Facility Measurement Uncertainty Analysis at NASA GRC

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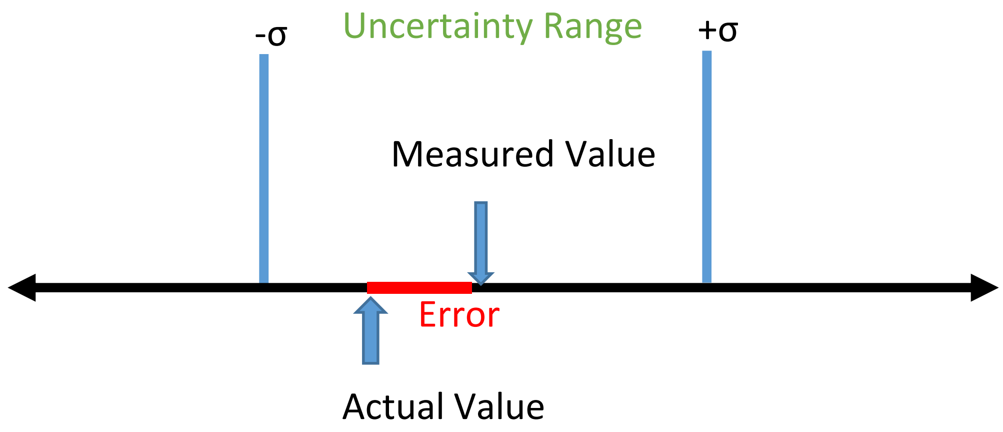
Importance of MUA

Understanding not just value, but also the process to obtain the value provides a greater understanding of the data acquired in the facilities.

Qualitative questions:	**Quantitative Answers:**
How good is the data?	+/- error limits on critical instruments *and* calculated values of interest
What are the facility's strengths and weaknesses?	Characterization of critical facility instruments and parameters
What instrumentation is best to measure…?	Quantification of instrumentation chain accuracy
What methods are best to measure…?	Determine percent contributions of uncertainty sources for clear understanding of where improvements should be made

Error Vs. Uncertainty

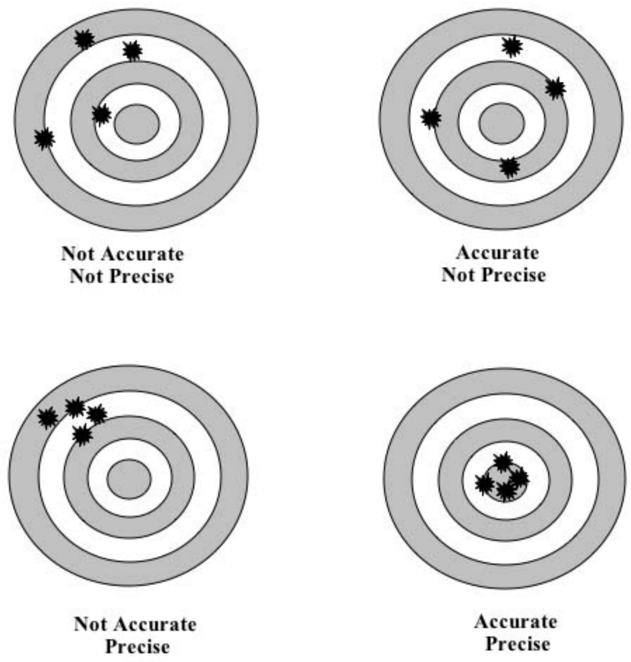
- Error of a measurement: the difference between the measured value and the unknown true value.
- Uncertainty of a measurement: an estimate of the range within which the actual value could fall, and the probability that it falls within that range[2].



[2] H. Coleman, W. Steele and H. Coleman, *Experimentation, validation, and uncertainty analysis for engineers*. Hoboken, N.J.: John Wiley & Sons, 2009.

Accuracy vs. Precision

- Accuracy:
  the ability to hit a specified point
- Precision:
  the ability to hit a consistent point.
- The two situations are not exclusive, you can have highly precise data which is not accurate and vise versa[3].

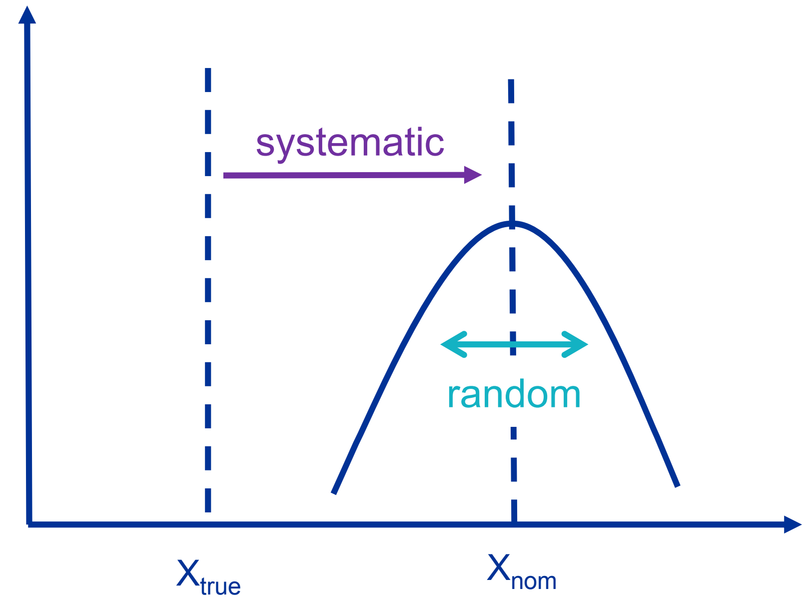


NOAA.gov

[3] Joint Committee for Guides in Metrology, 'Evaluation of Measurement Data — Guide to the Expression of Uncertainty in Measurement', JCGM/WG 1, 2008.

Uncertainty Type Classification

- **Type A**: evaluate by statistical analysis of observations
- **Type B**: evaluate by other means (based on calibration certificates, past experience, etc.)
- **Random**: the scatter of the results (repeatability, precision, scatter)
- **Systematic**: standard offset (bias, accuracy)



Customers looking to compare test results with CFD results are more concerned with systematic uncertainty effects.

Customers testing for the effect of model changes will be more concerned about random uncertainty effects.

Approaches to Uncertainty:

Statistical Process Control

- A quality control method which uses statistical techniques for regulation, characterization, and optimization of a process[3].
- Includes facility characterization and check standards
- Important for maintaining quality over time

Ground-up Analysis

- Analyze available data and spec. sheets to determine elemental uncertainties, then propagate through equations to values of interest.
- Powerful tool for determining both over-all and itemized uncertainty.
- Easy to implement “what if…?” scenario simulations for cost-benefit analysis for potential improvements

[3]J. Devore, *Probability and statistics for engineering and the sciences*. Monterey, Calif.: Brooks/Cole Pub. Co., 1982.

Approaches to Uncertainty, continued:

Statistical Process Control

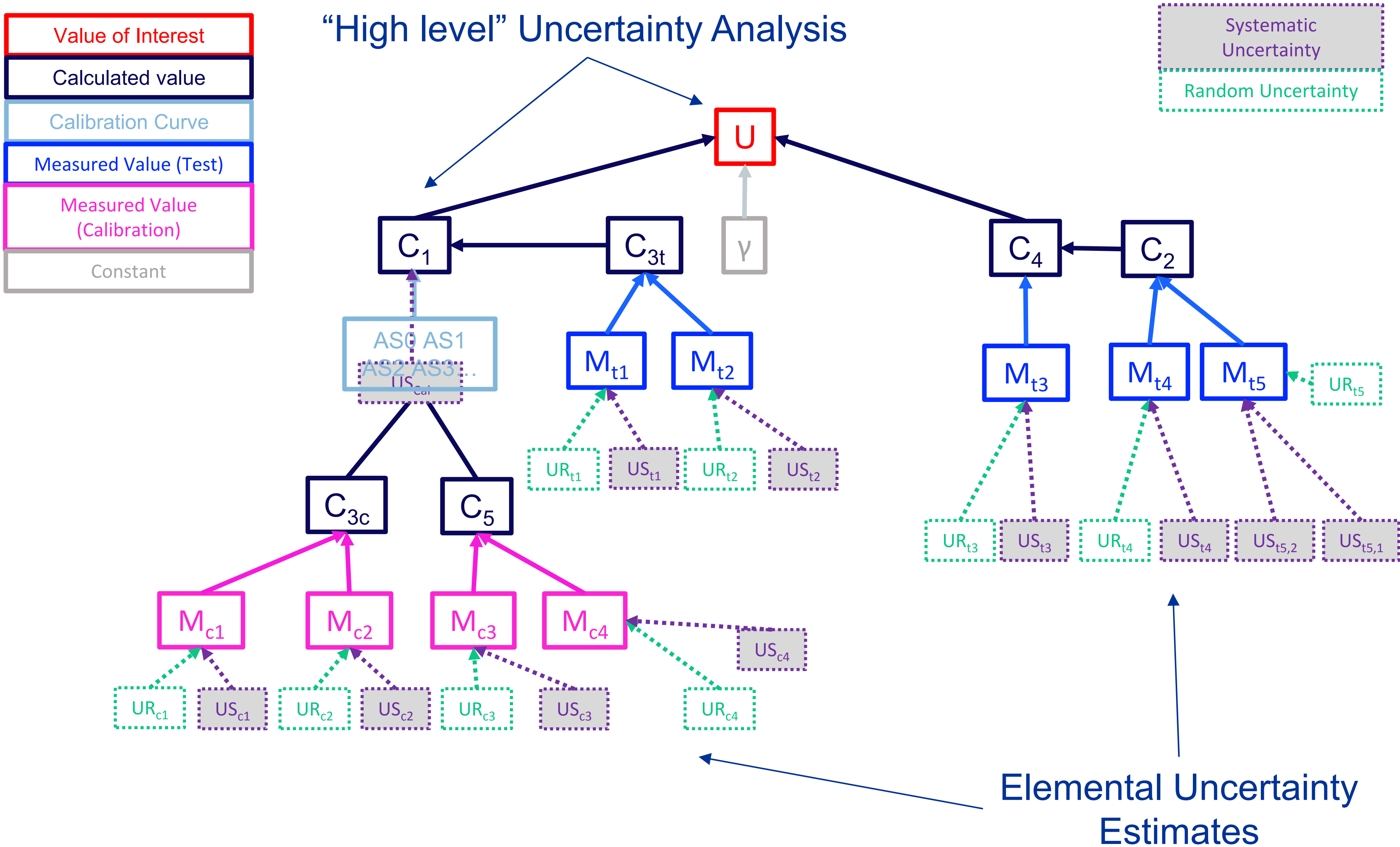
- Great at characterizing repeatability
- Ignores some systematic uncertainties
- Very difficult to separate out individual uncertainty sources
- Optimistic Results

Ground-up Analysis

- Output quality is based on input quality (elemental uncertainty estimates)
- Straight-forward process for adding new data as it becomes available
- Conservative Results

Ideally, both approaches should be implemented. When used together, uncertainty estimates are more accurate and better understood, and methods of reducing the uncertainty further are more apparent.

Analysis: Uncertainty Propagation

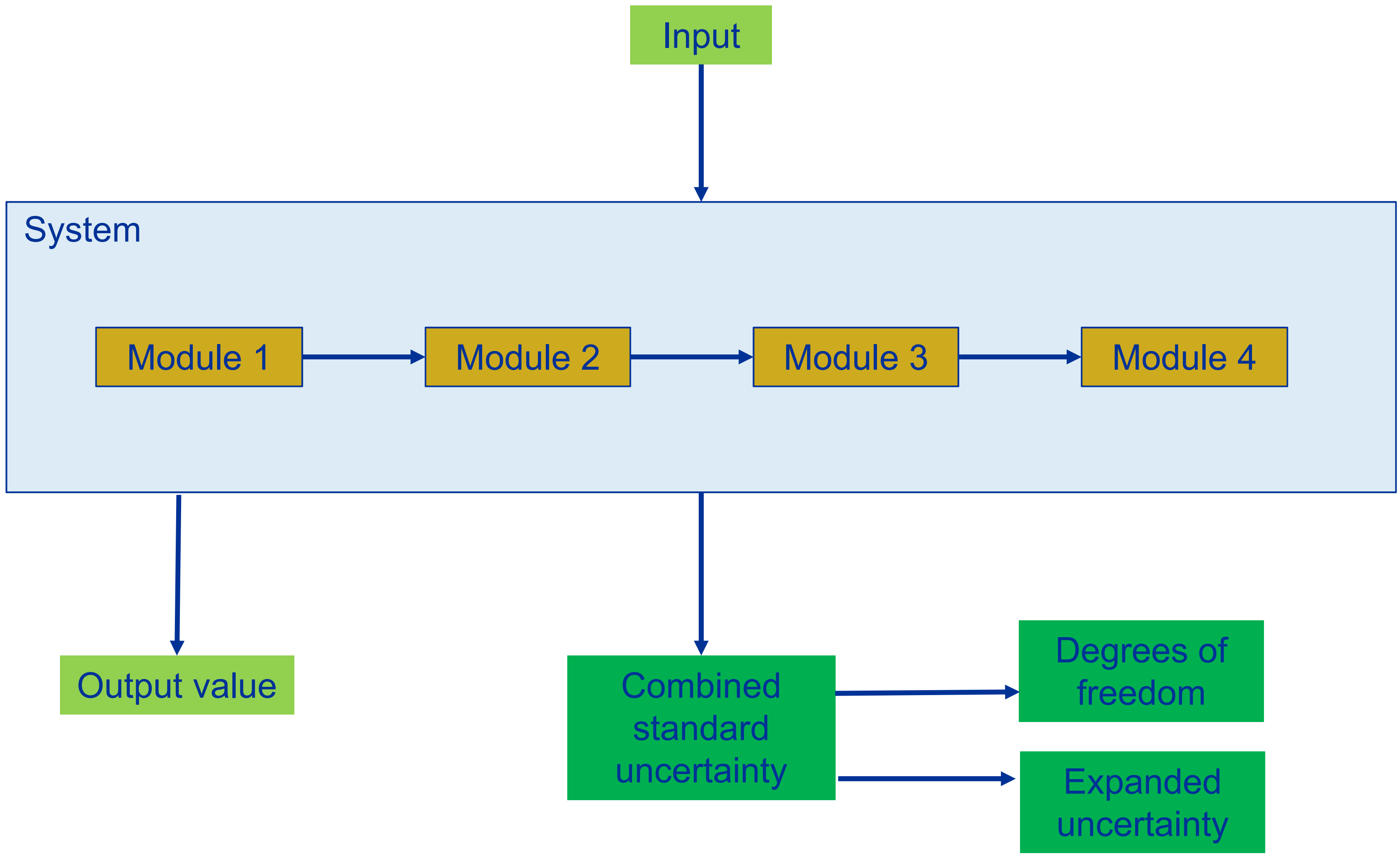


MANTUS

Measurement ANalysis Tool for Uncertainty in Systems

- A modular approach at modeling measurement systems.
- Based on NASA-HDBK-8739.19-3
- Each block represents a single piece of instrumentation in the signal measurement channel.
- The scope of the tool is to model and analyze a single, representative measurement channel such as one transducer or thermocouple connected to a data system.
- February 23, 2016: MANTUS Rev 2.0 released as a “beta” version (MANTUS 2.0) to GRC Facilities E-Team with a provided training course
  - Rolling release to “super users” to build modules for accessible library
  - Rolling release to standard users who will build systems from elements in the module library

MANTUS


Input
System
Module 1
Module 2
Module 3
Module 4
Output value
Combined standard uncertainty
Degrees of freedom
Expanded uncertainty

Estimate Elemental Uncertainties

- Systematic Uncertainties due to Instrumentation: MANTUS!
- Random Uncertainties of measured variables: Statistical analysis of data.
  - Population Standard deviation:

$$s_X = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(X_i - \bar{X})^2}$$

  - Must be measured over an appropriate time scale to capture desired random effects (back-to-back measurements are not considered distinct)
  - Estimate for small sample size:

$$s_x \cong \sigma_x = \frac{x_{max} - x_{min}}{d_2(n)}$$

- Other systematic considerations: spatial uniformity, calibration curves, etc.

