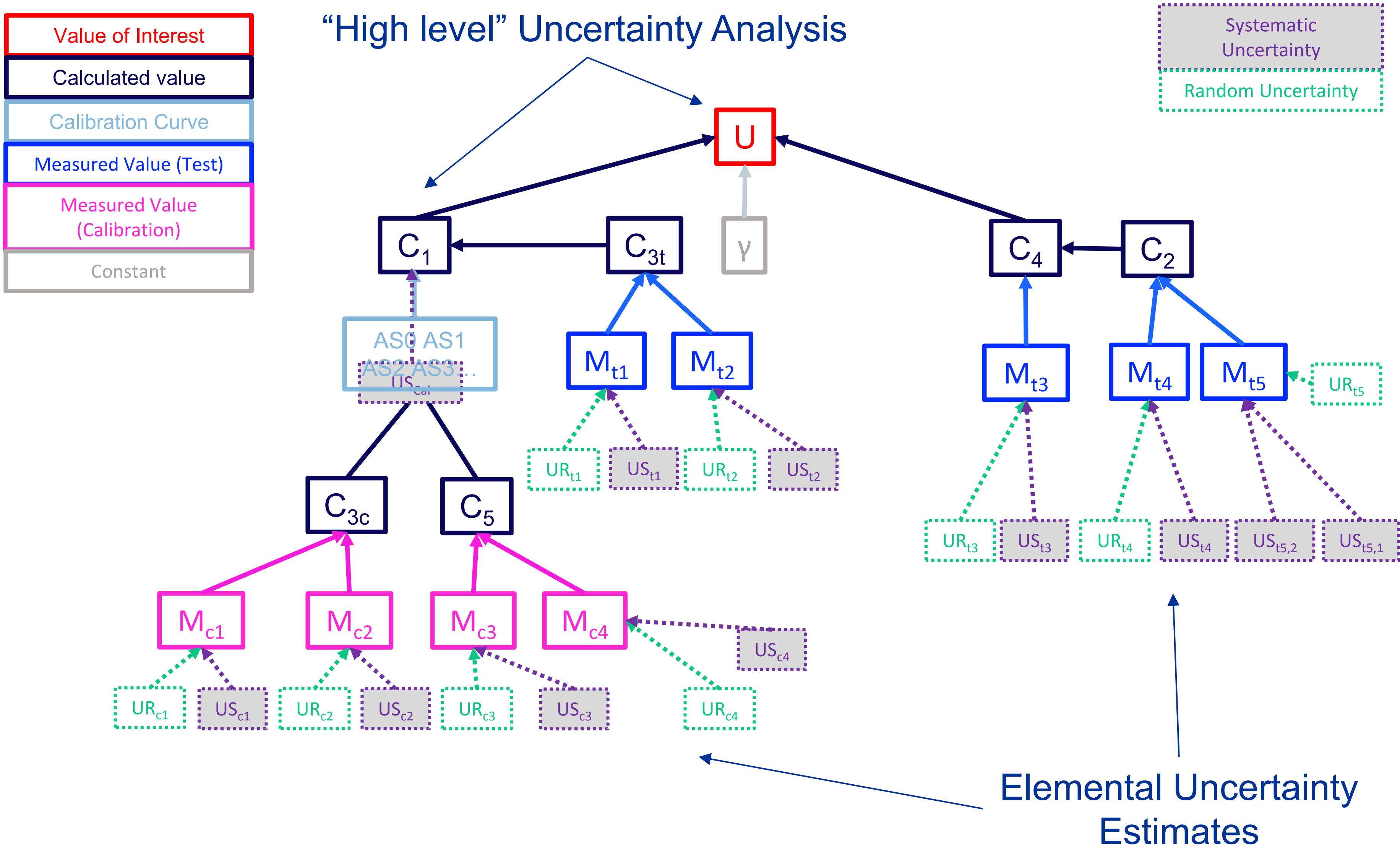
- Great at characterizing repeatability
- Ignores some systematic uncertainties
- Very difficult to separate out individual uncertainty sources
- Optimistic Results

Ground-up Analysis

- Output quality is based on input quality (elemental uncertainty estimates)
- Straight-forward process for adding new data as it becomes available
- Conservative Results

Ideally, both approaches should be implemented. When used together, uncertainty estimates are more accurate and better understood, and methods of reducing the uncertainty further are more apparent.

Analysis: Uncertainty Propagation

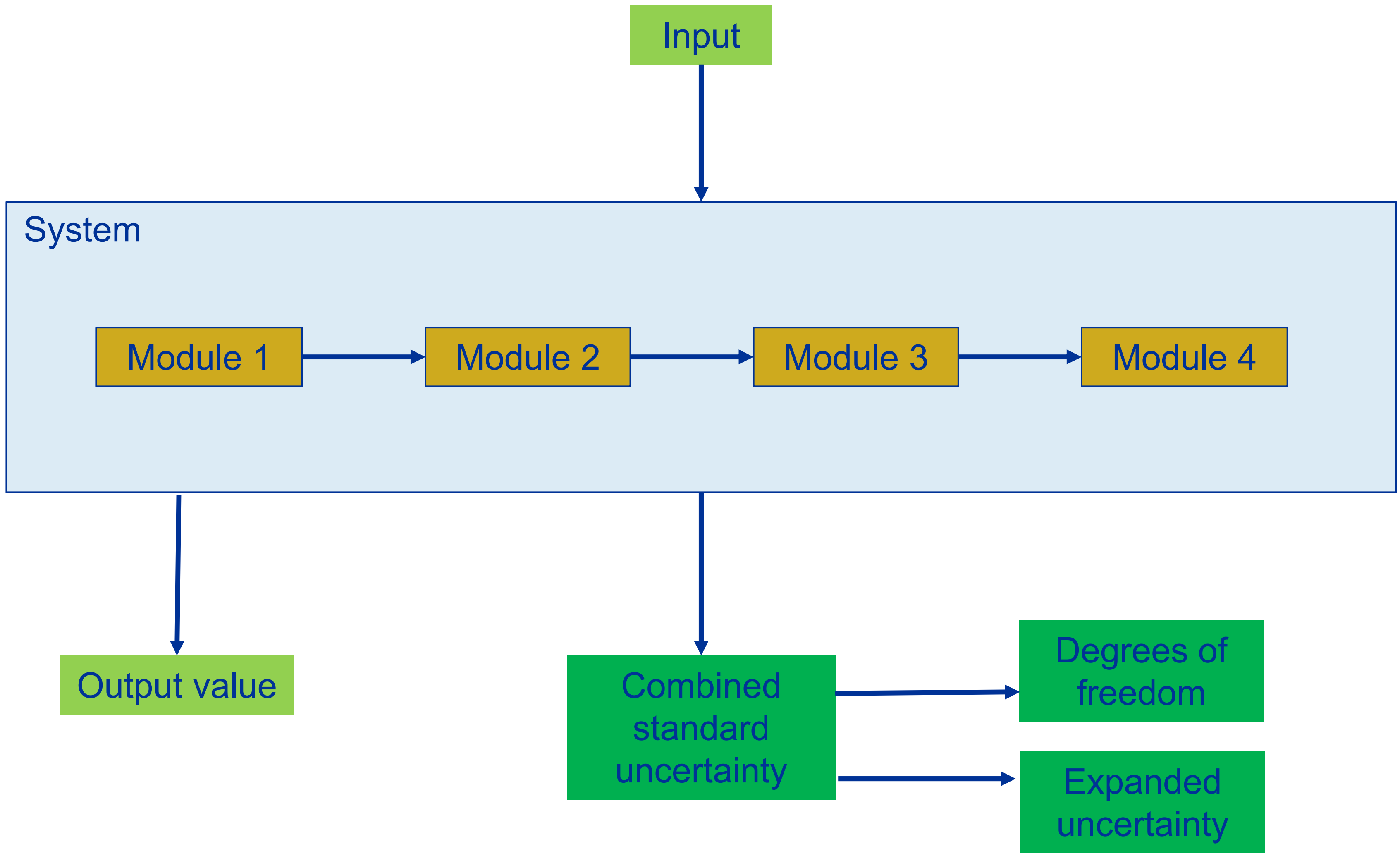


MANTUS

Measurement ANalysis Tool for Uncertainty in Systems

- A modular approach at modeling measurement systems.
- Based on NASA-HDBK-8739.19-3
- Each block represents a single piece of instrumentation in the signal measurement channel.
- The scope of the tool is to model and analyze a single, representative measurement channel such as one transducer or thermocouple connected to a data system.
- February 23, 2016: MANTUS Rev 2.0 released as a “beta” version (MANTUS 2.0) to GRC Facilities E-Team with a provided training course
  – Rolling release to “super users” to build modules for accessible library
  – Rolling release to standard users who will build systems from elements in the module library

MANTUS


Input
System
Module 1
Module 2
Module 3
Module 4
Output value
Combined standard uncertainty
Degrees of freedom
Expanded uncertainty

