

Filter Tuning Using the Chi-squared Statistic

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This paper examines the use of the Chi-square statistic as a means of evaluating filter performance. The goal of the process is to characterize the filter performance in the metric of covariance realism. The Chi-squared statistic is the value calculated to determine the realism of a covariance based on the prediction accuracy and the covariance values at a given point in time. Once calculated, it is the distribution of this statistic that provides insight on the accuracy of the covariance. The process of tuning an Extended Kalman Filter (EKF) for Aqua and Aura support is described, including examination of the measurement errors of available observation types, and methods of dealing with potentially volatile atmospheric drag modeling. Predictive accuracy and the distribution of the Chi-squared statistic, calculated from EKF solutions, are assessed.

Key Words: Covariance Analysis, Filter Tuning, Orbit Determination

Nomenclature

ε	: Error vector
$\chi^2_{3\,dof}$: 3 degree of freedom chi-squared
C	: 3x3 Positional covariance matrix

Subscripts

u	: radial
v	: in-track
w	: cross-track

1. Introduction

The Goddard Space Flight Center (GSFC) Flight Dynamics Facility (FDF) performs orbit determination (OD) for the Aqua and Aura satellites. Both satellites are located in low-Earth orbit (LEO), and are part of what is considered the “A-Train satellite” constellation. The FDF was recently tasked with delivering definitive covariance for each satellite.

The definitive covariance will be used to propagate the covariance further into the future, to be paired with a predictive ephemeris. The information will then be used to assess possible conjunctions. Continued existence of the spacecraft itself can rely on the covariance provided in these ephemerides being accurate. On the other hand, a covariance that is overly conservative could result in a high number of false positive conjunction events. Since each event requires consideration and mission planning, an oversized covariance could prove a substantial burden on the operations team. This need for accuracy was the impetus for the forthcoming analysis, which focuses on the validity of the covariance. As with any propagation, the error in the initial covariance state plays a major role in the error of the propagated state.

As the use of covariance in conjunction operations has become increasingly popular, there has been substantial research into many aspects of covariance realism. There have been a number of papers that examine the best methods for using a rigorous statistical analysis of the chi-squared value to determine whether a covariance can be deemed realistic ⁵⁾. Other work has focused on the practice of propagating a state

while accurately sizing the covariance ²⁾. This analysis attempts to examine how common filter tuning practices affect both the predictive accuracy of a solution, as well as the realism associated with the covariance.

2. The Chi-squared Statistic

The Chi-squared value is calculated in an attempt to characterize how well the covariance conforms to the actual error of a state. The value takes into account both the error in the state propagation, as well as the behavior of the covariance. Much of the technical information underlying the use of the chi-squared statistic for covariance realism in this analysis, was taken from a practical guide to covariance realism analysis published by M.D. Hejduk in Ref. 1.

The predictive error of a solution can be defined by comparing a predicted state to a truth state. For this analysis, the predictive ephemeris was generated as an output of the AGI Inc. Orbit Determination Toolkit (ODTK) software. The software takes the final definitive state, which occurs at 13:00:00 GMT on any given day, and propagates the state, along with the covariance, 47 hours into the future. The truth solution is considered the definitive state from ODTK at 12:00:00 GMT on a given day. Thus, the ephemeris compare occurs at 12:00:00 GMT on a given day, between a predictive state after 47 hours of propagation, and the definitive state. The error values are calculated in each of three components, radial, in-track and cross-track. They are grouped together into the vector of errors for calculation of the chi-squared statistic.

$$\varepsilon = [\varepsilon_u \ \varepsilon_v \ \varepsilon_w] \quad (1)$$

The magnitude of this error vector will be referred to subsequently as the prediction error. The magnitude of the error is affected by a multitude of factors including orbit regime, filter performance and environmental factors. For Aqua and Aura, the in-track component of the error dominates the error vector due to atmospheric drag. This is consistent with other covariance analysis conducted on the LEO regime ⁸⁾.

The covariance of the state is the other input to the chi-squared calculation. Collision avoidance operations utilize the

positional covariance of a spacecraft as an input to the probability of collision calculation. The covariance of the velocity is often ignored, the merits of which do not fall into the scope of this analysis. ODTK processing allows for the extraction of the entire 3x3 position covariance matrix, including correlation terms. The formula for the statistic is given as:

$$\chi^2_{3\text{ dof}} = \varepsilon C^{-1} \varepsilon^T \quad (2)$$

This statistic was calculated at 12:00:00 GMT for each day in the analysis span. This particular time was chosen to ensure the analysis followed the proposed concept of conjunction assessment operations as closely as possible.