Uncertainty Propagation Methods

Taylor Series Method

- Analytical method used to develop a model for system behavior.
- Sensitivity coefficients are calculated to define relationship between changes in variables and the resulting output.
- Elemental uncertainties are attributed to data reduction equation variables and combined accordingly.
- Uncertainty is combined for the whole system to produce a uncertainty estimate.

$$A = \pi r^2 \qquad U_A^2 = \left(\frac{\partial A}{\partial r}\right)^2 * b_r^2 + \left(\frac{\partial A}{\partial r}\right)^2 * s_r^2$$

Pros
- Fast for simple models
- Commonly used

Cons
- Analysis complication increases exponentially with complication of model.

Mass Flow:

$$m_{OR} = C_1\left(1 - \frac{C_2 P_D}{P^2}\right)\sqrt{\frac{P P_D}{T}}$$

$$P = P_{bar} - P_a,$$

$$P_a = \frac{1}{2}(P_{a1} + P_{a2}),$$

$$P_d = \frac{1}{2}(P_{d1} + P_{d2}),$$

$$T = \frac{1}{4}(T_1 + T_2 + T_3 + T_4)$$

Systematic Uncertainties:

- By Taylor series

$$b_{Pa} = \sqrt{\left(\frac{\partial P_a}{\partial P_{a1}}\right)^2 b_{Pa1,unc}^2 + \left(\frac{\partial P_a}{\partial P_{a2}}\right)^2 b_{Pa2,unc}^2 + 2\left(\frac{\partial P_a}{\partial P_{a1}}\right)\left(\frac{\partial P_a}{\partial P_{a2}}\right) b_{Pa1,corr} b_{Pa2,corr}}$$

$$b_P = \sqrt{\left(\frac{\partial P}{\partial P_a}\right)^2 b_{Pa}^2 + \left(\frac{\partial P}{\partial P_{bar}}\right)^2 b_{Pbar}^2}$$

$$b_{PD} = \sqrt{\left(\frac{\partial P_D}{\partial P_{D1}}\right)^2 b_{PD1,unc}^2 + \left(\frac{\partial P_D}{\partial P_{D2}}\right)^2 b_{PD2,unc}^2 + 2\left(\frac{\partial P_D}{\partial P_{D1}}\right)\left(\frac{\partial P_D}{\partial P_{D2}}\right) b_{PD1,corr} b_{PD2,corr}}$$

$$b_T = \sqrt{\begin{array}{l} \left(\frac{\partial T}{\partial T_1}\right)^2 b_{T1,unc}^2 + \left(\frac{\partial T}{\partial T_2}\right)^2 b_{T2,unc}^2 + \left(\frac{\partial T}{\partial T_3}\right)^2 b_{T3,unc}^2 + \left(\frac{\partial T}{\partial T_4}\right)^2 b_{T4,unc}^2 \\ +2\left(\frac{\partial T}{\partial T_1}\right)\left(\frac{\partial T}{\partial T_2}\right) b_{T1,corr} b_{T2,corr} + 2\left(\frac{\partial T}{\partial T_1}\right)\left(\frac{\partial T}{\partial T_3}\right) b_{T1,corr} b_{T3,corr} \\ +2\left(\frac{\partial T}{\partial T_1}\right)\left(\frac{\partial T}{\partial T_4}\right) b_{T1,corr} b_{T4,corr} + 2\left(\frac{\partial T}{\partial T_2}\right)\left(\frac{\partial T}{\partial T_3}\right) b_{T2,corr} b_{T3,corr} \\ +2\left(\frac{\partial T}{\partial T_2}\right)\left(\frac{\partial T}{\partial 4}\right) b_{T2,corr} b_{T4,corr} + 2\left(\frac{\partial T}{\partial T_3}\right)\left(\frac{\partial T}{\partial T_4}\right) b_{T3,corr} b_{T4,corr} \end{array}}$$

Uncertainty Propagation Methods (continued)

Monte Carlo Method

- Iterative method where a distribution of random numbers is applied to each elemental error source creating a synthetic error population
- The resulting sample of possible values is used in place of the original variable in the transfer function.
- With a sufficiently large number of iterations, the average of the calculated output represents the most likely result ("nominal" value).
- The standard deviation of the resulting outputs represents the standard uncertainty of the transfer function output.

Pros

- Simpler for more complex calculations
- Flexible for "what if" modeling

Cons

- Computation time

Random Population of radius(r)

r1
r2
r3
.
.
.
r(n)

$$\pi r^2 = A$$

Resulting Area(A)

A1
A2
A3
.
.
.
A(n)

$$\bar{A} = \frac{\sum A_i}{n}$$

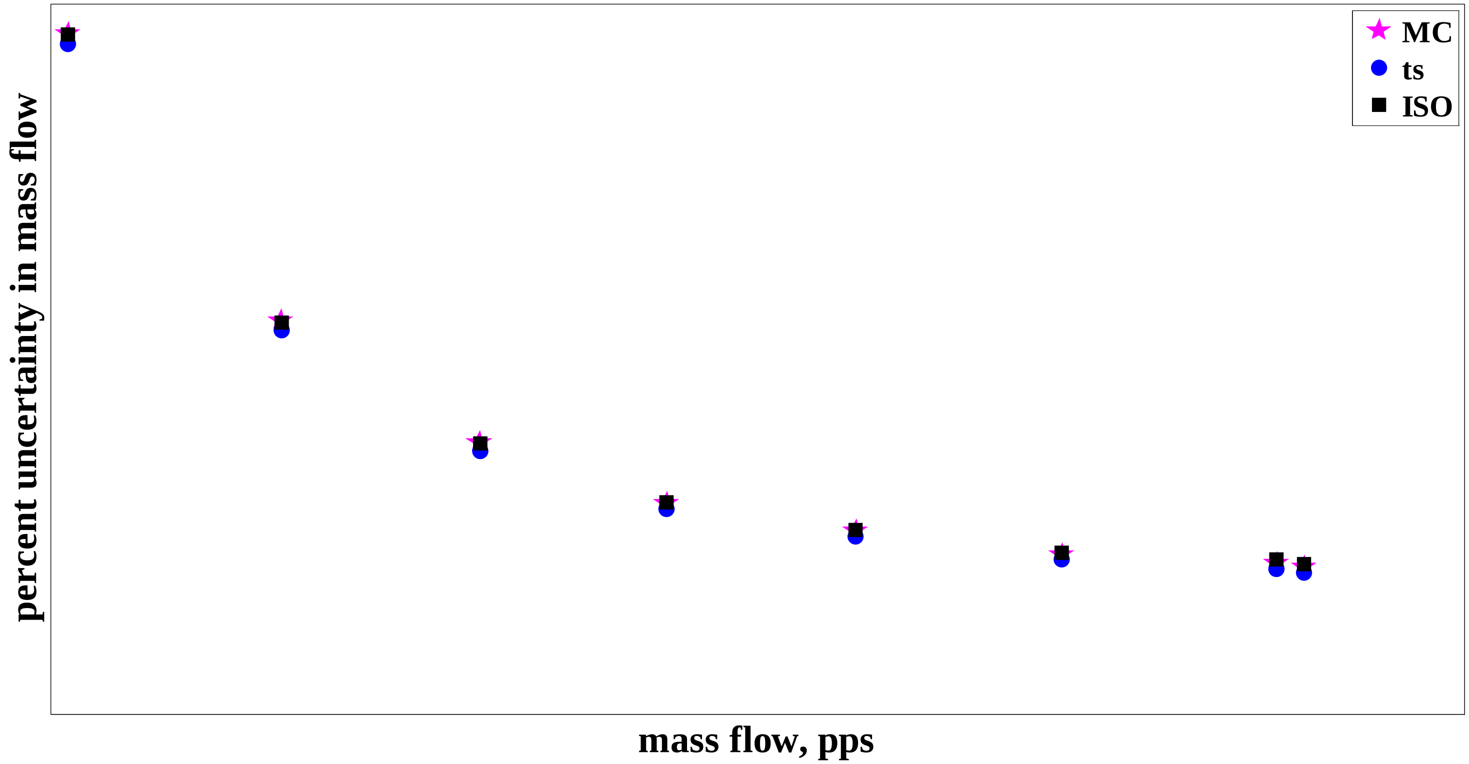
$$\bar{A} = A_{nom}$$

$$\sigma_A = \sqrt{\frac{\sum (A_i - \bar{A})^2}{n}}$$

$$\sigma_A = u_A$$

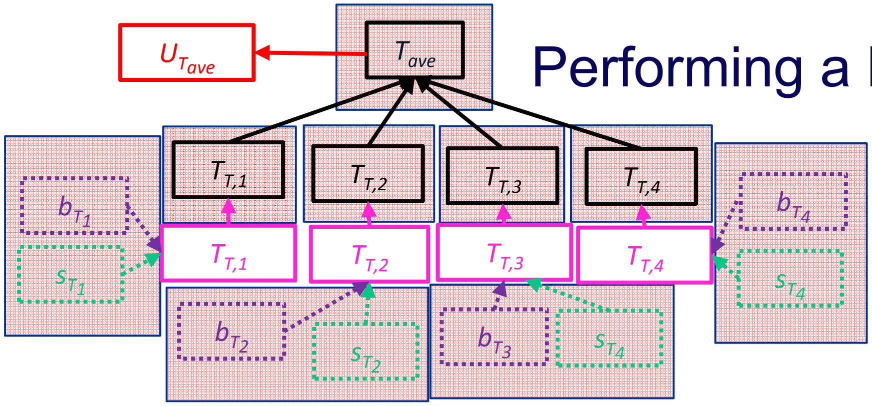
You get the same result

Example Method Comparison, mass flow



back

Performing a Monte Carlo Analysis



Input "true" values of variables

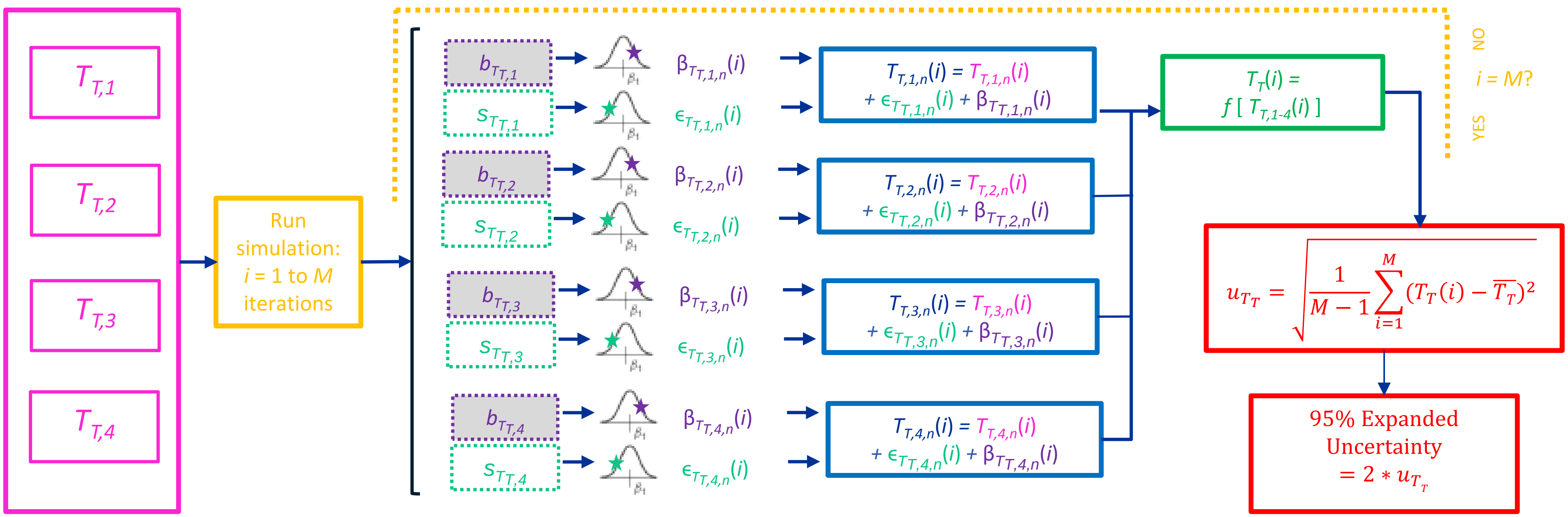
Input random and systematic uncertainties for each variable

Randomly generate an error along error distribution for each uncertainty source

Apply error to appropriate variables

Calculate result of value of interest from applicable data reduction equation

Calculate standard deviation of the value of interest



Presenting Results

- By “flagging” the uncertainties appropriately within the Monte Carlo code, the contribution of individual uncertainties or groups of uncertainties to the total uncertainty of the value of interest can be determined.
- Presenting the uncertainties as non-dimensional Uncertainty Percent Contributions (UPCs) in progressively smaller sub-groups is useful in determining the sources with the most impact.