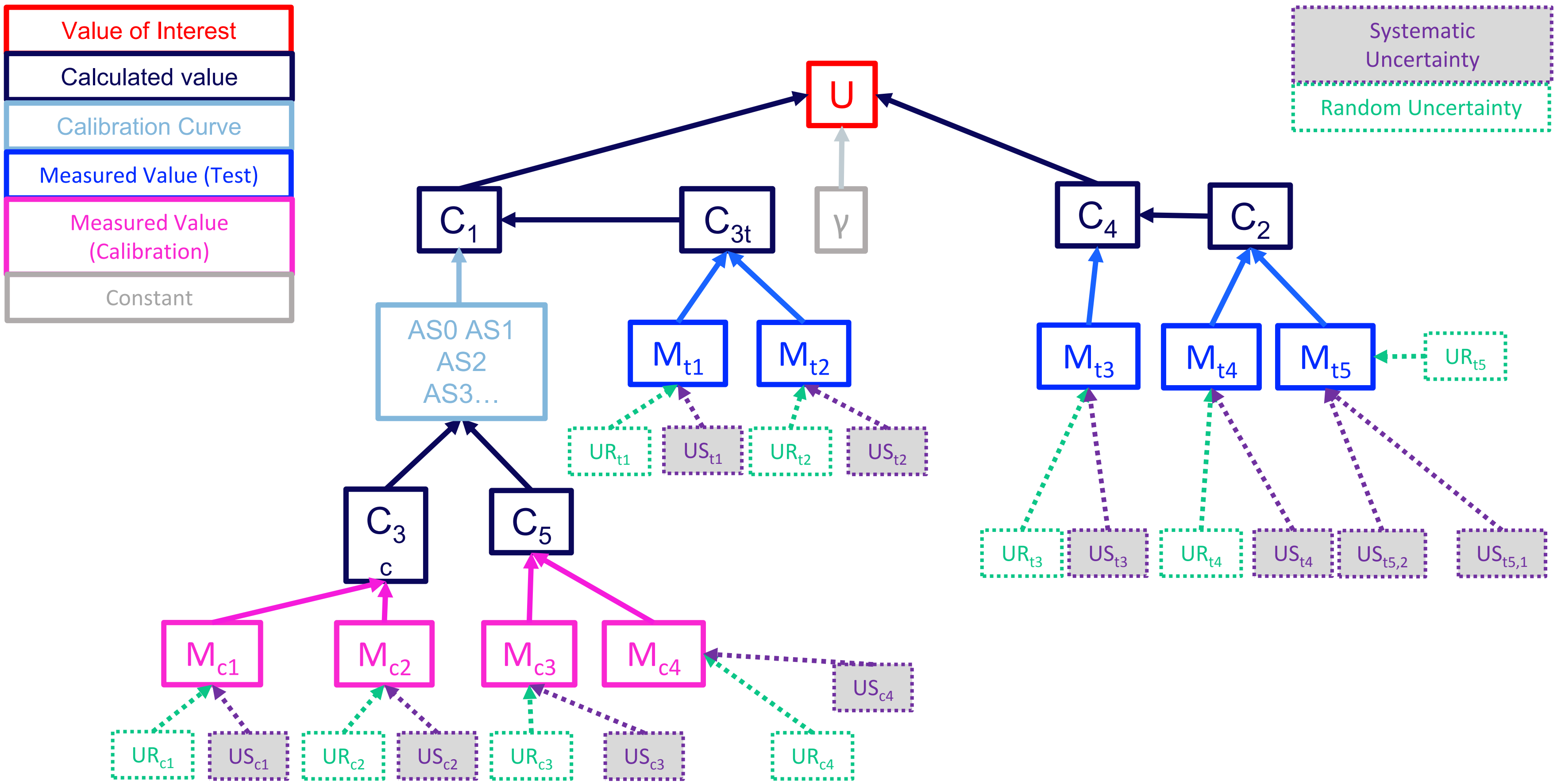
Thermocouple System Example

Input:
100 ºC

	Module	Combined Uncertainty (1σ)	% contribution from module
	Type E Thermocouple & Wire	1.202 ºC	97.2%
Nominal value:	100 ºC		
	Reference Oven	0.0000822 V	2.6%
Nominal value:	0.0023V		
	Signal Conditioner	0.1686 V	0.2%
Nominal value:	4.663 V		
	A/D Converter	539.7 Counts	0%
Nominal value:	14921 counts		
	ESCORT Unit Conversion	1.220 ºC Standard Uncertainty of system	0%
Output Value:	100 ºC		

Applied to MC

Analysis: Uncertainty Propagation



Estimate Elemental Uncertainties

- Systematic Uncertainties due to Instrumentation: MANTUS!
- Random Uncertainties of measured variables: Statistical analysis of data.
  - Population Standard deviation:

$$s_X = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(X_i - \bar{X})^2}$$

  - Must be measured over an appropriate time scale to capture desired random effects (back-to-back measurements are not considered distinct)
  - Estimate for small sample size:

$$s_x \cong \sigma_x = \frac{x_{max} - x_{min}}{d_2(n)}$$

- Other systematic considerations: spatial uniformity, calibration curves, etc.

Uncertainty Propagation Methods

Taylor Series Method

- Analytical method used to develop a model for system behavior.
- Sensitivity coefficients are calculated to define relationship between changes in variables and the resulting output.
- Elemental uncertainties are attributed to data reduction equation variables and combined accordingly.
- Uncertainty is combined for the whole system to produce a uncertainty estimate.

$$A = \pi r^2 \qquad U_A^2 = \left(\frac{\partial A}{\partial r}\right)^2 * b_r^2 + \left(\frac{\partial A}{\partial r}\right)^2 * s_r^2$$

Pros
- Fast for simple models
- Commonly used

Cons
- Analysis complication increases exponentially with complication of model.

Mass Flow:

$$m_{OR} = C_1 \left(1 - \frac{C_2 P_D}{P^2}\right) \sqrt{\frac{P P_D}{T}}$$

$$P = P_{bar} - P_a,$$

$$P_a = \frac{1}{2}(P_{a1} + P_{a2}),$$

$$P_d = \frac{1}{2}(P_{d1} + P_{d2}),$$

$$T = \frac{1}{4}(T_1 + T_2 + T_3 + T_4)$$

Systematic Uncertainties:

- By Taylor series

$$b_{Pa} = \sqrt{\left(\frac{\partial P_a}{\partial P_{a1}}\right)^2 b_{Pa1,unc}^2 + \left(\frac{\partial P_a}{\partial P_{a2}}\right)^2 b_{Pa2,unc}^2 + 2\left(\frac{\partial P_a}{\partial P_{a1}}\right)\left(\frac{\partial P_a}{\partial P_{a2}}\right) b_{Pa1,corr} b_{Pa2,corr}}$$

$$b_P = \sqrt{\left(\frac{\partial P}{\partial P_a}\right)^2 b_{Pa}^2 + \left(\frac{\partial P}{\partial P_{bar}}\right)^2 b_{Pbar}^2}$$

$$b_{PD} = \sqrt{\left(\frac{\partial P_D}{\partial P_{D1}}\right)^2 b_{PD1,unc}^2 + \left(\frac{\partial P_D}{\partial P_{D2}}\right)^2 b_{PD2,unc}^2 + 2\left(\frac{\partial P_D}{\partial P_{D1}}\right)\left(\frac{\partial P_D}{\partial P_{D2}}\right) b_{PD1,corr} b_{PD2,corr}}$$

$$b_T = \sqrt{\begin{array}{l} \left(\frac{\partial T}{\partial T_1}\right)^2 b_{T1,unc}^2 + \left(\frac{\partial T}{\partial T_2}\right)^2 b_{T2,unc}^2 + \left(\frac{\partial T}{\partial T_3}\right)^2 b_{T3,unc}^2 + \left(\frac{\partial T}{\partial T_4}\right)^2 b_{T4,unc}^2 \\ +2\left(\frac{\partial T}{\partial T_1}\right)\left(\frac{\partial T}{\partial T_2}\right) b_{T1,corr} b_{T2,corr} + 2\left(\frac{\partial T}{\partial T_1}\right)\left(\frac{\partial T}{\partial T_3}\right) b_{T1,corr} b_{T3,corr} \\ +2\left(\frac{\partial T}{\partial T_1}\right)\left(\frac{\partial T}{\partial T_4}\right) b_{T1,corr} b_{T4,corr} + 2\left(\frac{\partial T}{\partial T_2}\right)\left(\frac{\partial T}{\partial T_3}\right) b_{T2,corr} b_{T3,corr} \\ +2\left(\frac{\partial T}{\partial T_2}\right)\left(\frac{\partial T}{\partial 4}\right) b_{T2,corr} b_{T4,corr} + 2\left(\frac{\partial T}{\partial T_3}\right)\left(\frac{\partial T}{\partial T_4}\right) b_{T3,corr} b_{T4,corr} \end{array}}$$

8x6 Supersonic Wind Tunnel: Mach Number Equations

$$\Phi = \frac{P_{S,bal}}{P_{T,bm}}$$

$$P_{S,ts} = P_{T,bm}(B_0 + B_1\Phi + B_2\Phi^2 + B_3\Phi^3 + B_4\Phi^4 + B_5\Phi^5 + B_6\Phi^6)$$

Subsonic regime:

$$P_{T,ts} = A_0 + A_1 P_{T,bm} + A_2 P_{T,bm}^2$$

$$M_{ts} = \sqrt{\frac{2}{\gamma - 1}\left[\left(\frac{P_{S,ts}}{P_{T,ts}}\right)^{-\frac{\gamma-1}{\gamma}} - 1\right]}$$

Supersonic regime:

$$P_{T,2,ts} = P_{T,bm}(AS_0 + AS_1\Phi + AS_2\Phi^2 + AS_3\Phi^3 + AS_4\Phi^4 + AS_5\Phi^5 + AS_6\Phi^6)$$

$$\frac{P_{T,2,ts}}{P_{S,ts}} = \left[\frac{(\gamma + 1)M_{ts}^2}{2}\right]^{\frac{\gamma}{\gamma-1}} \left[\frac{\gamma + 1}{2\gamma M_{ts}^2 - (\gamma - 1)}\right]^{\frac{1}{\gamma-1}}$$

Uncertainty Propagation Methods (continued)

Monte Carlo Method

- Iterative method where a distribution of random numbers is applied to each elemental error source creating a synthetic error population
- The resulting sample of possible values is used in place of the original variable in the transfer function.
- With a sufficiently large number of iterations, the average of the calculated output represents the most likely result ("nominal" value).
- The standard deviation of the resulting outputs represents the standard uncertainty of the transfer function output.