The performance of the filter can be evaluated based on the behavior of this chi-squared statistic. In the ideal case, the value of chi-squared would be equal to three, showing that the covariance at any given time exactly matched the error in the solution. As with most real-world applications, this is not the case, and the most effective way to evaluate the metric is using a data set that contains a large number of independent data points. In each of the subsequent sections, the data examined will be the chi-squared values over a span of 91 days. The span covers from 9/03/15 until 12/01/15 for both spacecraft. During that span both spacecraft executed drag makeup maneuvers. In the current operations process, maneuvers are not modeled, which causes a discontinuity in the collection of the chi-squared statistic after a maneuver. In this analysis, calculation of the chi-square statistic requires two days of propagation uninterrupted by a maneuver. This resulted in each maneuver causing two days where the chi-square statistic was not valid and these dates were removed from the data set.

Table 1: Spacecraft maneuvers

Spacecraft	Maneuver Dates
Aqua	9/4, 9/26, 10/16, 11/21
Aura	9/3, 10/02, 11/11

The initial analysis utilized operational solutions produced from daily ODTK filter runs. Scripts were developed to automate the process of calculating the chi-squared statistic. The error vector and covariance are both pulled from the operational ephemeris file using AGI's Systems Tool Kit (STK). The report from STK allows the user to automatically convert the covariance into the radial, in-track and cross track components, reducing the need for manual coordinate conversion. The error vector and covariance matrix are then loaded into MATLAB for efficient matrix math. The script utilizes MATLAB to save the chi-squared value, along with the date, predictive error values and filter sigma values. The same process is utilized for evaluation of the tuned filters described later in this analysis.

Once the chi-squared values are collected for a given span, the focus turns to the distribution of the value over that span. The chi-squared distribution of a realistic three-dimensional covariance should match a multivariate distribution of the sum

of the squares of three independent variables. Thus, the distribution of chi-squared for the three degrees of freedom in orbit should have a mean value of 3, and a standard deviation of 2.333. The distribution of the statistic should follow accepted norms, an example being that 61 percent of all points should fall below the mean of 3. Comparing the actual distribution of the chi-squared value to these accepted norms provides insight into the realism of the covariance.

3. Evaluating the Baseline Scenario

The FDF has provided orbit determination support for both the Aqua and Aura spacecraft since they began their missions. The requirement for the FDF states that the FDF "must provide definitive orbit determination within 20 meters of the truth." The current ODTK scenario used for operations has been proven extremely reliable at meeting this requirement. Prior to the missions' request that covariance data be delivered, the FDF had no requirement to evaluate the covariance behavior of the operational filter. The request for a new product prompted an analysis aimed at evaluating the current state of the covariance produced by the operational scenario.

The chi-squared statistics for both spacecraft were collected from the operations filter, which will henceforth be referred to as the baseline filter. Figures 1 and 2 show the distribution of the calculated chi-squared statistic for both spacecraft. The red line on the chart, and all subsequent charts, represents the ideal 3 degree of freedom distribution mentioned in Section 2.

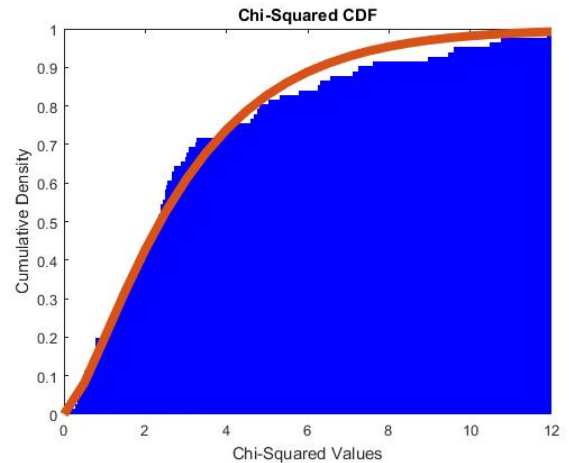


Figure 1: Aqua baseline chi-squared distribution

The data for Aqua in Fig. 1 shows that the cumulative density generally matches the ideal distribution. Upon further examination, the average chi-squared value was 3.29, and the standard deviation is 3.26. Both the average and standard deviation of the statistic are higher than desired, indicating there is room to improve the distribution through filter tuning.

The data for Aura in Fig. 2 shows that the covariance tends to exceed the actual error in the propagated state. The average chi-square value for Aura was 2.23, with a standard deviation of 2.41. The average value falls significantly below the desired

values for average, meaning that the filter sigma values tended to exceed the actual predictive error values. The effect is a covariance that would be too pessimistic, as opposed to one that would instill false confidence in a solution. Based on the baseline distribution for both spacecraft, it was clear that further filter tuning needed to be conducted in an attempt to improve performance.

