A Time-Regularized, Multiple Gravity-Assist Low-Thrust, Bounded-Impulse Model for Trajectory Optimization

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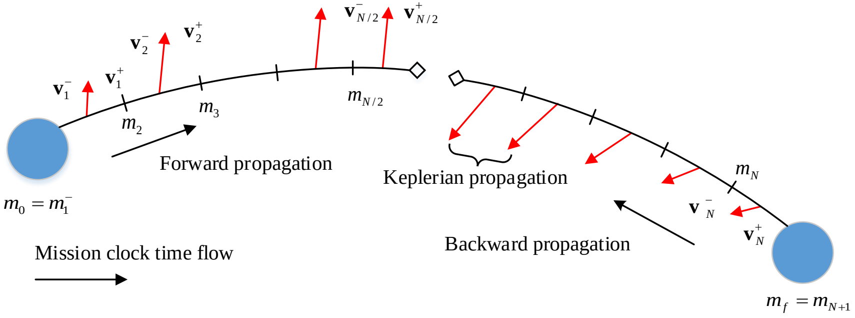
Bounded-Impulse Trajectory Models

- Integral aspect of preliminary mission design
- Provide an accurate mass budget, if not trajectory
- EMTG, MALTO, PaGMO, GALLOP
  - Sims-Flanagan transcription, (Sims and Flanagan, 1999)
- Medium-fidelity solutions used as initial guesses for flight-qualified solutions
  - Mystic, GMAT

$$\begin{bmatrix} \mathbf{r}_{k+1} \\ \mathbf{v}_{k+1} \end{bmatrix} = \begin{bmatrix} F & G \\ \dot{F} & \dot{G} \end{bmatrix} \begin{bmatrix} \mathbf{r}_k \\ \mathbf{v}_k \end{bmatrix} \tag{1}$$

Multiple Gravity-Assist Low-Thrust Model



- Sims-Flanagan and patched conic approximation, phase divided into N control segments
- Control nodes evenly spaced in time $\Delta t = \frac{\Delta t_{\mathrm{p}}}{N}$

Modeling High-Eccentricity with MGALT

- Equal temporal spacing of control nodes is non-optimal for increasing eccentricity.
- Apoapse raising is most efficiently achieved via maneuvering at periapse.
- Using additional segments results in an increase in problem dimension.
- Redistribution of the same number of nodes would be more desirable.
- This is typically achieved with a form of time regularization.

The Sundman Transformation

- Differential transformation introduced by Karl Sundman to regularize the three-body problem.
- A new variable χ replaces time as the independent propagation variable:

$$dt = \frac{r}{\sqrt{\mu}} d\chi \tag{2}$$

- This results in an even geometric distribution of control nodes around the orbit.

Multiple Gravity-Assist Low-Thrust Sundman

- The MGALT model encodes the phase flight time Δt_{p} as a decision variable
- MGALTS adds a total angular propagation variable $\Delta \chi_{\mathsf{p}}$
- Each segment is propagated for a portion of the total independent variable: $\Delta \chi = \frac{\Delta \chi_{\mathsf{p}}}{N}$
- The propagation time associated with each $\Delta \chi$ is computed using a universal Kepler solver in an inverse sense.
  - χ is transcendental in Kepler's equation, but is now selected by NLP solver $\implies$ iteration is no longer required in Kepler solver
- An additional nonlinear match point constraint is required to ensure match point epoch is continuous:

$$\mathbf{c}^{\dagger} = \begin{bmatrix} \mathbf{r}_B - \mathbf{r}_F \\ \mathbf{v}_B - \mathbf{v}_F \\ m_B - m_F \\ t_B - t_F \end{bmatrix} = \mathbf{0} \tag{3}$$

- Analytic derivatives of the match point constraint are still possible to compute

Calculating Maximum Impulse Magnitudes

- For MGALT, the maximum permissible Δv in a segment is computed as follows:

$$\Delta v_{\text{max}} = \frac{N_{\text{active}} D \, T_{\text{max}} \Delta t}{m_k} \tag{4}$$

- For MGALTS, this can only be computed after the segment propagation time Δt has been determined
- Impulses shifted from the center of the segments to the boundary
- The maneuver at the match point can *potentially* be twice the size of the others
  - This potential reduces as the number of segments increases.