Pros
- Simpler for more complex calculations
- Flexible for "what if" modeling

Cons
- Computation time

Random Population of radius(r)

r1
r2
r3
.
.
.
r(n)

$$\pi r^2 = A$$

Resulting Area(A)

A1
A2
A3
.
.
.
A(n)

$$\bar{A} = \frac{\sum A_i}{n}$$

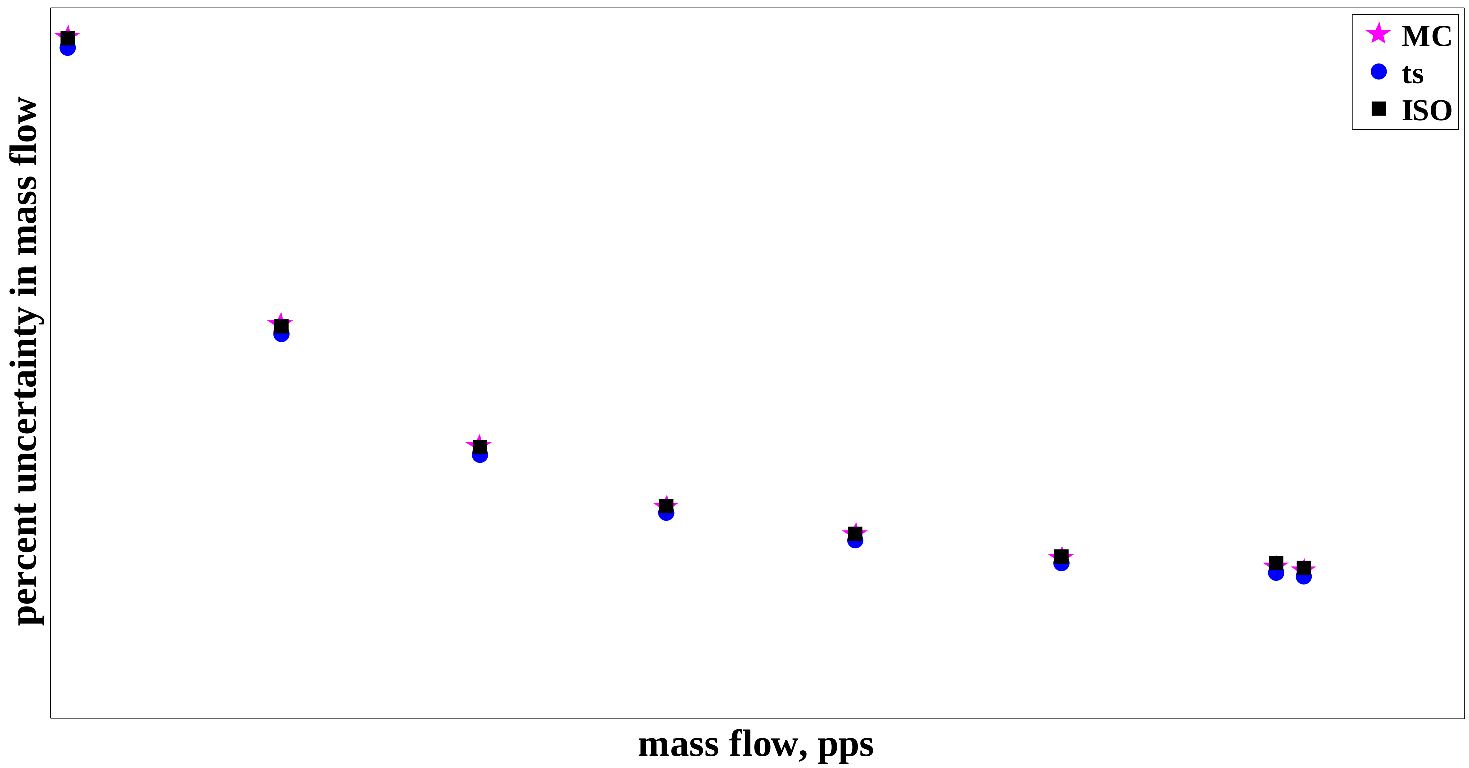
$$\bar{A} = A_{nom}$$

$$\sigma_A = \sqrt{\frac{\sum (A_i - \bar{A})^2}{n-1}}$$

$$\sigma_A = u_A$$

You get the same result

Example Method Comparison, mass flow



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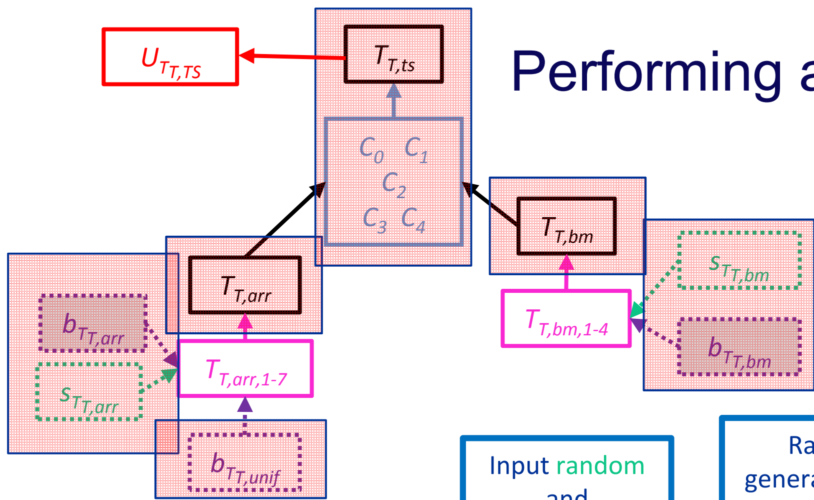
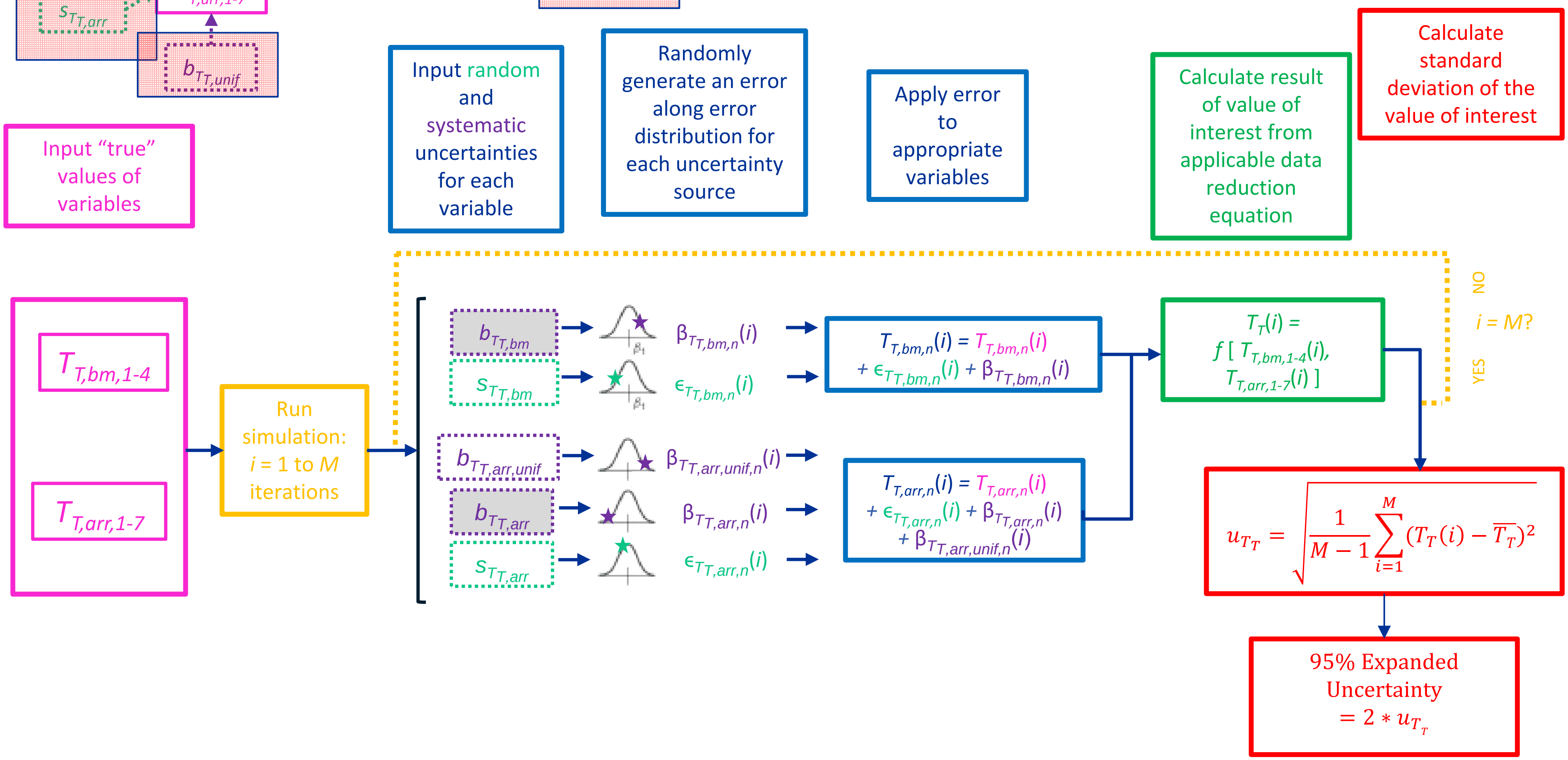
Performing a Monte Carlo Analysis





Presenting Results

- By "flagging" the uncertainties appropriately within the Monte Carlo code, the contribution of individual uncertainties or groups of uncertainties to the total uncertainty of the value of interest can be determined.
- Presenting the uncertainties as non-dimensional Uncertainty Percent Contributions (UPCs) in progressively smaller sub-groups is useful in determining the sources with the most impact.

  - Customers looking to compare test results with CFD results are more concerned with **systematic** uncertainty. These uncertainties can result in a bias in measurements and calculated variables from an expected outcome.