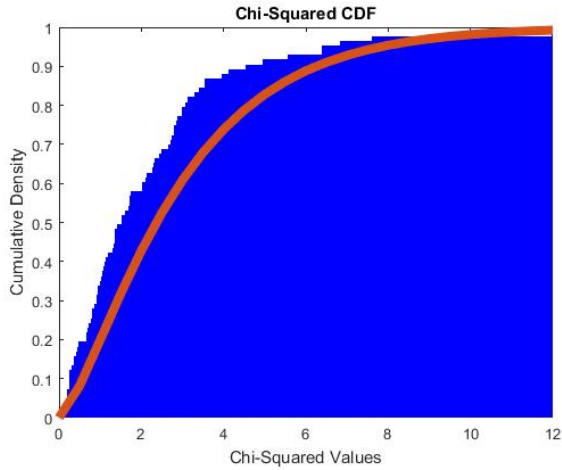


Figure 2: Aura baseline chi-squared distribution

4. Measurement Tuning

The formulation of a definitive orbit is drastically affected by the way measurements are processed. Both missions utilize multiple tracking assets, including both ground-based and space-based tracking networks. The ODTK Filter contains a multitude of settings that can be adjusted to effect the overall filter performance. The following section focuses on the tuning of the settings that dictate the weighting of the measurements used in the ODTK scenario, and how they affect both the predictive accuracy and the covariance of the spacecraft state.

The baseline scenario was tuned with an emphasis on the

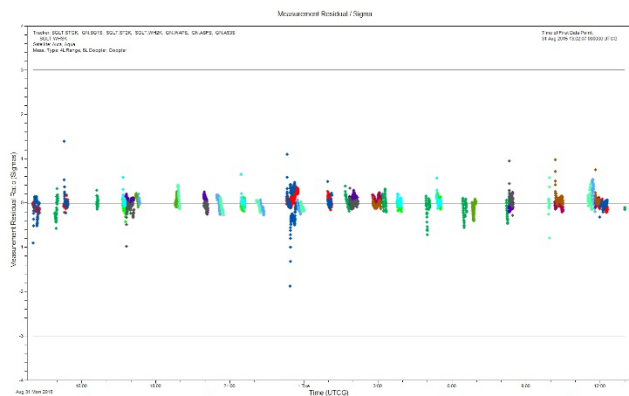


Figure 3: Baseline scenario residual ratios

definitive solution, and minimizing the need for operator intervention. One of the considerations that was made in the initial setup was that the majority of the tracking data should be accepted into the solution, allowing for improved likelihood of

convergence. This led to large noise values associated with the measurements. The residual ratios in Fig. 3 show that all of the data falls within the 3-sigma bounds, but there is a large gap between the majority of the measurements and the 3-sigma lines at the top and bottom of the figure. While helpful for maintaining convergence, this white space can also be associated with added uncertainty being associated with each measurement.

Both the Aqua and Aura spacecraft receive tracking from ground assets each day to assist in the orbit determination process. Both spacecraft transponders facilitate two-way Doppler measurements. In ODTK, the settings for tuning ground station measurements are tied to the ground station object. Therefore, modifying the settings in the ground station object are the most effective way to modify the error associated with this measurement type in the orbit determination process. The baseline scenarios set the white noise sigma (WNS) values for these Doppler measurements at 5 cm per second. Tuning these stations involved reducing these values, re-processing the tracking data measurements, and examining the residual ratio values. Inspecting the residual ratio plots, as well as histograms of the station performance gave insight into how WNS changes affected the processing of the measurements from the station. Through this process, the WNS values for all of the stations were significantly reduced. Each station ended with a different value, but on average the noise was reduced to about 0.5 cm per second. The measurements are only part of all of the tracking data incorporated into the orbit solution.

The Aqua and Aura spacecraft are both consistent users of NASA's Space Network (SN). Tracking passes taken on the SN provide two-way range and Doppler measurements for use in orbit determination. The tracking data itself actually comes from measurements taken by a given tracking data relay satellite (TDRS) spacecraft located in geosynchronous orbit. The fact that the object collecting the tracking measurement is itself in orbit adds another layer of processing complexity. The accuracy of the TDRS spacecraft state feeds into the accuracy of the measurement. This means that the error associated with the SN data has two components which can be tuned to modify the total error associated with these measurements.

One component, which is directly analogous to ground-based measurements, is the actual noise associated with the physical uncertainties of capturing the measurement. This component is changed in ODTK by manipulating the settings associated with the TDRS Space to Ground Link Terminal (SGLT), which is actually the final collection point of the SN data. Each SGLT can have different noise settings depending on the type of measurement, allowing for separate tuning of range and Doppler measurements. Initial efforts focused on varying these values to affect the distribution of residuals. After a number of changes showed little effect, it was clear that the dominant source of noise for this measurement lay in another area.