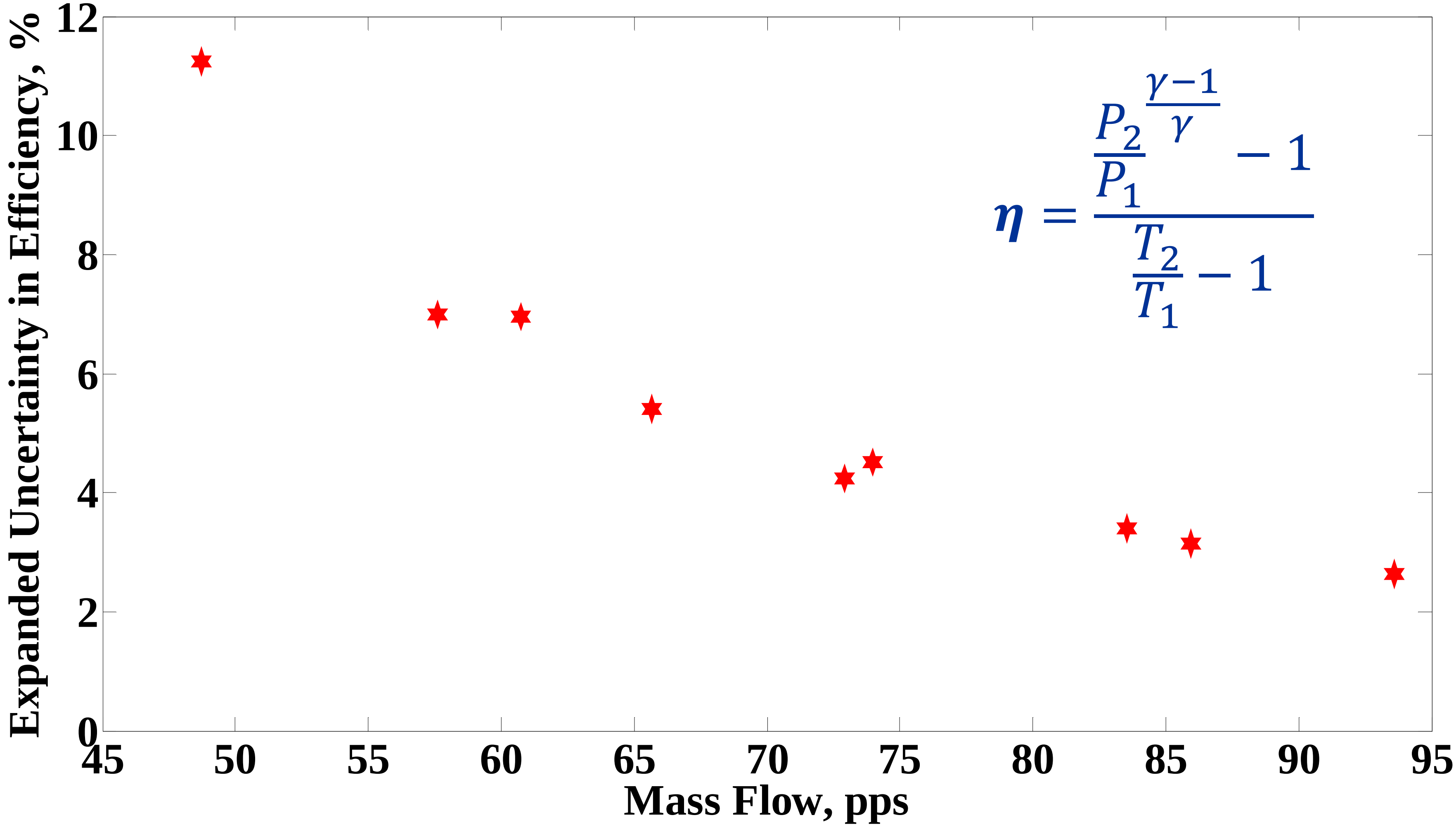
  - Customers looking to compare test results with CFD results are more concerned with **systematic** uncertainty. These uncertainties can result in a bias in measurements and calculated variables from an expected outcome.

  - Customers testing for the effect of model changes will be more concerned about **random** uncertainty. These uncertainties can result in scatter about a mean value, and can be reduced by increasing sample size.

Example Results

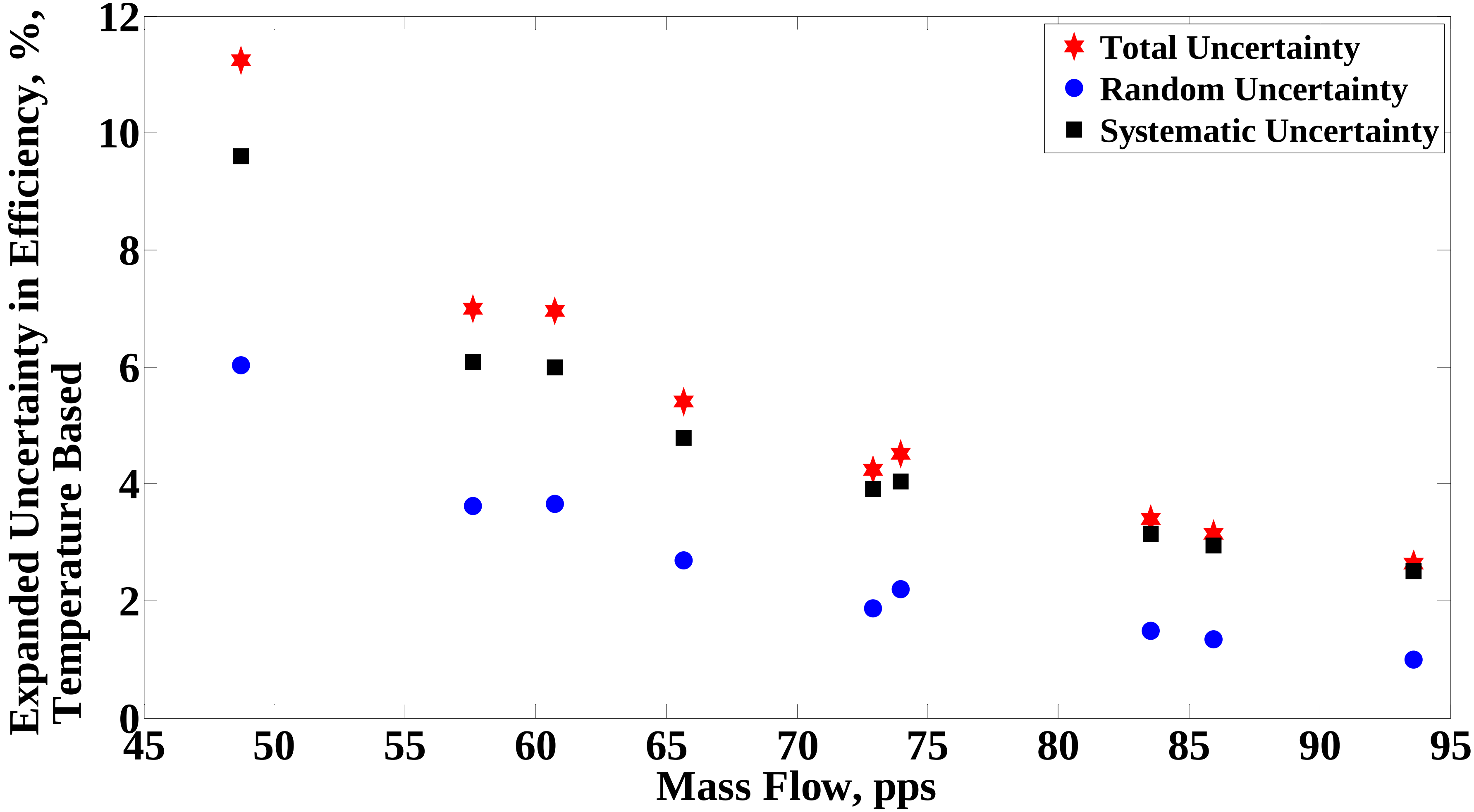
Efficiency

Adiabatic Efficiency Example: Temperature Based

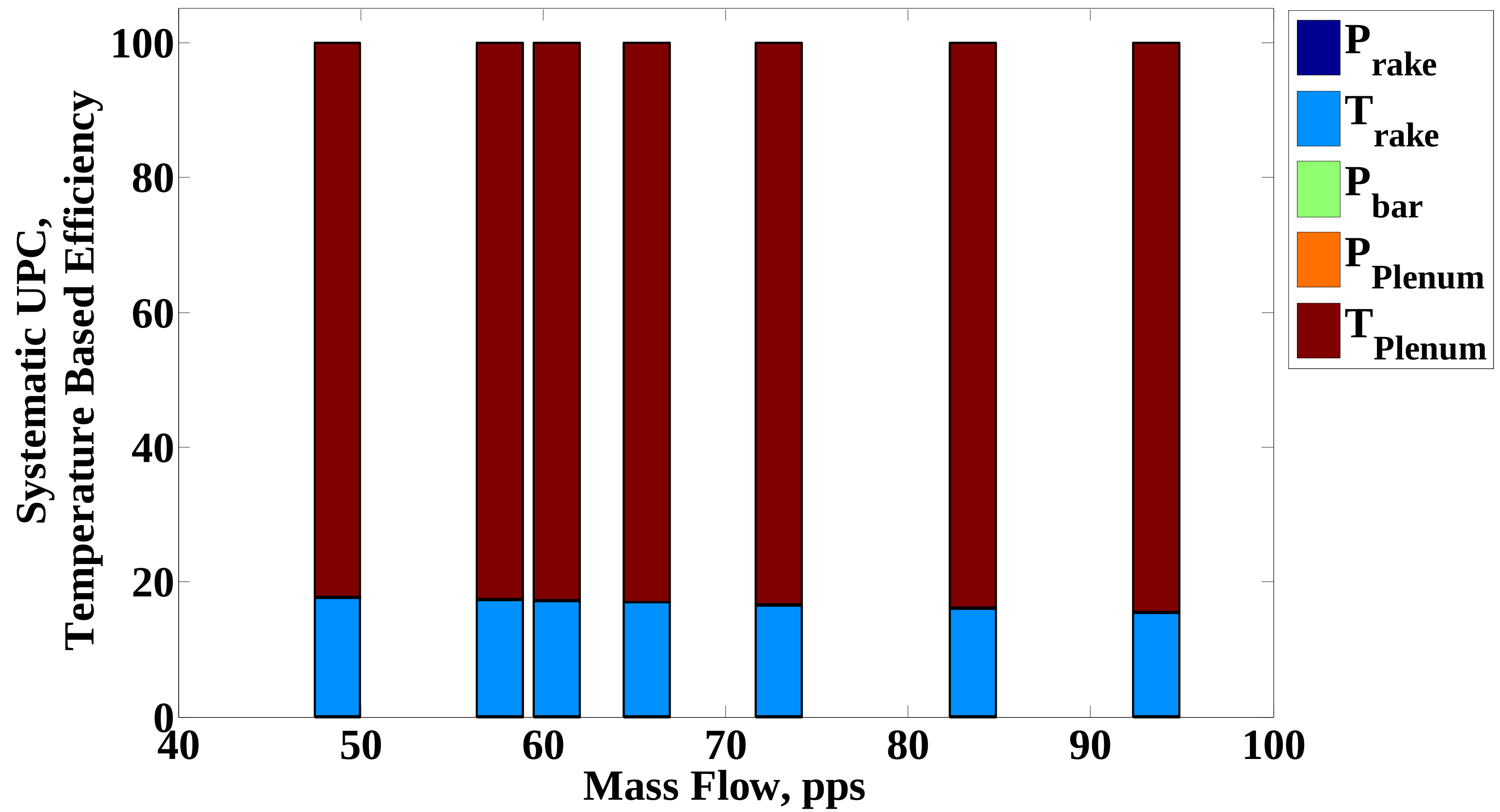


$$\eta = \frac{\frac{P_2}{P_1}^{\frac{\gamma-1}{\gamma}} - 1}{\frac{T_2}{T_1} - 1}$$

