Analysis and Characterization of Damage and Failure Utilizing a Generalized Composite Material Model Suitable for Use in Impact Problems

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ABSTRACT

A material model which incorporates several key capabilities which have been identified by the aerospace community as lacking in state-of-the art composite impact models is under development. In particular, a next generation composite impact material model, jointly developed by the FAA and NASA, is being implemented into the commercial transient dynamic finite element code LS-DYNA. The material model, which incorporates plasticity, damage, and failure, utilizes experimentally based tabulated input to define the evolution of plasticity and damage and the initiation of failure as opposed to specifying discrete input parameters (such as modulus and strength). The plasticity portion of the orthotropic, three-dimensional, macroscopic composite constitutive model is based on an extension of the Tsai-Wu composite failure model into a generalized yield function with a non-associative flow rule. For the damage model, a strain equivalent formulation is utilized to allow for the uncoupling of the deformation and damage analyses. In the damage model, a semi-coupled approach is employed where the overall damage in a particular coordinate direction is assumed to be a multiplicative combination of the damage in that direction resulting from the applied loads in the various coordinate directions. Due to the fact that the plasticity and damage models are uncoupled, test procedures and methods to both characterize the damage model and to covert the material stress-strain curves from the true (damaged) stress space to the effective (undamaged) stress space have been developed. A methodology has been developed to input the experimentally determined composite failure surface in a tabulated manner. An analytical approach is then utilized to track how close the current stress state is to the failure surface.

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INTRODUCTION

As composite materials are gaining increased use in aircraft components where impact resistance is critical (such as the turbine engine fan case), the need for accurate material models to simulate the deformation, damage and failure response of polymer matrix composites under impact conditions is gaining in importance. While there are several material models currently available within commercial transient dynamic codes such as LS-DYNA [1] to analyze the impact response of composites, areas have been identified where the predictive capabilities of these models can be improved. Most importantly, the existing models often require extensive correlation based on structural level impact tests, which significantly limits the ability to use these models as predictive tools. Furthermore, most of the existing models apply either a plasticity based approach (such as that used by Sun and Chen [2]) or a continuum damage mechanics approach (such as that used by Matzenmiller et al [3]) to simulate the nonlinearity that takes place in the composite response. As documented in detail by Goldberg, et al [4, 5], either of these approaches can capture certain aspects of the actual composite behavior. However, optimally, combining a plasticity based deformation model (to capture the rate dependence and significant nonlinearity, particularly in shear, observed in the composite response) with a damage model (to account for the nonlinear unloading and strain softening observed after the peak stress is reached) can provide advantages over using one approach or the other. In addition, the input to current material models generally consists of point-wise properties (such as the modulus, failure stress or failure strain in a particular coordinate direction) that leads to curve fit approximations to the material stress-strain curves. This type of approach either leads to models with only a few parameters, which provide a crude approximation at best to that actual material response, or to models with many parameters which require a large number of complex tests to characterize. An improved approach is to use tabulated data, obtained from well-defined set of straightforward experiments. Using tabulated data allows the actual material response data to be entered in discretized form, which permits a more accurate representation of the actual material response.

To begin to address these needs, a new composite material model is being developed and implemented for use within the commercial transient dynamic finite element code LS-DYNA. The material model is meant to be a fully generalized model suitable for use with any composite architecture (laminated or textile). The deformation model is based on extending the commonly used Tsai-Wu composite failure model [6] to a strain hardening plasticity model with a non-associative flow rule. For the damage model, a strain equivalent formulation is used in which the deformation and damage calculations can be uncoupled. A significant feature in the developed damage model is that a semi-coupled approach has been utilized in which a load in a particular coordinate direction results in damage (and thus stiffness reduction) in multiple coordinate directions. While different from the approach used in many existing damage mechanics models [3] in which a load in a particular coordinate direction only leads to a stiffness reduction in the load direction, this approach has the potential to more accurately reflect the damage behavior that actually takes place, particularly for composites with more complex fiber architectures. To model the failure response of the composite, the stress (or strain) based three dimensional failure envelope for the composite is entered in a tabulated fashion. A set of stress (or strain) invariants and stress (or strain) ratios are then used to determine if the current stress (or strain) state is inside or outside of the failure envelope.

In the following sections of this paper, a brief summary of the plasticity based deformation model is presented. The key aspects of the damage model are then described, along with details of the procedures that are required to characterize the damage model. The fundamental aspects of the failure model along with a set of proposed procedures to characterize the failure model are then discussed.