The other major component that affects the SN measurements is the state of the TDRS spacecraft at the time of observation. In ODTK, the state can be drawn from an ephemeris, or it can be determined in the real time as another spacecraft. The FDF in particular is uniquely suited to run in the latter configuration, since it provides the orbit determination for the TDRS fleet. The baseline ODTK filter is a multi-mission filter that incorporates the Aqua, Aura, Terra satellites along with five of the TDRS spacecraft. The TDRS orbit determination is based on ground-based and Doppler measurements. By running the Aqua and Aura missions in the same filter, the filter has the most up-to-date state of each TDRS spacecraft when it processes the SN measurements. The filter also has access to the covariance state of the TDRS spacecraft at that time, which feeds into the calculation of the noise that should be associated with the measurement. This proved to be the most significant challenge associated with accurately tuning the noise values associated with the SN measurements. For the purposes of this analysis, the limiting factor in the reduction of noise in SN measurements was the covariance of the TDRS. The majority of the time spent tuning for this section of the analysis was spent adjusting the noise measurements associated with the TDRS OD.

Reducing this covariance followed the same process described for the ground-based measurements associated with Aqua and Aura. The end result was also similar, a significant reduction in the noise values associated with the measurements. The increased confidence in the measurements shrank the covariance of the TDRS spacecraft, reducing the noise

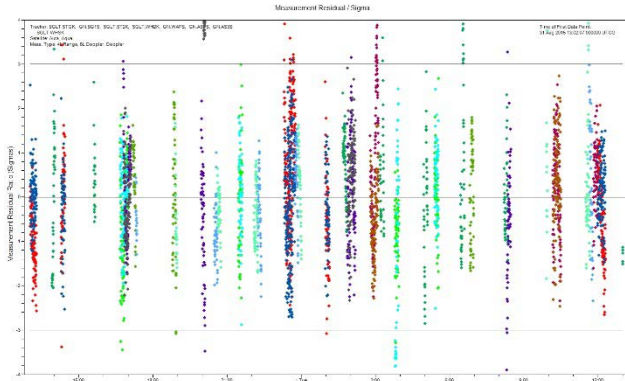


Figure 4: Residual ratios from tuned scenario

associated with the SN tracking data. Further tuning was accomplished using the SGLT measurement noise settings mentioned previously. The final results of the tuning can be seen in Fig. 4, which shows the same time span as Fig. 3. The amount of white space within the three sigma lines is significantly reduced. The noise reduction allows the tracking data measurements to have more weight, reducing the uncertainty in the solution.

This smaller covariance had a significant impact on the chi-squared distribution for both spacecraft. Figure 5 shows the distribution for Aqua is now well below the ideal curve, illustrating that the covariance is no longer capturing the actual error in the solution. This type of covariance would be considered dangerous for operations, since it is displaying a

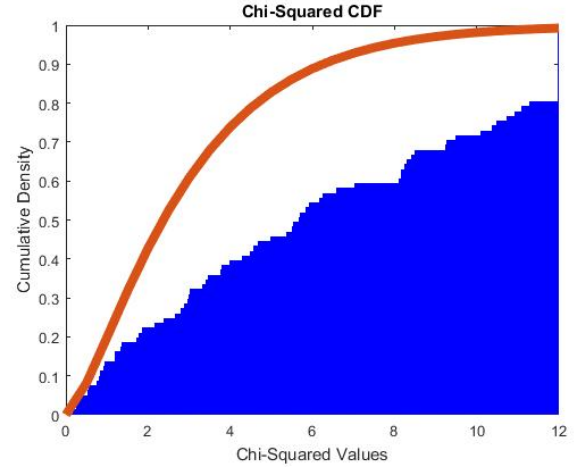


Figure 5: Aqua chi-squared distribution after tuning

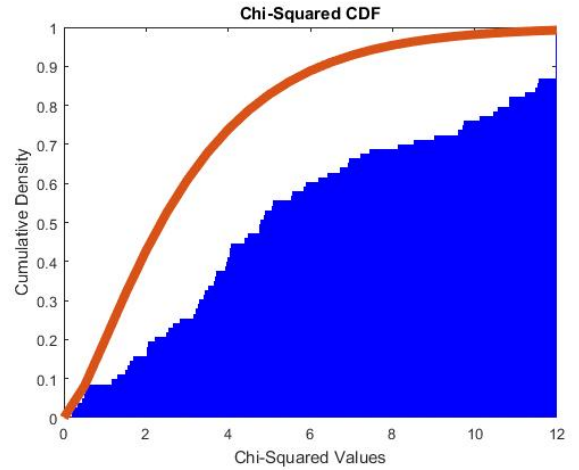


Figure 6: Aura chi-squared distribution after tuning

false confidence in the solution. If the actual state were outside the covariance, then possible conjunction events could be seen as less hazardous than they are. The data for Aura, shown in Fig. 6, shows the same pattern, although the data is slightly closer to the desired distribution. The new values can be directly related to the covariance getting smaller, due to the improved assumed performance of the tracking measurements themselves⁶⁾. A smaller covariance tends to indicate a better definitive state, which can often be seen in improved performance in the propagation. Indeed, for both spacecraft, the predictive error fell as a result of these changes. The average predictive accuracy of the Aqua propagation fell from 205 meters to 184 meters, and the same metric for Aura fell from 195 to 183 meters.