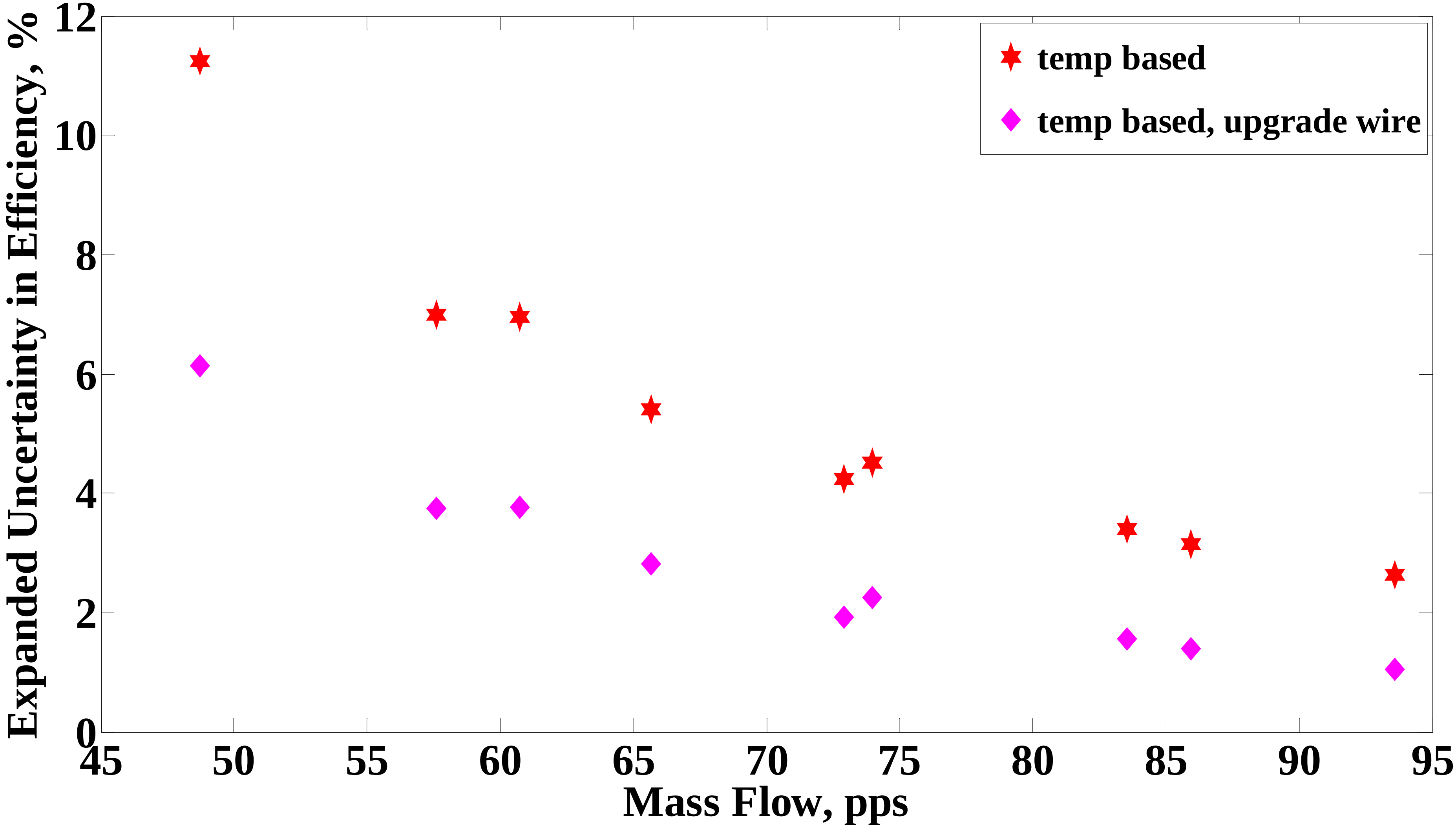
Temperature Based Uncertainty: Random vs. Systematic



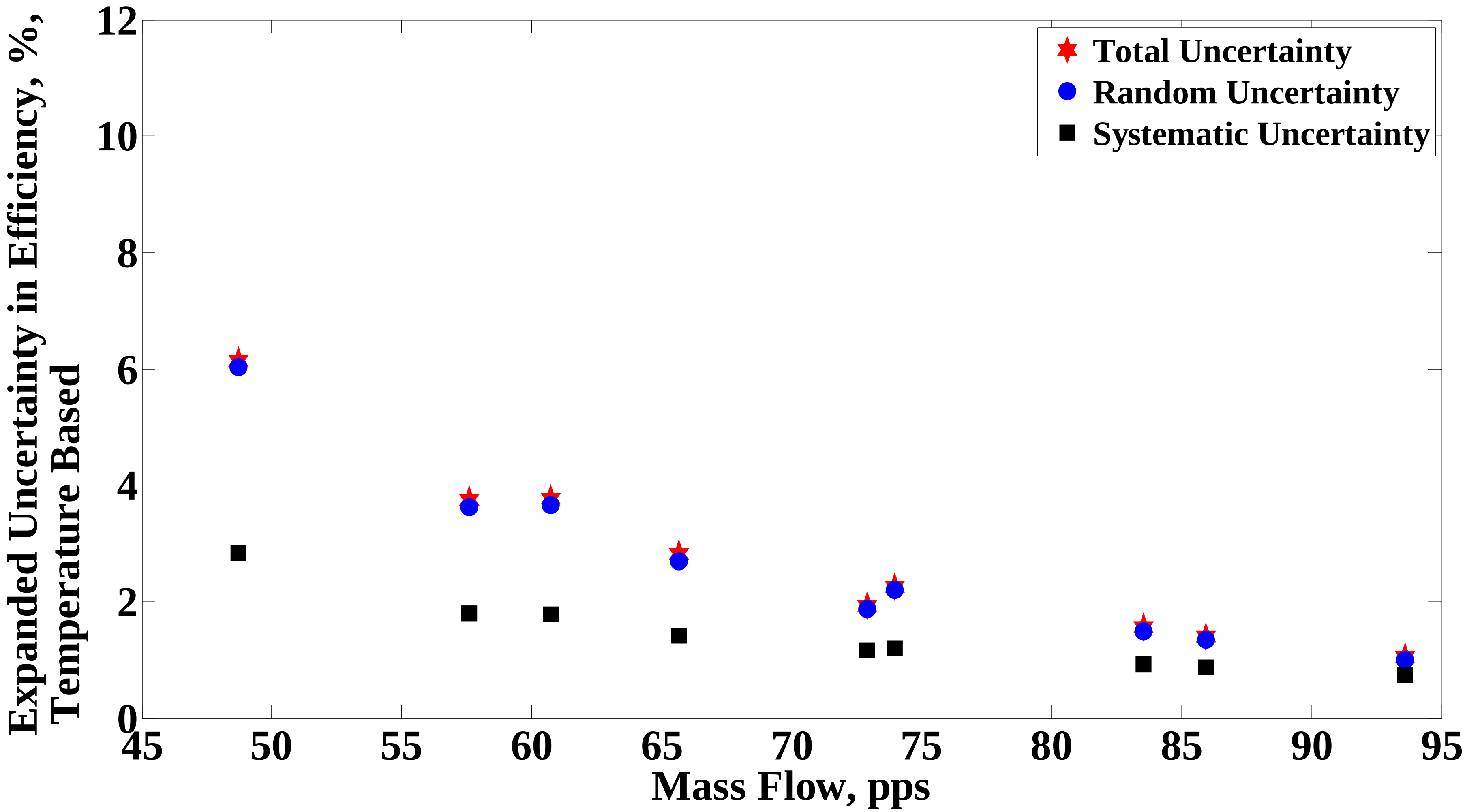
Contributors to Systematic Uncertainty in Efficiency



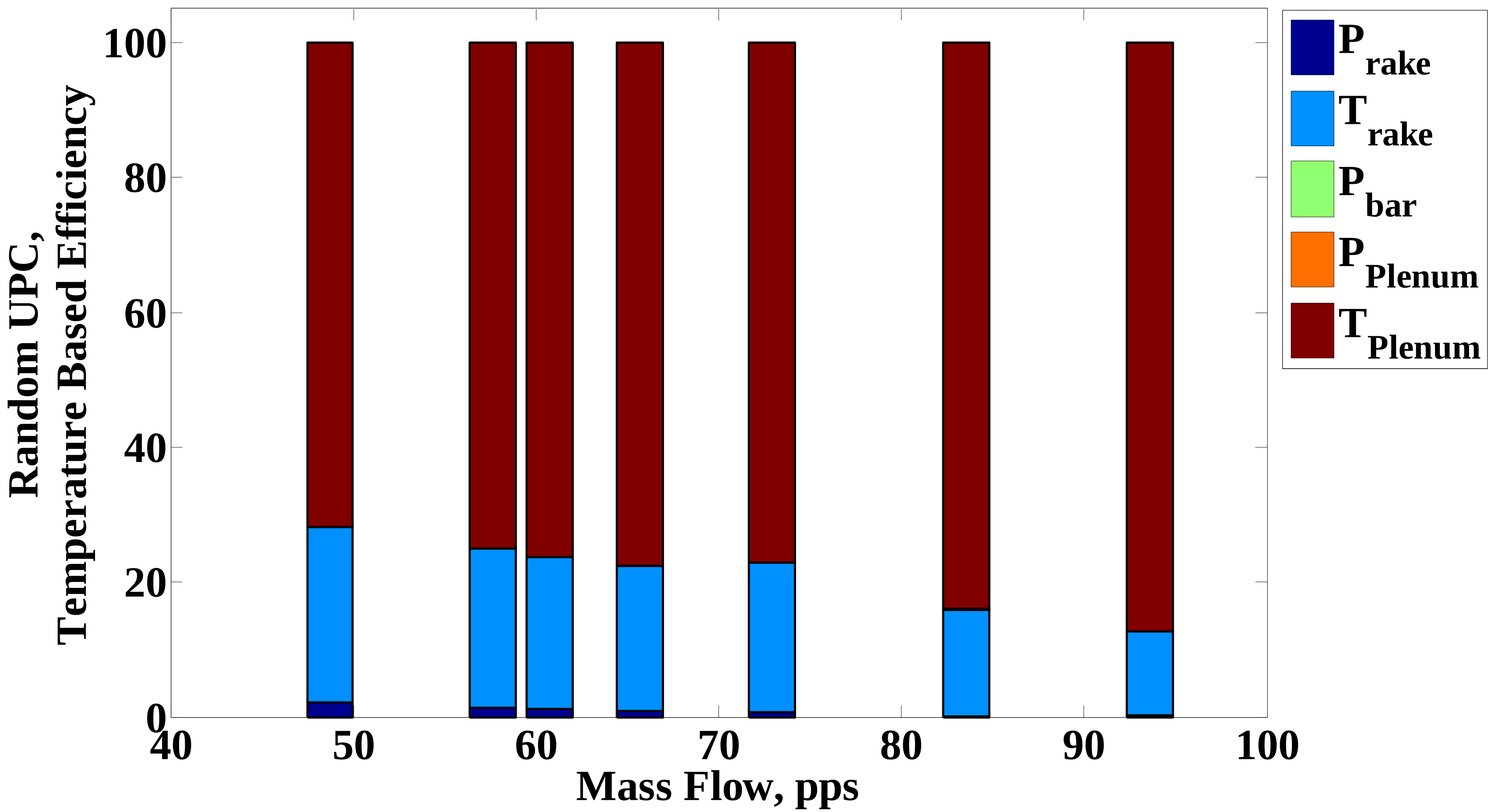
What if the TC wire was calibrated?



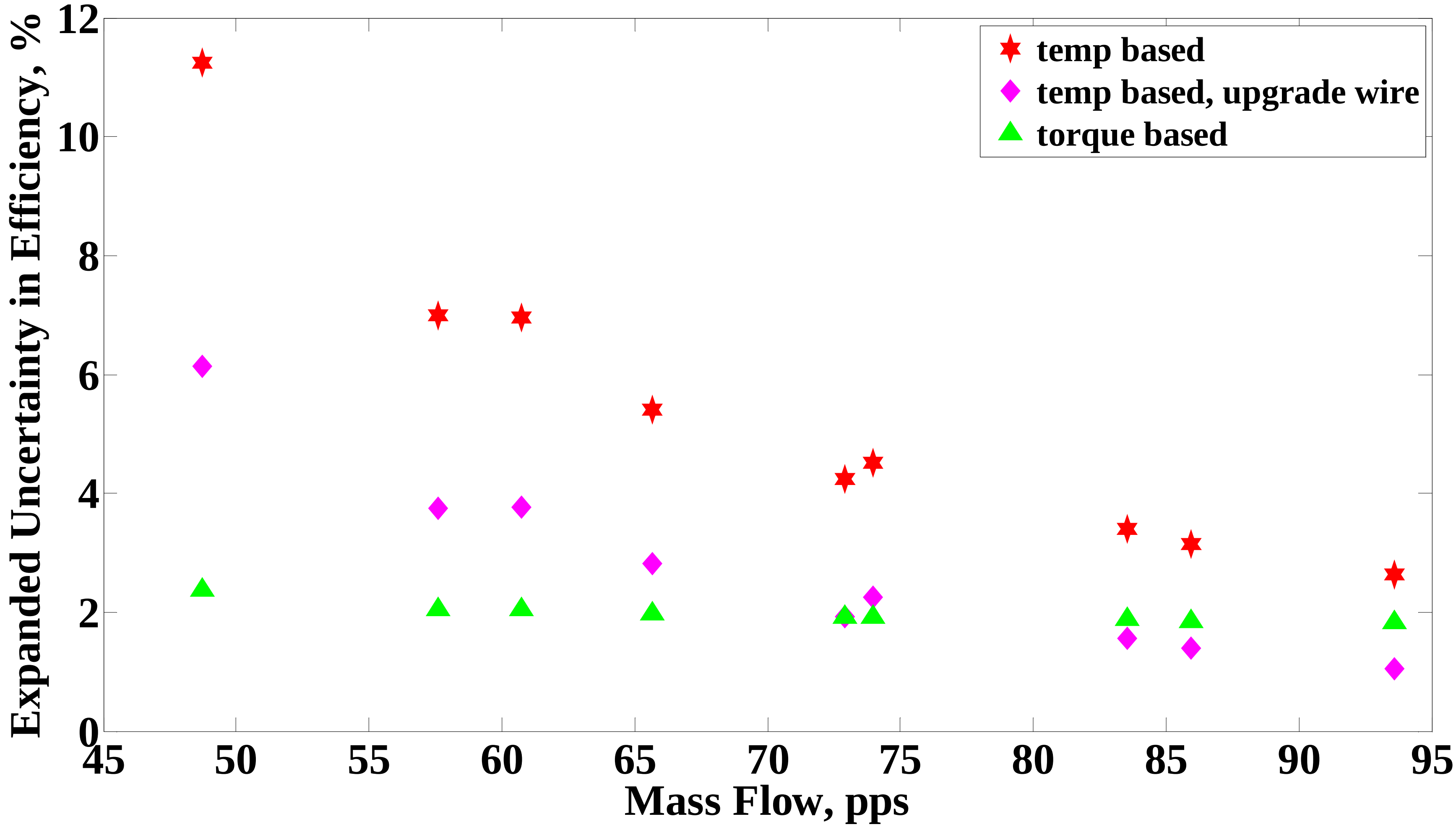
Temperature Based Uncertainty with Calibrated wire: Random vs. Systematic



