National Aeronautics and Space Administration

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SPACE LAUNCH SYSTEM

January 11, 2017

Using Dispersed Modes During Model Correlation

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SLS SPACE LAUNCH SYSTEM

Outline

- **Introduction**
  - Dispersions
  - Best Model Estimate
- **Model Dispersions**
  - Using MAT cards
    - Uncertainty distributions
  - Dispersion-to-test comparison metrics
    - Frequency
    - Mode shapes – MAC vs XOR
  - Pareto front
  - Using Attune
- **Examples**
  - TAURUS
  - Ares I-X
- **Conclusions/Future Work**

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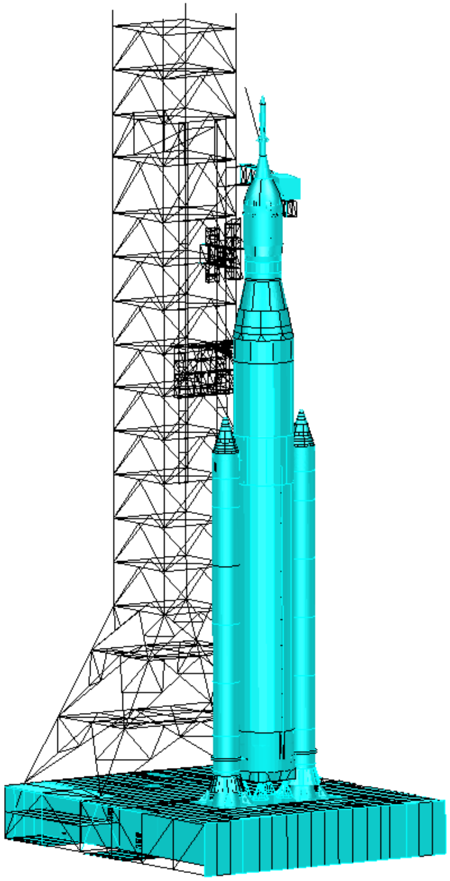
- Rodney Rocha (JSC/ES611)

Introduction

- **Model correlation procedure typically takes several months to complete for a large modal test**
  - SLS timeframe only allows for less than 2 weeks

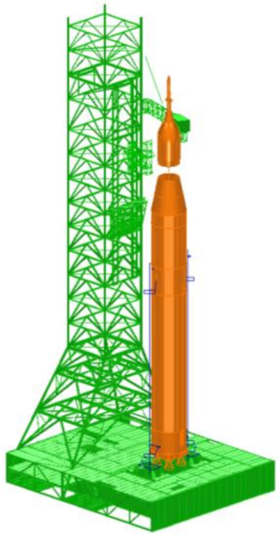
- **Best Model Estimate (BME) process developed to perform a QUICK model correlation**
  - Model that most closely represents test data
  1. Prior to test, create thousands of pretest model dispersions
  2. Post test, select one dispersion that best matches the test data → provides a coarse optimization
  3. Using single iteration of Attune (from ATA Engineering), perform optimization to further refine model

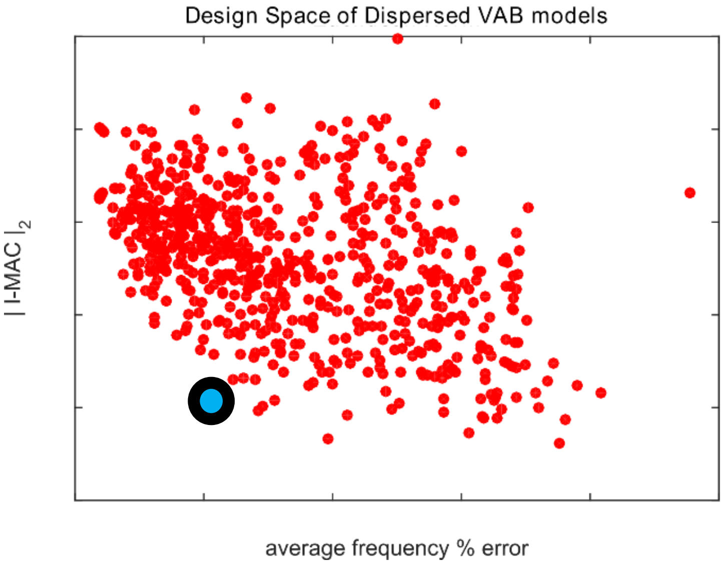
- **BME is a QUICK model tuning effort and not a full model correlation**
  - Full model correlation will occur after test but will not finish until much later



BME Creation - Dispersions

- **Candidate set is created with dispersions of the nominal test configuration model**
  - 1000's of dispersions are created ahead of the test
    - Enables the quick turnaround
- **Each of the model dispersions are compared against test data to find the dispersion that most closely matches test**
  - Mode frequencies
  - Mode shapes
- **The best match is further refined using optimization tool**
  - Refined model is BME
- **Presentation will show two examples of the BME method**
  - TAURUS-T model
  - Ares I-X





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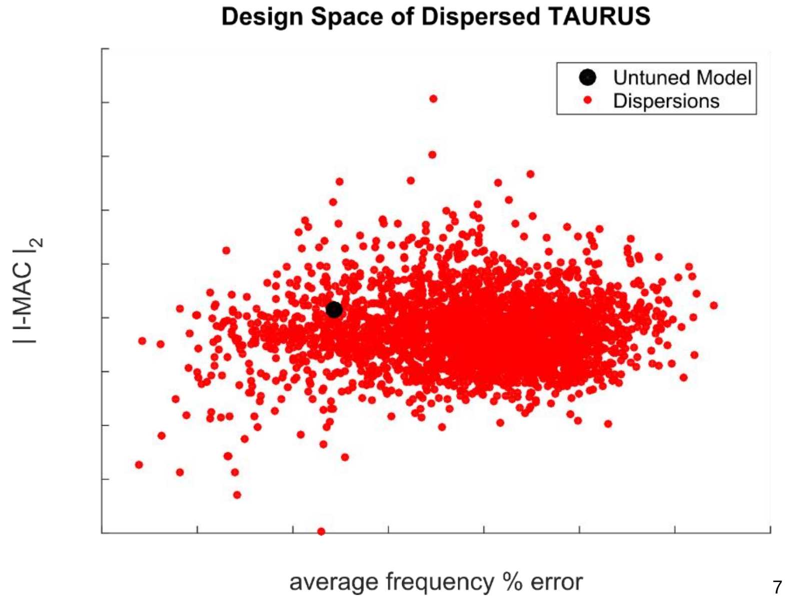
Model Dispersions – Evaluation

- **Uncertainty factors applied to NASTRAN bulk data file MAT cards (elastic modulus or spring stiffness)**
  - Perform sensitivity work to select variables
- **Create dispersions by perturbing nominal**
  - MAT cards: uniform distribution of ±20%
  - Create many dispersions of the test model to cover the design space
    - Limited by computational resources and time
  - Group parameters to reduce number of variables
- **Dispersions are compared against two objectives**
  - Frequency error
    - Average absolute relative error between selected test modes and analysis modes
  - Mode shape error
    - Norm of XOR/MAC error (accuracy of eigenvectors)
      - Euclidean norm
      - Root sum square of the diagonals (used in future)

$$g_1 = \frac{1}{N}\sum_{i=1}^{N} W_i \left| \frac{f_i^{test} - f_i^{model}}{f_i^{test}} \right| * 100$$

$$g_2 = \| [I]\{W\} - [MAC]\{W\} \|$$



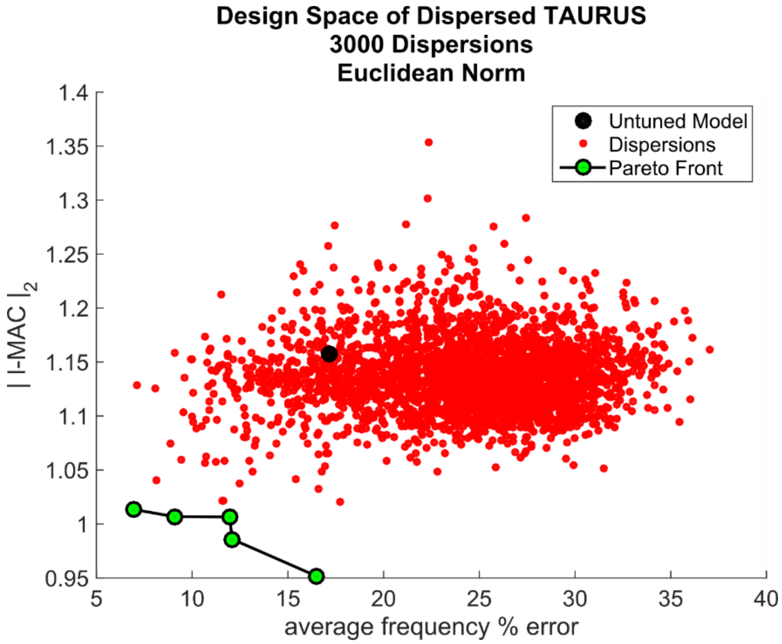
BME Selection – Pareto Front

- **Select model dispersions that best matches data**
  - For this example, “test data” is a dispersed model
  - Selection based on two “objectives”
    - Frequency error
    - Mode shape error
  - Will likely lead to best set of dispersions
  →Pareto Front

$$g_1 = \sum_{i=1}^{f<6Hz} W_i \left| \frac{f_i^{test} - f_i^{model}}{f_i^{test}} \right|$$