The problem introduced by this reduced covariance can be addressed in different ways. Further tuning other parameters associated with the filter, such as drag forces, could more accurately size the covariance. Another method would be to add some of the measurement noise back into the solution, expanding the covariance again. Changing these settings could also add or detract from the propagation accuracy benefits that

resulted from the reduced measurement noise.

5. Drag Modelling

There are various techniques for effectively modeling the drag force on spacecraft. The LEO orbit of the Aqua and Aura spacecraft result in the drag force being the predominant perturbation for both objects. Effectively quantifying any variations in the drag will enhance the orbit determination, as well as the propagation accuracy. The ODTK software provides many different techniques for modeling drag, with input variables ranging from the actual spacecraft area to the behavior of Earth's atmosphere. Uncertainties, half-life values, and different modeling techniques can be modified for each of these inputs. The result is a highly customizable set of parameters that can be used to carefully calculate the behavior of the spacecraft due to drag.

The drag parameters of the baseline filter were set to give the most accurate definitive orbit for both Aqua and Aura. The baseline scenario utilizes the Jacchia-Roberts atmospheric density model as the basis for drag calculations. The spacecraft themselves are modelled as spherical cannonballs, with an area approximating their actual drag area. The definitive orbit solutions have the obvious advantage of including tracking data, which helps the filter calculate the effect that drag is having on the vehicle. The filter estimates a ballistic coefficient correction to model the drag force on both vehicles. Since the area is static, it is the ballistic coefficient correction that captures the effect of drag.

For the purpose of this paper, an attempt was made to better model the drag forces on the spacecraft using an advanced spacecraft model. The enhancement utilized a box and wing model to compute the solar radiation pressure and drag areas at each time step during the filter process, and during the forward propagation. Spacecraft dimensions used by the plugin were populated using mission design documents. The models assumed that the solar array was always pointing in the direction of the sun, even in eclipse. The goal of the implementation was mainly to improve the forward propagation of the state. To isolate the effect of the drag modeling, all of the measurement weighting parameters mentioned previously were unchanged from the baseline filter.

The results of implementing the box and wing model for both spacecraft were not favorable. Both spacecraft showed increased error in the 47-hour prediction with the advanced area model implemented. In the case of the Aqua spacecraft, this made the chi-squared distribution further from ideal. For the Aura spacecraft, it would seem that the covariance was improved, since it was closer to the expected distribution. However, this improvement was due to the fact that the model actually introduced more propagation error into the solution, making it conform more closely to the filter sigma values. Further work is needed to determine exactly the cause of the degraded performance. Evaluating whether the sun pointing assumption for the solar array is accurate, tuning the atmospheric density approximations and even further

examining the behavior of the flux values over the time period are all areas that could explain these results^{9,10}. The outcome of this run further illustrates that the covariance realism is not an all-encompassing metric, and must be evaluated with other characteristics of the filter. It is possible to obtain a good chi-square distribution, but still have plenty of room for performance improvement in other aspects of the filter.

6. Continued Measurement Tuning

Further tuning runs were completed to attempt to reach the goal of a realistic covariance for both spacecraft. As the drag tuning attempted in section 5 only seemed to cause detrimental effects in filter performance, those changes were abandoned and the focus was on tuning the measurement noise values to obtain a realistic covariance. The effects of these changes on the chi-square statistic, and on the predictive accuracy were the main metrics analyzed for the analysis.

From section 4, the measurement tuning run yielded an average chi-squared value of 9.88 for Aqua and 7.99 for Aura, indicating that the covariance tended to be much smaller than the actual error in propagation. One method of expanding the covariance is to relax these measurement values, reducing the certainty of the state. This would result in a larger covariance in the definitive state, and in turn a larger covariance when the state is propagated 47 hours into the future. For the next run, all of the measurement noise values from the final measurement tuned filter were increased by 50 percent. This configuration yielded a chi-square average of 6.50 and a prediction error average of 186.52 meters. This chi-square value was still higher than desired, although the number did decrease, so the desired effect was achieved. The next run involved relaxing the noise values even further. All of the values were increased again by 50% to further expand the covariance. The chi-squared value again trended in the correct direction, but the predictive accuracy for both spacecraft deteriorated again.