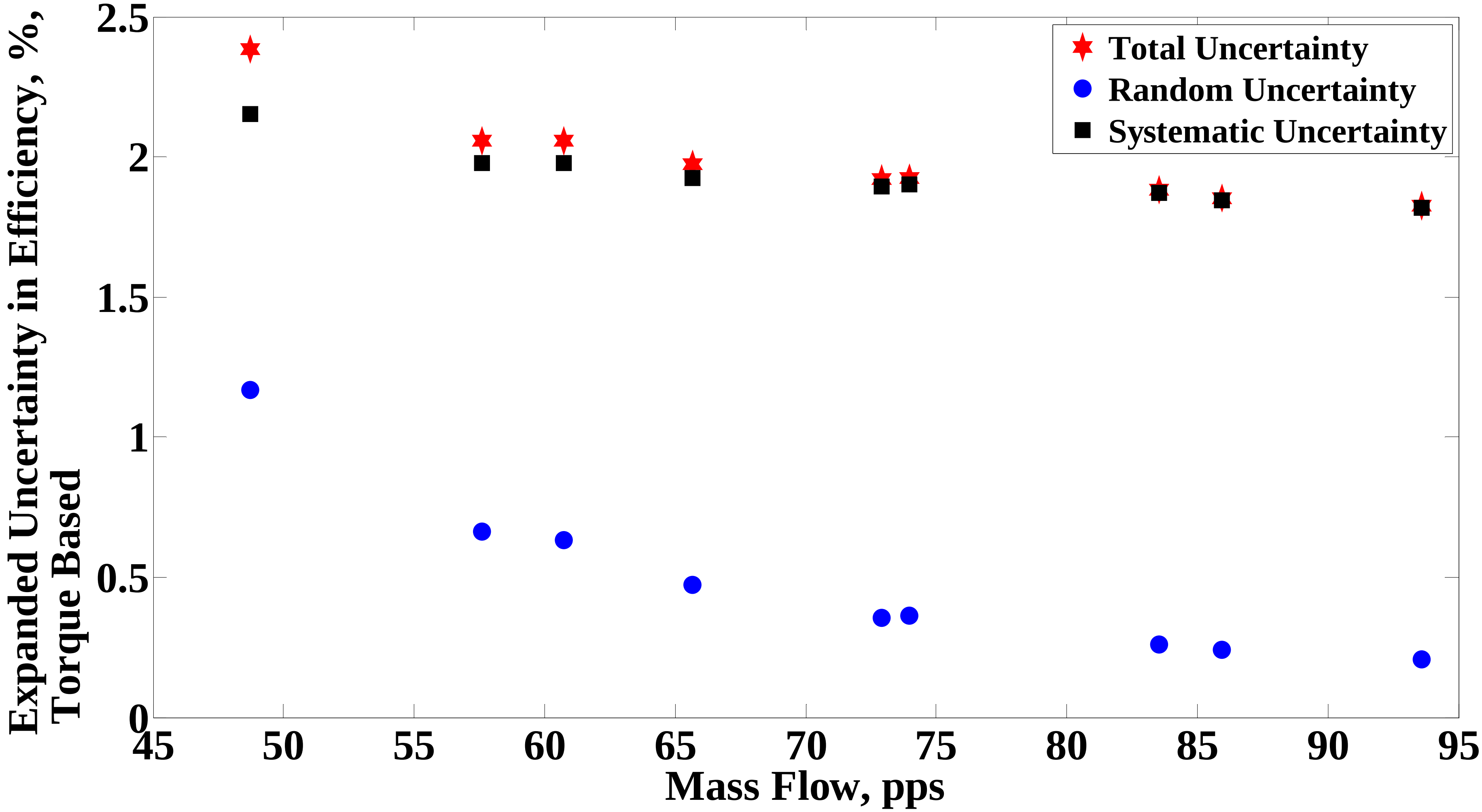
Contributors to Random Uncertainty in Efficiency



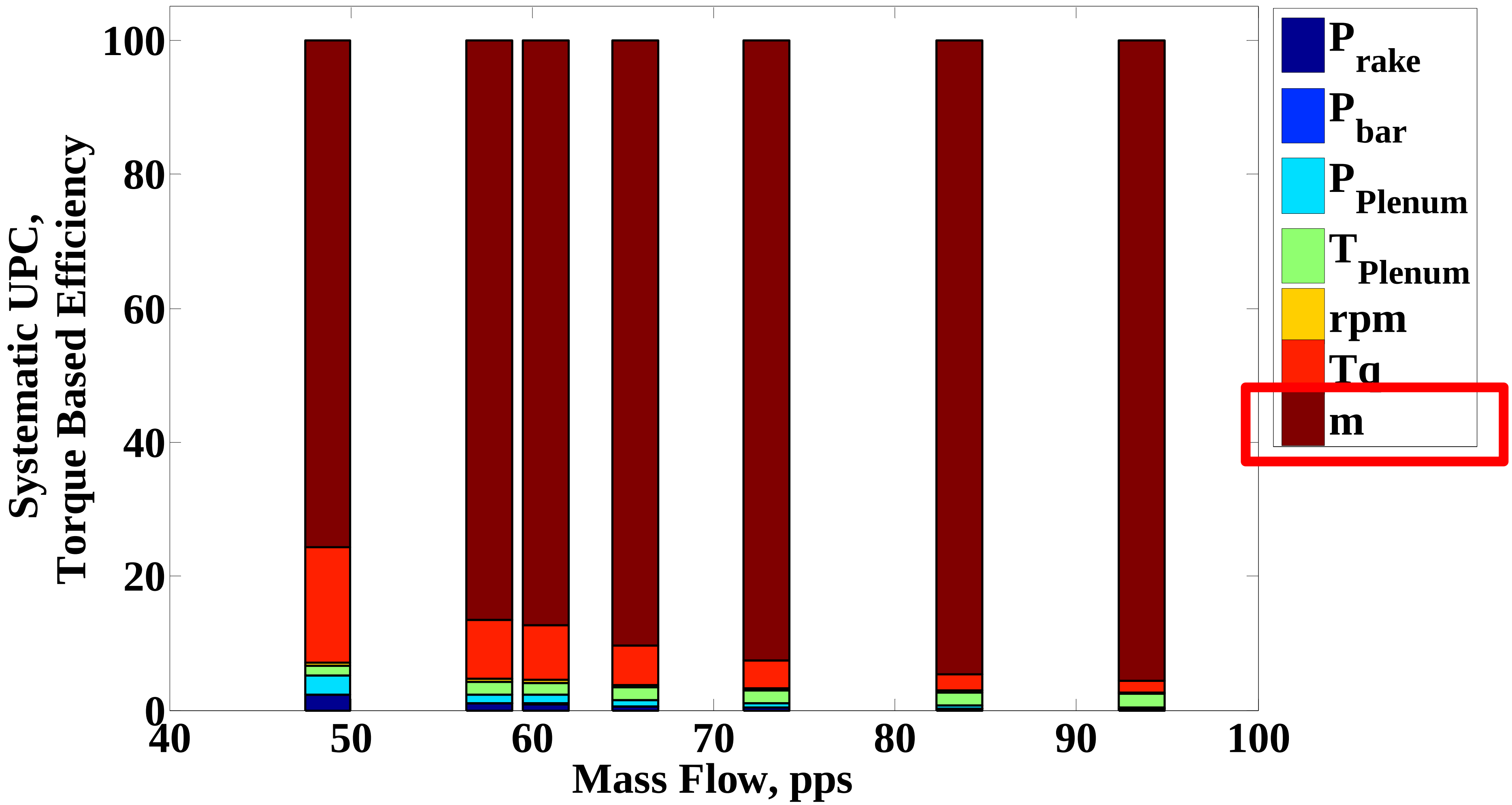
Adiabatic Efficiency Example: Torque Based



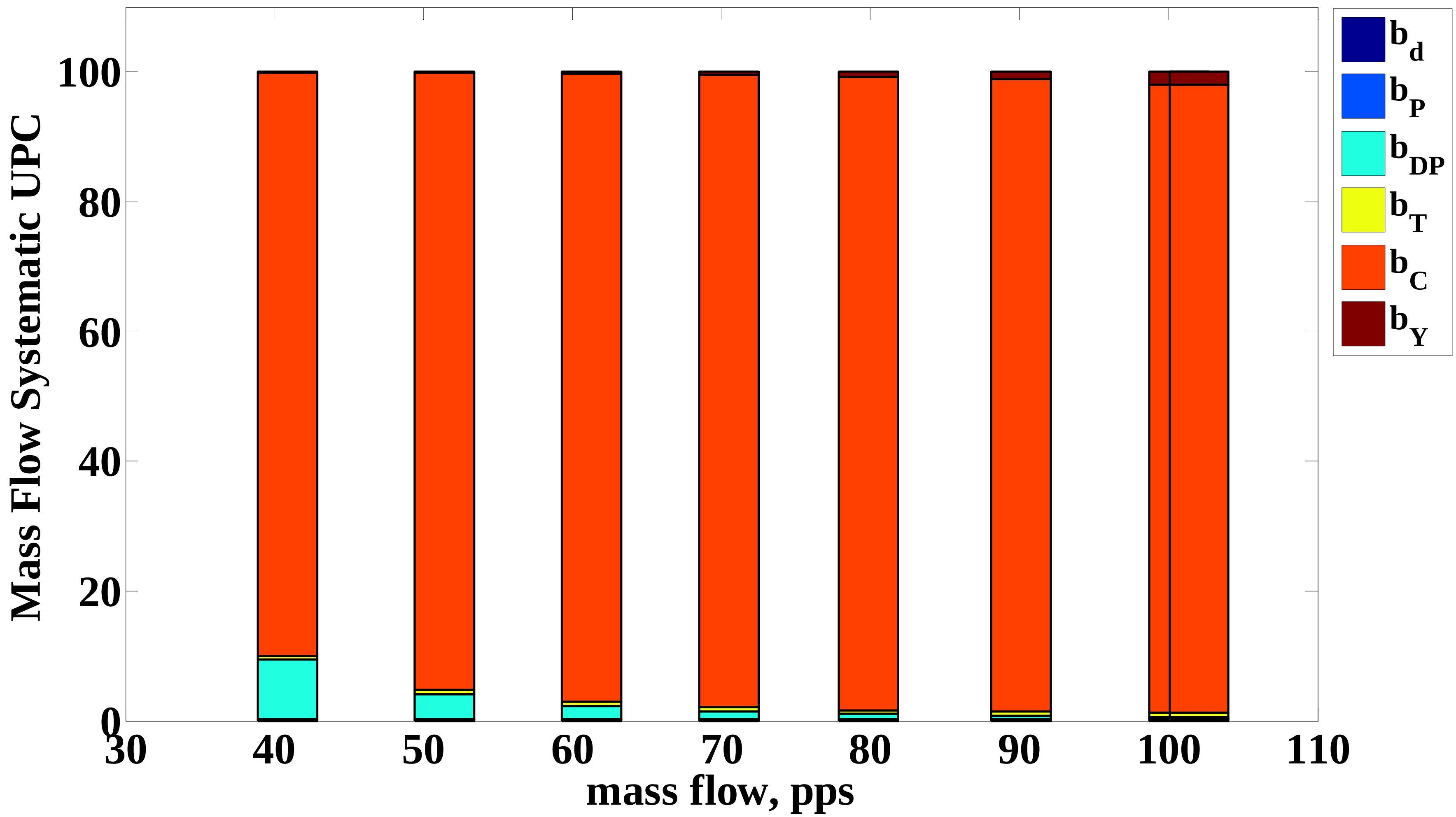
$$\eta = \frac{m\frac{\gamma R T_1}{\gamma - 1}\left[\frac{P_2}{P_1}^{\frac{\gamma-1}{\gamma}} - 1\right]}{\omega T_q}$$