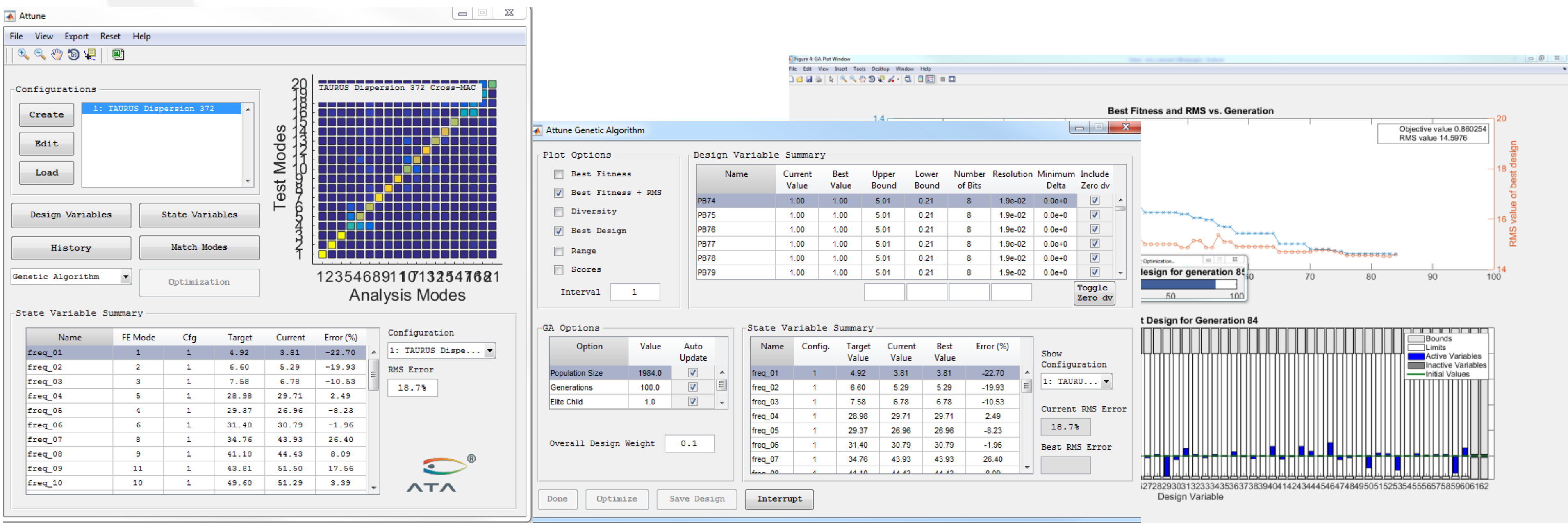
$$g_2 = \| [I]\{W\} - [MAC]\{W\} \|$$

- **Pareto front is a natural outcome of multiobjective optimization**
- **No solution is THE optimal solution**
- **Set of non-dominated optimal solutions**
- **Pareto BME discrimination factors**
  - Closeness to nominal model
  - How models do when different weighting factors are used
  - Using different mode shape error norm



BME Selection – Attune

- **After selection of the best single dispersion, Attune is used to for further model tuning**
  - Use same mode shapes and frequencies as was used in the model dispersion process
  - Variables and groups will be the same as used in model dispersions
  - Upper and lower bounds on variables will be closer to nominal than the dispersion uniform distributions
    - Attune rewards these designs in objective function
  - Use of cross-orthogonality or MAC built in
  - Can weight or de-weight modes as desired
    - Will keep same weights as dispersions



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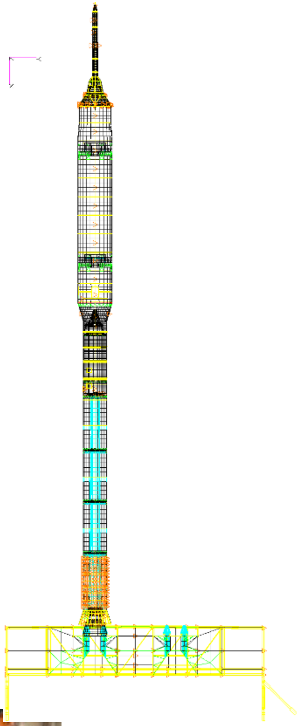
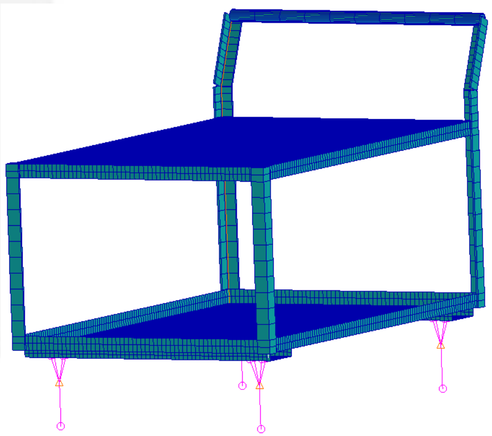
Examples of BME Process

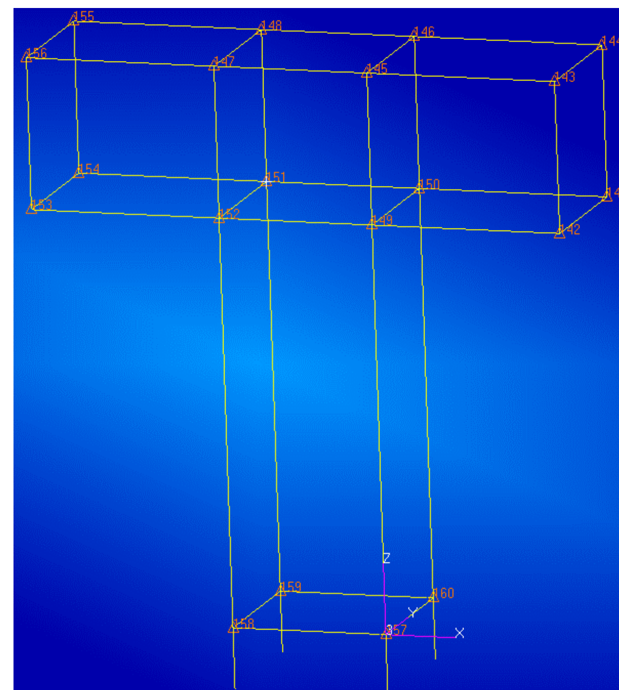
- **The BME dispersion process has been implemented on three models of increasing rocket-ness**
  - ET40 cart model **(in back-up charts)**
    - EV31 intern, Megan Hathcock, built a finite element model, performed modal tests, and correlated the nominal model to the test data
    - Simple, already have test data, have correlated model
  - JSC TAURUS-T
    - Correlated model and model description from Rodney Rocha
    - Test data provided by Michael Grygier
    - Simple, test data provided
    - Sent a tuned version of the TAURUS-T back to JSC for use in the transient response method
  - Ares I-X
    - Actual launch vehicle with test data
    - Nominal model and optimized model provided by Mercedes Reaves at LaRC
    - Test data provided by Dan Lazor (primary curve fitter for I-X)