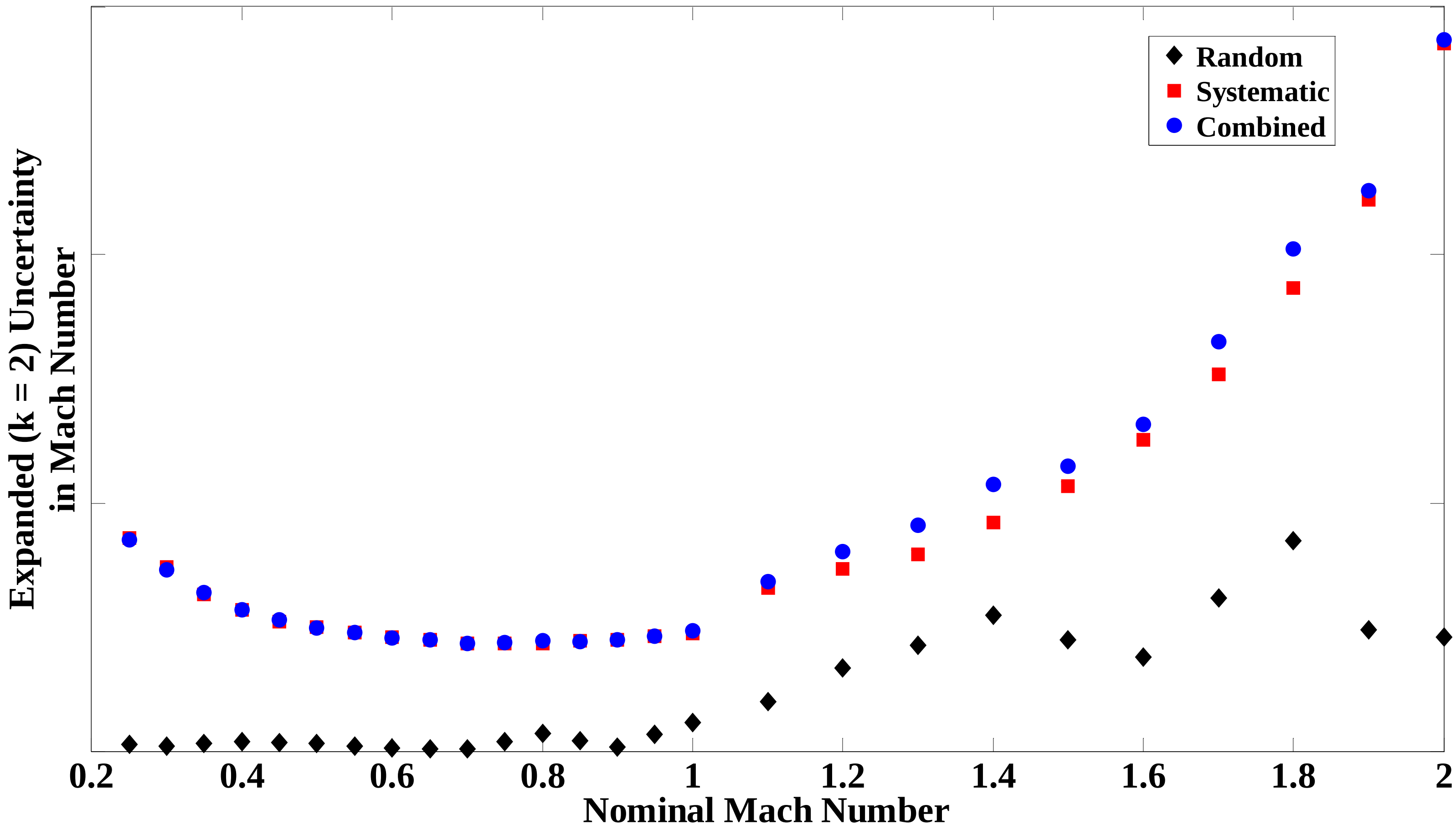
  - Customers testing for the effect of model changes will be more concerned about **random** uncertainty. These uncertainties can result in scatter about a mean value, and can be reduced by increasing sample size.

Example Results

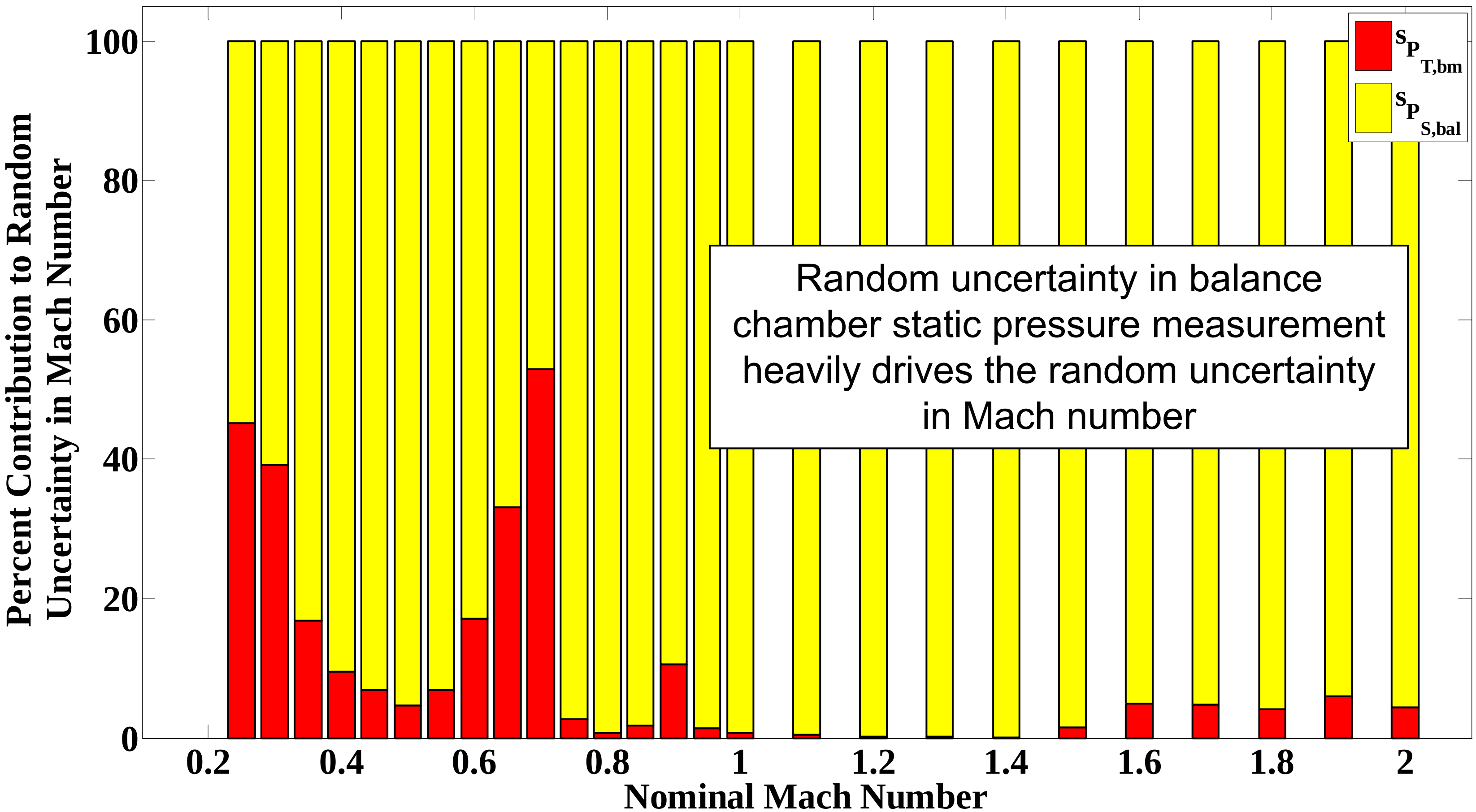
8x6 Supersonic Wind Tunnel:
Test Section Mach Number

Mach Number Uncertainty Results

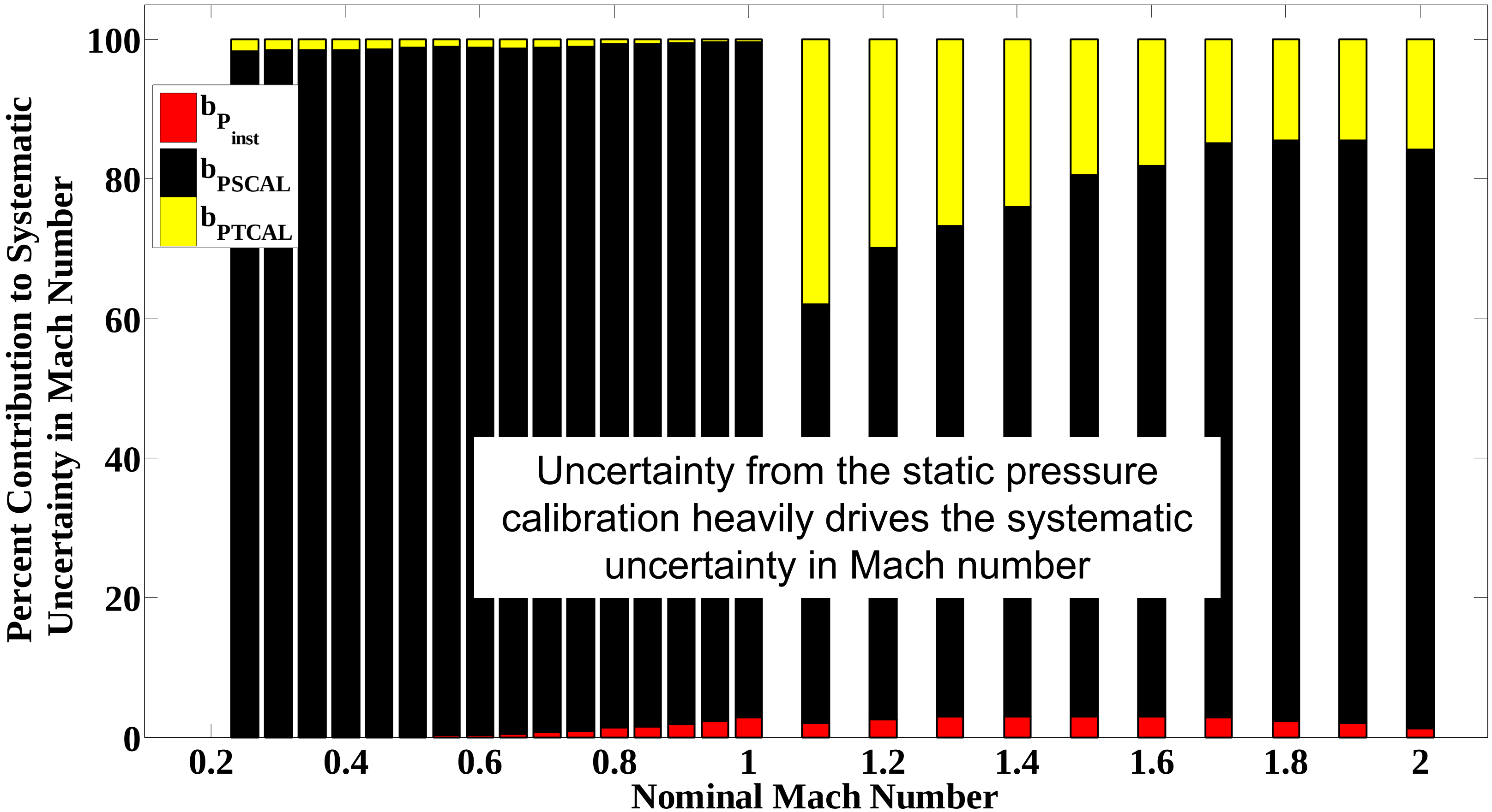
- Random and systematic uncertainty results are broken out separately. They add as root-sum-squares to obtain the combined uncertainty.