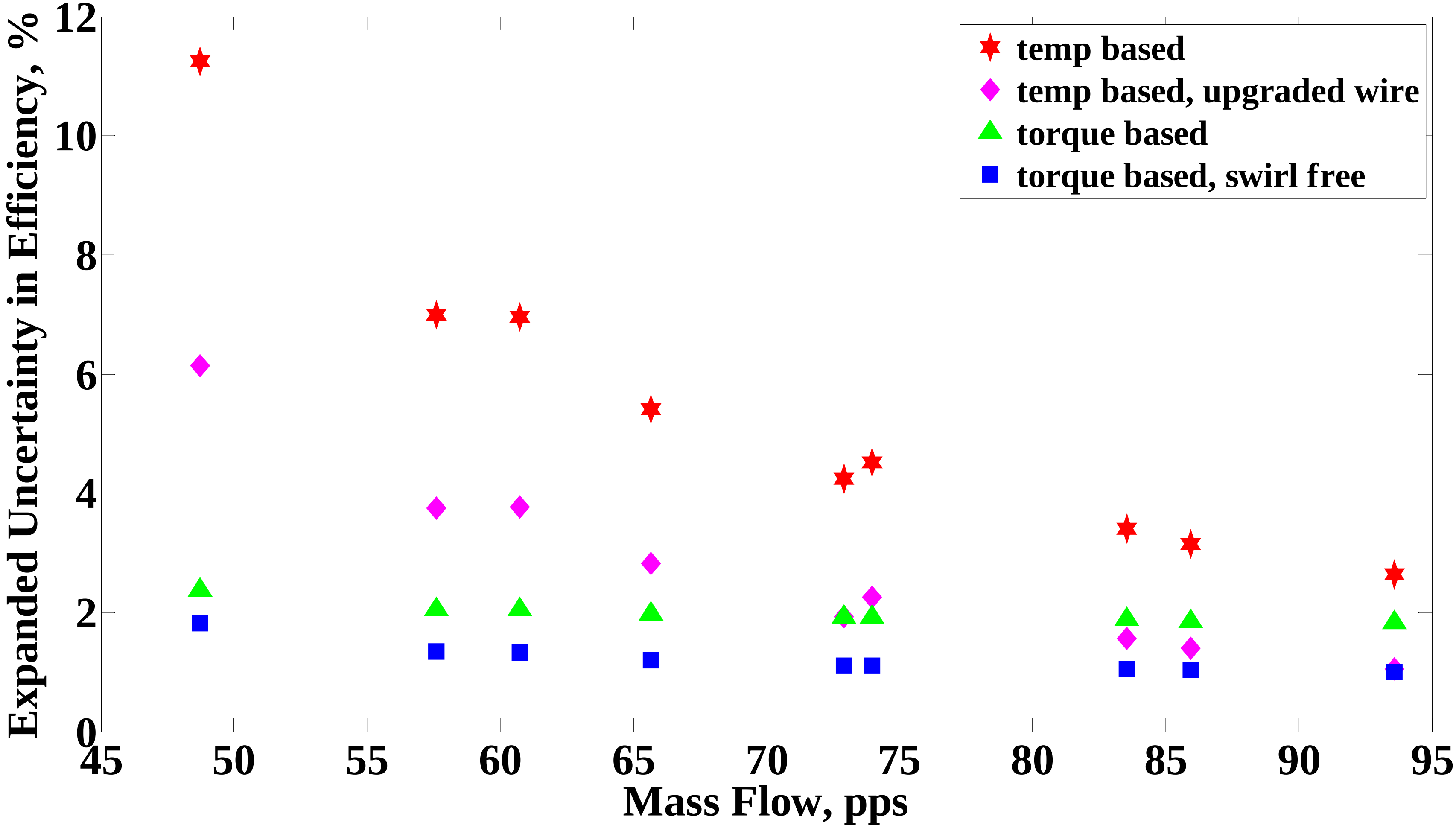
Contributors to Systematic Uncertainty



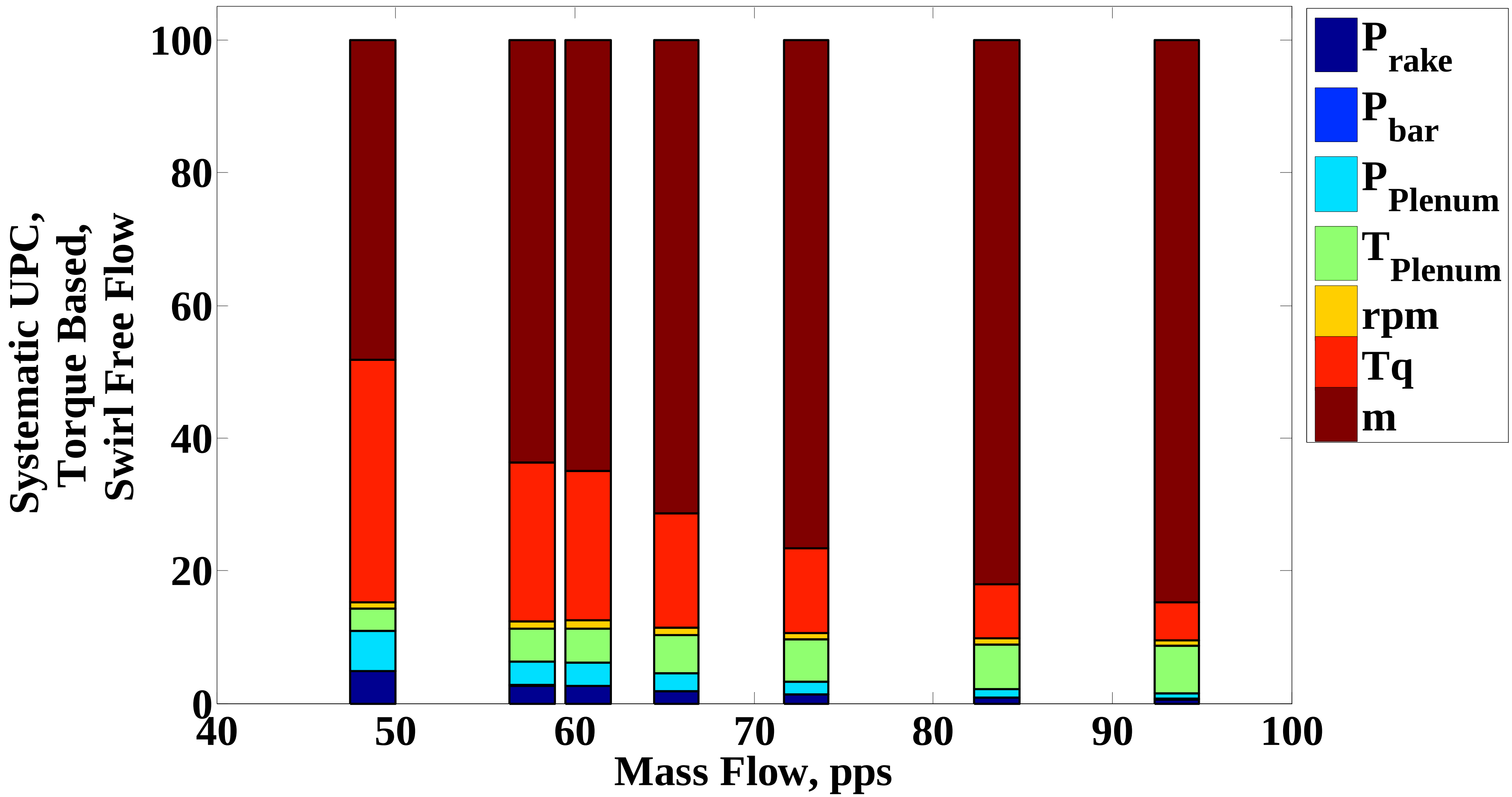
Contributions to Systematic Uncertainty in Mass Flow



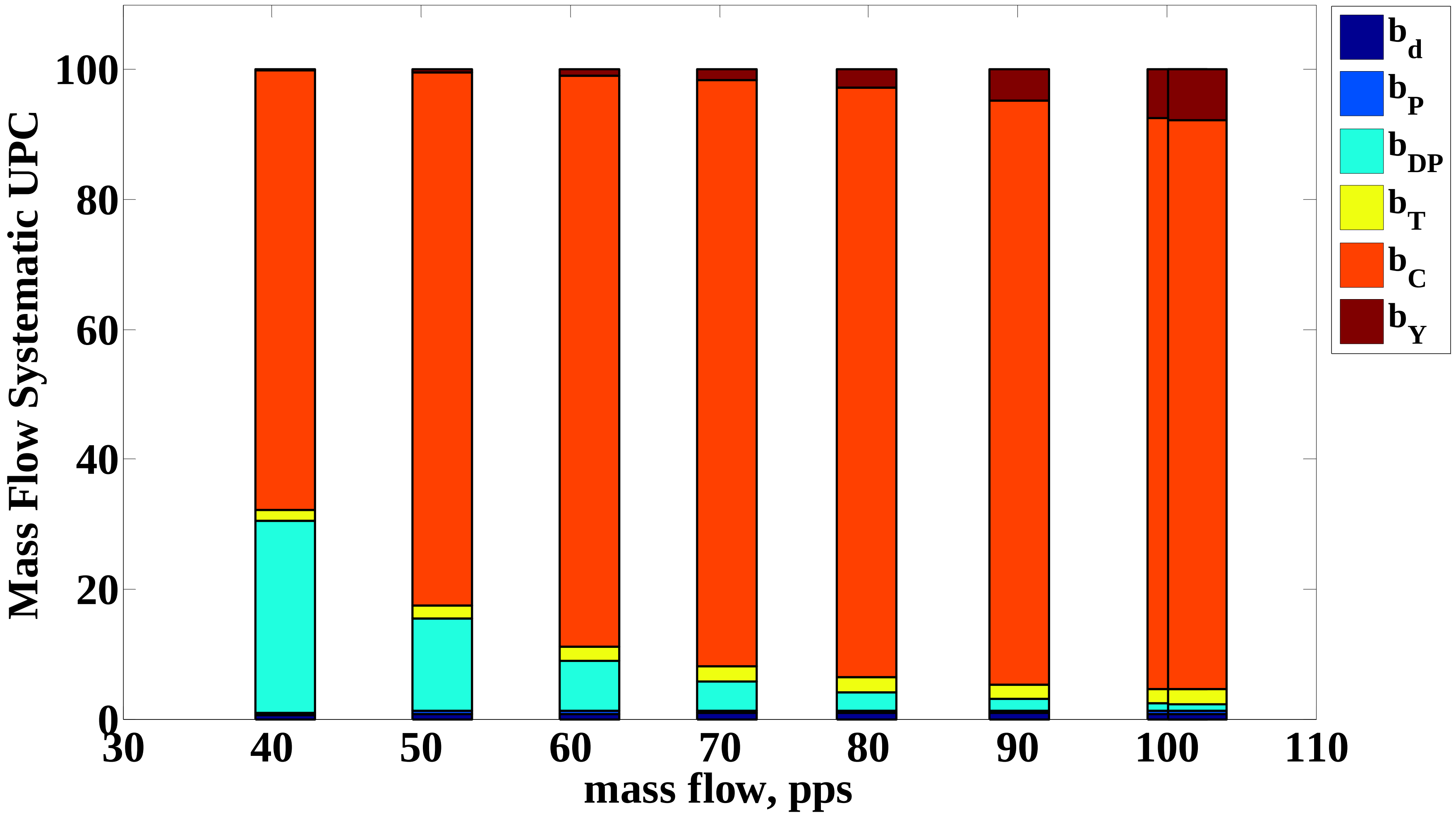
What if the flow was fully developed and swirl free?



Contributors to Systematic Uncertainty



Systematic Uncertainty in Mass Flow



Conclusions

- A thorough understanding of facility uncertainty requires both statistical process control and a bottom-up analysis of uncertainty propagation
- A rigorous analysis of uncertainty propagation provides
  - A quantitative understanding of the quality of the data
  - An understanding of the uncertainty sources
  - An understanding of the different aspects of uncertainty (repeatability vs bias)
- Utilizing a Monte Carlo approach allows for ease of implementation in complicated math models or where a lot of correlations are present.
- The Monte Carlo also allows a straight forward process for investigating potential scenarios for facility improvement.

References

- Joint Committee for Guides in Metrology, 'Evaluation of Measurement Data — Guide to the Expression of Uncertainty in Measurement', JCGM/WG 1, 2008.
- National Aeronautics and Space Administration, 'Measurement Uncertainty Analysis Principles and Methods', NASA, Washington DC, 2010.
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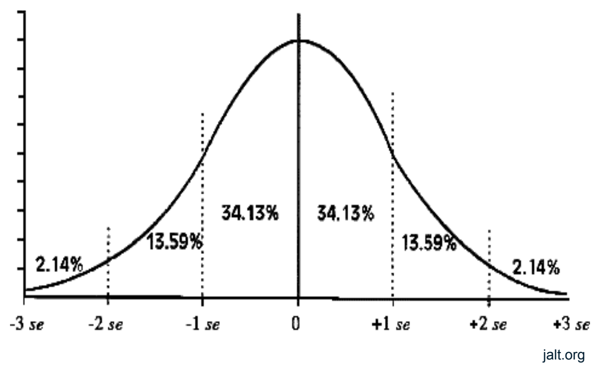
- American Society of Mechanical Engineers. “Test Uncertainty”. Standard ASME PTC,19.1-2013, 2014.
- American Institute of Aeronautics and Astronautics. “Assessment of Experimental Uncertainty with Application to Wind Tunnel Testing”. Standard AIAA S-071A-1999,1999
- J.L. Devore. Probability and Statistics for Engineering and the Sciences. Brooks/Cole,Fifth edition, 2000.

Supplemental Slides

Confidence Intervals and Degrees of Freedom

- Confidence interval
  - The probabilistic determination of an outcome. Often expressed as the percentage area under a distribution curve.

- Degrees of freedom
  - Quantification of the independence of a data set.
  - Defined most commonly as sample size – 1, (n-1)



Standard uncertainty	x	Coverage factor (k)	=	Expanded uncertainty
1.220 ºC	x	2 (for 95% coverage)	=	2.440 ºC

4 temperature measurements in the bellmouth

$b_{T_{T,bm}}$ → $T_{T,bm,1\text{-}4}$

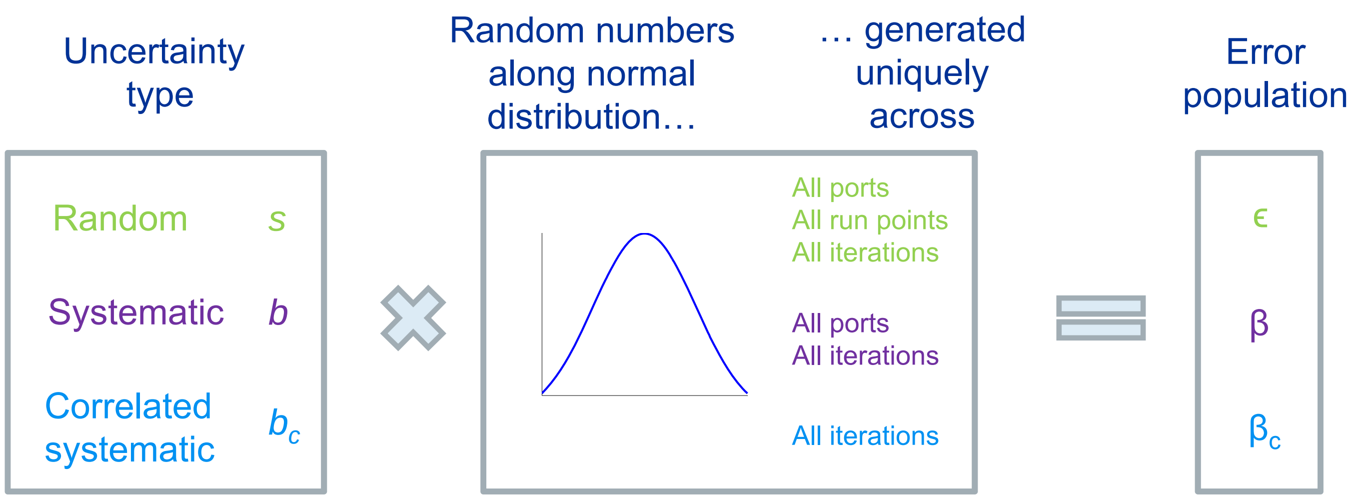
Generated from random number population with normal distribution, mean=0, and σ=1