The chi-squared values were trending in the correct direction with the inflated noise values, however that was directly correlated to an increase in the average propagation error. This tactic for inflating the covariance was clearly enhancing the realism of the covariance, but sacrificing accuracy in both the definitive and predictive states of the spacecraft. To proceed in obtaining a realistic covariance, another means of inflating the covariance was chosen.

7. Injecting Process Noise

The covariance of the filter can be adjusted using many different techniques. The techniques in sections 4, 5 and 6 focused on adjusting the final covariance state of the definitive data. The problem with this solution was that it degraded the definitive state accuracy, in turn degrading the accuracy of the 47-hour prediction. Another approach was selected to inflate the covariance for the subsequent runs. The most direct approach for inflating the covariance is adding noise during the propagation. As mentioned previously, this technique has been explored for other missions. The major difference in this case

is that when the process noise is applied directly to the satellite object, it can affect the definitive accuracy of the orbit state.

ODTK allows the user to inject process noise as an unmodeled force for each spacecraft. The filter configuration described at the end of section 4 was used as the starting point for this process. These results provided the best predictive accuracy, but extremely high chi-squared values. Knowing this, a large amount of process noise was injected to attempt to properly size the covariance. ODTK allows for the injection of process noise in the radial, in-track or cross-track directions. This format is consistent with the reference frame used for comparisons, and is useful for avoiding complications associated with Cartesian covariance propagation⁷⁾. The first run applied 1 cm per second of process noise in the in-track direction for both Aqua and Aura spacecraft.

The chi-squared values produced by this filter configuration showed movement in the desired direction. This progress was also accompanied by a number of negative effects to the overall filter performance. The predictive error in the position for both spacecraft increased by nearly a factor of two. The chi-squared statistic itself, while closer to the desired average, showed a very high standard deviation. Inspection of the filter sigma values in the in-track direction showed that they were consistently about 1600 meters for both spacecraft. All of these effects seem to point to the fact that covariance was over inflated by the 1 cm per second process noise. The next run would need to scale back the amount of process noise to improve the behavior of the covariance.

The in-track process noise for the next run was scaled based on two previous values from the analysis. For in ideal filter, the sigma and predictive error values should have a one to one relationship. The effect of this relationship can be seen from inspection of the chi-squared calculation. If it is assumed that the covariance matrix off-diagonal terms are insignificant compared to the diagonal values, they can be set to zero. Eq. 1 can then be simplified to directly relate to the filter sigma and the predictive error values.

$$\frac{\varepsilon_u^2}{\sigma_u^2} + \frac{\varepsilon_v^2}{\sigma_v^2} + \frac{\varepsilon_w^2}{\sigma_w^2} = \chi_{3\,dof}^2 \quad (3)$$

Thus, if the predictive error to filter sigma relationship is 1-to-1, it would be expected that the chi-squared value would be the desired value of 3. If it is assumed that the average error in the propagation is roughly 200 meters, as it was for the final measurement tuned filter, then ideally, the filter sigma values should have an average value of 200 meters. This lead to the in-track process noise value being reduced by a factor of eight, in an attempt to obtain this average filter sigma value. This process was repeated until the filter sigma values were reduced to the desired level. At the end of this process, the in-track process noise was set at 0.075 cm per second and the filter sigma values were reduced to an average of 230 meters. With the drastic reduction in the added process noise, the predictive accuracies returned to the improved levels noted in section 4. Each iteration showed improvement in the chi-squared

distribution, but overall the results remained far under the curve of an ideal distribution. With the in-track sigma value approaching the desired level, further effort was put into determining why the chi-squared values remained higher than expected.

As shown in Eq. 3, the relationship between the predictive error values and the filter sigma values in a given component can provide further insight into the performance of the filter. For an accurately tuned covariance, the error value in a given component divided by the sigma value in that component, henceforth referred to as the error over the sigma (EOS), should have a normal distribution around one³⁾. To further diagnose issues with the chi-squared distribution, this value was calculated for each component, each day. The most direct way to examine the distribution of these values is the cumulative density function (CDF). Upon inspection of the CDF in each component, both of the spacecraft showed similar behavior. The radial and in-track components showed a relatively normal distribution, with roughly 50 percent of the values falling on either side of 1. When the cumulative density function for the cross-track was plotted, it showed a distinct problem. The predictive error exceeded the sigma value in roughly 70 percent of all of the data points. It was clear that the covariance in this component was not large enough to capture the actual predictive error. This was in turn driving up the chi-squared value in a significant portion of the cases.

Process noise in the cross-track component was added to the filter for both spacecraft. The initial value used was 0.001 cm per second. This value was used as an initial guess because it was significantly smaller than the value used for the in-track error, which on average was 100 times larger than the cross-track error. Additionally, the in-track error was further reduced to 0.065 cm per second, to further tune that component.