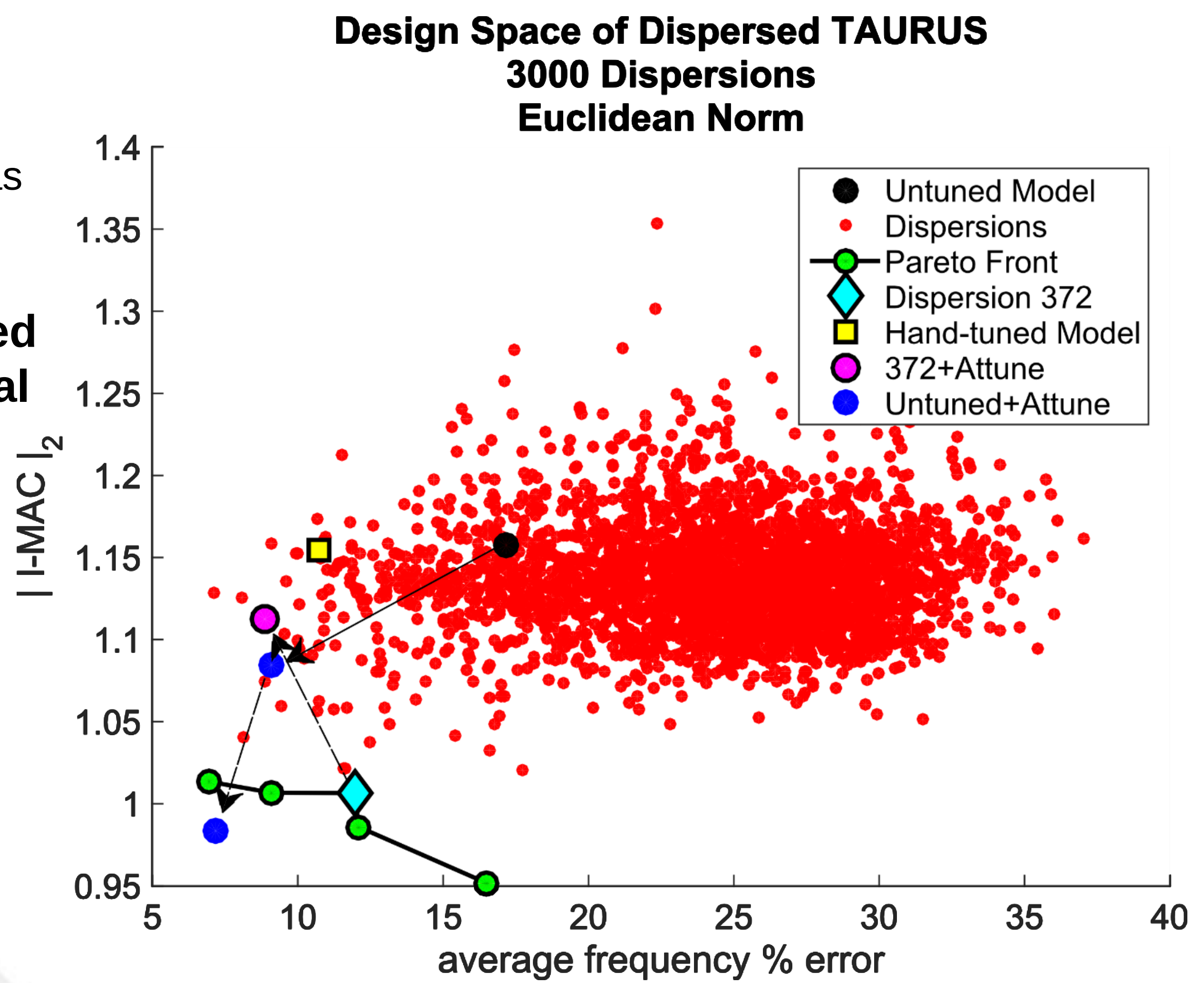
Example: TAURUS

- **Test Article Unit for Rectified Utility Systems Testing (TAURUS-T)**
  - Unistrut structure 70” wide and 80” in height
  - Bolted to floor (rough approximation of cantilever)
  - Joints modeled with springs since unistrut bar shear centers do not quite align
  - Joint stiffness approximated by modeling the brackets that connect the bars
- **Test modes come from hammer tests**
- **Correlated model provided**
- **Nominal model created from correlated model by changing joint springs**
  - Correlated model falls within the range of uncertainties during dispersion process

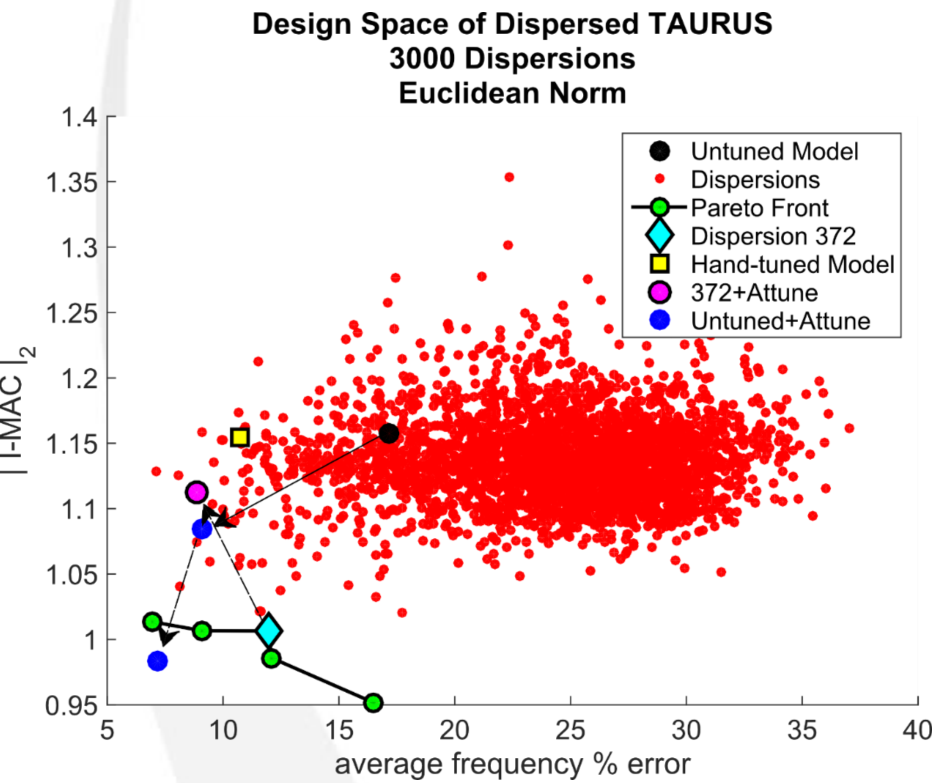


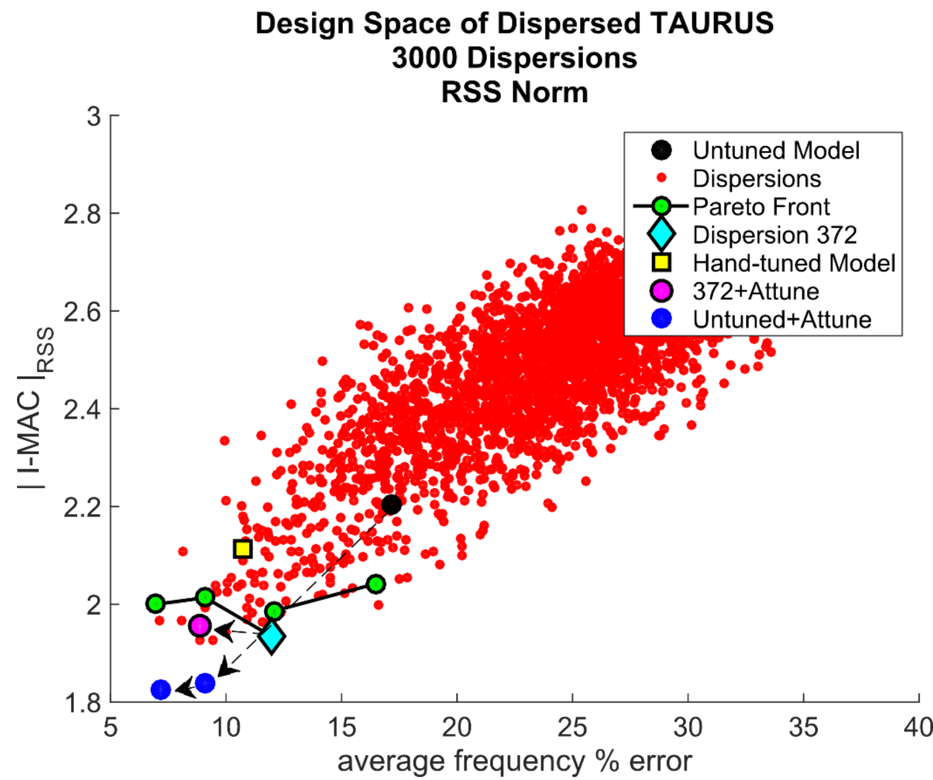
Example: TAURUS

- Comparing to the first 20 test modes
- RSS norm of the mode shape error used as tie-breaker for Pareto front
  - Dispersion 372 chosen as starting point for Attune
- Dispersions outperform hand-tuned model and the nominal model after one iteration of Attune