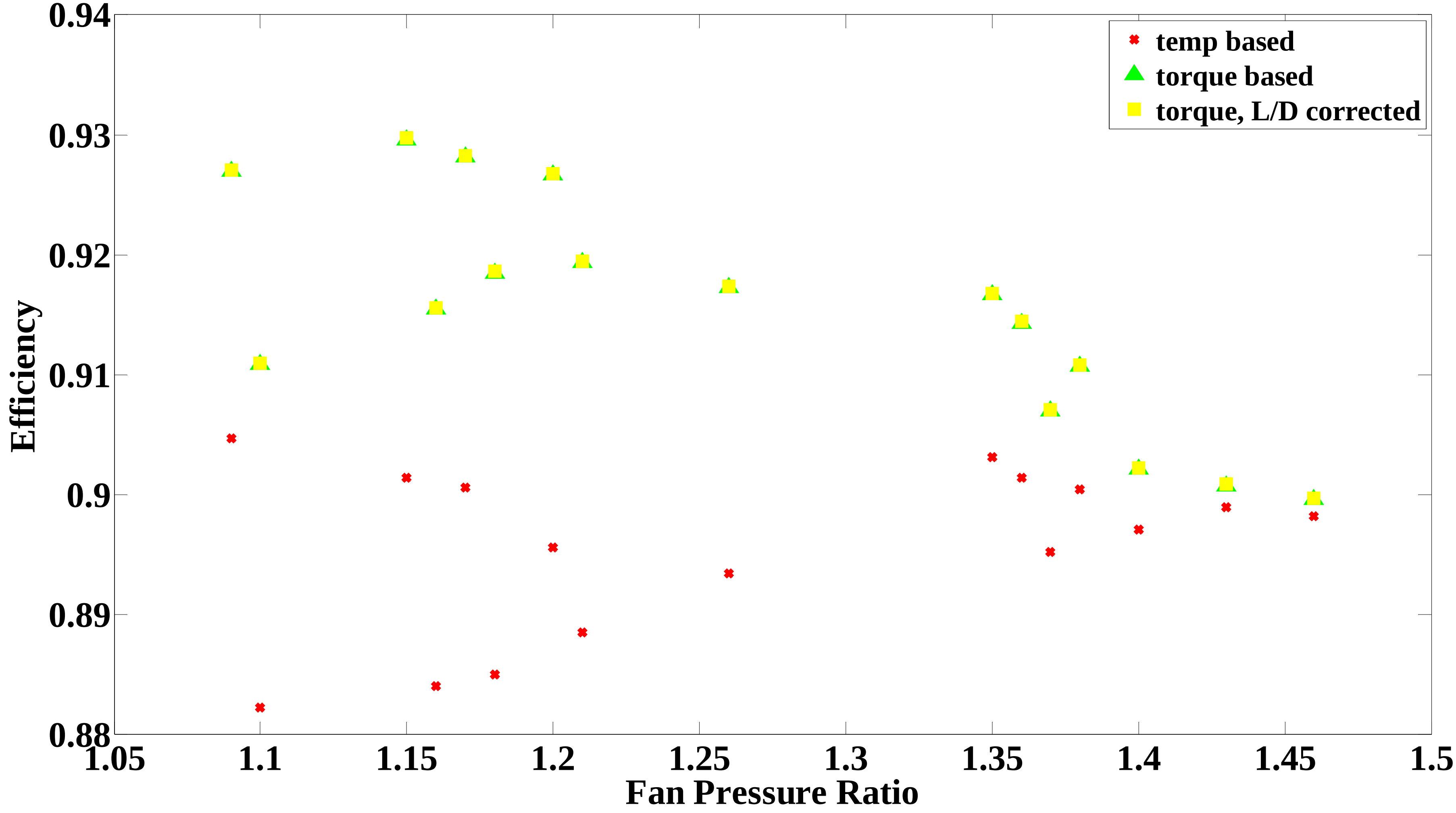
			Monte Carlo ITERATION 1			Monte Carlo ITERATION 2		
TC name	Measured Temp, °C	Standard uncertainty, °C	Random number	Error, °C	Perturbed measured temp, °C	Random number	Error, °C	Perturbed measured temp, °C
$T_{T,bm(1)}$	100	1.22	-0.40	-0.50	99.50	0.51	0.64	100.63
$T_{T,bm(2)}$	100	1.22	-1.08	-1.33	98.67	-1.62	-2.00	98.03
$T_{T,bm(3)}$	100	1.22	1.07	1.32	101.32	0.97	1.20	101.18
$T_{T,bm(4)}$	100	1.22	0.10	0.13	100.13	1.77	2.19	102.16

Standard uncertainty × Random number = Error

Monte Carlo Analysis: Populating Errors

- Appropriately populating errors is *critical* to the integrity of the Monte Carlo approach to error propagation.
- If errors are populated correctly, correlated errors are inherently handled within the data reduction.
  - Taylor Series approach requires correlations to be handled overtly.





0.94
0.93
0.92
0.91
0.9
0.89
0.88
Efficiency
1.05
1.1
1.15
1.2
1.25
1.3
1.35
1.4
1.45
1.5
Fan Pressure Ratio
temp based
torque based
torque, L/D corrected

Example Results

Mass Flow

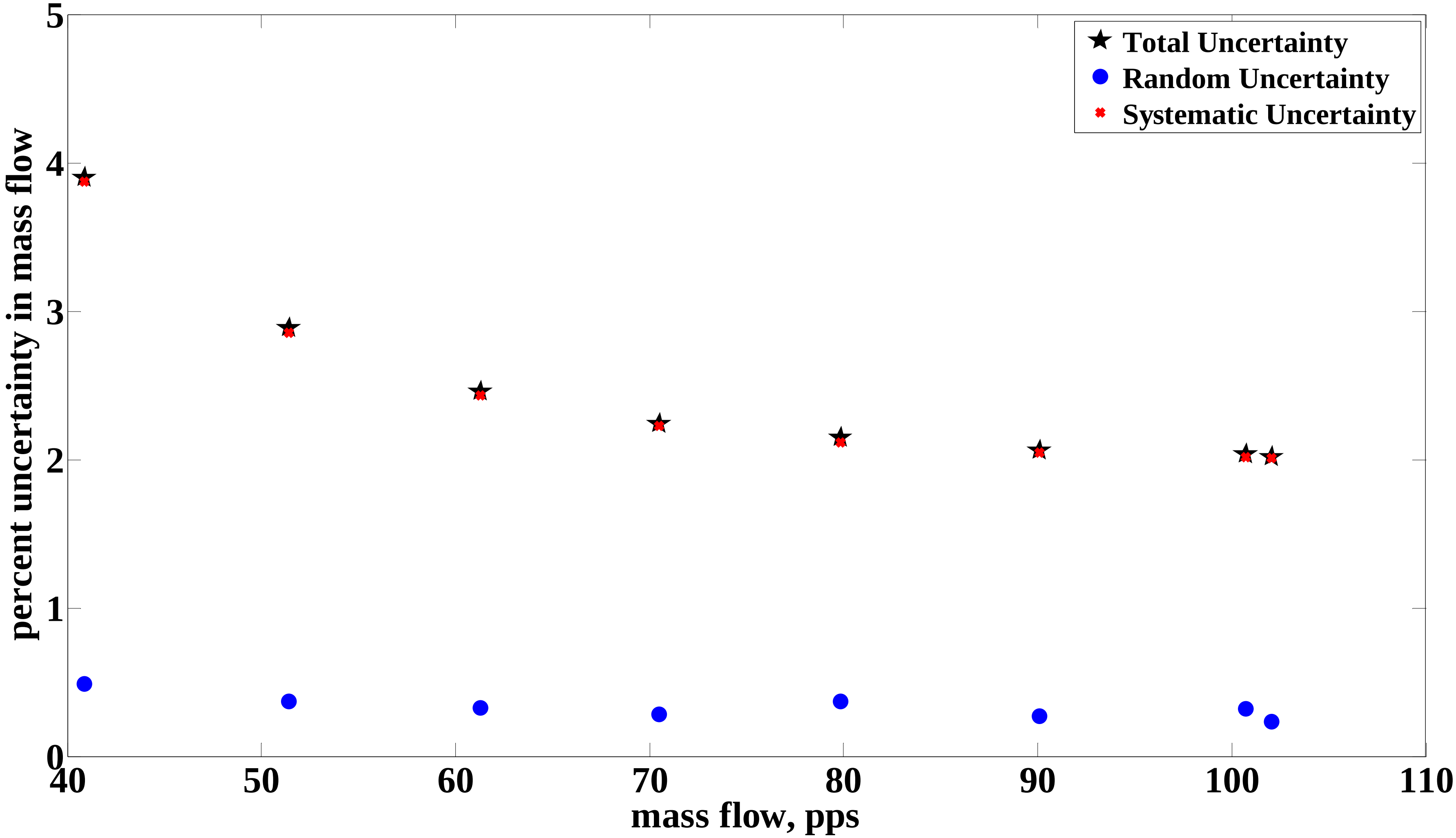
Mass Flow Calculation

$$m_{OR} = C_1\left(1 - \frac{C_2 P_D}{P^2}\right)\sqrt{\frac{P P_D}{T}}$$

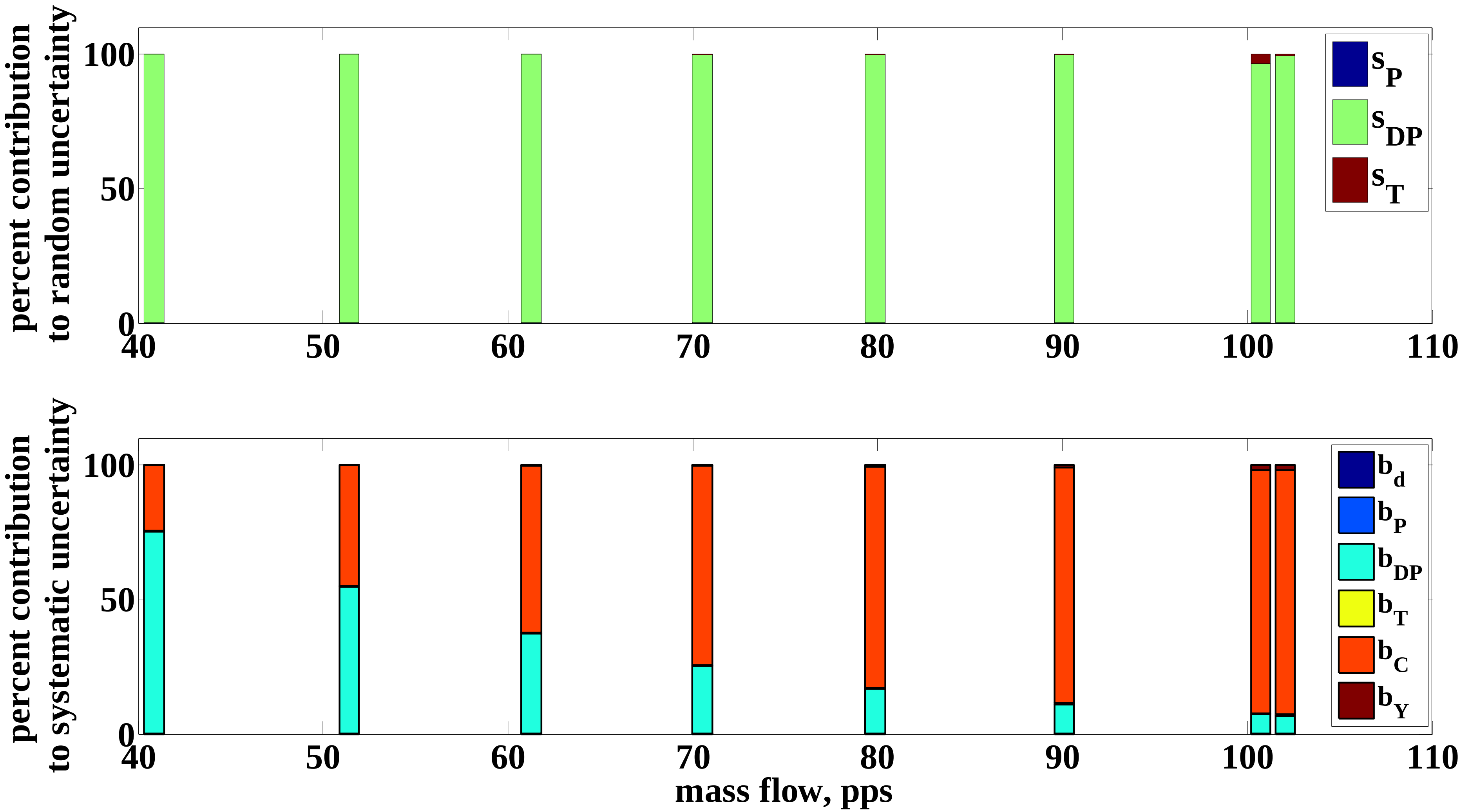
$$m_{OR} = 0.5202\left(\frac{C Y d^2 F_a}{\sqrt{1-\beta^2}}\right)\sqrt{\frac{144 P P_D}{RT}}$$

C and Y have systematic uncertainties defined by ISO and ASME Standards. These values increase if the facility does not have proper Length to Diameter ratios.