True Anomaly as the Independent Variable

- In order to achieve a different control node distribution, consider the generalized Sundman transformation

$$dt = c\, r^{\gamma} d\chi \qquad c \in \mathbb{R} \quad \text{and} \quad \gamma \in \{\mathbb{R} : \gamma \geq 0\} \tag{5}$$

- Universal Kepler propagator based on $\{U_n(\chi;\alpha)\}$, derivations based on $\gamma = 1$ corresponding to the eccentric anomaly.
- Require a relationship between Δf and $\Delta\chi$

$$\Delta\chi = \begin{cases} \dfrac{2r_k \sin\left(\frac{1}{2}\Delta f\right)}{\sqrt{p}\,\cos\left(\frac{1}{2}\Delta f\right) - \sigma_k \sin\left(\frac{1}{2}\Delta f\right)} & ;\alpha = 0 \\ \dfrac{2}{\sqrt{\alpha}}\tan^{-1}\left[\dfrac{\sqrt{\alpha}r_k \sin\left(\frac{1}{2}\Delta f\right)}{\sqrt{p}\,\cos\left(\frac{1}{2}\Delta f\right) - \sigma_k \sin\left(\frac{1}{2}\Delta f\right)}\right] & ;\alpha > 0 \\ \dfrac{2}{\sqrt{-\alpha}}\tanh^{-1}\left[\dfrac{\sqrt{-\alpha}r_k \sin\left(\frac{1}{2}\Delta f\right)}{\sqrt{p}\,\cos\left(\frac{1}{2}\Delta f\right) - \sigma_k \sin\left(\frac{1}{2}\Delta f\right)}\right] & ;\alpha < 0 \end{cases} \tag{6}$$

Rendezvous with Comet 45P/Honda-Mrkos-Pajdušáková

Parameter	Value
Maximum allowed initial mass	4000 kg
Launch window	5 years
Maximum flight time	unbounded
Earliest allowed launch date	August 27^{th}, 2016
Latest allowed arrival date	January 1^{st}, 2030
Launch vehicle	Atlas V (555) NLS-2
Maximum launch C3	21.7156 km^2/s^2
Launch declination bounds	$[-28.5^\circ, 28.5^\circ]$
Thruster	2 x NEXT TT11 high thrust
Throttle logic	minimum number of thrusters
Thruster duty cycle	0.95
Solar array BOL power (launch at 1 A.U.)	40.0 kW
Power margin (δ_{power})	15%
Post-launch checkout coast	60 days
Number of segments per phase	200
Ephemeris	SPICE
Objective function	maximize final mass

- Solar electric propulsion model adheres to Discovery class mission proposal requirements

Rendezvous with Comet 45P: MGALT

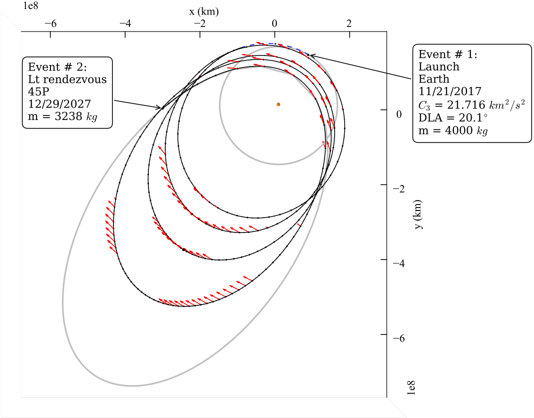


Figure: Rendezvous with 45P using MGALT with 200 segments, $\Delta t_{\text{mission}} = 3690$ days

Rendezvous with Comet 45P: MGALTS

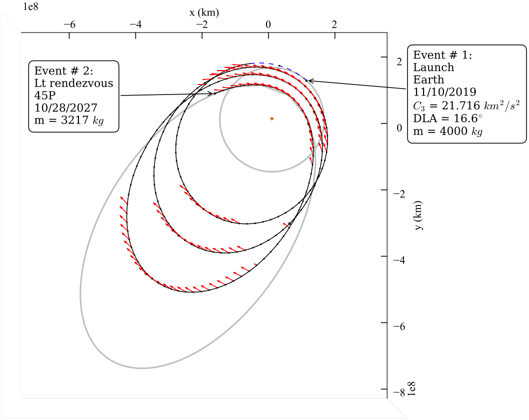
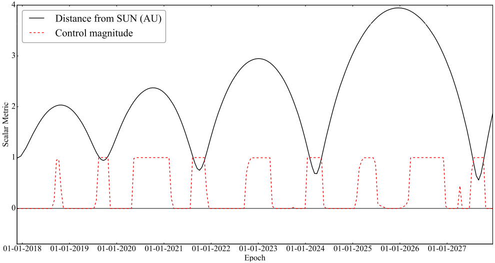
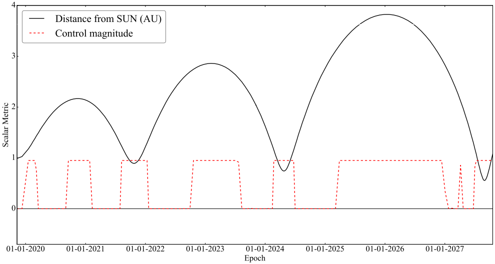