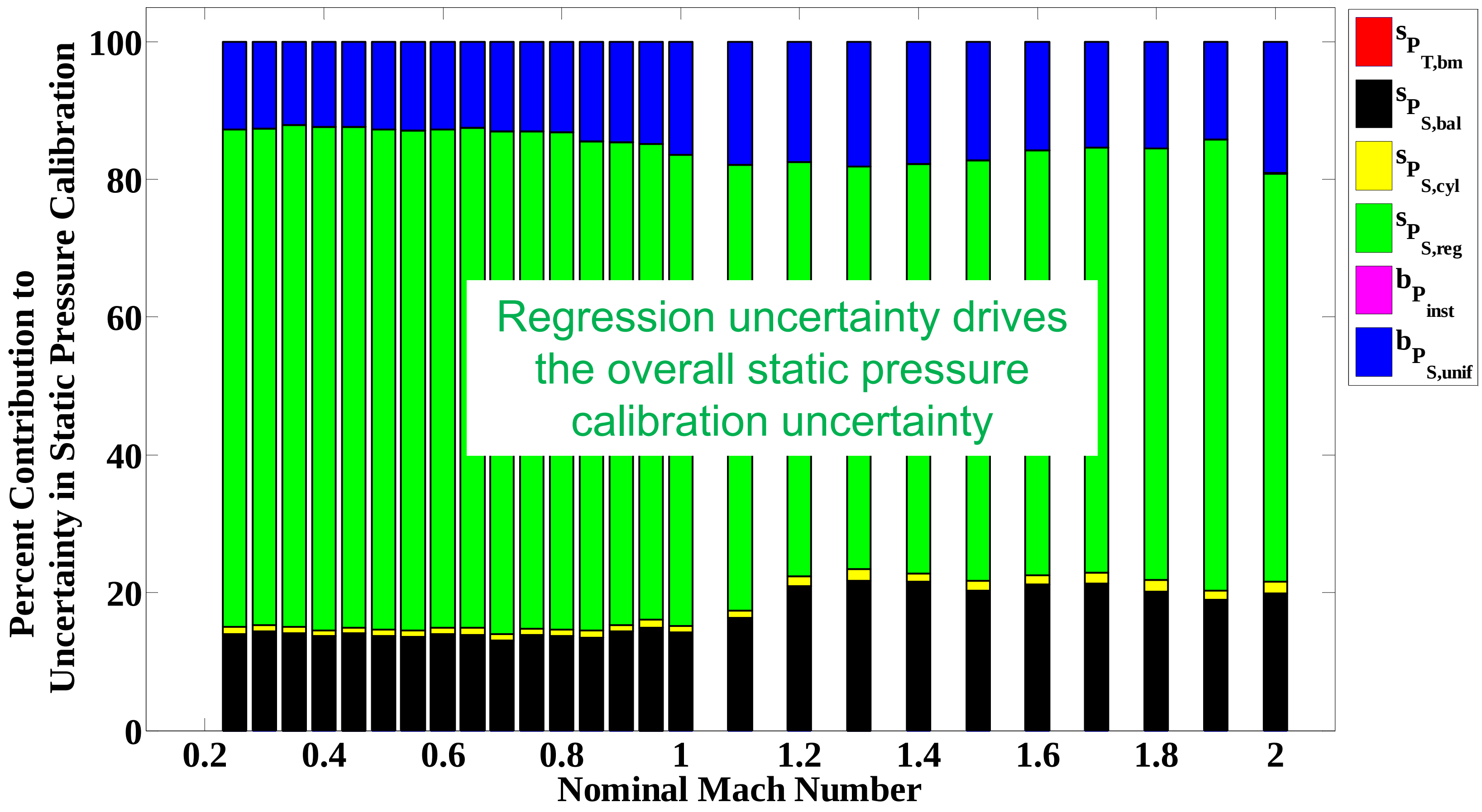
UPC to Random Uncertainty in Mach Number (Tunnel Configuration 1)



UPC of Systematic Uncertainty of Mach Number



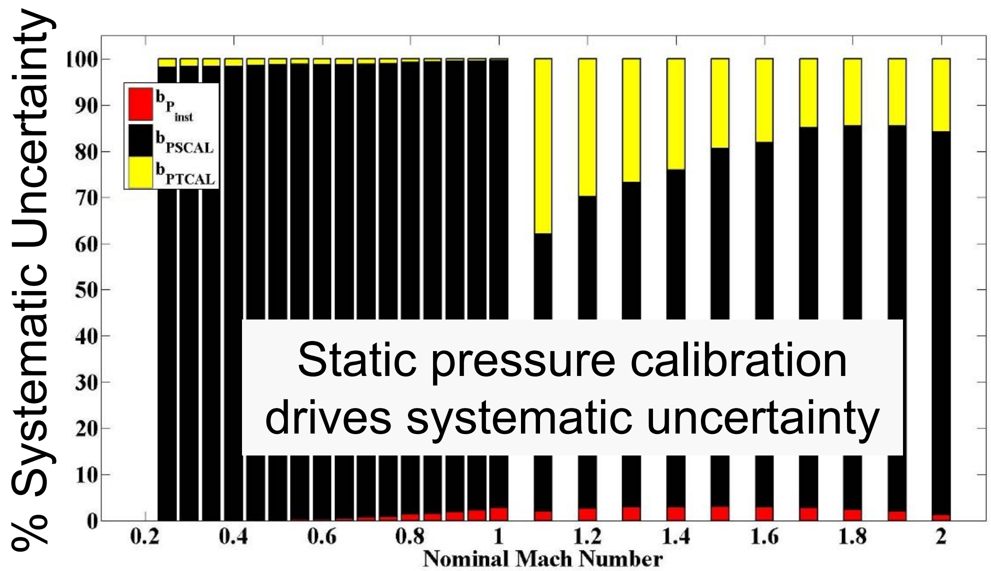
UPC of Static Pressure Calibration Uncertainty



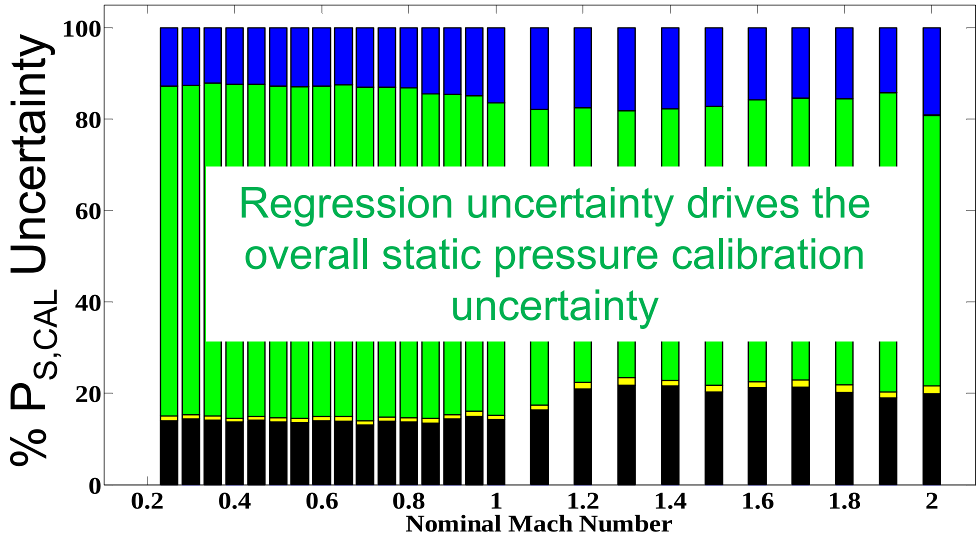
What-If Improvement Scenarios

Mach Number Uncertainty Improvement

- Studying the UPC results can give an idea of what scenarios should be explored to provide substantial changes and uncertainty improvements.



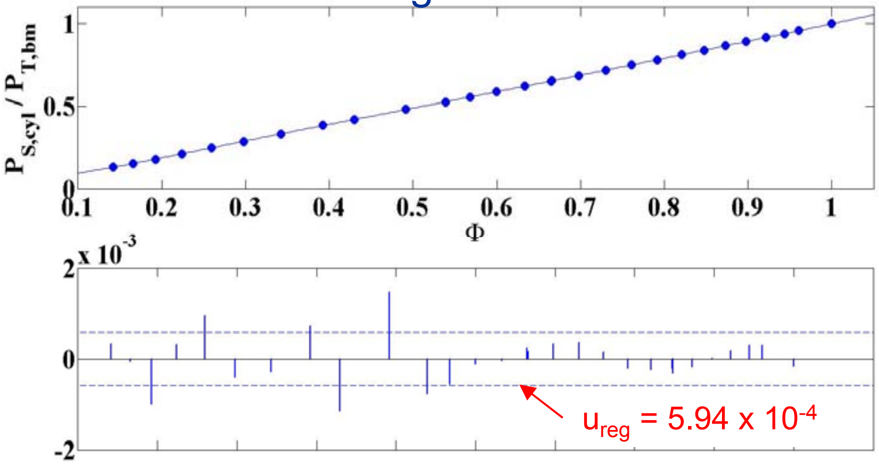
- In this case, scenarios that might improve uncertainty from the static pressure calibration, particularly the regression model, should be considered.



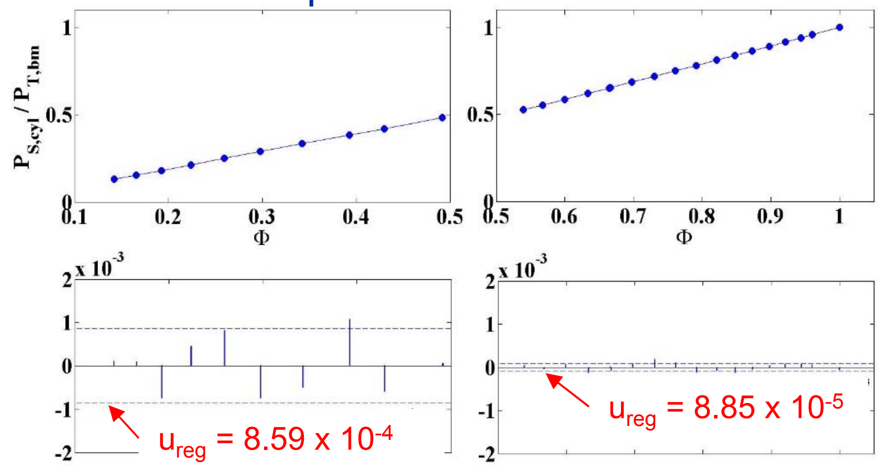
Mach Uncertainty Scenario 1a: Split Static Pressure Calibration Curve by Flow Regime

- Residual characteristics are different for the subsonic and supersonic portions of the calibration curve.
- The least squares process "correlates" these data points so the regression uncertainty must be applied across the entire range of the curve.
- This artificially inflates uncertainty results in the subsonic regime and deflates results in the supersonic regime.
- Since regression uncertainty drives the static pressure calibration uncertainty, it is of interest to see the uncertainty impact of splitting the calibration curves by flow regime.