Example: TAURUS





- **Optimizer makes the model "worse" by one metric while improving it in another metric**
- **Attune using different metrics than when we started**
  - Need to be consistent

Example: Ares I-X

- **Ares I-X**
  - Actual launch vehicle with test data
  - Nominal model provided by Mercedes Reaves at LaRC
  - Test data provided by Dan Lazor (primary curve fitter for I-X)

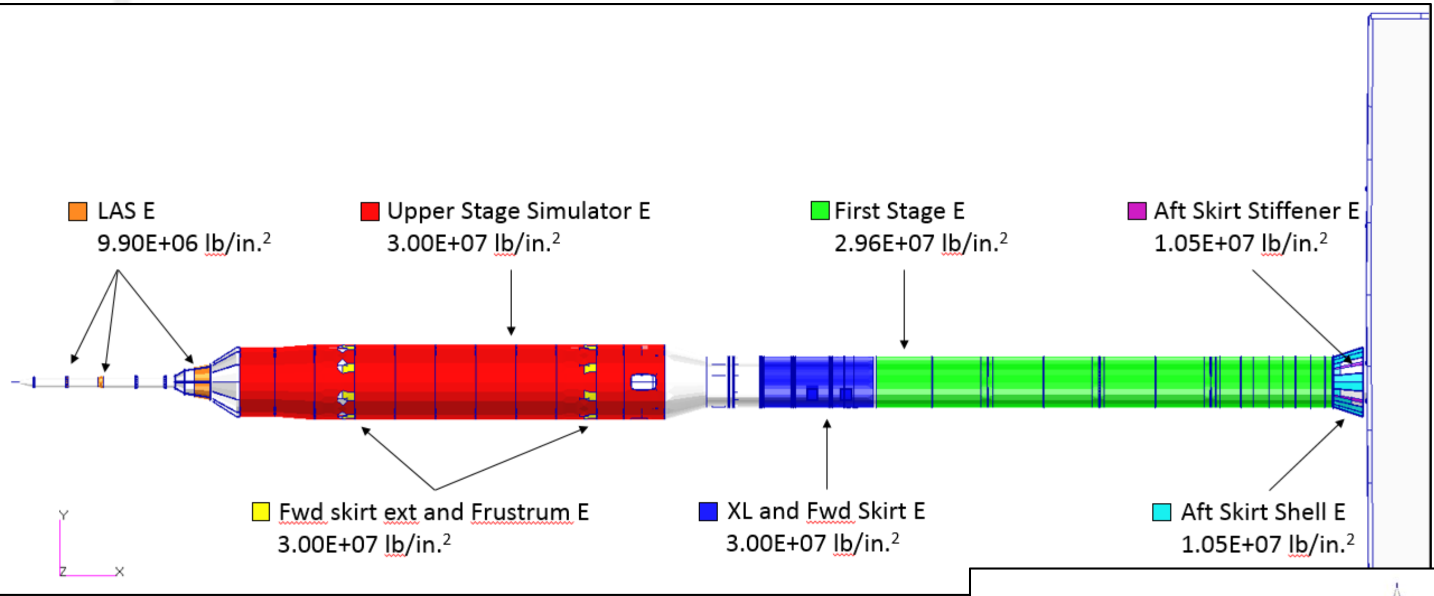
- **Able to compare BME dispersion process to the optimized Ares I-X model found in the literature**
  - L.G. Horta, M.C. Reaves, R.D. Buehrle, J.D. Templeton, D.R. Lazor, J.L. Gaspar, R.A. Parks, P.A. Bartolotta, "Finite Element Model Calibration for Ares I-X Flight Vehicle," Journal of Experimental Mechanics, Vol. 51, 2011, pp. 1251–1263.
  - Used same variables as Horta, et al.

- **Used first seven test modes in the dispersion comparisons**
  - Torsion modes and axial modes difficult to match to test data

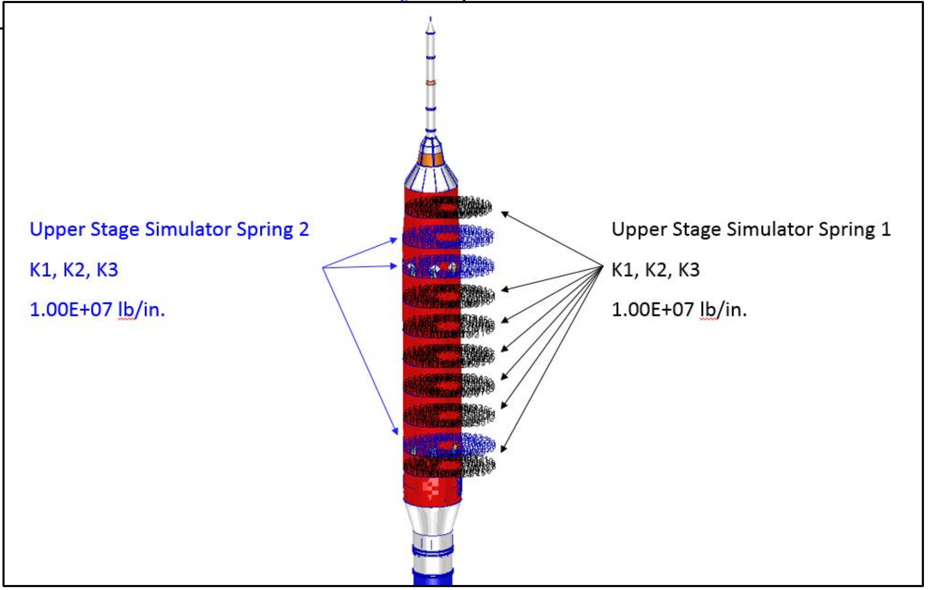
- **Used XOR instead of MAC**
  - Attempting different mode shape metrics