Figure: Rendezvous with 45P using MGALTS with 200 segments, $\Delta t_{\text{mission}} = 2909$ days

Rendezvous with Comet 45P: Comparison





- MGALT requires an additional revolution of the sun
- Additional nodes at periapse affords MGALTS more control authority
- MGALTS TOF is substantially less

Rendezvous with Comet 45P: MGALT More Nodes!

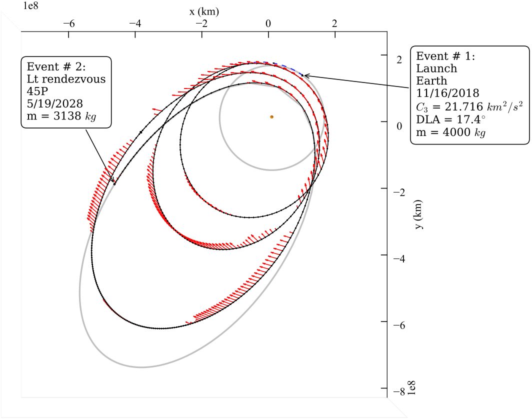


Figure: Rendezvous using MGALT with 400 segments, $\Delta t_{\text{mission}} = 3472$ days

- MGALTS can achieve the same transfer using far fewer control nodes

Rendezvous with Comet 45P: MGALTS True Anomaly

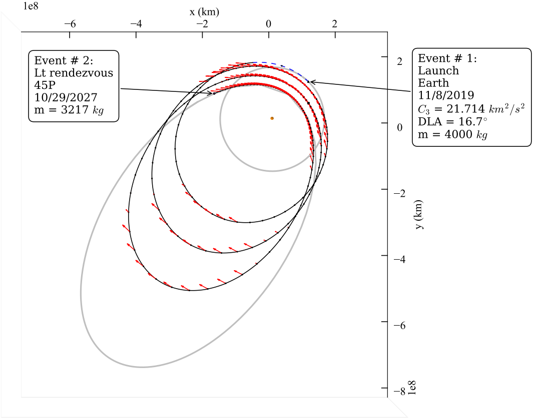


Figure: Rendezvous using the MGALTS true anomaly-based transcription with 200 segments, $\Delta t_{\text{mission}} = 2913$ days

Rendezvous with Comet 45P: MGALTS True Anomaly 2

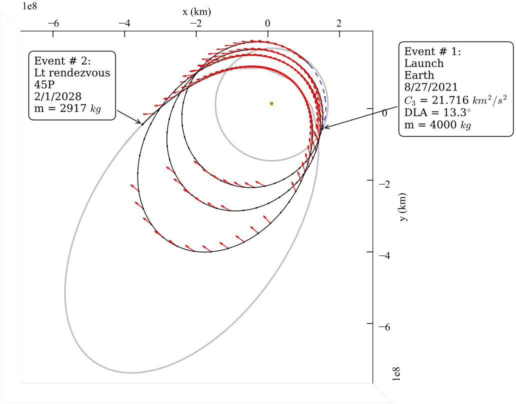


Figure: MGALTS true anomaly, 200 segments, $\Delta t_{\text{mission}} = 2350$ days

- Using true anomaly as the independent variable, nodes are now concentrated at periapse
- Short flight time pump-up maneuvers are accessible
- Useful inside an outer loop that is capable of exploring wide flight time ranges

Summary

- A medium-fidelity low-thrust model has been developed that solves the time-regularized Kepler problem exactly.
- Straightforward extension of the MGALT model in use by many preliminary design tools
- Kepler propagation is made iteration free at the expense of one additional decision variable and constraint per phase.
- Especially useful for trajectories requiring large eccentricity increases
  - e.g. Pump-up/down maneuvers
- MGALTS can perform certain transfers using fewer control nodes than MGALT.
- Certain solution families can only reasonably be accessed using MGALTS.

Thank You

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