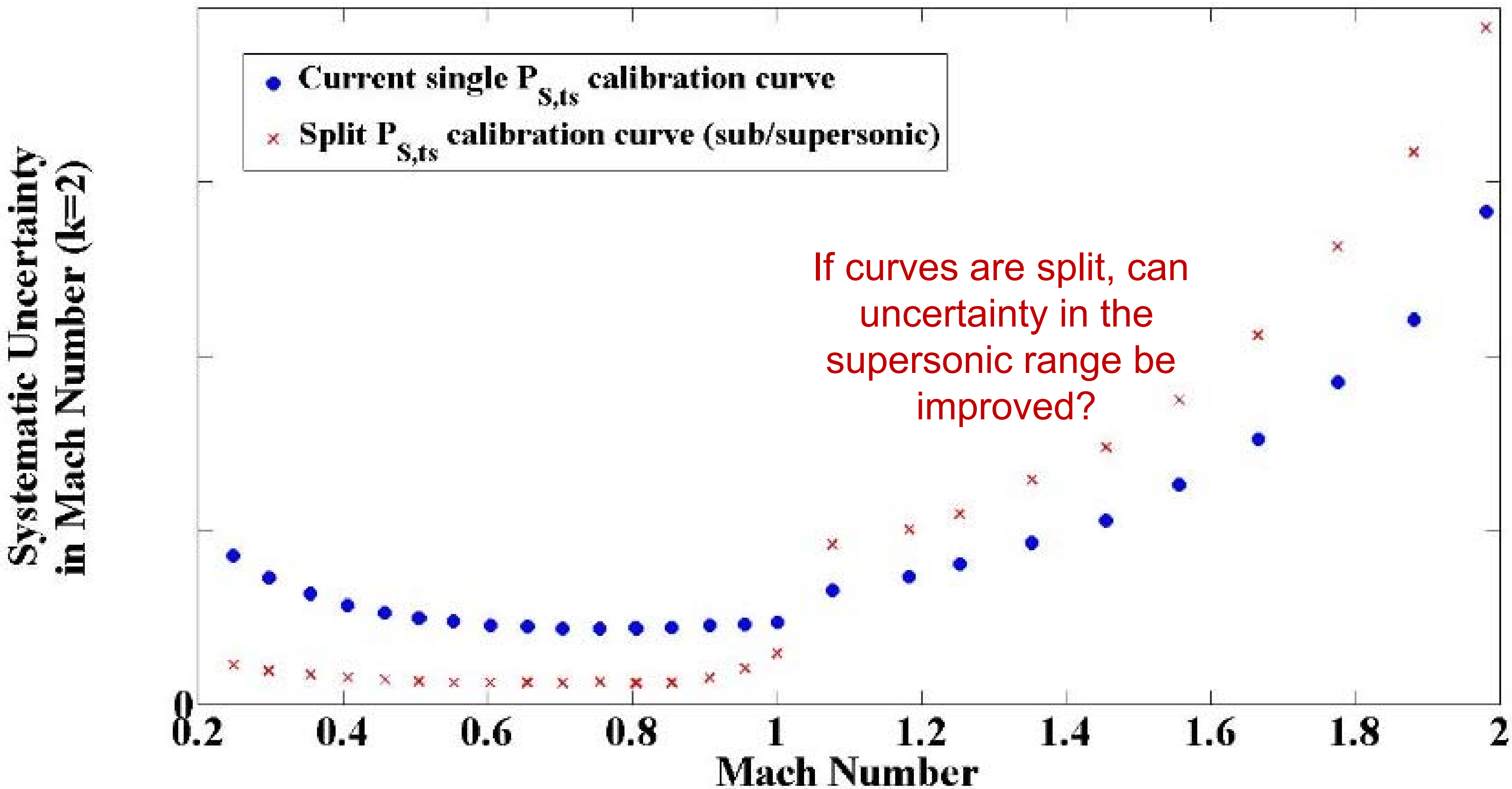
Current single calibration curve



Split calibration curves

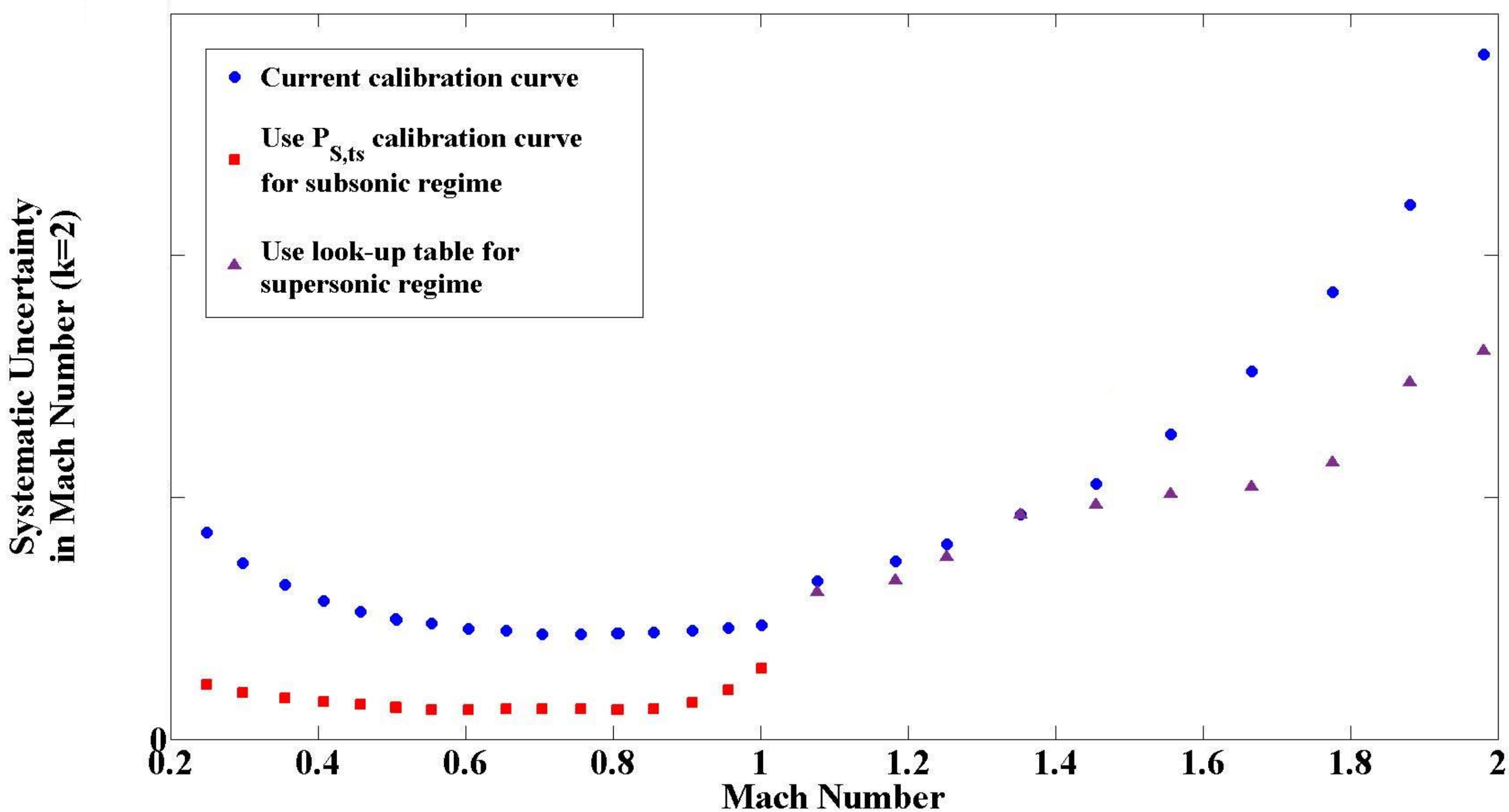


Mach Number Uncertainty Scenario 1a: Results



- For both flow regimes, this scenario indicates a more representative uncertainty result. Significant improvement in the subsonic regime suggests the calibration curve should be split.

Scenario 1b: Use look-up table for supersonic flow regime



Results provide uncertainty results that are 30-80% lower for most tunnel set points.

Use of look-up tables for the supersonic range will be implemented for both static and total pressure calibrations to improve uncertainty for future testing in 8x6SWT.

Total Temperature Uncertainty Scenario 2: Replace current temperature instrumentation & wires with higher accuracy hardware

- Researchers for a current test requested that we use MANTUS to quantify the effect of a thermocouple system upgrade on temperature measurement uncertainty in 8x6SWT

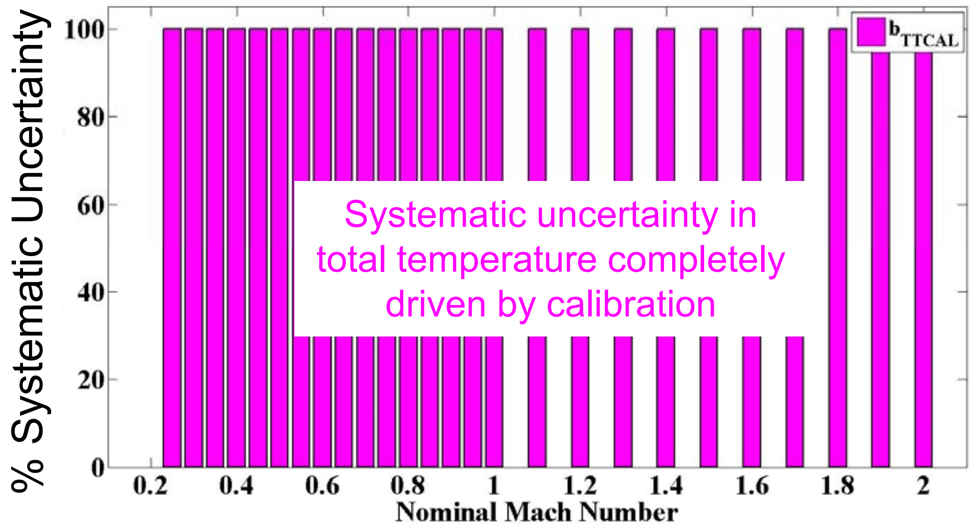
	Uncertainty (95% confidence)
Original TC/wire system	4.3 °F
NEW TC/wire system	0.5 °F