Example: Ares I-X Variables

- **Used same variables as Horta, et al.**

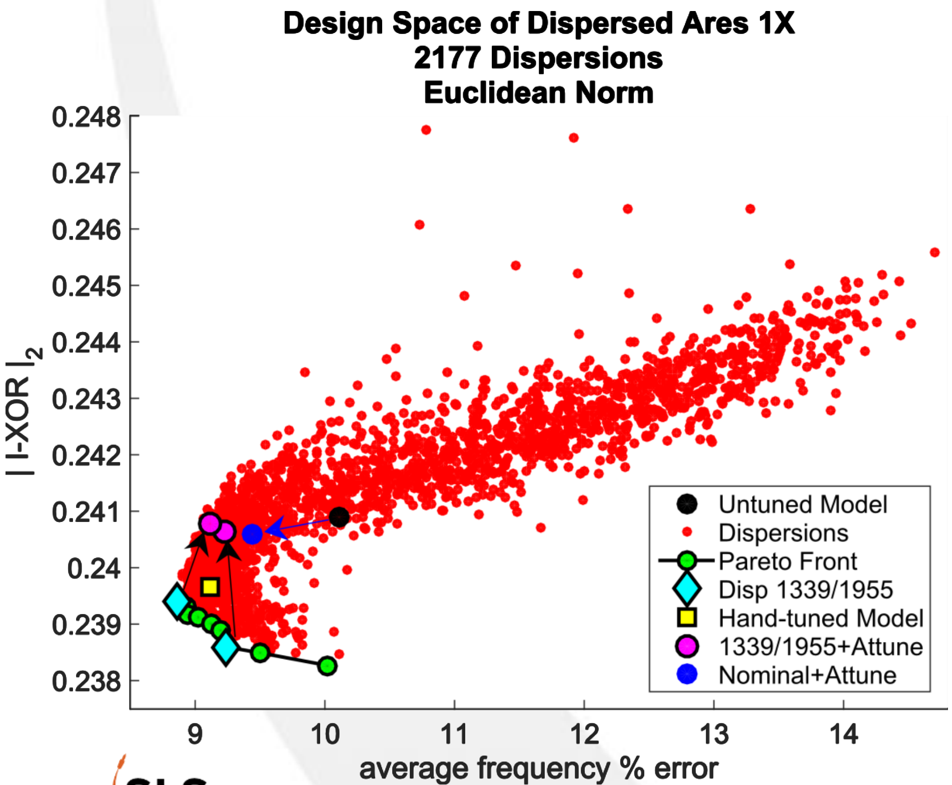


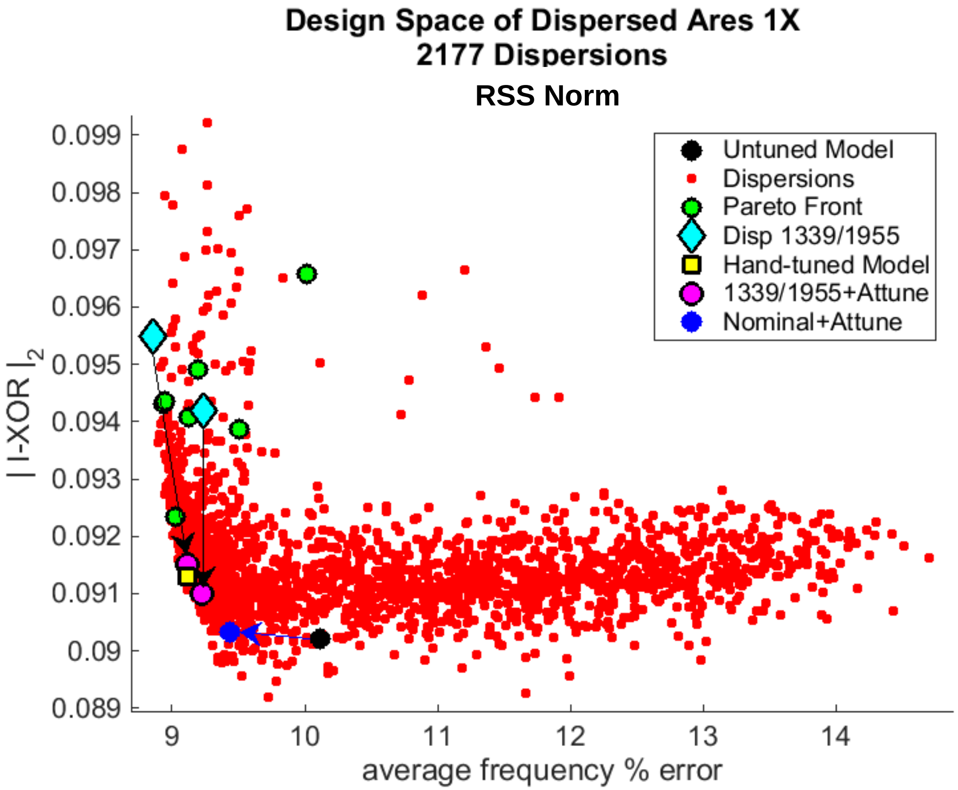
- **Young's modulus of different parts of the vehicle**
  - ±20% uniform distribution
- **Upper stage simulator springs**
  - ±100% uniform distribution



Example: Ares I-X Dispersions

- Using Euclidean norm, the Pareto front outperforms the nominal model and the hand-tuned model
- Using Attune, the Pareto designs get "worse"
- Using the RSS norm of the XOR shows that the optimized model of Horta, et al. is Pareto-optimal and is on par with dispersions + Attune
- Seems to be a limit on how well the Ares I-X model can be tuned





Example: Effect on Non-target Modes

- **Since we only target a few modes to select the BME, what is the effect on the non-target modes that may have an effect on the loads analysis**

- **During BME process, only first 7 test modes are kept for the Ares I-X comparisons, so we compare test modes 8-14**
  - 5 of 7 modes improve in frequency error, only marginally worse for other two
  - 4 of 7 improve XOR diagonal, worst case is decrease from 0.80 to 0.74 (test mode 13)

- **Need to check for other models, but maybe rising tide raises all boats**
  - Improve the physics for some modes, improve the physics for all modes?

Disp 1

			Model Modes													
	Freq		0.18	0.22	1.02	1.17	2.66	1.87	3.25	3.58	3.50	4.78	4.84	3.49	6.01	6.24	
		Mode	1	2	3	4	6	5	7	10	9	13	14	8	16	17	% diff freq
Test Modes	0.18	1	0.98	0.03	0.04	0.02	0.00	0.06	0.04	0.80	0.04	0.15	0.04	0.16	0.28	0.00	4.7
	0.22	2	0.08	0.97	0.02	0.00	0.01	0.06	0.02	0.05	0.00	0.02	0.11	0.05	0.04	0.06	1.0
	1.06	3	0.00	0.00	0.98	0.01	0.00	0.01	0.07	0.21	0.01	0.04	0.00	0.02	0.06	0.01	3.8
	1.19	4	0.00	0.01	0.01	0.91	0.10	0.01	0.00	0.00	0.29	0.00	0.04	0.05	0.01	0.01	1.2
	1.84	5	0.00	0.01	0.02	0.10	0.89	0.09	0.03	0.02	0.11	0.02	0.07	0.02	0.00	0.10	44.9
	2.06	6	0.01	0.01	0.03	0.02	0.05	0.85	0.05	0.04	0.04	0.04	0.02	0.12	0.01	0.03	9.2
	3.45	7	0.03	0.00	0.09	0.01	0.01	0.04	0.89	0.01	0.00	0.08	0.02	0.18	0.01	0.03	6.0
	3.64	8	0.75	0.02	0.20	0.19	0.00	0.05	0.08	**0.77**	**0.35**	**0.02**	**0.15**	**0.09**	**0.07**	**0.00**	1.8
	3.67	9	0.53	0.05	0.12	0.34	0.05	0.06	0.03	**0.57**	**0.51**	**0.01**	**0.25**	**0.11**	**0.07**	**0.01**	4.9
	4.61	10	0.13	0.07	0.03	0.03	0.03	0.00	0.01	**0.03**	**0.09**	**0.79**	**0.51**	**0.01**	**0.09**	**0.10**	3.8
	4.78	11	0.11	0.11	0.03	0.03	0.04	0.01	0.01	**0.03**	**0.15**	**0.62**	**0.71**	**0.00**	**0.07**	**0.12**	1.3
	6.18	12	0.09	0.06	0.27	0.02	0.13	0.00	0.05	**0.03**	**0.10**	**0.01**	**0.06**	**0.35**	**0.42**	**0.31**	43.5
	6.41	13	0.28	0.02	0.04	0.03	0.03	0.04	0.10	**0.10**	**0.03**	**0.02**	**0.01**	**0.08**	**0.80**	**0.20**	6.2
	6.66	14	0.04	0.11	0.20	0.16	0.22	0.05	0.04	**0.03**	**0.02**	**0.00**	**0.22**	**0.08**	**0.11**	**0.27**	6.3

Disp 1339 +Attune

			Model Modes													
	Freq		0.18	0.22	1.03	1.19	2.66	1.87	3.34	3.70	3.61	4.81	4.86	3.51	6.26	6.60	
		Mode	1	2	3	4	6	5	7	10	9	13	14	8	16	18	% diff freq
Test Modes	0.18	1	0.98	0.03	0.04	0.02	0.00	0.06	0.03	0.81	0.03	0.14	0.04	0.10	0.30	0.06	2.2063
	0.22	2	0.08	0.97	0.02	0.01	0.01	0.06	0.02	0.06	0.00	0.02	0.10	0.04	0.04	0.07	2.4
	1.06	3	0.00	0.00	0.98	0.01	0.00	0.01	0.06	0.20	0.01	0.03	0.00	0.01	0.04	0.01	2.5
	1.19	4	0.00	0.00	0.01	0.91	0.11	0.01	0.00	0.00	0.29	0.00	0.03	0.03	0.03	0.07	0.1
	1.84	5	0.00	0.01	0.02	0.10	0.89	0.09	0.03	0.03	0.10	0.02	0.06	0.01	0.02	0.06	45.1
	2.06	6	0.01	0.01	0.03	0.02	0.05	0.85	0.05	0.06	0.05	0.03	0.02	0.12	0.00	0.04	9.2
	3.45	7	0.00	0.00	0.10	0.01	0.01	0.04	0.88	0.01	0.01	0.09	0.02	0.19	0.01	0.01	3.1
	3.64	8	0.75	0.01	0.21	0.19	0.01	0.05	0.08	0.78	0.35	0.02	0.15	0.06	0.08	0.02	1.4
	3.67	9	0.54	0.04	0.13	0.34	0.04	0.06	0.04	0.58	0.50	0.00	0.25	0.04	0.09	0.01	1.9
	4.61	10	0.13	0.07	0.03	0.03	0.03	0.00	0.00	0.03	0.09	0.81	0.51	0.01	0.04	0.05	4.4
	4.78	11	0.11	0.11	0.03	0.03	0.04	0.01	0.00	0.03	0.15	0.61	0.72	0.01	0.04	0.06	1.7
	6.18	12	0.11	0.05	0.27	0.00	0.13	0.00	0.05	0.06	0.12	0.02	0.07	0.34	0.49	0.12	43.1
	6.41	13	0.32	0.02	0.03	0.03	0.03	0.04	0.08	0.11	0.04	0.02	0.01	0.08	0.74	0.18	2.3
	6.66	14	0.04	0.11	0.20	0.13	0.22	0.05	0.04	0.03	0.01	0.00	0.25	0.08	0.11	0.35	0.8