DEFORMATION MODEL

A complete description of the deformation model is given in Goldberg et al [4, 5]. A summary of the key features of the model is presented here. In the deformation model, a general quadratic three-dimensional orthotropic yield function based on the Tsai-Wu failure model is specified as follows, where 1, 2, and 3 refer to the principal material directions

$$
\begin{aligned}
f(\sigma) = -1 + \begin{pmatrix} F_1 & F_2 & F_3 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{12} \\ \sigma_{23} \\ \sigma_{31} \end{pmatrix} \\
+ \begin{pmatrix} \sigma_{11} & \sigma_{22} & \sigma_{33} & \sigma_{12} & \sigma_{23} & \sigma_{31} \end{pmatrix} \begin{pmatrix} F_{11} & F_{12} & F_{13} & 0 & 0 & 0 \\ F_{12} & F_{22} & F_{23} & 0 & 0 & 0 \\ F_{13} & F_{23} & F_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & F_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & F_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & F_{66} \end{pmatrix} \begin{pmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{12} \\ \sigma_{23} \\ \sigma_{31} \end{pmatrix}
\end{aligned} \tag{1}
$$

In the yield function, σ_{ij} represents the stresses and F_{ij} and F_k are coefficients that vary based on the current values of the yield stresses in the various coordinate directions. By allowing the coefficients to vary, the yield surface evolution and hardening in each of the material directions can be precisely defined. The values of the normal and shear coefficients can be determined by simplifying the yield function for the case of unidirectional tensile and compressive loading in each of the coordinate directions along with shear tests in each of the shear directions. In the above equation, the stresses are the current value of the yield stresses in the normal and shear directions. To determine the values of the off-axis coefficients (which are required to capture the stress interaction effects), the results from 45° off-axis tests in the various coordinate directions can be used. The values of the off-diagonal terms in the yield function can also be modified as required in order to ensure that the yield surface is convex [5].

A non-associative flow rule is used to compute the evolution of the components of plastic strain. The plastic potential for the flow rule is shown below

$$h = \sqrt{H_{11}\sigma_{11}^2 + H_{22}\sigma_{22}^2 + H_{33}\sigma_{33}^2 + 2H_{12}\sigma_{11}\sigma_{22} + 2H_{23}\sigma_{22}\sigma_{33} + 2H_{31}\sigma_{33}\sigma_{11} + H_{44}\sigma_{12}^2 + H_{55}\sigma_{23}^2 + H_{66}\sigma_{31}^2} \quad (2)$$

where σij are the current values of the stresses and H_{ij} are independent coefficients, which are assumed to remain constant. The values of the coefficients are computed based on average plastic Poisson's ratios [4, 5]. The plastic potential function in Equation (2) is used in a flow law to compute the components of the plastic strain rate, where the usual normality hypothesis from classical plasticity [7] is assumed to apply and the variable $\dot{\lambda}$ is a scalar plastic multiplier.

$$\dot{\boldsymbol{\varepsilon}}^{p} = \dot{\lambda} \frac{\partial h}{\partial \boldsymbol{\sigma}} \quad (3)$$

Given the flow law, the principal of the equivalence of plastic work [7] can be used to determine that the plastic potential function h can be defined as the effective stress and the plastic multiplier can be defined as the effective plastic strain rate.

To compute the current value of the yield stresses needed for the yield function, in the developed model tabulated stress-strain curves are used to track the yield stress evolution. The user is required to input twelve stress versus plastic strain curves. Specifically, the required curves include uniaxial tension curves in each of the normal directions (1,2,3), uniaxial compression curves in each of the normal directions (1,2,3), shear stress-strain curves in each of the shear directions (1-2, 2-3 and 3-1), and 45 degree off-axis tension curves in each of the 1-2, 2-3 and 3-1 planes. The 45 degree curves are required in order to properly capture the stress interaction effects. By utilizing tabulated stress-strain curves to track the evolution of the deformation response, the experimental stress-strain response of the material can be captured exactly without any curve fit approximations. The required stress-strain data can be obtained either from actual experimental test results or by appropriate numerical experiments utilizing stand-alone codes. Currently, only static test data is considered. Future efforts will involve adding strain rate and temperature dependent effects to the computations. To track the evolution of the deformation response along each of the stress-strain curves, the effective plastic strain is chosen to be the tracking parameter. Using a numerical procedure based on the radial return method [7] in combination with an iterative approach, the effective plastic strain is computed for each time/load step. The stresses for each of the tabulated input curves corresponding to the current value of the effective plastic strain are then used to compute the yield function coefficients.