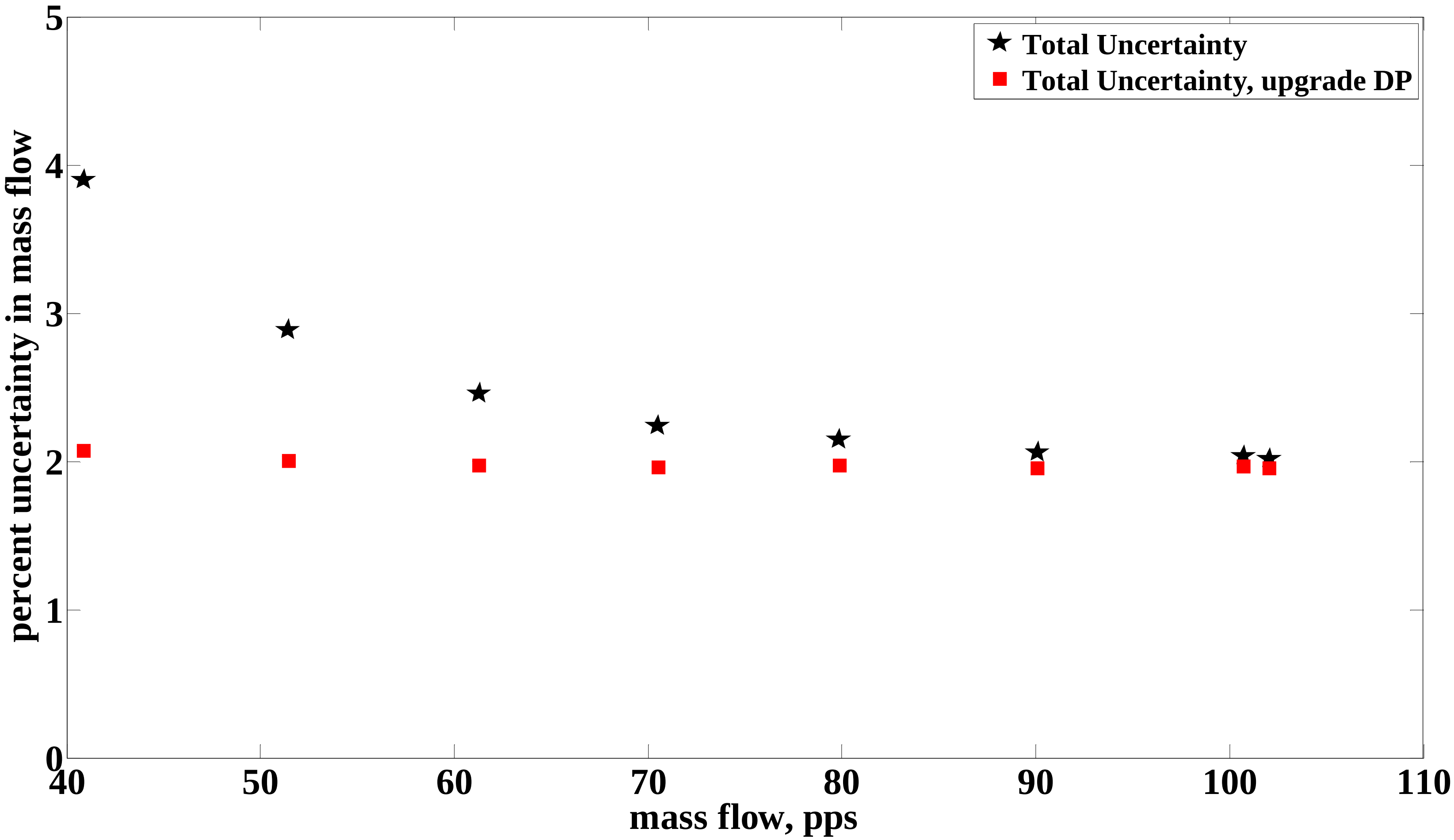
Orifice Plate Mass Flow Example



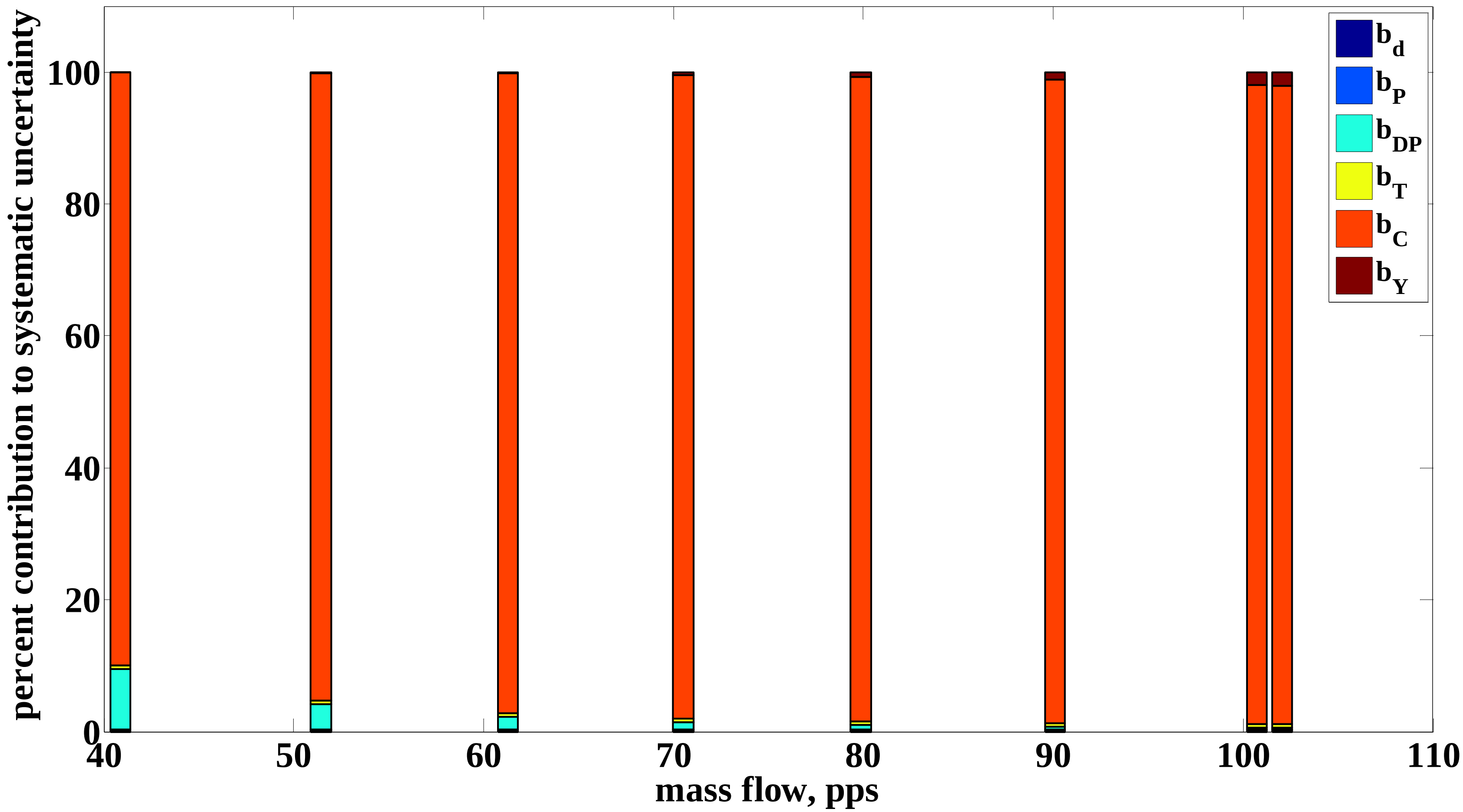
Contributors to Mass Flow Uncertainty



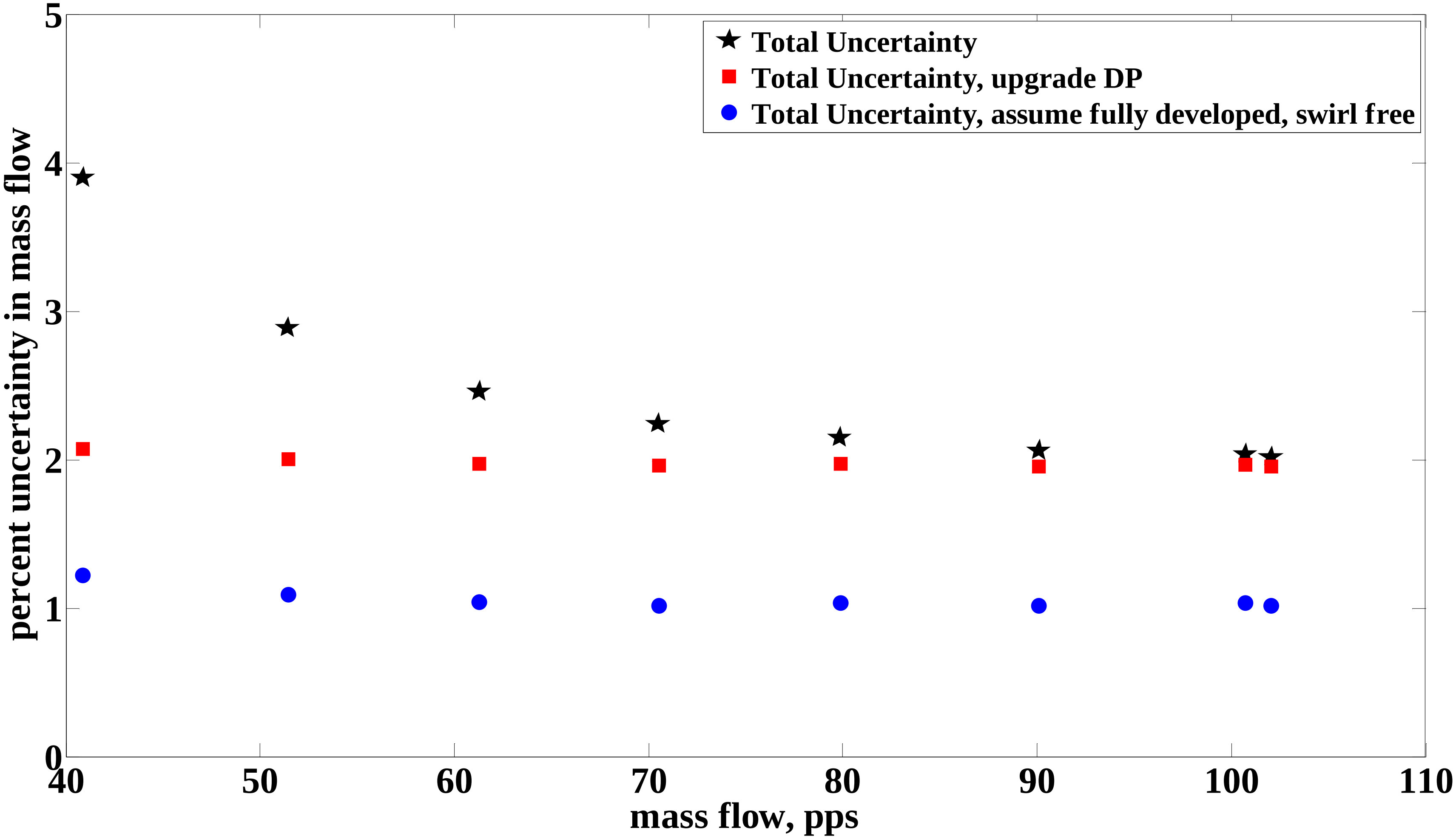
What if we upgrade the differential pressure transducers?



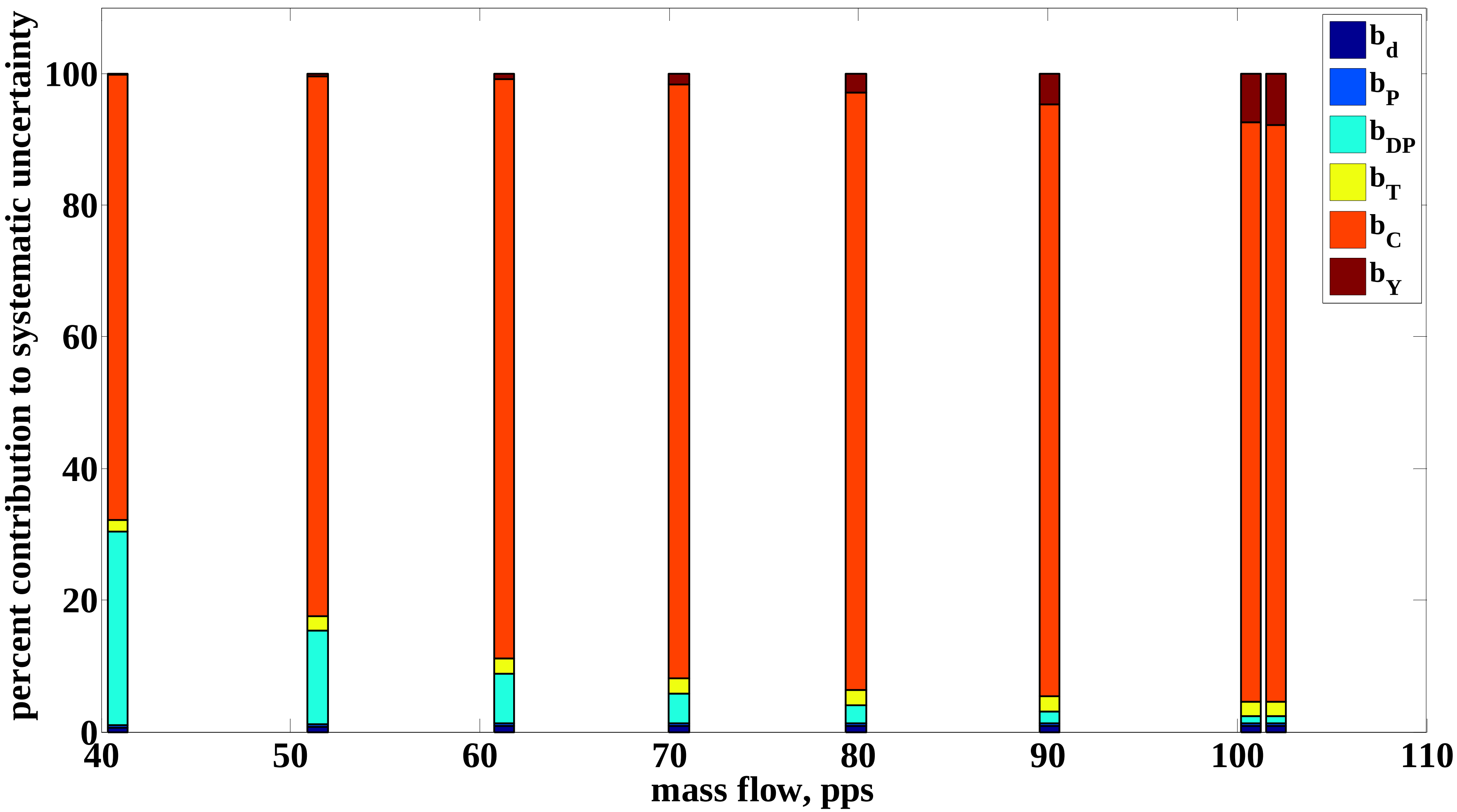
Contributors to Systematic Uncertainty



What if we correct the L/D ratios in the piping?



Contributors to Uncertainty



Thermocouple System Example

Input: 100 ºC

	Module	Combined Uncertainty (1σ)	% contribution from module
	Type E Thermocouple & Wire	1.202 ºC	97.2%
Nominal value:	100 ºC		
	Reference Oven	0.0000822 V	2.6%
Nominal value:	0.0023V		
	Signal Conditioner	0.1686 V	0.2%
Nominal value:	4.663 V		
	A/D Converter	539.7 Counts	0%
Nominal value:	14921 counts		
	ESCORT Unit Conversion	1.220 ºC Standard Uncertainty of system	0%
Output Value:	100 ºC		

Applied to MC