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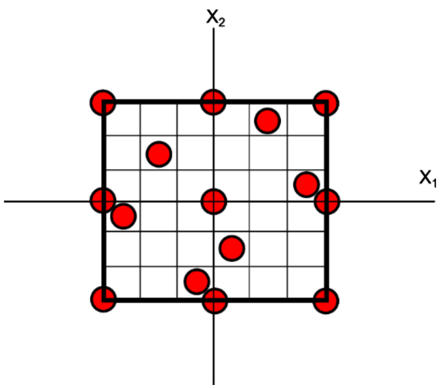
Conclusions and Future Work

- **Using model dispersions as a starting point allows us to quickly adjust a model to reflect new test data**
  - The analyst does a lot of work before the test to save time post-test
  - Creating 1000s of model dispersions to provide “coarse tuning,” then use Attune to provide the “fine tuning”
- **Successful model tuning on three structures**
  - TAURUS
  - Ares I-X
  - Cart (in backup charts)
- **Mode weighting factors, matrix norm method, and XOR vs. MAC all play key roles in determining the BME**
- **The BME process will be used on future tests**
  - ISPE modal test (ongoing work)
  - SLS modal test (mid 2018)

BACK UP

Model Dispersions – Overview

- **Uncertainty factors applied to bulk data file M* cards (Young's modulus or spring stiffness)**
  - Piggyback off of prior sensitivity work to select variables



- **Create dispersions by applying uncertainty factors**
  - M* cards have uniform distribution of ±20% of nominal values
  - Groups of parameters used to reduce number of variables
  - Create many dispersions for the test
    - Goal is 5000 dispersions for each structure
    - Limited by computational resources and time

- **Future Work**
  - Optimal Latin Hypercube design of experiments used to generate the dispersion designs
    - FAC Viana, SURROGATES Toolbox User's Guide, Gainesville, FL, USA, version 3.0 ed., 2011, available at https://sites.google.com/site/srgtstoolbox/.
    - Jin, R., Chen, W., and Sudjianto, A., "An efficient algorithm for constructing optimal design of computer experiments," Journal of Statistical Planning and Inference, Vol. 134, 2005, pp. 268–287.
    - Bates, S. J., Sienz, J., and Toropov, V. V., "Formulation of the optimal Latin hypercube design of experiments using a permutation genetic algorithm," 45th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics and Materials Conference, AIAA, Palm Springs, CA, USA, Apr 2004, pp. AIAA–2004–2011.
  - Investigating how to augment the LHS design space if we get extra time or computational power

MPCV							MSA	LVSA		Forward Skirt		LOx Tank			Intertank				LH2 Tank						Engine Section	
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
0.848	1.177	1.095	1.011	1.158	0.892	1.137	0.907	0.943	1.069	0.913	1.139	0.847	1.045	0.993	0.933	0.943	1.145	1.016	1.137	0.802	1.197	1.042	0.834	1.152	1.133	1.092
0.896	1.052	1.195	1.164	0.931	0.903	1.166	1.144	1.150	1.037	1.008	1.015	0.810	0.876	0.944	1.157	1.187	1.089	1.190	0.846	0.865	0.960	1.134	0.891	1.117	0.837	0.941
0.820	0.822	1.168	1.025	1.044	0.802	1.174	1.194	1.029	0.934	1.057	1.148	0.935	1.145	0.901	1.104	0.984	1.131	0.915	1.040	0.825	0.928	1.170	0.855	1.111	1.196	1.001
1.040	0.899	0.929	1.146	0.940	1.137	0.861	0.903	0.864	1.193	0.969	1.113	0.927	1.163	1.092	1.165	1.197	0.902	0.999	1.080	0.846	1.085	0.952	1.049	0.826	1.155	0.814
1.107	0.813	0.892	0.924	1.078	1.083	0.924	1.113	1.134	1.027	1.167	0.866	1.149	0.962	1.109	0.824	1.194	0.945	0.826	0.956	1.044	0.846	1.084	1.060	0.927	1.097	0.887

Model Dispersions – Tracking modes

Frequency error calculation

- Frequency error determined from mode shape matching

$$MAC = \frac{(\phi_t^T \phi_m)^2}{(\phi_t^T \phi_t)(\phi_m^T \phi_m)} \qquad XOR = \phi_t^T M_{AA} \phi_m \qquad g_1 = \frac{1}{N}\sum_{i=1}^{N}\left(W_i \left|\frac{f_i^{test} - f_i^{model}}{f_i^{test}}\right| * 100\right)$$

Mode shape error calculation

- Calculation of an initial MAC or XOR comparison
  - Note: Guyan mass matrix pulled out of model using Attune DMAP
- MAC matrices are sorted to keep highest values on diagonal
  - Use in-house codes sortMAC.m and sortXOR.m
  - Details in back-up charts

MAC Original Cart Model

Test Modes			Model Modes							
	Freq (Hz)		14.6	16.1	22.6	25.5	26.5	42	46.7	53.3
		Mode	1	2	3	4	5	6	7	8
	23.8	1	0.24	0	0	0	0.69	0	0	0
	24.3	2	0	0.62	0	0	0.01	0.01	0.03	0.03
	29.1	3	0	0	0.17	0	0	0.62	0	0
	40.7	4	0.42	0	0	0	0.18	0.03	0	0
	45.3	5	0	0	0.10	0	0	0.09	0.01	0
	47.4	6	0	0.03	0	0.62	0	0	0.05	0.10
	51	7	0.01	0	0	0.03	0	0	0	0.48
	53.8	8	0	0	0	0.04	0	0	0.01	0.27

MAC Original Cart Model Sorted

Test Modes			Model Modes							
	Freq (Hz)		26.5	16.1	42	14.6	22.6	25.5	53.3	46.7
		Mode	5	2	6	1	3	4	8	7
	23.8	1	0.69	0	0	0.24	0	0	0	0
	24.3	2	0.01	0.62	0.01	0	0	0	0.03	0.03
	29.1	3	0	0	0.62	0	0.17	0	0	0
	40.7	4	0.18	0	0.03	0.42	0	0	0	0
	45.3	5	0	0	0.09	0	0.1	0	0	0.01
	47.4	6	0	0.03	0	0	0	0.62	0.1	0.05
	51	7	0	0	0	0.01	0	0.03	0.48	0
	53.8	8	0	0	0	0	0	0.04	0.27	0.01

MAC/XOR Sort Details

- MAC = $\frac{(\varphi_T' * \varphi_M)^2}{\varphi_T' * \varphi_T * \varphi_M' * \varphi_M}$
- **MAC matrices are sorted to keep the highest values along the diagonal**

MAC Original Model

	Model Modes									
	Freq (Hz)		14.6	16.1	22.6	25.5	26.5	42	46.7	53.3
		Mode	1	2	3	4	5	6	7	8
Test Modes	23.8	1	0.24	0	0	0	0.69	0	0	0
	24.3	2	0	0.62	0	0	0.01	0.01	0.03	0.03
	29.1	3	0	0	0.17	0	0	0.62	0	0
	40.7	4	0.42	0	0	0	0.18	0.03	0	0
	45.3	5	0	0	0.10	0	0	0.09	0.01	0
	47.4	6	0	0.03	0	0.62	0	0	0.05	0.10
	51	7	0.01	0	0	0.03	0	0	0	0.48
	53.8	8	0	0	0	0.04	0	0	0.01	0.27

MAC Original Model Sorted

	Model Modes									
	Freq (Hz)		26.5	16.1	42	14.6	22.6	25.5	53.3	46.7
		Mode	5	2	6	1	3	4	8	7
Test Modes	23.8	1	0.69	0	0	0.24	0	0	0	0
	24.3	2	0.01	0.62	0.01	0	0	0	0.03	0.03
	29.1	3	0	0	0.62	0	0.17	0	0	0
	40.7	4	0.18	0	0.03	0.42	0	0	0	0
	45.3	5	0	0	0.09	0	0.1	0	0	0.01
	47.4	6	0	0.03	0	0	0	0.62	0.1	0.05
	51	7	0	0	0	0.01	0	0.03	0.48	0
	53.8	8	0	0	0	0	0	0.04	0.27	0.01