Filter tuning continued in this pattern for 6 additional runs. Each time the process noise was changed based on the distribution of the error over the sigma value. If the component distribution was 60 percent or higher favoring either the predictive error, or the filter sigma, it was adjusted accordingly. Otherwise, if the value fell between 40 and 60 percent it was left unchanged. Table 2 below shows a subset of the runs, and the effect on both predictive accuracy and the chi-squared statistic.

Table 2: Filter performance results

Run	Satellite	Process Noise (cm/sec)	Average Predicted Error (meters)	Chi-squared Average
7	Aqua	AT: 0.075 CT: 0.001	185	8.77
	Aura	AT: 0.075 CT: 0.001	202	6.50
8	Aqua	AT: 0.060 CT: 0.001	185	8.77
	Aura	AT: 0.075 CT: 0.002	202	6.50
9	Aqua	AT: 0.060 CT: 0.003	191	8.20
	Aura	AT: 0.075 CT: 0.005	203	5.56
10	Aqua	AT: 0.075 CT: 0.300	179	6.59
	Aura	AT: 0.075 CT: 0.400	204	4.84
11	Aqua	AT: 0.050 CT: 0.300	181	6.63
	Aura	AT: 0.060 CT: 0.400	189	5.19
12	Aqua	AT: 0.030 CT: 0.300	193	7.59
	Aura	AT: 0.030 CT: 0.400	185	5.79

The last row of Table 2 shows the final result of the tuning, when all of the EOS distribution for both satellites fell in the desired EOS range. The breakdown of the EOS distribution by component for Aqua was 51 percent below a value of one for the radial component, 58 percent for in-track, and 51 percent for cross-track. For Aura, the breakdown was 46 percent below one for radial, 59 percent for in-track and 54 percent for cross-track. Even with these values falling close to their expected distribution, the chi-squared values of both spacecraft were still elevated. The final distribution from run twelve for Aqua can be seen in Fig. 7, and for Aura in Fig. 8. Both spacecraft showed a chi-squared distribution closer to the norm when compared to the first tuned run. However, both curves still fall under the desired distribution.

Insight comes from examining the standard deviation of both the predictive error and the filter sigma values. From these, it is clear that the propagation error has a much higher standard deviation than the filter sigma values. The final run for Aqua showed the standard deviation of the in-track error was 158 meters, while the standard deviation of the in-track filter sigma was only 44 meters. Clearly the sigma values were not scaling adequately with the error values. This caused large values in the EOS calculations, which can be directly related to inflated chi-squared values.

The results from the table also shed some light on the relationship between the propagation error and the chi-squared value. The best performance for each are not necessarily seen

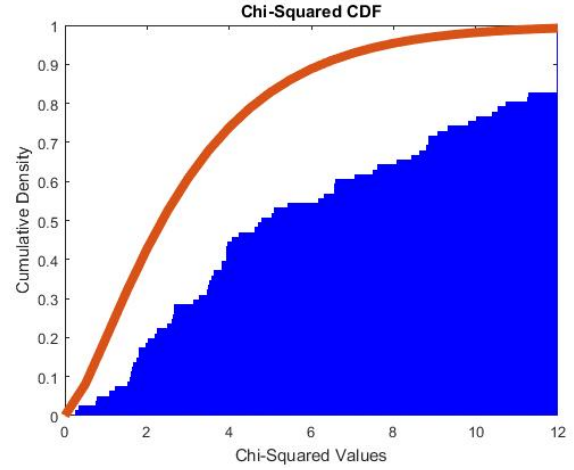


Figure 7: Aqua chi-squared distribution from run twelve

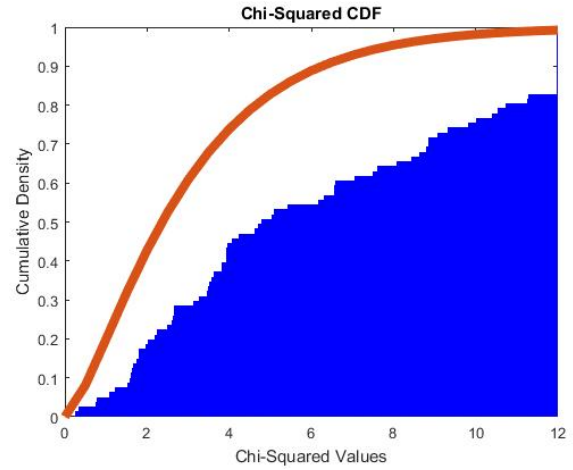


Figure 8: Aura chi-squared distribution from run twelve

under the same conditions. This is most apparent when inspecting the results for Aura. The best chi-squared value for Aura occurred in Run 10 and resulted in a propagation error of 204 meters. The optimal propagation performance of 185 meters however occurred in run 12, which resulted in an average chi-squared value of 5.79. The table shows a trade-off between the two performance metrics.