DAMAGE MODEL OVERVIEW

The deformation portion of the material model provides the majority of the capability of the model to simulate the nonlinear stress-strain response of the composite. However, in order to capture the nonlinear unloading and local softening of the stress-strain response often observed in composites [8], a complementary damage law is required. In the damage law formulation, strain equivalence is assumed, in which for every time step the total, elastic and plastic strains in the actual and effective stress spaces are the same [9]. The utilization of strain equivalence permits the plasticity and damage calculations to be uncoupled, as all of the plasticity computations can take place in the effective stress space.

The first step in the development of the damage model is to relate the actual stresses to a set of effective stresses by use of a damage tensor **M**

$$\boldsymbol{\sigma} = \mathbf{M}\boldsymbol{\sigma}_{eff} \tag{4}$$

The effective stress rate tensor can be related to the total and plastic strain rate tensors by use of the standard elasto-plastic constitutive equation

$$\dot{\boldsymbol{\sigma}}_{eff} = \mathbf{C}\left(\dot{\boldsymbol{\varepsilon}} - \dot{\boldsymbol{\varepsilon}}_p\right) \tag{5}$$

where **C** is the standard elastic stiffness matrix and the actual total and plastic strain rate tensors are used due to the strain equivalence assumption.

Given the usual assumption that the actual stress tensor and the effective stress tensor are symmetric, the actual stresses can be related to the effective stresses in the following manner, where the damage tensor **M** is assumed to have a maximum of 36 independent components.

$$\begin{pmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{12} \\ \sigma_{23} \\ \sigma_{13} \end{pmatrix} = [\mathbf{M}] \begin{pmatrix} \sigma_{11}^{eff} \\ \sigma_{22}^{eff} \\ \sigma_{33}^{eff} \\ \sigma_{12}^{eff} \\ \sigma_{23}^{eff} \\ \sigma_{13}^{eff} \end{pmatrix} \tag{6}$$

In order to maintain a one to one relationship between the effective stresses and the actual stresses (i.e. to ensure that a uniaxial load in the actual stress space does not result in a multi-axial load in the effective stress space), the damage tensor is assumed to be diagonal, leading to the following form.

$$[\mathbf{M}] = \begin{bmatrix} M_{11} & 0 & 0 & 0 & 0 & 0 \\ 0 & M_{22} & 0 & 0 & 0 & 0 \\ 0 & 0 & M_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & M_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & M_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & M_{66} \end{bmatrix} \tag{7}$$

However, an implication of a diagonal damage tensor is that loading the composite in a particular coordinate direction only leads to a stiffness reduction in the direction of the load due to the formation of matrix cracks perpendicular to the direction of the load. However, as discussed in detail in Goldberg, et al [5], in actual composites, particularly those with complex fiber architectures, a load in one coordinate direction can lead to stiffness reductions in multiple coordinate directions. To maintain a diagonal damage tensor while still allowing for the damage interaction in at least a semi-coupled sense, each term in the diagonal damage matrix should be a function of the plastic strains in each of the normal and shear coordinate directions, as follows for the example of the M_{11} term for the plane stress case

$$M_{11} = M_{11}\left(\varepsilon_{11}^{p}, \varepsilon_{22}^{p}, \varepsilon_{12}^{p}\right) \tag{8}$$

Note that plastic strains are chosen as the "tracking parameter" due to the fact that, within the context of the developed formulation, the material nonlinearity during loading is simulated by use of a plasticity based model. The plastic strains therefore track the current state of load and deformation in the material. To explain this concept of damage coupling further, assume a load is applied in the 1 direction to an undamaged specimen. The undamaged modulus in the 1 direction is E_{11} and the undamaged modulus in the 2 direction is E_{22}. The stress-strain response of the material is assumed to become nonlinear (represented in the current model by the accumulation of plastic strain) and damage is assumed to occur. The original specimen is unloaded and reloaded elastically in the 1 direction. Due to the damage, the reloaded specimen has a reduced modulus in the 1 direction of E_{11}^{d11}. The reduced area and modulus are a function of the damage induced by the loading and resulting nonlinear deformation in the 1 direction (reflected as plastic strain) as follows:

$$E_{11}^{d11} = \left(1 - d_{11}^{11}\left(\varepsilon_{11}^{p}\right)\right)E_{11} \tag{9}$$

where d_{11}^{11} is the damage in the 1 direction due to a load in the 1 direction. Alternatively, if the damaged specimen was reloaded elastically in the 2 direction, due to the assumed damage coupling resulting from the load in the 1 direction, the reloaded specimen would have a reduced modulus in the 2 direction of E_{22}^{d11}. The reduced modulus is also a function of the damage induced by the load and resulting nonlinear deformation in the 1 direction as follows:

$$E_{22}^{d11} = \left(1 - d_{11}^{22}\left(\varepsilon_{11}^{p}\right)\right)E_{22} \tag{10}$$

where d_{11}^{22} is the damage in the 2 direction due to a load in the 1 direction. Similar arguments can be made and equations developed for the situation where the original specimen is loaded in the 2 direction.

For the case of multiaxial loading, the semi-coupled formulation needs to account for the fact that as the load is applied in a particular coordinate direction, the loads are acting on damaged areas due to the loads in the other coordinate directions, and the load in a particular direction is just adding to the damaged area. For example, if one loaded the material in the 2 direction first, the reduced modulus in the 1 direction would be equal to E_{11}^{d22}. If one would then subsequently load the material in the 1 direction, the baseline modulus in the 1 direction would not be equal to the original modulus E_{11}, but instead the reduced modulus E_{11}^{d22}. Therefore, the loading in the 1 direction would result in the following further reduction in the modulus in the 1 direction:

$$
\begin{aligned}
E_{11}^{d11} &= \left(1 - d_{11}^{11}\left(\varepsilon_{11}^{p}\right)\right)E_{11}^{d22} = \left(1 - d_{11}^{11}\left(\varepsilon_{11}^{p}\right)\right)\left(1 - d_{22}^{11}\left(\varepsilon_{22}^{p}\right)\right)E_{11} \\
A_{11}^{d11} &= \left(1 - d_{11}^{11}\left(\varepsilon_{11}^{p}\right)\right)A_{11}^{d22} = \left(1 - d_{11}^{11}\left(\varepsilon_{11}^{p}\right)\right)\left(1 - d_{22}^{11}\left(\varepsilon_{22}^{p}\right)\right)A_{11}
\end{aligned} \tag{11}
$$

These results suggest that the relation between the actual stress and the effective stress should be based on a multiplicative combination of the damage terms as opposed to an additive combination of the damage terms. For example, for the case of plane stress, the relation between the actual and effective stresses could be expressed as follows:

$$
\begin{aligned}
\sigma_{11} &= \left(1 - d_{11}^{11}\right)\left(1 - d_{22}^{11}\right)\left(1 - d_{12}^{11}\right)\sigma_{11}^{eff} \\
\sigma_{22} &= \left(1 - d_{11}^{22}\right)\left(1 - d_{22}^{22}\right)\left(1 - d_{12}^{22}\right)\sigma_{22}^{eff} \\
\sigma_{12} &= \left(1 - d_{11}^{12}\right)\left(1 - d_{22}^{12}\right)\left(1 - d_{12}^{12}\right)\sigma_{12}^{eff}
\end{aligned} \tag{12}
$$

where for each of the damage terms the subscript indicates the direction of the load which initiates the particular increment of damage and the superscript indicates the direction in which the damage takes place.

CHARACTERIZATION OF DAMAGE MODEL

There are two primary items needed for model characterization and input for the damage portion of the material model. First, the values of the various damage parameter terms d_{ij}^{kl} need to be defined in a tabulated manner as a function of the effective plastic strain. Similar to the deformation model, the values of the damage parameters are defined in a tabulated, discretized form in order to reflect the actual material behavior in the most accurate manner possible. The values are tabulated as a function of the effective plastic strain in order to provide a unified framework to simultaneously track the evolution of multiple damage parameters under multiaxial loading conditions. Note that for the case of uniaxial loading the effective plastic strain equals the uniaxial plastic strain, which maintains consistency with the formulation described above. In addition to characterizing the damage parameters, the various input stress-strain curves need to be converted into plots of effective (undamaged) stress versus effective plastic strain in order to carry out the calculations required by the deformation (plasticity) model. As an example of how this process could be carried out, assume that a material is loaded unidirectionally in the 1 direction. At multiple points, once the actual stress-strain curve has become nonlinear, the total strain (ε_{11}), actual stress (σ_{11}), and average local, damaged modulus E_{11}^{d11} can be measured. Assuming that the original, undamaged modulus E_{11} is known, since the damage in the 1 direction is assumed to be only due to load in the 1 direction (due to the uniaxial load), the damage parameters and effective stress in the 1 direction can be computed at a particular point along the stress-strain curve as follows:

$$\begin{aligned}
1-d_{11}^{11} &= \frac{E_{11}^{d11}}{E_{11}} \\
M_{11} &= 1-d_{11}^{11} \\
\sigma_{11}^{eff} &= \frac{\sigma_{11}}{M_{11}} \\
\varepsilon_{11}^{p} &= \varepsilon_{11} - \frac{\sigma_{11}^{eff}}{E_{xx}}
\end{aligned} \tag{13}$$

These values need to be determined at multiple points, representing different values of plastic strain, in order to fully characterize the evolution of damage as the plastic strain increases.

With this information, an effective stress versus plastic strain $\left(\varepsilon_{11}^{p}\right)$ plot can be generated. From this plot, the effective plastic strain corresponding to the plastic strain in the 1 direction at any particular point can be determined by using the equations shown below, which are based on applying the principal of the equivalence of plastic work [7] in combination with Equation (2), simplifying the expressions for the case of unidirectional loading in the 1 direction [4]:

$$\begin{aligned}
h &= \sqrt{H_{11}\left(\sigma_{11}^{eff}\right)^{2}} \\
\varepsilon_{e}^{p} &= \int \frac{\sigma_{11}^{eff}\, d\varepsilon_{11}^{p}}{h}
\end{aligned} \tag{14}$$

where ε_{e}^{p} is the effective plastic strain and $d\varepsilon_{11}^{p}$ is the increment of plastic strain in the 1 direction. From this data, plots of the effective stress in the 1 direction versus the effective plastic strain as well as plots of the damage parameter d_{11}^{11} as a function of the effective plastic strain can be generated. By measuring the damaged modulus in the other coordinate directions at each of the measured values of plastic strain in the 1 direction, the value of the damage parameters $d_{11}^{22}, d_{11}^{12}, d_{11}^{33}$, etc. can be determined as a function of the plastic strain in the 1 direction, and thus as a function of the effective plastic strain. Ongoing efforts will involve developing and carrying out an appropriate experimental test matrix to characterize and validate the model for a series of representative composite materials.

FAILURE MODEL

A wide variety of failure models have been developed for composites. In models such as the Tsai-Wu failure model [6], a quadratic function of the macroscopic stresses is defined in which the coefficients of the failure function are related to the tensile, compressive and shear failure stresses in the various coordinate directions. This model, while mathematically simple and easy to implement numerically, assumes that the composite failure surface has an ellipsoidal (in 2D) or ovoid (in 3D) shape. In reality, composite failure surfaces often are not in the form of simple shapes. More complex models, such as the Hashin model [10], also utilize quadratic combinations of the macroscopic failure stresses, but utilize only selective terms in the quadratic function in order to link the macroscopic stresses to local failure modes such as fiber or matrix failure. However, an overall quadratic form to the failure functions (albeit in a piecewise fashion) are still assumed. This approach was extended in models such as those developed by Puck et al [11], Pinho et al [12] and Maimi et al [13], in which complex equations were developed to predict local failure mechanisms in terms of macroscopic level stresses. In this manner, the failure response and complex failure surfaces present in actual composites could be more accurately represented. However, in these advanced models, very complex tests are often required to characterize the model parameters and the applicability of the models may be limited to specific composite architectures with specific failure mechanisms.

Within the context of transient dynamic analyses, failure models based on tabulated input are becoming more widely used. For example, in a model recently implemented in LS-DYNA for the impact analysis of metals [14], the effective plastic failure strain of the material is defined to be a function of the triaxiality, t, and the Lode parameter, μ, which are defined below:

$$
\begin{aligned}
t &= \frac{\frac{1}{3}(\sigma_{11}+\sigma_{22}+\sigma_{33})}{\sqrt{3J_2}} \\
\mu &= \frac{27J_3}{2\left(\sqrt{3J_2}\right)^3}
\end{aligned}
\qquad (15)
$$