MAC/XOR Sort Details

MAC Original Model									
	Model Modes								
	Mode	1	2	3	4	5	6	7	8
Test Modes	1	0.24	0	0	0	0.69	0	0	0
	2	0	0.62	0	0	0.01	0.01	0.03	0.03
	3	0	0	0.17	0	0	0.62	0	0
	4	0.42	0	0	0	0.18	0.03	0	0
	5	0	0	0.10	0	0	0.09	0.01	0
	6	0	0.03	0	0.62	0	0	0.05	0.10
	7	0.01	0	0	0.03	0	0	0	0.48
	8	0	0	0	0.04	0	0	0.01	0.27
Row Position		4	2	3	6	1	3	6	7

Duplicate MAC									
	Model Modes								
	Mode	1	2	3	4	5	6	7	8
Test Modes	1	0.24	0	0	0	0.69	0	0	0
	2	0	0.62	0	0	0.01	0.01	0	0.03
	3	0	0	0	0	0	0.62	0	0
	4	0.42	0	0	0	0.18	0.03	0	0
	5	0	0	0.10	0	0	0.09	0	0
	6	0	0.03	0	0.62	0	0	0	0.10
	7	0.01	0	0	0.03	0	0	0	0.48
	8	0	0	0	0.04	0	0	0.01	0.27
Row Position		4	2	5	6	1	3	8	7

- **sortMAC.m identifies the maximum value i( ) in each column, returns the row position, and creates a duplicate MAC matrix.**
- **It then compares row positions and looks for repeats ( ). In this case both 3 and 6 have duplicates.**
- **It compares the duplicate values and zeroes ( ) out the smaller one in the duplicate MAC matrix.**
- **The function then finds the max in each column again and repeats the process until there are no duplicates in the max position array ( ).**

MAC/XOR Sort Details

Sorting Array								
Model Modes	1	2	3	4	5	6	7	8
Max Position	4	2	5	6	1	3	8	7

Sorting Array								
Model Modes	5	2	6	1	3	4	8	7
Max Position	1	2	3	4	5	6	7	8

MAC Original Model Sorted

			Model Modes							
	Freq (Hz)		26.5	16.1	42	14.6	22.6	25.5	53.3	46.7
		Mode	5	2	6	1	3	4	8	7
Test Modes	23.8	1	0.69	0	0	0.24	0	0	0	0
	24.3	2	0.01	0.62	0.01	0	0	0	0.03	0.03
	29.1	3	0	0	0.62	0	0.17	0	0	0
	40.7	4	0.18	0	0.03	0.42	0	0	0	0
	45.3	5	0	0	0.09	0	0.1	0	0	0.01
	47.4	6	0	0.03	0	0	0	0.62	0.1	0.05
	51	7	0	0	0	0.01	0	0.03	0.48	0
	53.8	8	0	0	0	0	0	0.04	0.27	0.01

- **Once there are no duplicates, the function creates a sorting array by concatenating the model mode numbers and max position arrays ( ).**
- **The function then sorts the sorting array by max position and then arranges the original MAC matrix and model frequencies by the Model Modes matrix.**

BME Example: ET40 Cart

- **Second example for BME process is small cart in ET40**
  - Initially chosen as a way to test out EV31 operational modal analysis codes
  - Model developed and correlated in-house

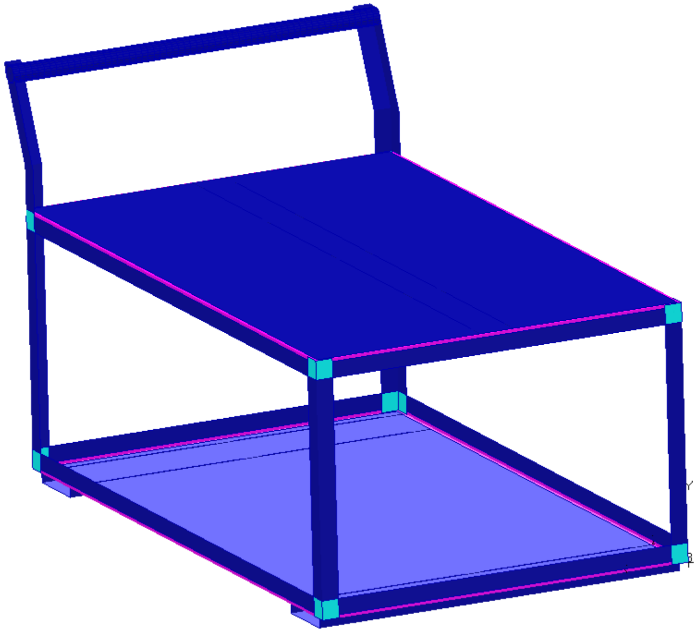
  - Hand-tuning process took 300+ iterations to get final model
  - Dispersions + Attune process saves time and effort while achieving similar results

BME Example: ET40 Cart Modes

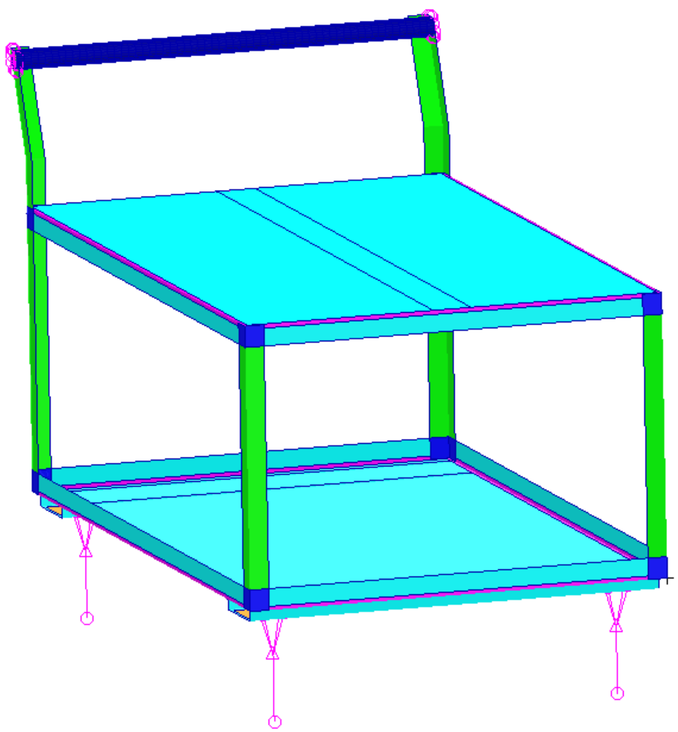
- First nominal model mode (7.5 Hz) is wheel rotation mode and not captured in test due to lack of sensors
- Similarly, second model mode (9.1 Hz) is wheel mode
- Modes 3-10 used to compare dispersions to test data
- Data from roving accelerometer test with cart in “grounded” configuration
  - Wheels are constrained to prevent motion during testing (duct tape and bolts)

Mode #	Nominal Model Modes (Hz)	Hand-Tuned Model Modes (Hz)	Test Modes (Hz)
1	7.5	1.56	—
2	9.1	15.05	—
3	14.59	22.07	23.8
4	16.07	25.43	24.3
5	22.56	31.39	29.1
6	25.48	43.13	40.7
7	26.49	47.27	45.3
8	42.04	47.41	47.4
9	46.65	50.18	51
10	53.26	56.74	53.8

BME Example: ET40 Cart Variables

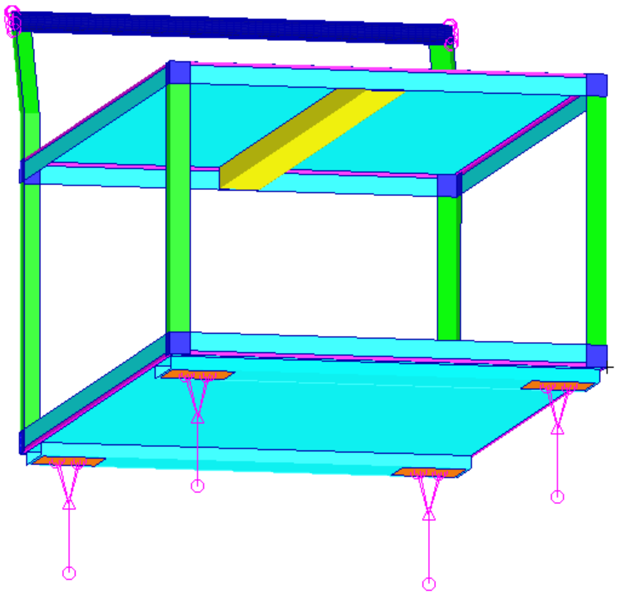


Cart Material Properties				
Color	Group	Property	Int. Value	Units
(blue)	Mat 1	E	2.90E+07	psi
		Rho	0.283	lb/in.^{3}
(magenta)	Mat 2	E	2.90E+07	psi
		Rho	0.20874	lb/in.^{3}
(cyan)	Mat 3	E	2.90E+07	psi
		Rho	0.283	lb/in.^{3}
Dispersed ± 20%				