8. Conclusion

The filter tuning process contains a wealth of variables and techniques that can be adjusted for a desired effect. This was evident in the two different techniques that were applied in an attempt to balance a realistic covariance while maximizing prediction accuracy. Both the measurement weighting and the process noise had an appreciable effect on the predictive accuracy and the covariance sizing. Utilizing both methods allows the user to ensure that neither single technique is being overused to produce the desired result.

The original ODTK scenario showed that the chi-squared statistic for the Aqua spacecraft was close to what is considered a realistic covariance. However, upon further analysis it can be seen that this was facilitated by the inflated measurement noise

associated with the tracking data. Reducing the measurement noise improved the accuracy of the solution but inflated the chi-squared statistic. To keep the improved accuracy, process noise was added to inflate the covariance to the desired level. This combination of techniques allowed for improved metrics in two different filter performance areas. This leads to the observation that while an accurate chi-squared distribution is a desirable goal, it does not implicitly imply a filter is operating in the optimal configuration. Balancing the inverse relationship between propagation error and the chi-squared value has been the most challenging aspect of the tuning process.

The dynamics affecting a spacecraft also play a large role in the difficulty associated with achieving a realistic covariance. In the case of this analysis, the primary force perturbing both of the orbits was the drag force due to Earth's atmosphere. This resulted in the in-track component being the dominant error component in all of the predictions. The average error in this component was an order of magnitude higher than in either the radial or cross-track directions. This made accurately inflating the covariance in these less dominant components a challenge. In reality, appropriately sizing the in-track is the most important aspect for collision operations. Diligence must be taken to ensure that the physical implications of the covariance and predictive error values are well understood in the context of the orbit. While the chi-squared metric can aid in tuning a number of aspects of the filter, appropriate knowledge of the orbit and the critical values must be a focus of the tuning process.

Further effort will be undertaken to refine the process noise for the Aqua and Aura ODTK scenario. Techniques to increase the standard deviation of these values need to be investigated to improve the performance of the chi-squared statistic. One possible technique to explore would be to associate the atmospheric density calculations with a certain amount of added noise, to accurately scale the growth of the covariance. The goal will continue to be a realistic covariance, with an emphasis on a normal distribution of the predictive error and filter sigma values. The process will also need to be repeated based on different environmental factors, such as periods of maximum solar activity.

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References

1. M. D. Hejduk, "Covariance Realism Evaluation Approaches." Flight Dynamics Support Services II Technical Memorandum. 10 July 2015. FDSS-II-16-0049.
2. M. Duncan, A. Long, "Realistic Covariance Prediction for the Earth Science Constellation." AAS/AIAA Astrodynamics Specialists Conference and Exhibit. Keystone, Colorado. 21-24 August 2006. AIAA 2006-6293.
3. D. A. Vallado, J. Seago, "Covariance Realism." Advances in the Astronautical Sciences 09-304. 2009.
4. S. Slojkowski, J. Lowe, J. Woodburn, "Orbit Determination for the Lunar Reconnaissance Orbiter Using an Extended Kalman Filter." 25th International Symposium on Space Flight Dynamics. Munich, Germany. 19 October 2015.
5. J.L. Foster, J.C. Frisbee, "Position Error Covariance Matrix Scaling Factors for Early Operational ISS Debris Avoidance." NASA, Johnson Space Center/DM33.
6. M.T. Majji, "Updated Jth Moment Extended Kalman Filtering for Estimation of Nonlinear Dynamic Systems." Paper AIAA 2008-7386, Proceedings of the AAS/AIAA Astrodynamics Specialist Conference, Honolulu, HI.
7. C. Sabol, T. Sukut, K. Hill, K. Alfrend, B. Wright, Y. Li, P. Schumacher, "Linearized Orbit Covariance Generation and Propagation Analysis Via Simple Monte Carlo Simulations". Advances in the Astronautical Sciences. January 2010.
8. D. Vallado, S. Alfano, "Curvilinear Coordinates for Covariance and Relative Motion Operations". 2011 AAS/AIAA Astrodynamics Specialist Conference. Gridwood, AK. July 31-August 4. AAS 11-464.
9. K. Alfrend, D. Lee, M. Wilkins, "Characterizing Orbit Uncertainty Due to Atmospheric Drag Uncertainty". 14th International Symposium on Space Flight Dynamics. Foz do Iguacu, Brazil. 8 February 1999.
10. M. Hejduk, L. Newman, R. Besser, D. Pachura, "Predicting Space Weather Effects on Close Approach Events". Advanced Maui Optical and Space Surveillance Technologies Conference. 15 September 2015.