Thermocouple/wire system changes result:

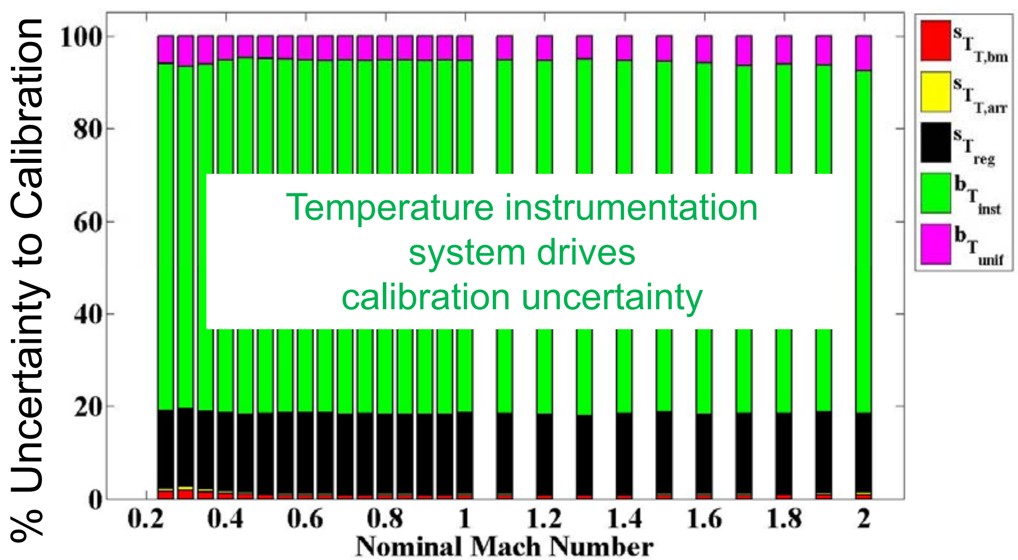
~85% decrease in instrument uncertainty

Calibrated Free Stream Total Temperature Uncertainty Improvement

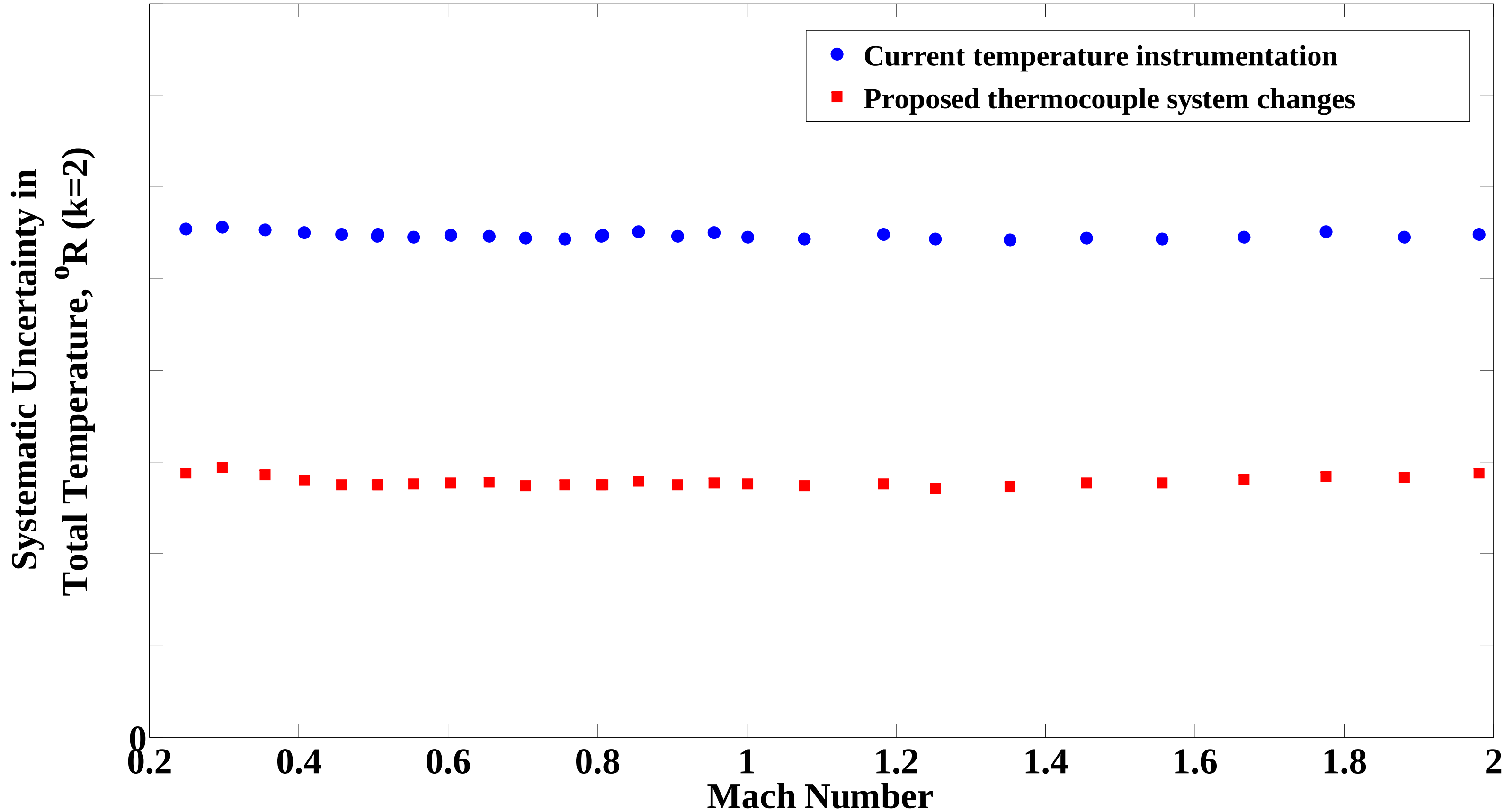
- Similar to Mach number uncertainty, by analyzing UPC plots for the calibrated total temperature, the drivers to uncertainty can be determined so that useful scenarios can be developed.



- In this case, the temperature instrumentation system is a clear driver to uncertainty in the calibrated free stream total temperature.



Total Temperature Uncertainty Scenario 3: Results



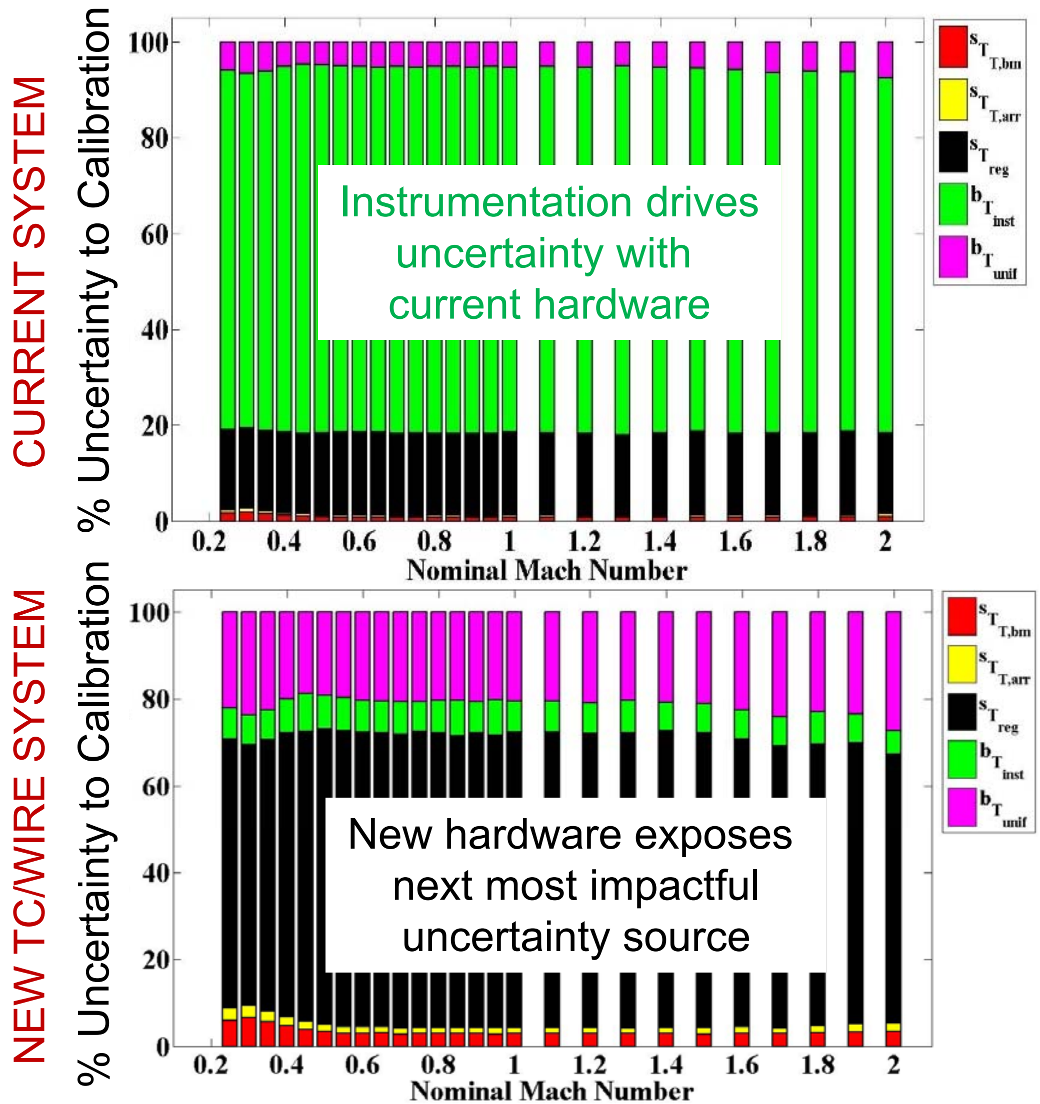
Free stream total temperature simulation results*:

50% decrease in uncertainty with new TC/wire system

*Note: simulated result using former temperature data; actual results likely lower

If thermocouple accuracy is 85% better, why is the calibrated free stream temperature result only 50% better?