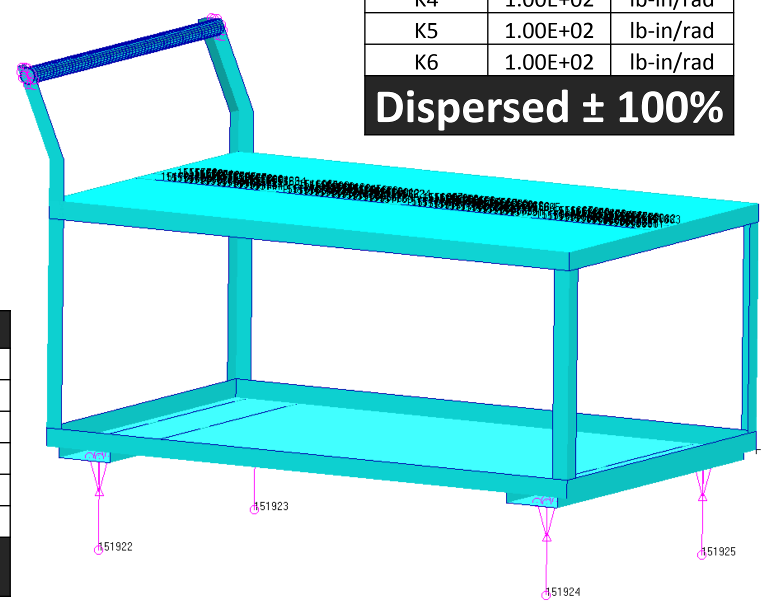
Cart Physical Properties			
Color	Property	Int. Value	Units
(yellow)	Top Beam Thickness	0.103	in.
(green)	Leg Thickness	0.2	in.
(cyan)	Shelve Thickness	0.12	in.
(dark blue)	Handle Diameter (OD/ID)	0.68	in.
		0.61	in.
(orange)	Wheel Plate Thickness	0.34	in.
(magenta)	Edge Thickness	0.12	in.
(blue)	Corner Thickness	0.16	in.
Dispersed ± 30%			

BME Example: ET40 Cart Variables



Cart Physical Properties			
Color	Property	Int. Value	Units
	Top Beam Thickness	0.103	in.
	Leg Thickness	0.2	in.
	Shelve Thickness	0.12	in.
	Handle Diameter (OD/ID)	0.68	in.
		0.61	in.
	Wheel Plate Thickness	0.34	in.
	Edge Thickness	0.12	in.
	Corner Thickness	0.16	in.
Dispersed ± 30%			

Top Bar Springs		
K1	1.00E+02	lb/in.
K2	1.00E+02	lb/in.
K3	1.00E+02	lb/in.
K4	1.00E+02	lb-in/rad
K5	1.00E+02	lb-in/rad
K6	1.00E+02	lb-in/rad
Dispersed ± 100%		



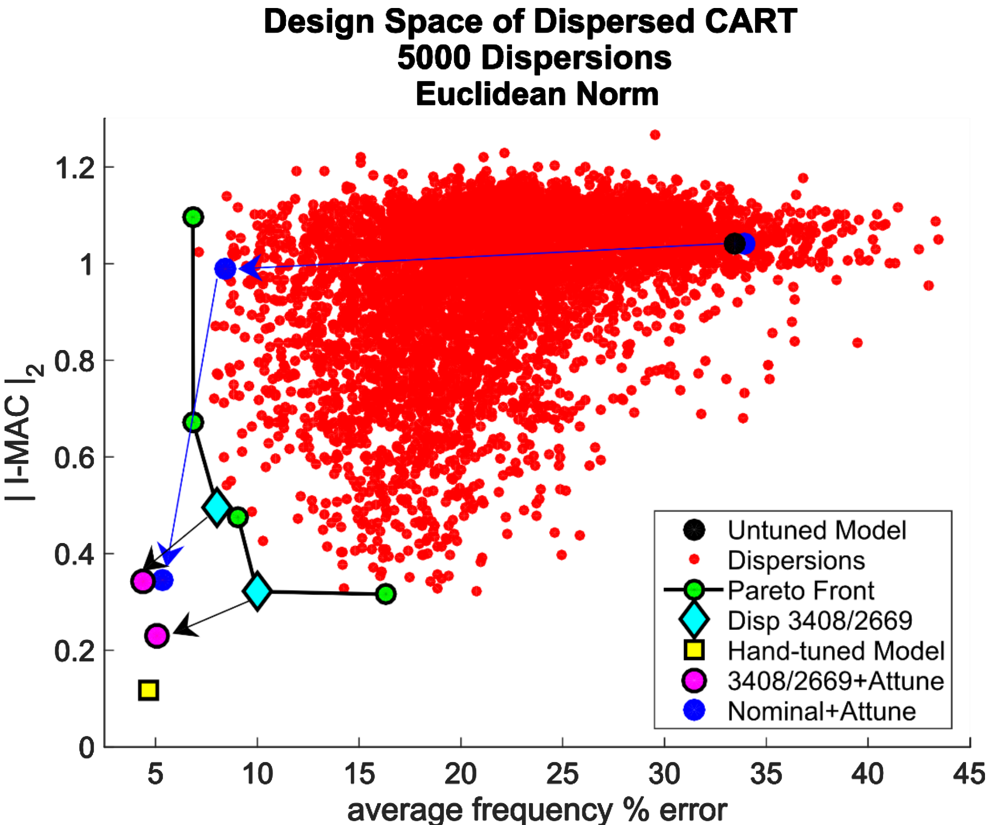
Back Wheel Springs		
K1	1.00E+03	lb/in.
K2	1.00E+03	lb/in.
K3	1.00E+03	lb/in.
K4	1.00E+04	lb-in/rad
K5	1.00E+04	lb-in/rad
K6	1.00E+04	lb-in/rad
Dispersed ± 100%		

Front Wheel Springs		
K1	1.00E+03	lb/in.
K2	1.00E+03	lb/in.
K3	1.00E+03	lb/in.
K4	1.00E+03	lb-in/rad
K5	1.00E+03	lb-in/rad
K6	1.00E+03	lb-in/rad
Dispersed ± 100%		

BME Example: ET40 Cart Dispersions

- **Starting Pareto points chosen by engineering judgment**
  - Chose points with good MAC to avoid mode swapping

- **Hand-tuned model performs the best**
- **Pareto points + 1 iteration of Attune (magenta) perform nearly as well as the hand tuned model while saving time**
- **Nominal model with three iterations of Attune is dominated by the Pareto+1 Attune points**



Example: TAURUS

- The range of joint spring stiffnesses chosen such that the correlated model is a possible dispersion model
- 3000 dispersions were created by applying an uncertainty factor between 0.05-20 for each of the spring degrees of freedom
- Young's modulus and density uncertainty factors between 0.95-1.05

	Nominal	min	max
k1-k3	1.0e5	3.3e4	1.8e6
k4-k5	1.0e5	2.4e4	9.3e5

Mac: Test vs Hand Tuned Model													
	Model Modes												
		Freq(Hz)	3.36	5.02	5.65	27.85	27.06	30.88	38.46	43.82	47.14	49.92	
	Freq(Hz)	Mode	1	2	3	5	4	6	7	8	9	10	Freq % Diff
Test Modes	4.92	1	0.99	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	31.74
	6.60	2	0.00	0.99	0.00	0.00	0.00	0.01	0.00	0.02	0.00	0.00	24.00
	7.58	3	0.00	0.00	0.99	0.01	0.00	0.00	0.00	0.00	0.00	0.00	25.50
	28.99	4	0.00	0.00	0.00	0.47	0.32	0.16	0.02	0.00	0.00	0.00	3.91
	29.37	5	0.00	0.00	0.00	0.20	0.75	0.00	0.00	0.00	0.00	0.00	7.89
	31.40	6	0.00	0.01	0.00	0.11	0.06	0.76	0.03	0.00	0.00	0.01	1.66
	34.76	7	0.00	0.00	0.00	0.00	0.00	0.04	0.94	0.01	0.00	0.00	10.65
	41.10	8	0.00	0.01	0.00	0.00	0.00	0.09	0.01	0.90	0.00	0.02	6.60
	43.81	9	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.55	0.00	7.62
	49.60	10	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.06	0.61	0.64

Example: Ares I-X XOR vs MAC

- Using MAC instead of cross-orthogonality will change the Pareto front