In these expressions, J_2 is the second invariant of the deviatoric stress tensor and J_3 is the third invariant of the deviatoric stress tensor. The advantage of using the triaxiality and Lode parameter as the independent variables for defining the failure surface is that these terms can completely describe the load state of the material. Under plane stress loading conditions, the triaxiality can vary from a value of ⅔ for the case of biaxial tension to a value of -⅔ for the case of uniaxial compression. Any other load conditions yield triaxiality values within these bounds. For example, the case of uniaxial tension yields a triaxiality value of ⅓, the case of uniaxial compression yields a triaxiality value of -⅓, the case of pure shear yields a triaxiality value of 0 and the case of plane strain yields a triaxiality value of 1/√3 under plane stress conditions. The Lode parameter generalizes the process for the case of three dimensional loading conditions. The Lode parameter can vary between values of -1 and 1 depending on the state of stress that is applied to the material. Therefore, the load state of the material can be completely described by using these two parameters. By plotting the effective plastic strain of the material as a function of the triaxiality and the Lode parameter, the failure surface can be completely defined. These relationships are defined by using input curves and tables based on rigorously obtained experimental data as opposed to analytical functions, thus providing a tabulated representation of the failure surface. Advantages of this approach are that the experimental failure surface can be precisely defined in the model input and the variation in the material failure response due to varying load conditions can be accounted for in a systematic manner. Furthermore, the actual, experimentally obtained failure surface is used in the model, and a specific shape to the failure surface is not assumed. The failure model and relevant input are based on stress and strain invariants, which maintains the generality of the model.

For the composite failure model developed in this work, the approach applied in the tabulated failure model for metals described above is adapted. The use of stress and strain invariants in composite failure models has been investigated by researchers such as Mayes and Hansen [15] and Feng [16]. Due to the fact that the failure response of composites can be brittle in certain material directions, instead of using the effective plastic strain as the dependent variable, a stress invariant first identified by Fleischer [17] is used as the dependent variable, defined as follows for the plane stress case (but generalizable for the full three dimensional case):

$$\bar{\sigma} = \sqrt{\sigma_{11}^2 + \sigma_{22}^2 + 2\sigma_{12}^2} \tag{16}$$

This invariant can be considered to be like a "radius" from the origin to the failure surface. The factor of 2 in front of the shear stress term reflects the symmetry of the stress tensor. This invariant can be plotted in terms of the triaxiality to define the failure surface under plane stress conditions. However, to account for the orthotropy of composite materials, the "radius" versus triaxiality curve needs to be defined for a variety of fiber orientation angles in order to fully describe the failure behavior of the composite. The angle can also be thought of as the angle between the first principal direction of the stress tensor and the structural axis system. An important point to note is that the failure surface (and the corresponding invariants) can be constructed on the basis of failure stresses or failure strains. To generalize to a three dimensional loading condition, the stress (or strain) invariant needs to be plotted as a function of the triaxiality and Lode parameter for a variety of fiber orientation angles.

To characterize the failure model, the stress invariant defined in Equation (16) needs to be determined experimentally at the point of failure for a variety of loading conditions, expressed as appropriate values of the triaxiality and Lode parameter, for a variety of fiber orientation angles. These values need to then be expressed in a tabulated form. To utilize the failure model within a computational algorithm, given a computed stress state, the angle of the first principal stress direction and the baseline axis system, computed using the first eigenvector of the stress tensor, needs to be determined. Next, the triaxiality and Lode parameter of the stress condition need to be determined. Having then fully defined the "location" of the current load condition within the stress space, the stress invariant defined in Equation (16) needs to be computed. If the value of the invariant for the computed stresses for the selected angle, triaxiality and Lode parameter is less than the value at failure, failure is deemed to not have occurred. If the value of the invariant is equal to or greater than the corresponding failure value of the invariant for the particular load state, failure is considered to have occurred.

CONCLUSIONS

A generalized composite model suitable for use in polymer composite impact simulations has been developed. The model utilizes a plasticity based deformation model obtained by generalizing the Tsai-Wu failure criteria. A strain equivalent damage model has also been developed in which loading the material in a particular loading direction can lead to damage in multiple coordinate directions. A systematic approach to characterizing the parameters in the damage model has been developed. A methodology for utilizing a general, tabulated failure model has been formulated.

Future efforts will included developing the detailed numerical algorithm to implement the damage and failure models into the material model being developed for inclusion within the LS-DYNA computer code. The failure model will also be refined to include appropriate techniques for element removal once the failure criteria has been satisfied. Methods to account for delamination failure will also be formulated. Extensive sets of verification and validation studies will also be undertaken in order to fully exercise the developed model.

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