- Test section total temperature is calculated using a calibration curve
- Uncertainty from measurements taken during the calibration are fossilized into the curve with several contributing elemental uncertainties; in this case, regression uncertainty begins to drive uncertainty once instrumentation is optimized.



Conclusions

- A thorough understanding of facility uncertainty requires both statistical process control and a bottom-up analysis of uncertainty propagation
- A rigorous analysis of uncertainty propagation provides
  - A quantitative understanding of the quality of the data
  - An understanding of the uncertainty sources
  - An understanding of the different aspects of uncertainty (repeatability vs bias)
- Utilizing a Monte Carlo approach allows for ease of implementation in complicated math models or where a lot of correlations are present.
- The Monte Carlo also allows a straight forward process for investigating potential scenarios for facility improvement.

References

- Joint Committee for Guides in Metrology, 'Evaluation of Measurement Data — Guide to the Expression of Uncertainty in Measurement', JCGM/WG 1, 2008.
- National Aeronautics and Space Administration, 'Measurement Uncertainty Analysis Principles and Methods', NASA, Washington DC, 2010.
- H. Coleman, W. Steele and H. Coleman, *Experimentation, validation, and uncertainty analysis for engineers*. Hoboken, N.J.: John Wiley & Sons, 2009.
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- J. Devore, *Probability and statistics for engineering and the sciences*. Monterey, Calif.: Brooks/Cole Pub. Co., 1982.
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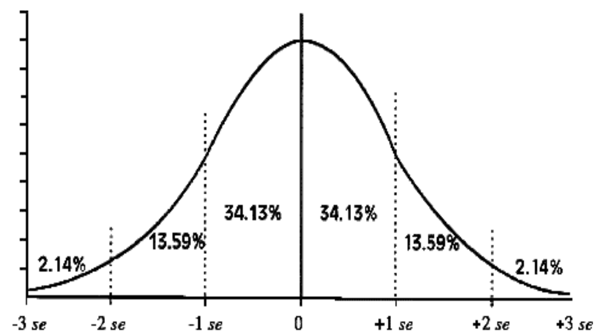
- American Institute of Aeronautics and Astronautics. “Assessment of Experimental Uncertainty with Application to Wind Tunnel Testing”. Standard AIAA S-071A-1999,1999
- J.L. Devore. Probability and Statistics for Engineering and the Sciences. Brooks/Cole,Fifth edition, 2000.

Supplemental Slides

Confidence Intervals and Degrees of Freedom

- Confidence interval
    - The probabilistic determination of an outcome. Often expressed as the percentage area under a distribution curve.

- Degrees of freedom
    - Quantification of the independence of a data set.
    - Defined most commonly as sample size – 1, (n-1)



jalt.org

Standard uncertainty x Coverage factor (k) = Expanded uncertainty

1.220 ºC x 2 (for 95% coverage) = 2.440 ºC

4 temperature measurements in the bellmouth

$b_{T_{T,bm}}$ → $T_{T,bm,1-4}$

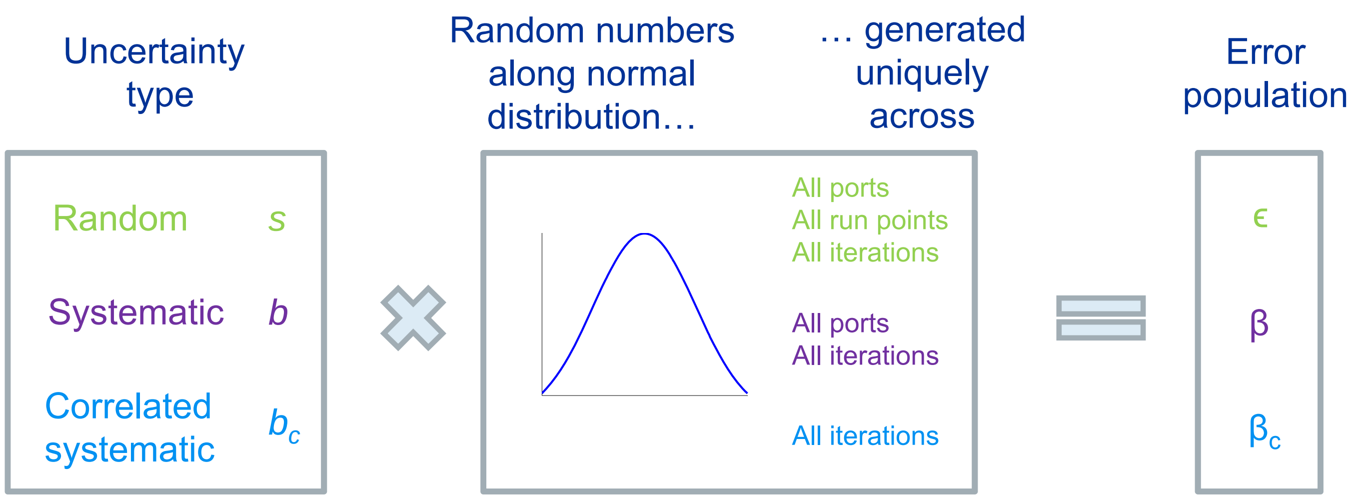
Generated from random number population with normal distribution, mean=0, and σ=1

			Monte Carlo ITERATION 1			Monte Carlo ITERATION 2		
TC name	Measured Temp, °C	Standard uncertainty, °C	Random number	Error, °C	Perturbed measured temp, °C	Random number	Error, °C	Perturbed measured temp, °C
$T_{T,bm(1)}$	100	1.22	-0.40	-0.50	99.50	0.51	0.64	100.63
$T_{T,bm(2)}$	100	1.22	-1.08	-1.33	98.67	-1.62	-2.00	98.03
$T_{T,bm(3)}$	100	1.22	1.07	1.32	101.32	0.97	1.20	101.18
$T_{T,bm(4)}$	100	1.22	0.10	0.13	100.13	1.77	2.19	102.16

Standard uncertainty × Random number = Error

Monte Carlo Analysis: Populating Errors

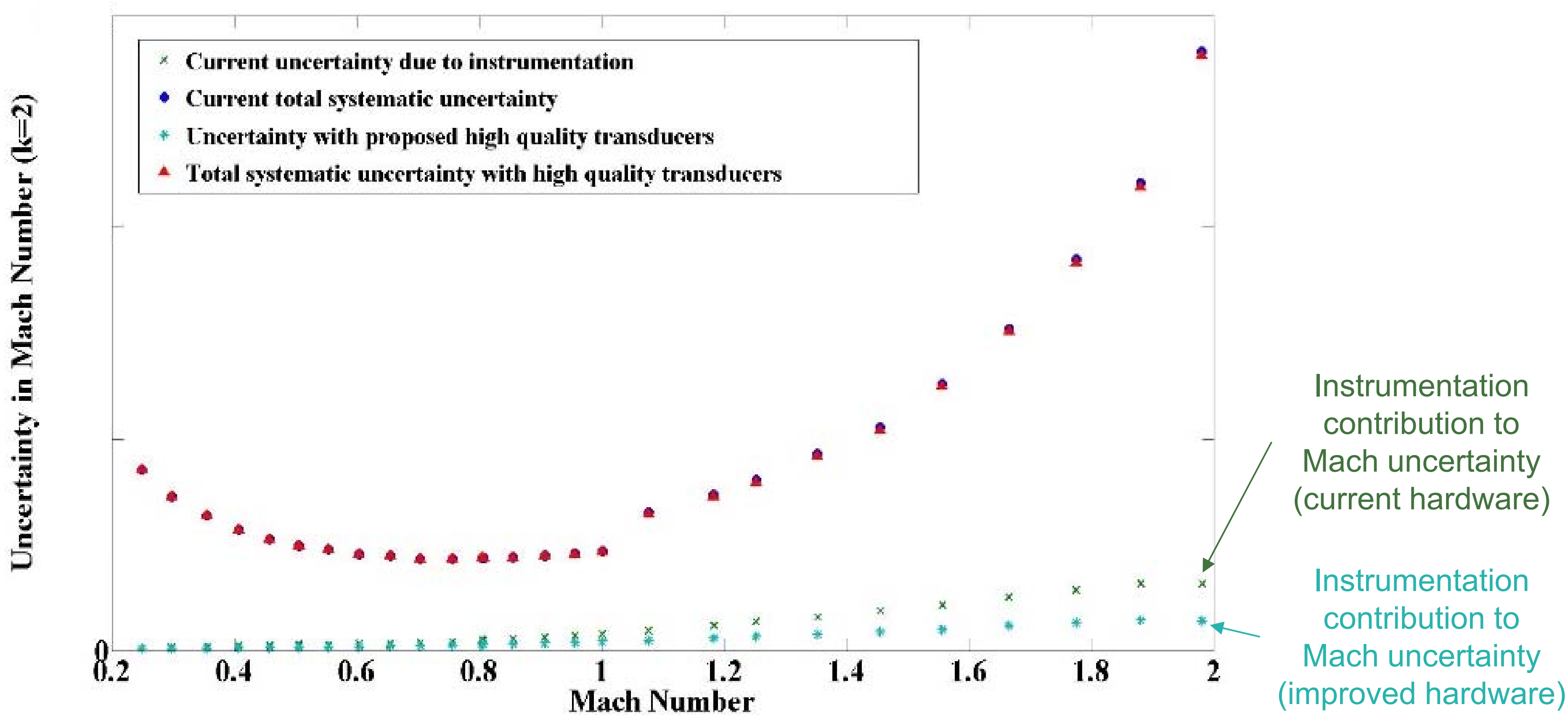
- Appropriately populating errors is *critical* to the integrity of the Monte Carlo approach to error propagation.
- If errors are populated correctly, correlated errors are inherently handled within the data reduction.
  - Taylor Series approach requires correlations to be handled overtly.



Mach Number Uncertainty Scenario 2: Replace current pressure instrumentation with higher accuracy instrumentation

- Often when facilities are interested in improving uncertainty, improving the quality of instrumentation is high on the list.
- This scenario explores the effect of improving instrumentation such that the instrument system uncertainty is 0.02% reading.
- Is the benefit (magnitude of decrease in Mach uncertainty) proportional to the cost of such a change?

Mach Number Uncertainty Scenario 2: Results



- Even though the instrumentation contribution to Mach is 50% lower, no appreciable change in systematic Mach number uncertainty is observed with this scenario because instrumentation is a small contributor to Mach number uncertainty.